THE CHARGE FORM FACTOR, THE QUADRUPOLE MOMENT AND THE PHOTODISINTEGRATION OF ^{6}Li

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The root mean square radius, the charge form factor, the charge density, the quadrupole moment and the bremsstrahlung weighted cross section for the photodisintegration of ^{6}Li, are calculated using a polarised cluster model wave function, which is modified to take into account, in its relative motion part, the requirement of a shell model node. A set of parameters, in the modified cluster model wave function, which account for the available experimental data for the afore-said quantities, is determined.

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1. Introduction

The ^{6}Li nucleus is very interesting from both the theoretical and experimental view points. It is a stable nucleus, containing, on the one hand sufficiently many nucleons to exhibit many important general features of nuclear phenomena and on the other hand, sufficiently few nucleons so that detailed calculations can be made even with nuclear forces containing a strong repulsive core. There is abundant experimental evidence for the alpha-deuteron cluster structure of ^{6}Li [1]. In a series of papers, Wildermuth et al. [2] successfully studied the low-lying states of ^{6}Li and other light nuclei, within the framework of the standard cluster model. Cheon [3] proposed a modification to the standard

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alpha-deuteron cluster model, in which the clusters are deformed in the direction of their relative axis, neglecting the effect of antisymmetrization of the wave function. On the basis of the dynamics, the structure of the ^{6}Li ground state wave function should take into account the requirement of a shell model node [4, 5]. We call Cheon's wave function as the polarised cluster model wave function and its modification, which incorporates the shell model node, as the modified polarised cluster model.

Our aim here is to study the root mean square radius, the charge form factor, the charge density, the quadrupole moment and the bremsstrahlung weighted cross section for the photodisintegration of ^{6}Li, using the standard, polarised and the modified polarised cluster models, to ascertain the importance of the role of the relative motion wave function, avoiding fitting procedures (e.g. Cheon [3]), or variational procedures (e.g. Tang et al. [2], which is the basis for the later work of Jain and Sarma [6]) and to obtain a set of parameters which will account for the available experimental data.

2. *The wave functions*

The wave function of ^{6}Li can be written [2] as an antisymmetrized product of the wave functions for an alpha-cluster, a deuteron cluster and their relative motion part:

$$\Psi = \mathscr{A}[\Phi_j(\alpha)\Phi_k(\mathrm{d})\chi(\boldsymbol{R}_\alpha - \boldsymbol{R}_\mathrm{d})], \tag{1}$$

where Φ_j and Φ_k describe the alpha and the deuteron clusters, respectively, $\chi(\boldsymbol{R} = \boldsymbol{R}_\alpha - \boldsymbol{R}_\mathrm{d})$ refers to the relative motion between the clusters and $\mathscr{A}$ the antisymmetrization operator which takes into account the exchange of all pairs of particles with the same spin-isospin assignment. The nucleons belonging to the alpha and the deuteron clusters may be assumed to be in the 1s- and 1p-shells, respectively, from the shell model stand point. Lodhi [7] proposed a wave function which is similar in form to that of Wildermuth et al. [2] but with a different choice of parameters. We adopt Lodhi's choice, according to which the alpha and deuteron length parameters, a_s and a_p, may be varied to a certain extent, but the separation parameter, belonging to the relative motion part of the wave function is defined uniquely in terms of a_s and a_p. The standard cluster model wave function is:

$$\Psi = N\mathscr{A} \exp\left[-\frac{\alpha_0}{2}\sum_{i=1}^{4} \varrho_i^2 - \frac{\alpha_1}{2}\sum_{j=5}^{6} \varrho_j^2 - \tfrac{2}{3}\beta R^2\right] R^2 Y_0^0(\hat{R})\xi(1234;56), \tag{2}$$

where

$$\varrho_i = \boldsymbol{r}_i - \boldsymbol{R}_\alpha, \quad \varrho_j = \boldsymbol{r}_j - \boldsymbol{R}_\mathrm{d},$$

$$\boldsymbol{R}_\alpha = z + \tfrac{1}{3}\boldsymbol{R}, \quad \boldsymbol{R}_\mathrm{d} = z - \tfrac{2}{3}\boldsymbol{R},$$

$$\boldsymbol{R} = \boldsymbol{R}_\alpha - \boldsymbol{R}_\mathrm{d}, \quad z = \tfrac{1}{3}(2\boldsymbol{R}_\alpha + \boldsymbol{R}_\mathrm{d}),$$

$$\alpha_0 = a_\mathrm{s}^2, \quad \alpha_1 = a_\mathrm{p}^2, \quad \beta = \tfrac{1}{3}(\alpha_0 + 2\alpha_1),$$

$\xi(1234; 56)$ is the charge-spin function, $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$ being the position coordinates of the four 1s-nucleons belonging to the alpha particle and the two 1p-nucleons belonging to the deuteron, respectively. The normalization of the wave function for the ground state, considering the exchange of nucleons between the clusters becomes:

$$N^2 = 6! \int (\Psi_0 - 2\Psi_1 + \Psi_2)^* \Psi_0 d\tau, \tag{3}$$

where

$$\Psi_0 = \Psi(1234; 56), \qquad \Psi_1 = \Psi(5234; 16), \qquad \Psi_2 = \Psi(5634; 12),$$

correspond to the no-exchange, one-nucleon exchange and the two-nucleon exchange wave functions, respectively.

The standard cluster model wave function (2) gives rise to a spherical charge density distribution and hence to a zero quadrupole moment for ^{6}Li. But, the quadrupole moment of ^{6}Li has been measured [8] to be -0.08 ± 0.008 fm^2. Several attempts have been made to explain satisfactorily the charge form factor and the quadrupole moment of ^{6}Li. Ciofi degli Atti [9] introduced short-range correlations in the ground state wave function, Wong and Lin [10] resorted to an intermediate coupling shell model, Bouten et al. [11] and Radhakant et al. [12] used a wave function obtained from projected Hartree-Fock calculations, but all these investigators report that the parameter set adjusted to give a good fit to the charge form factor results in a quadrupole moment that is almost ten times larger than the experimental value. Cheon [7] proposed a modification for the standard cluster model wave function, in which the alpha and deuteron clusters are deformed in the direction of their relative axis and his unantisymmetrized wave function has the form:

$$\Psi_0(1234; 56) = N \exp\left[-\frac{\alpha_0}{2} \sum_{i=1}^{4} \left(\varrho_i^2 - \delta \frac{\bar{\alpha}_0}{\alpha_0} \varrho_{iz}^2 \right) - \frac{\alpha_1}{2} \sum_{j=5}^{6} \left(\varrho_j^2 - \delta \frac{\bar{\alpha}_1}{\alpha_1} \varrho_{jz}^2 \right) \right.$$
$$\left. -\tfrac{2}{3}(\beta - \delta\bar{\beta}) R^2 \right] R^2 Y_0^0(\hat{R}) \xi(1234; 56), \tag{4}$$

where

$$\alpha_0 = \bar{\alpha}_0(1+\delta/3), \qquad \alpha_1 = \bar{\alpha}_1(1+\delta/3), \qquad \beta = \bar{\beta}(1+\delta/3),$$

with δ being the deformation parameter of the two clusters. With this polarised cluster model wave function (4), Cheon was able to account for the charge form factor and the quadrupole moment of ^{6}Li. However, following the arguments of Kudeyarov et al. [5], that the antisymmetrization effect is not important in the case of the isolation parameter $\kappa = \beta/\alpha_0 = 0.45$, Cheon neglected the nucleon exchange between the clusters. But this aspect of the problem has been adequately dealt with by Kudeyarov et al. [5] and by Jain [6].

In the case of ^{6}Li, Hasegawa and Nagata [4] derived by Schrödinger equation for the relative motion between the clusters using the resonating group method, and obtained the 2s-wave function as the ground state of ^{6}Li, having a node at the inner part as well as a long tail. Also, Okai and Park [13] have shown that though the relative motion part of the wave function changes as one goes from the shell model to the cluster model, its shell model nodes remain at the same points and that these nodes are almost energy independent. Thus, on the basis of the dynamics, in the absence of antisymmetrization, the structure of the ground state wave function should incorporate the requirement of a shell model node in its relative motion part. The form used for the relative motion part in the standard cluster model (2) is:

$$\chi(R) = \frac{R^2}{N_0} \exp\left(-\tfrac{2}{3}\beta R^2\right),$$

and in the polarised cluster model (4) it is:

$$\chi(R) = \frac{R^2}{N_0} \exp\left[-\tfrac{2}{3}(\beta-\delta\beta)R^2\right],$$

where N_0 is the appropriate normalisation factor. In the modified polarised cluster model wave function, the explicit form of $\chi(R)$, which incorporates the shell model node has the form [5]:

$$\chi(R) = \frac{1}{N_0}\left(1-\tfrac{8}{9}\beta_0 R^2\right) \exp\left[-\tfrac{2}{3}(\beta-\delta\beta)R^2\right], \tag{5}$$

with $\beta_0 = \alpha_0$. Thus, our modification to Cheon's polarised cluster model wave function takes into account the role of the Pauli principle in the interaction between the clusters in a simple way. In the following sections, we use these cluster model wave functions to study the r.m.s. radius, the charge form factor, the charge density, the quadrupole moment and the bremsstrahlung weighted photodisintegration cross section for ^{6}Li.

3. *The r.m.s. radius*

It is well-known [1, 2, 7] that the ^{6}Li nucleus has a large r.m.s. radius which cannot be accounted for within the framework of a harmonic oscillator shell model, and that the cluster model successfully reproduces the experimental value for the r. m. s. radius. Consider ^{6}Li to be described by one of the wave functions described in Section 2. Its r.m.s. radius is given by:

$$\langle r^2\rangle^{1/2} = \left(\tfrac{1}{6}\int \Psi^* \sum_{i=1}^{6} r_i^2 \Psi_0 d\tau\right)^{1/2}, \tag{6}$$

where Ψ is the totally antisymmetrized wave function, Ψ_0 is the unantisymmetrized wave function and $d\tau$ stands for $\prod_{i=1}^{6} d\boldsymbol{r}_i$, $\boldsymbol{r}_i$ being the position vector of the i-th nucleon. Explicitly,

$$R_{\text{r.m.s.}} = \langle r^2\rangle^{1/2} = \left[\int \Psi^* \tfrac{1}{6}\left(\sum_{i=1}^{6} \varrho_i^2 + \tfrac{4}{3}R^2\right)\Psi_0 d\tau\right]^{1/2},$$

$$= \left[\frac{6!}{N^2}(R_0 - 2R_1 + R_2)\right]^{1/2}, \tag{7}$$

where the normalization constant N, defined by (3), can be written as:

$$N^2 = 6!(A_0 - 2A_1 + A_2). \tag{8}$$

The explicit expressions for A_0, A_1, A_2, R_0, R_1 and R_2 are given in the Appendix, to correct for the errors (typographical and other) in the paper of Lodhi [7], which are non-trivial errors, since the range of the parameters in the wave function, giving the experimental value of the r.m.s. radius, is very different from what he obtained.

When we use the polarised cluster model wave function (4), which is unantisymmetrized, we get for the r.m.s. radius:

$$R_{\text{r.m.s.}} = \tfrac{1}{2}\left\{\left(\frac{3}{\alpha_0} + \frac{1}{\alpha_1}\right)\left(\frac{3-\delta}{3-2\delta}\right) + \frac{7}{3\beta}\left(\frac{3+\delta}{3-2\delta}\right)\right\}^{1/2}, \tag{9}$$

which when $\delta = 0$, reduces to the value of $(R_0/A_0)^{1/2}$ given in the Appendix (A10).

If the relative motion part of the wave function is given by (5), then

$$R_{\text{r.m.s.}} = \tfrac{1}{2}\left\{\left(\frac{3}{\alpha_0} + \frac{1}{\alpha_1}\right)\left(\frac{3-\delta}{3-2\delta}\right) + \frac{1}{a_0}\frac{1}{\beta}\delta'\left[1 - \tfrac{10}{3}\frac{\beta_0}{\beta}\delta' + \tfrac{35}{9}\left(\frac{\beta_0}{\beta}\delta'\right)^2\right]\right\}^{1/2}, \tag{10}$$

where

$$\delta' = \frac{3+\delta}{3-2\delta} \quad \text{and} \quad a_0 = 1 - 2\frac{\beta_0}{\beta}\delta' + \tfrac{5}{3}\left(\frac{\beta_0}{\beta}\delta'\right)^2. \tag{11}$$

4. The charge form factor and the charge density

The charge form factor and the quadrupole moment are quantities which depend on the entire internal structure of the nucleus rather than on any specific portion of it. The study of the charge form factor gives information about the charge distribution of the nucleus. From electron scattering experiments, the elastic form factor of ^{6}Li has been reported upto a momentum squared, q^2, of 6.9 fm^{-2}, by Suelzle et al. [13]. A diffraction minimum in the ^{6}Li form factor has been reported at $q^2 = 8$ fm^{-2} by Whitney et al. [14], which has been corroborated by Li et al. [15] who determined the charge form factor upto 13 fm^{-2}. Several attempts [16] have been made to explain the experimental features of the form factor. While Cheon [3] neglects the effects of antisymmetrization, the effect of antisymmetrizing his phenomenological polarised cluster model wave function has been shown to be important in the study of the form factor by Jain [6], who finds that for $q^2 > 7$ fm^{-2}, the inclusion of exchange terms increases the charge form factor by about a factor of ten. However, Kudeyarov et al. [5], in their study of elastic and inelastic form factors, find that the role of antisymmetrization is more significant in the case of magnetic form factors and that an unantisymmetrized cluster model wave function, which contains a shell model node in its relative motion part, is capable of producing the factor of ten rise in the charge form factor at high momentum transfers. This clearly emphasizes the role of the relative motion function in the study of the static moments of ^{6}Li. Since our aim is to obtain a set of parameters in the cluster model wave function which reproduce all the properties mentioned earlier for $q^2 \leqslant 6\,\mathrm{fm}^{-2}$, and further at large q^2, nucleon-nucleon correlations at small distances [17] and mesonic corrections [18] seem to become essential, we neglect the effect of antisymmetrization in the case of the charge form factor and the quadrupole moment.

The charge form factor of ^{6}Li is defined as:

$$F(q^2) = \tfrac{1}{3} \sum_{i=1,3,5} \int \Psi_0^* \exp\left[i\boldsymbol{q} \cdot (\boldsymbol{r}_i - \boldsymbol{z})\right] \Psi_0 d\tau = \tfrac{1}{3}(2B_0 + B_1), \tag{12}$$

where

$$B_0 = \int \Psi_0^* \exp\left[i\boldsymbol{q} \cdot (\boldsymbol{\varrho}_1 + \tfrac{1}{3}\boldsymbol{R})\right] \Psi_0 d\tau,$$

$$B_1 = \int \Psi_0^* \exp\left[i\boldsymbol{q} \cdot (\boldsymbol{\varrho}_5 - \tfrac{2}{3}\boldsymbol{R})\right] \Psi_0 d\tau,$$

and Ψ_0 is the polarised cluster model wave function (4). We find it convenient to evaluate these integrals in the Cartesian coordinate system and we get:

$$B_0 = \left[1 + P_1 q^2 + P_2 q^4\right] \exp\left[-A(q_x^2 + q_y^2) - Bq_z^2\right], \tag{13}$$

and

$$B_1 = \left[1 + P_1' q^2 + P_2' q^4\right] \exp\left[-A'(q_x^2 + q_y^2) - B'q_z^2\right], \tag{14}$$

where

$$P_1 = -\frac{\delta'}{36\beta}, \quad P_2 = \tfrac{3}{20} P_1^2, \quad P_1' = 4P_1, \quad P_2' = 16P_2,$$

$$A = \frac{3}{16\alpha_0} + \frac{\delta'}{48\beta}, \quad B = \frac{3\delta'}{16\alpha_0} + \frac{\delta'}{48\beta}, \quad A' = \frac{1}{8\alpha_1} + \frac{\delta'}{12\beta},$$

$$B' = \frac{\delta'}{8\alpha_1} + \frac{\delta'}{12\beta}. \tag{15}$$

The form factor at zero momentum transfer, $F(0)$, is normalized to be 1, for a charged particle, as usual. The normalized probability density $\varrho(\boldsymbol{r})$ can be obtained from the knowledge of $F(q^2)$ by Fourier transforming (12) to obtain:

$$\varrho(\boldsymbol{r}) = \frac{1}{(2\pi)^3} \int F(q^2) \exp\left[-i\boldsymbol{q} \cdot \boldsymbol{r}\right] d\boldsymbol{q} \tag{16}$$

and it is completely determined if $F(q^2)$ is known for all values of q^2. Using the above expressions for $F(q^2)$, we obtain for the charge density:

$$\varrho(\boldsymbol{r}) = \frac{\pi^{3/2}}{3(2\pi)^3} \left[2 \left\{ \exp\left(- \frac{x^2+y^2}{4A} - \frac{z^2}{4B} \right) \left[a_1 + a_2(x^2+y^2) + a_3 z^2 \right.\right.\right.$$
$$\left.\left.\left. + a_4(x^2+y^2)^2 + a_5(x^2+y^2)z^2 + a_6 z^4\right] \right\} + \{\text{primed}\} \right], \tag{17}$$

with:

$$a_1 = 1+\frac{P_1}{2}\left(\frac{2}{A}+\frac{1}{B}\right)+\frac{P_2}{4}\left(\frac{8}{A^2}+\frac{4}{AB}+\frac{3}{B^2}\right),$$

$$a_2 = -\frac{1}{4A^2}\left[P_1+P_2\left(\frac{4}{A}+\frac{1}{B}\right)\right], \quad a_3 = -\frac{1}{4B^2}\left[P_1+P_2\left(\frac{2}{A}+\frac{3}{B}\right)\right],$$

$$a_4 = P_2/16A^4, \quad a_5 = P_2/8A^2B^2, \quad a_6 = P_2/16B^4,$$

and {primed} is similar in form to the flower bracketed expression in (17) with A, B, P_1, P_2, replaced by their primed quantities A', B', P_1' and P_2'. If the polarised cluster model wave function is modified with $\chi(R)$, given by (5), then in the expressions (12 to 14) for $F(q^2)$ and (17) for $\varrho(\boldsymbol{r})$, the values of P_1 and P_2 to be used are:

$$P_1 = 36X^2\beta_0(1-60\beta_0X)/a_0, \quad P_2 = 324X^4\beta_0^2/a_0, \tag{18}$$

with $X = \delta'/36\beta$ and a_0 given by (11), instead of the values for P_1 and P_2 given in (15).

5. *The quadrupole moment*

The quadrupole moment of a nucleus gives information about the departure of the nucleus from its spherical shape. Due to the deformation, assumed to be along the z-axis, we arrived at the non-spherical probability distribution, (16) for $\varrho(\boldsymbol{r})$. It can be verified that when δ is set equal to zero, $\delta' = 1$, $A = B$ and $A' = B'$ and $\varrho(\boldsymbol{r})$ becomes a function of $r^2 = (x^2+y^2+z^2)$, i.e. $\varrho(\boldsymbol{r})$ becomes spherically symmetric and it is well known that for a spherically symmetric charge density, the quadrupole moment is zero.

The classical quadrupole moment is defined as:

$$Q = \int \varrho(\boldsymbol{r})(3z^2-r^2)d\boldsymbol{r}. \tag{19}$$

Explicitly,

$$Q = \tfrac{1}{3}\Big[2\Big\{a_1(D-C)+a_2C(D-2C)+\frac{a_3}{2}D(3D-C)+a_4C^2(D-3C)$$
$$+a_5CD(\tfrac{3}{2}D-C)+\tfrac{3}{4}a_6D^2(5D-C)\Big\}+\{\text{primed}\}\Big], \tag{20}$$

with $C = 4A$, $D = 4B$ and the quantity {primed} is similar in form to the flower bracketed quantity in (20) with A, B, C, D, P_1 and P_2 replaced by A', B', C', D', P_1' and P_2', respectively. As in the case of the charge form factor and the charge density, for the polarised cluster model, P_1 and P_2 are given by (15) and for the modified polarised cluster model, P_1 and P_2 are given by (18).

6. *The photodisintegration*

In the case of some of the nuclei, such as ^{3}H, ^{3}He and ^{4}He, the bremsstrahlung weighted cross section, which is the energy weighted electric dipole cross section for the nuclear photoeffect, is directly proportional to the mean square radius of the target nucleus.

However, this is not true in the case of ^{6}Li, since the cluster model ground state wave function is not completely symmetric in the space coordinates of all the nucleons. Therefore, a study of the bremsstrahlung cross section for the photodisintegration of ^{6}Li, defined by:

$$\sigma_b = \int (\sigma/W) dW, \tag{21}$$

where σ is the electric dipole cross section for a given photon energy W, is expected to provide additional information about the cluster model. The cross section σ_{0n} for photon absorption is given by [18]:

$$\sigma_{0n} = \frac{2\pi^2 e^2 \hbar}{mc} f_{0n} \tag{22}$$

where f_{0n} is the oscillator strength for an E1 transition between the discrete states 0 and n and is defined by:

$$f_{0n} = \frac{2mW}{\hbar^2} |\bar{Z}_{0n}|^2 \tag{23}$$

where $W = E_n - E_0$, and $\bar{Z}_{0n}$ is the component of the displacement of the nucleon. Using the closure property, we have for the bremsstrahlung cross section:

$$\sigma_b = \sum_n \sigma_{0n}/W = \frac{4\pi^2 e^2}{\hbar c} |\bar{Z}_{00}|^2. \tag{24}$$

Following Dellafiore and Brink [19] equation (24) can be written as:

$$\sigma_b = \frac{4\pi^2}{3} \frac{e^2}{\hbar c} \tfrac{9}{4} \langle R_{pn}^2 \rangle, \tag{25}$$

where $\boldsymbol{R}_{pn} = \boldsymbol{R}_p - \boldsymbol{R}_n$, $\boldsymbol{R}_p$ and $\boldsymbol{R}_n$ represent the centres of mass of the protons and the neutrons, respectively and the expectation value is in the ground state of the nucleus. In the coordinate system, defined in Section 2, the operator:

$$R_{pn}^2 = \tfrac{1}{9}\left[\sum_{i=1}^{6} \boldsymbol{r}_i \left(\frac{1+\tau_3^i}{2}\right) - \sum_{i=1}^{6} \boldsymbol{r}_i \left(\frac{1-\tau_3^i}{2}\right)\right]^2$$
$$= \tfrac{4}{9}(\varrho_1 + \varrho_3 + \varrho_5)^2, \tag{26}$$

using 1, 3 and 5 as the proton indices and

$$\langle R_{pn}^2 \rangle = \frac{6!}{N^2}(B_{pn}^0 - 2B_{pn}^1 + B_{pn}^2), \tag{27}$$

where the normalization constant N is defined by (8). For the standard cluster model wave function (2), a straightforward calculation yields:

$$B_{pn}^0 = \frac{A_0}{3\alpha_0}\left(2 + \frac{1}{z}\right), \tag{28}$$

$$B^1_{\mathrm{pn}} = \tfrac{4}{9}\frac{A_1}{\alpha_0}\left[\tfrac{3}{4}\left(\frac{11+4z}{2+3z}\right)+\left(\frac{1-z}{2+3z}\right)^2\left(1-\frac{q_1^2}{4p_1^2}\right)^{-4}\left(1+\tfrac{2}{3}\frac{q_1^2}{4p_1^2}\right)^{-1}\right.$$
$$\left.\times\left\{\frac{7\cdot 5}{p_1}\left(1+\tfrac{4}{3}\frac{q_1^2}{4p_1^2}\right)-\frac{q_1^2}{4p_1^2}\left(25+\tfrac{5}{2}\frac{q_1^2}{p_1^2}\right)\right\}\right], \tag{29}$$

$$B^2_{\mathrm{pn}} = \frac{A_2}{3\alpha_0}\left(\frac{5+z}{1+z}\right), \tag{30}$$

where the expressions for A_0, A_1, A_2, p_1 and q_1 are given in the Appendix.

The use of the polarised cluster model wave function (4) yields for B^0_{pn} the expression:

$$B^0_{\mathrm{pn}} = \frac{A_0}{3\alpha_0}\left(2+\frac{1}{z}\right)\left(\frac{3-\delta}{3-2\delta}\right). \tag{31}$$

Since the operator R^2_{pn} given by (26) is independent of the relative coordinate, $\boldsymbol{R}$ $(=\boldsymbol{R}_\alpha-\boldsymbol{R}_d)$, the modification of the inter cluster wave function to include the shell model node, will not change the expression for $\langle R^2_{\mathrm{pn}}\rangle$.

7. *Results and discussion*

We calculate the r.m.s. radius, the charge form factor, the charge density, the quadrupole moment and the bremsstrahlung weighted cross section for the photodisintegration of ^{6}Li, using the cluster model wave functions of Section 2. While Cheon [3] obtains the

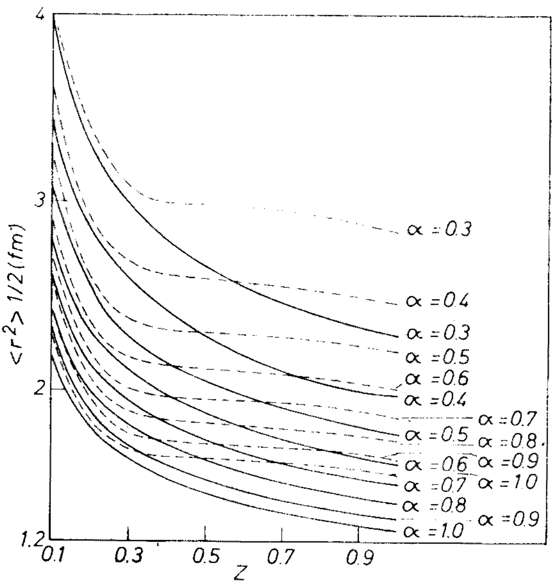

Fig. 1. The r.m.s. radius of ^{6}Li as a function of z for various values of α_0, using the unantisymmetrize and antisymmetrized standard cluster model wave function (2); ——— unantisymmetrized, - - - - antisymmetrized

charge form factor and the quadrupole moment by a χ^2-fit, others [2, 6, 11, 12] resort to variational procedures. Our aim is to first determine the range of parameters α_0 and $z(=\alpha_1/\alpha_0)$ which give the r.m.s. radius of ^{6}Li. We then compute the charge form factor, for the range of parameters already found, and obtain unique values of α_0 and z which give a good agreement with the data up to $q^2 \leqslant 6\ \text{fm}^{-2}$. These studies are made with the standard cluster model, as well as the modified polarised cluster model in which the deformation parameter δ is set to zero. Having determined the unique values of α_0 and z, we calculate the quadrupole moment given by (20). Here, the only unknown parameter is the deformation parameter δ, and so we obtain its unique value, which will reproduce the experimental value of Q.

TABLE I

Values of α_0 and z in the standard cluster model wave function for which the value of the r.m.s. radius of ^{6}Li is 2.54 ± 0.05 fm

α_0 (fm^{-2})	z in the standard cluster model wave function	
	Unantisymmetrized	Antisymmetrized
0.3	0.6 ± 0.04	—
0.4	0.32 ± 0.04	0.66 ± 0.2
0.5	0.205 ± 0.015	0.22 ± 0.02
0.6	0.14 ± 0.01	0.16 ± 0.01

The r.m.s. radius of ^{6}Li has been computed by varying the parameters α_0 and z within the ranges $0.3 \leqslant \alpha_0 \leqslant 1.0$ and $0.1 \leqslant z \leqslant 1.0$. In Fig. 1, we plot the r.m.s. radius as a function of z for various values of α_0, using the unantisymmetrized and antisymmetrized standard cluster model wave function (2). Throughout the range of z, for all values of α_0, we find that the effect of antisymmetrization is to enhance the value of the r.m.s. radius. The standard cluster model wave function will correspond to the single oscillator shell model wave function in the lowest configuration, when we set $\alpha_0 = \alpha_1 = \beta$. In our model, since the alpha cluster within ^{6}Li is expected to be larger than the free alpha particle, we note that the value of the width parameter should be restricted to less than the free alpha particle width of $0.58\ \text{fm}^{-2}$. From Fig. 1, we can find for a given value of α_0, the range of the parameter z for which the experimental value of 2.54 ± 0.05 fm, for the r.m.s. radius of ^{6}Li, can be obtained. In Table I, we list some values of α_0 and the ranges of the parameter z. As stated earlier, the expressions for A_0, A_1 and A_2, R_0, R_1 and R_2, given in the Appendix correct the errors in the corresponding expressions (21), (22) and (23), (24), (25) and (26), respectively, obtained by Lodhi [6], who concludes that "an acceptable value of $\langle r^2 \rangle^{1/2}$ lying between 2.4 fm to 2.8 fm is obtained for the set of parameters α and z, such that: $0.58\ \text{fm}^{-2} \geqslant \alpha \geqslant 0.34\ \text{fm}^{-2}$ and $0.4 \geqslant z \geqslant 1.0$". Clearly, our range of parameters given in Table I is considerably more restricted, though the shape of the curves is similar to those obtained by Lodhi. In Fig. 2, we plot the r.m.s. radius given by (10), for the modified polarized cluster model, as a function of z for different values of α_0 and for $\delta = 0.0$ and $\delta = -0.03$.

The point to be noted is that the use of an inter-cluster (or, relative motion part of the) wave function with a shell model node in it, enhances the r.m.s. radius for all values of α_0 and z. This effect is similar to the effect of antisymmetrization, observed in Fig. 1.

For these values of α_0 and z, we calculate the quadrupole moment (20). The only unknown parameter δ which accounts for the experimental value of $Q = -0.08$

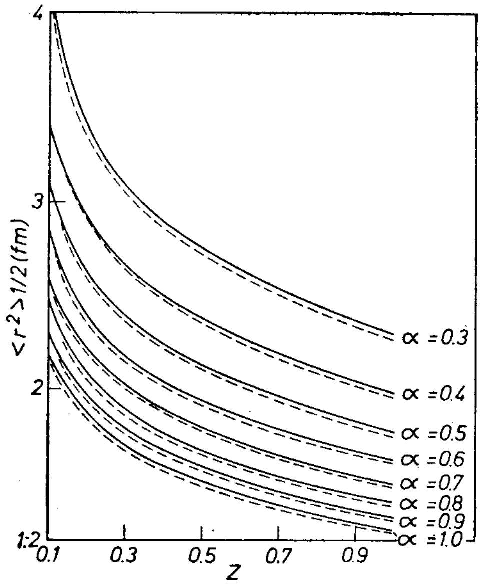

Fig. 2. The r.m.s. radius of ^{6}Li, for the modified polarised cluster model, as a function of z for different values of α_0; ——— $\delta = 0.0$ and ------ $\delta = -0.03$

± 0.009 fm^{-2} [13] for ^{6}Li, is now determined. For $\alpha_0 = 0.455$ fm^{-2} and $z = 0.3$ which, as we will see later, give a good fit for the charge form factor up to $q^2 \leqslant 6$ fm^{-2}, the value of δ is only -0.035. The negative value of δ means that the deformation of the clusters is of the oblate type. For a different choice of parameters made by Cheon [3] and Jain and Sarma [6], the value of δ required is -0.147 and -0.16, respectively.

Eq. (12) defines actually the point distribution form factor $F_{pt}(q^2)$ and the charge from factor of the nucleus is given by:

$$F(q^2) = F_{pt}(q^2)F^{p}_{ch}(q^2),$$

where the realistic proton form factor is given by [21]:

$$F^{p}_{ch}(q^2) = \frac{1.249}{1+q^2/15.6} - \frac{0.7892}{1+q^2/26.6} + \frac{0.5819}{1+q^2/8.19} - 0.0326. \quad (32)$$

In Fig. 3, we plot the form factor squared for ^{6}Li as a function of q^2 up to 6 fm^{-2}, for $\alpha_0 = 0.455$ fm^{-2}, $z = 0.3$ and $\delta = -0.035$. Curves *1* and *2* correspond to the polarised cluster model and the modified polarised cluster model and it can be seen that curve *2*, which takes into account the requirement of the shell model node in the inter-cluster wave function, is in better agreement with the experimental data [13]. In Fig. 4, we plot $|F(q^2)|^2/\exp(-1.763\, q^2)$ in order to visualize the difference in the models, as has been done before [3, 13]. Here, curves *1* and *2* correspond to $\delta = 0$ and $\delta = -0.035$ for the standard and the polarised cluster model wave functions, respectively. Curves *3* and *4* correspond to the same values of δ but for the modified polarised cluster model wave function. We

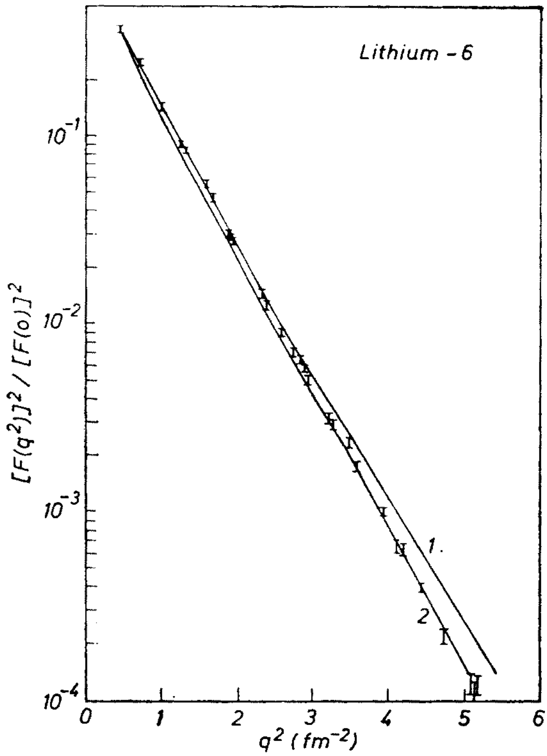

Fig. 3. The charge form factor of ^{6}Li, in the polarised cluster model (curve *1*) and the modified polarised cluster model (curve *2*)

note that curve *4* fits very well the experimental data for $q^2 < 7\ \mathrm{fm}^{-2}$, where the effect of antisymmetrization is not significant [5, 6].

In Fig. 5, we plot the average $r^2\varrho(r)$ using the set of parameters: $\alpha_0 = 0.455\ \mathrm{fm}^{-2}$, $z = 0.3$ and $\delta = -0.035$. Curve *1* is for the modified polarised cluster model and curve *2* is the best fit of Burleson and Hofstadter [23] to their experimental data. Burleson and Hofstadter [23] observe that a phenomenological charge distribution which gives a satisfactory fit to the elastic form factor of ^{6}Li, has a fairly long tail, which is essential if one is interested in the study of cluster-knockout reactions [22]. Radhakant and Ullah [12] proposed a modified radial wave function, which can explain the effects due to the long tail of the wave function and their charge density is shown as curve *3* in Fig. 5, which also agrees with the fit of Burleson and Hofstadter. We find that the charge density cannot be accounted for by either the standard or the polarised cluster model which contain the peripheral form for the intercluster wave function. It is satisfying to note that our charge density (curve *1*) using the modified polarised cluster model wave function is consistent with the best fitting phenomenological model (curve *2*) and the other attempt due to Radhakant and Ullah (curve *3*).

The bremsstrahlung weighted cross section σ_b for the photodisintegration is computed by varying $0.3\ \mathrm{fm}^{-2} \leqslant \alpha_0 \leqslant 1.0\ \mathrm{fm}^{-2}$ and $0.1 \leqslant z \leqslant 1.0$ and it is plotted in Fig. 6, as

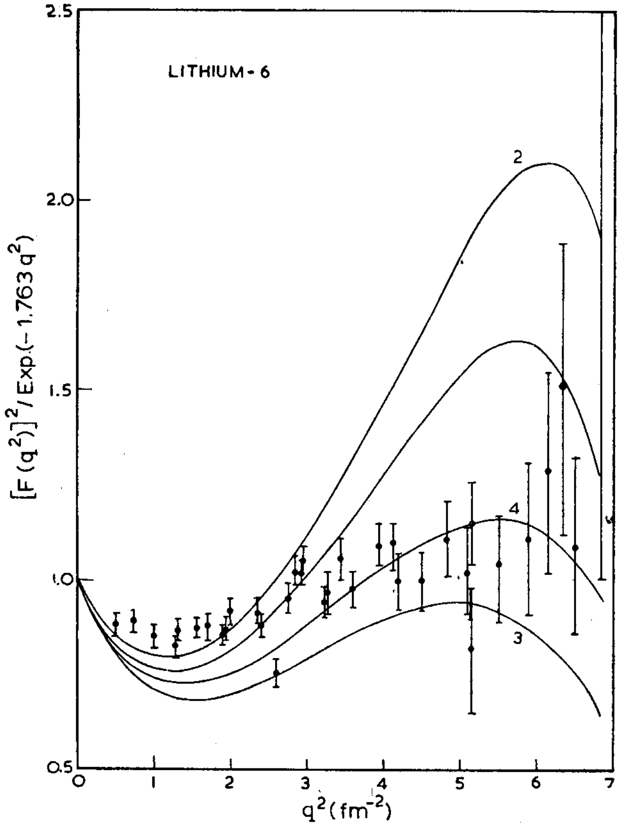

Fig. 4. The charge form factor of ^{6}Li, in the standard cluster model with $\delta = 0.0$ (curve *1*), $\delta = -0.035$ (curve *2*); and the modified polarised cluster model with $\delta = 0.0$ (curve *3*), $\delta = -0.035$ (curve *4*)

a function of z for several values of α_0, using the unantisymmetrized standard cluster model wave function. In Fig. 7, σ_b is plotted for the same set of parameters using the antisymmetrized standard cluster model wave function. Experimental results for the bremsstrahlung weighted cross section of ^{6}Li are due to Costa et al. [24], who measured σ_b as 3.8 ± 0.3 mb, from the photoneutron yield at photon energies of 5 MeV to 97 MeV. They also indicate that the inclusion of (γ, t) reaction enhances the value of σ_b to 5.1 ± 0.8 mb. In Table II, we list the various sets of parameters and the corresponding values of the r.m.s. radius and σ_b they yield. The parameters of Tang et al. [2] as well as those of Burleson and Hofstadter [23] assume the alpha cluster to be the larger cluster. The choice of $\beta = 2.616\ \mathrm{fm}^{-2}$

by Burleson and Hofstadter, to get the value of 2.54 fm for the r.m.s. radius is unrealistic, since it indicates a very small separation of the centres of mass of the two clusters. The set of parameters obtained by Jackson [25] and Wood [26] are based on the assumption that the deuteron cluster is the larger cluster. The last row in Table II gives our set of parameters, which not only reproduce the right value of the r.m.s. radius, but also account

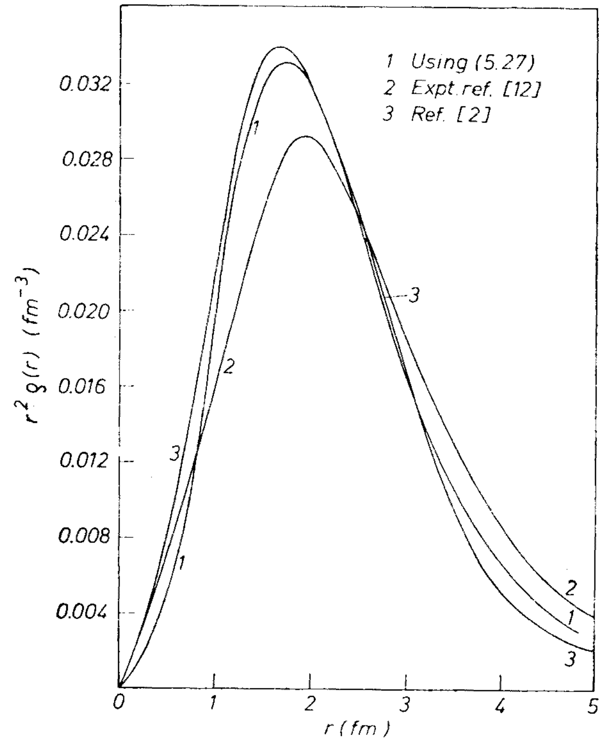

Fig. 5. The charge density distribution of ^{6}Li. Curve *1* is for the modified polarised cluster model, in which the average charge density is plotted; curve *2* is the best fit of Burleson and Hofstadter [23] to their experimental data and curve *3* that of Radhakant and Ullah [12]

for the charge form factor, the charge density and the quadrupole moment of ^{6}Li. For the parameters of Wood [26] we get her unantisymmetrized wave function values for the r.m.s. radius and σ_b, but we are unable to reproduce her values for the antisymmetrized case. For our choice of parameters (last row of Table II), we find that the bremsstrahlung weighted cross section is higher than the present experimental value. This is not very disturbing since one can expect the value of σ_b to increase when other photonuclear reactions are included in the determination of the bremsstrahlung weighted cross section or when the total photonuclear absorption cross section is available upto higher photon energies.

In conclusion, we find that for the set of parameters: $\alpha_0 = 0.455$ fm^{-2}, $z = 0.3$ and $\delta = -0.035$, the r.m.s. radius, the charge form factor, the charge density, the quadrupole moment and the bremsstrahlung weighted cross section for the photodisintegration of

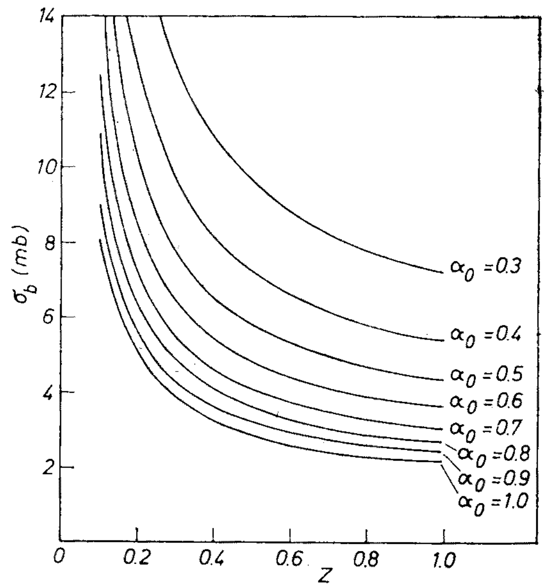

Fig. 6. The bremsstrahlung weighted cross section σ_b for the photodisintegration of ^{6}Li, using the unantisymmetrized standard cluster model wave function

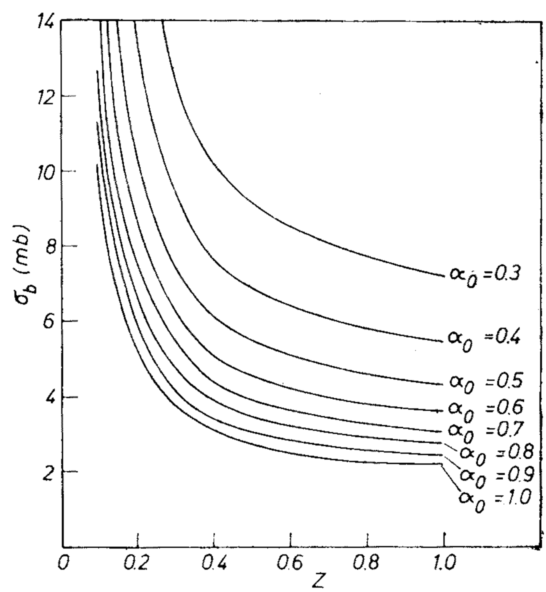

Fig. 7. The bremsstrahlung weighted cross section σ_b for the photodisintegration of ^{6}Li, using the antisymmetrized standard cluster model wave function

TABLE II

Results for the r.m.s. radius, the bremsstrahlung weighted cross section using the unantisymmetrized and antisymmetrized standard cluster model wave function for ^6Li

Ref.	α_0 (fm^{-2})	α_1 (fm^{-2})	β (fm^{-2})	$R'_{r.m.s.}$ (fm)	$R_{r.m.s.}$ (fm)	σ'_b (mb)	σ_b (mb)
[2]	0.433	0.659	0.329	1.97	2.02	4.43	3.86
[23]	0.142	0.873	2.616	2.41	2.54	10.99	11.47
[25]	0.338	0.218	0.199	2.51	2.54	7.58	6.40
[26]	0.3	0.189	0.226	2.53	2.52	8.63	5.85
this work	0.3	0.189	0.226	2.53	2.96	8.63	8.2
	0.5	0.2	0.3	2.12	2.32	6.5	6.1
	0.455	0.137	0.243	2.42	2.5	8.13	7.83

Note: The prime denotes the result for the case of the unantisymmetrized wave function of the standard cluster model.

^6Li, are in reasonable agreement with the available experimental data and that the inclusion of the shell-model node in the inter-cluster wave function is crucial in the study of the charge form factor. However, it will be worthwhile to study the effect of antisymmetrization of the modified polarised cluster model wave function on all the static properties of the ^6Li nucleus discussed here and use the same in the study of cluster-knock-put reactions on ^6Li.

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APPENDIX

Here we give the values of A_0, A_1 and A_2 which occur in the expression for the normalization constant (8) and the values of R_0, R_1 and R_2, which occur in the expression for the r.m.s. radius (7) of ^6Li. Introducing the parameters:

$$z = \alpha_1/\alpha_0 \quad \text{and} \quad \kappa = \beta/\alpha_0 \tag{A1}$$

(where κ is called the cluster isolation parameter), to simplify the expressions, we get:

$$A_0 = 2^9(0.01514)\pi^{13/2}\alpha_0^{-19/2}z^{-3/2}\kappa^{-7/2}, \tag{A2}$$

$$A_1 = 2^9(3.58)\pi^{13/2}\alpha_0^{-19/2}p_1^{-5}(2+3z)^{-3/2}\left(1-\frac{q_1^2}{4p_1^2}\right)^{-7/2}\left(1+\frac{q_1^2}{6p_1^2}\right), \tag{A3}$$

where

$$p_1 = \frac{3+11z+3z^2}{3(2+3z)} + \kappa, \tag{A4}$$

$$q_1 = -\frac{2(3+2z+3z^2)}{3(2+3z)}, \tag{A5}$$

and

$$A_2 = 2^9(3.58)\pi^{13/2}\alpha_0^{-19/2}p_2^{-5}(1+z)^{-3}\left(1-\frac{q_2^2}{4p_2^2}\right)^{-7/2}\left(1+\frac{q_2^2}{6p_2^2}\right), \tag{A6}$$

where

$$p_2 = \tfrac{5}{3}+\kappa \quad \text{and} \quad q_2 = \tfrac{8}{3}. \tag{A7}$$

For the polarised cluster model wave function (4), we get:

$$N_0^2 = A_0 = 2^9(0.01514)\pi^{13/2}\alpha_0^{-19/2}z^{-3/2}\kappa^{-7/2}\left(\frac{3+\delta}{3-2\delta}\right)^{7/2}. \tag{A8}$$

For the modified polarised cluster model wave function:

$$N_0^2 = A_0 = 2^9(0.007176)\pi^{13/2}\alpha_0^{-19/2}z^{-3/2}\kappa^{-7/2}\left(\frac{3+\delta}{3-2\delta}\right)^{7/2}$$

$$\times\left\{1-\frac{2\beta_0}{\beta}\left(\frac{3+\delta}{3-2\delta}\right)+\tfrac{5}{3}\left(\frac{\beta_0}{\beta}\,\frac{3+\delta}{3-2\delta}\right)^2\right\}. \tag{A9}$$

For the expressions R_0, R_1 and R_2, in (7), we obtain:

$$R_0 = \frac{A_0}{12\alpha_0}\left(9+\frac{3}{z}+\frac{7}{\kappa}\right), \tag{A10}$$

$$R_1 = \frac{A_1}{6\alpha_0}\left[\tfrac{9}{2}\left(\frac{3+2z}{2+3z}\right)+\left(1-\frac{q_1^2}{4p_1^2}\right)^{-1}\left(1+\frac{q_1^2}{4p_1^2}\right)^{-1}\right.$$

$$\left.\times\left\{\tfrac{15}{2}\,\frac{P}{p_1}\left(1+\frac{q_1^2}{3p_1^2}\right)+Q\,\frac{q_1}{8p_1^2}\left(25+\frac{5q_1^2}{2p_1^2}\right)\right\}\right], \tag{A11}$$

where

$$P = \frac{2(31+66z+63z^2)}{9(2+3z)^2}, \tag{A12}$$

$$Q = \frac{4(1+30z+9z^2)}{9(2+3z)^2}, \tag{A13}$$

and

$$R_2 = \frac{A_2}{6\alpha_0}\left[\frac{3}{2}\frac{(5+z)}{(1+z)} + \frac{16}{9}\left(1-\frac{q_2^2}{4p_2^2}\right)^{-1}\left(1+\frac{q_2^2}{6p_2^2}\right)^{-1} \times\left\{\frac{15}{2p_2}\left(1+\frac{q_2^2}{3p_2^2}\right)-\frac{q_2^2}{2p_2^2}\left(25+\frac{5q_2^2}{2p_2^2}\right)\right\}\right]. \tag{A14}$$

Note: The parameter Q defined in (A13) and used in (A11) has nothing to do with the quadrupole moment Q of Section 5.

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