PROBABILITY DISTRIBUTION OF THE NUMBER OF INTERACTING NUCLEONS OF THE INCIDENT NUCLEUS IN COLLISIONS OF RELATIVISTIC NUCLEI

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A method is given for obtaining the probability distribution of the number of interacting nucleons of the incident nucleus in collisions of relativistic nuclei. Cross sections and average pion multiplicities in collisions with various total charge of non-interacting fragments of the incident nucleus are used as input data. The method is applied to He+Ta and C+Ta collisions at a momentum of $p/A = 4.2$ GeV/c. The obtained distributions are compared with calculations using the multiple scattering model.

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Collisions of light relativistic nuclei with tantalum target have been studied in the Dubna 2 m propane bubble chamber. The results on multiplicity of negative pions produced in these reactions have been published in several papers [1–4]. Here we attempt to determine the probability distribution, P_ν, of the number of interacting nucleons of the incident nucleus in inelastic d+Ta, ^{4}He+Ta and ^{12}C+Ta collisions at 4.2 GeV/c per nucleon. In papers [3, 4] some information on this distribution has been already obtained based on the average value, $\langle \nu_i \rangle$, and dispersion, D_ν, of the P_ν distribution, which could be obtained from projectile fragmentation data in the model-independent way [4, 5]. These quantities are given in Table I together with the total numbers, N_{tot}, of interactions used in the analysis. We note that for He and C projectiles colliding with Ta the experimental values of the dispersion D_ν are significantly greater than it would have been expected for the flat P_ν distribution (P_ν = const) which, together with almost the same average value, $\langle \nu_i \rangle$, suggests that small and large values of ν_i would be enhanced.

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It can be shown that higher moments of the P_ν distribution could also be obtained from the projectile fragmentation data in the model-independent way [5]. In the case of an incident nucleus with charge Z, the moments of the P_ν distribution up to the Z-th order can be calculated. This would give more detailed information on the shape of the P_ν distribution, but, providing only Z constraints, does not allow one to obtain P_ν for $\nu = 1, 2, \ldots A$.

TABLE I

Total numbers of interactions of relativistic nuclei used in the analysis and average values and dispersions of the number of interacting nucleons of the incident nucleus [5]. Note that these data differ slightly from those given in our earlier papers [3, 4]

Reaction	N_{tot}	$\langle \nu_i \rangle$	D_ν^2
d+Ta	316	1.60±.04	0.24±.02
He+Ta	1408	2.86±.10	1.64±.09
C+Ta	1103	6.60±.30	16.8 ±1.0

The P_ν distribution ($\nu = 1, 2, \ldots A$) can be obtained if the projectile fragmentation data are supplemented with data on pion multiplicities in groups of events with a given number of interacting protons. The relations between unknown probabilities P_ν ($\nu = 1, 2, \ldots A$) and measured probabilities of the interaction of various numbers of protons of the projectile nucleus, W_n ($n = 0, 1, \ldots Z$), provide us with $Z+1$ linear equations:

$$W_0 = \sum_{\nu=1}^{N} P_\nu C_N^\nu / C_A^\nu \quad \text{and} \quad W_n = \sum_{\nu=n}^{N+n} P_\nu C_Z^n C_N^{\nu-n} / C_A^\nu,$$

where C are the binomial coefficients and N is the total number of neutrons in the incident nucleus [4, 5]. Further $Z+1$ equations of similar structure can be obtained using the experimental values of average pion multiplicities, $\langle n_- \rangle_n$, in subsamples of events with various numbers of interacting protons and assuming that pion production occurs in independent interactions of the nucleons of the projectile nucleus (evidence for such a mechanism follows from our previous observations [3, 4]). Thus we have altogether $2(Z+1)$ equations with A unknowns, which constitute an overdetermined system for light projectile nuclei with $A = 2Z$.

In the case of the deuteron incident upon tantalum, one obtains $P_1 = 0.44, P_2 = 0.56$. In the case of the ^{4}He nucleus incident upon tantalum, the system of $2(Z+1) = 6$ equations can be solved using standard minimization procedure. One obtains: $P_1 = 0.21, P_2 = 0.20$, $P_3 = 0.13$, $P_4 = 0.41$. However, the quality of the fit is poor, and a better solution is obtained dropping one of the six equations, namely that for average multiplicity in the case of two interacting protons of the ^{4}He nucleus because in this equation the experimental values of multiplicities which enter the coefficients have the biggest uncertainties. Then we obtain: $P_1 = 0.19$, $P_2 = 0.17$, $P_3 = 0.14$, $P_4 = 0.49$ with $\chi^2/\text{NDF} = 0.79/1$. This distribution is shown in Fig. 1a.

In the case of incident carbon nuclei the standard method of solution fails, the solutions being highly unstable often yielding negative values of P_ν. Therefore we tried to obtain a physically sensible solution using the method of regularization described in Refs. [6, 7].

The basis of this method is the requirement of regularity of the P_ν distribution (minimizing the integral $\int (dz/ds)^2 ds$, where $z(s)$ is the unknown solution), which seems to be justified in our case. The method of regularization is applied to solve the Fredholm equation of the type

$$A\{u(x), z(s)\} = \int_a^b K(x, s)z(s)ds = u(x) \qquad c \leqslant x \leqslant d,$$

where $K(x, s)$ is the kernel of the integral equation, $u(x)$ is a known function, and $z(s)$ is unknown. If the solutions of the above equation are unstable it is the so-called incorrect inverse problem and in this case it is necessary to use some additional information about the solution — the regularity of $z(s)$.

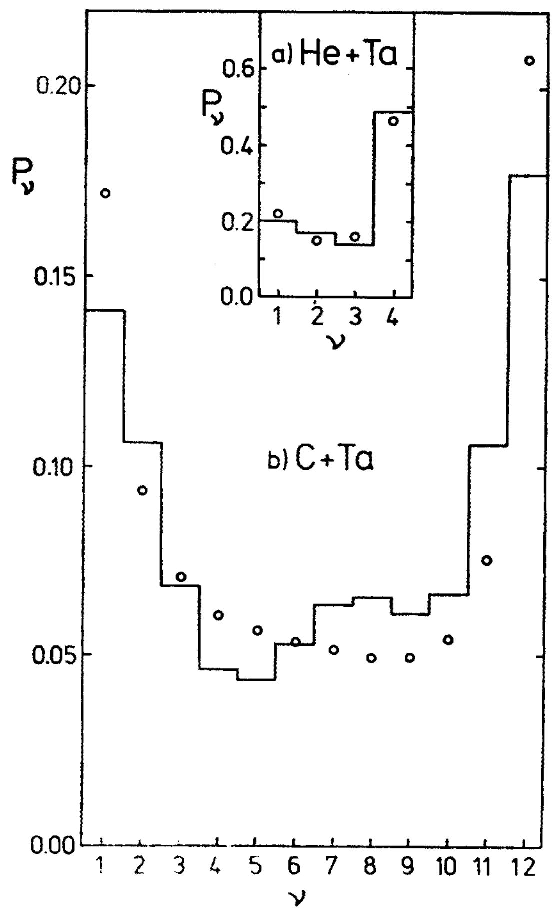

Fig. 1. Probability distribution of the number of nucleons of the projectile nucleus interacting with the target for a) He+Ta and b) C+Ta collisions at $p/A = 4.2$ GeV/c. Results of the multiple scattering model calculation are denoted by small circles. A somewhat different version of this figure, based on the preliminary analysis of the data, was given in Refs. [8, 9]

The condition of regularity requires that

$$\Omega[z(s)] = \int_a^b (dz/ds)^2 ds \text{ should be minimum.}$$

Then, using the Lagrange method for conditional extremum we obtain the function $M^\alpha[z, u]$ to minimize in the form

$$M^\alpha[z, u] = \int_c^d \{\int_a^b K(x, s)z(s)ds - u(x)\}^2 dx + \alpha\Omega[z(s)], \quad \alpha > 0,$$

where α is a free parameter. In order to choose a proper value of α one can use the condition

$$\|\int_a^b K(x, s)\bar{z}(s)ds - u(x)\| \leq \delta,$$

where $\bar{z}(s)$ is a regular solution, and δ characterizes the experimental errors of $u(x)$. It means that the obtained solution $\bar{z}(s)$ should reproduce the input data $u(x)$ within the limits of errors. In our case, the algebraization of integrals in the formula for M^α is straightforward, as $u(x)$ is a discrete function ($u(x) = W_n$ for $n = 0, 1, \ldots Z$ and $u(x) = W_n \cdot \langle n_- \rangle_n$ for the following $Z+1$ equations). With the value of $\alpha = 0.7$ we obtain the distribution shown in Fig. 1b. The result is not very sensitive to the chosen value of α between about 0.2 and 1.0. Let us remark that the used procedure does not yield a reliable estimate of errors of P_ν.

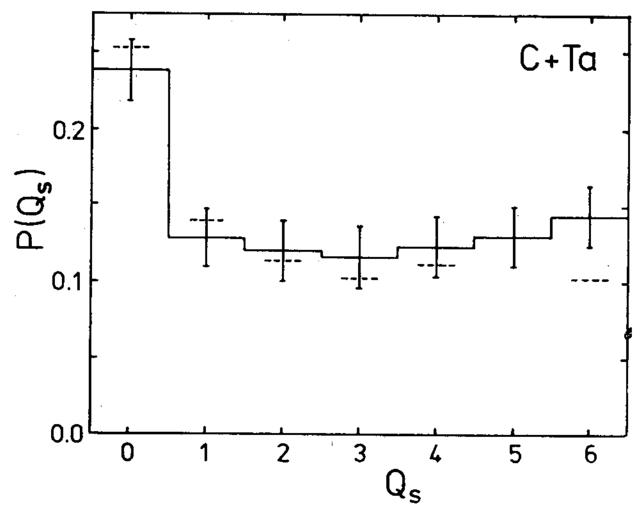

Fig. 2. Probabilities of occurring events with various total charge of non-interacting fragments in C+Ta collisions at $p/A = 4.2$ GeV/c. Values obtained from the P_ν distribution shown in Fig. 1 are denoted with horizontal dashed lines

The obtained P_ν distribution reproduces well the input data, which is shown in Figs. 2

and 3. Probabilities of occurring events with various total charge of non-interacting fragments are displayed in Fig. 2 and average multiplicities of negative pions in groups of events with given total charge of non-interacting fragments — in Fig. 3. The total charge of non-interacting fragments Q_s and the number of interacting protons n_p are connected by the relation $Q_s = Z - n_p$. The values calculated from the obtained P_v distribution are shown with short horizontal dashed lines in both figures. The agreement is satisfactory.

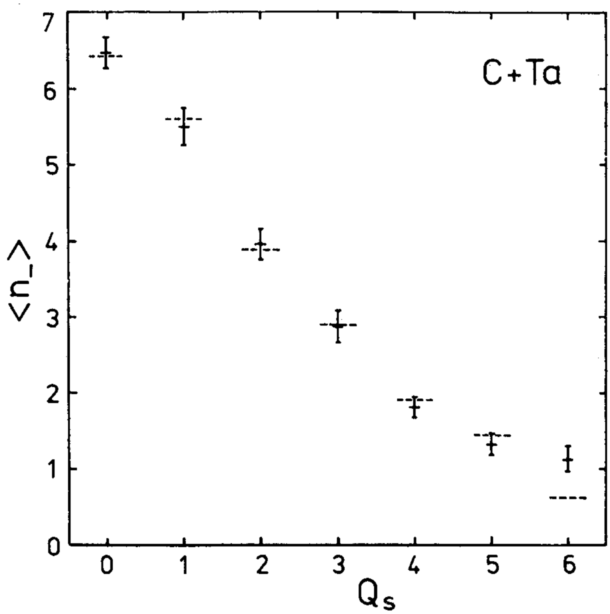

Fig. 3. Average multiplicities of negative pions in groups of events with given total charge of non-interacting fragments in C+Ta collisions at $p/A = 4.2$ GeV/c. Values obtained using the P_v distribution shown in Fig. 1 are denoted with horizontal dashed lines

The obtained P_v distributions are also in fair agreement with the results of theoretical calculation by Shabelsky and Cheplakov using the multiple scattering (Glauber-type) model. In this calculation the Gaussian density distribution was used for helium and carbon nuclei, and the Saxon-Woods density distribution for the tantalum nucleus (the same distribution was assumed for protons and neutrons). Similar calculations for helium nuclei incident upon various targets have been described in Ref. [10].

The obtained results mean that in interactions of light nuclei with a heavy target two classes of collisions are the most frequent: the "peripheral" ones in which only one nucleon of the projectile interacts in the target, and the "central" ones in which all or almost all nucleons of the projectile interact.

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