SEQUENTIAL MASS GENERATION AND CLASSIFICATION OF THE RELATIONS BETWEEN QUARK MASSES AND WEAK MIXING ANGLES

By A. Białas

Institute of Physics, Jagellonian University, Cracow*

(Received April 13, 1981)

Relations between quark masses and weak mixing angles suggested by the assumption of sequential mass generation are discussed. A classification of these relations according to additional conditions needed for their derivation is presented. In this classification the relation $\sin\theta_3/\sin\theta_2 \approx \sqrt{\frac{m_u m_b}{m_s m_c}}$ turns out to be singled out as the most direct consequence of the assumption of sequential mass generation.

PACS numbers: 12.40.–y

1. Introduction

The possibility of finding relations between quark masses and weak mixing angles was investigated by many authors [1–57]. In the present paper we also consider this problem, starting from the idea of "sequential" mass generation [12, 26, 33, 49]. We propose a particular formulation of this general idea and develop a classification of the relations which can be obtained from it.

In sequential mass generation it is assumed that at the tree level of fundamental Lagrangian only the top and bottom quarks have non-vanishing masses, whereas $m_c = m_s = m_u = m_d = 0$. The first order perturbative corrections generate non-vanishing m_c and m_s, but one has still $m_u = m_d = 0$ in this approximation. Second order corrections generate non-vanishing m_u and m_d. Naturally, such a general formulation does not give specific quantitative predictions for measurable quantities. Other, more precise proposals are needed. Several specific examples have already been investigated [12, 26, 33, 49]. In this paper we discuss a possibility which has not been considered yet. It emphasizes geometrical aspects of the problem and may therefore be of some interest. However, it does not pretend to be a unique solution and should be looked at as yet another possibility in the search for the correct model of the origin of fermion masses and fermion mixing.

* Address: Instytut Fizyki, Uniwersytet Jagielloński, Reymonta 4, 30-059 Kraków, Poland.

We analyze the mass matrices of the general form

$$M_{ik} = \sum_{\mu=1}^{3} a_i(\mu)a_k^*(\mu) \tag{1.1}$$

i.e. hermitian matrices with non-negative eigenvalues. This form is chosen, because it is suggestive of the models with composite Higgs bosons [58, 59]. Our approach is different from e.g. that of Ref. [12], [26] and [33] which considered the hermitian matrices with one negative and two positive eigenvalues.

Eq. (1.1) expresses the mass matrix in terms of three vectors $\mathbf{a}(\mu)$. We formulate the assumption of sequential mass generation by requiring that at each step of approximation one new independent direction is available for construction of the vectors $\mathbf{a}(\mu)$. Thus, at the tree level all vectors $\mathbf{a}(\mu)$ must be parallel to $\mathbf{a}(3)$ (some of them may vanish). This implies, as required, non-vanishing m_t and m_b whereas all other masses vanish. In the first order approximation one new direction is available, so that $\mathbf{a}(2)$ and $\mathbf{a}(1)$ acquire components orthogonal to $\mathbf{a}(3)$, but all three vectors are lying in one plane, so that one eigenvalue of M is still vanishing. Finally, in the second order approximation the vector $\mathbf{a}(1)$ is moved out of the $\mathbf{a}(2)$ $\mathbf{a}(3)$ plane so that all three eigenvalues of M become different from zero.

The more detailed description of this construction of the matrix M is given in the next two sections. As the next step, we then study the relations between the mass matrices M and M' of down- and up- quark families, which follow from the relations between the vectors $\mathbf{a}(\mu)$ defining the matrix M and the vectors $\mathbf{a}'(\mu)$ defining the matrix M'. In particular, we propose a classification of these relations by distinguishing:

Relations of class A which follow from the assumption that the vectors $\mathbf{a}(2)$, $\mathbf{a}(3)$, $\mathbf{a}'(2)$, $\mathbf{a}'(3)$ all lie in the same plane;

Relations of class B which follow from the additional assumption that the vectors $\mathbf{a}(3)$ and $\mathbf{a}'(3)$ are parallel to one another;

Relations of class C which require other constraints. This classification is invariant with respect to arbitrary unitary transformations in the space of three quark families.

Relations of *class A* should be valid if only 3-rd and 2-nd families are mixed in the first order of mass generation. They are thus fundamental for the very idea of sequential mass generation. Relations of *class B* test the physical meaning of our construction given by Eq. (1.1). Finally, the relations of *class C* are rather ad hoc conditions reflecting possible symmetries of the theory. They are not important for testing the assumption of sequential mass generation.

We show in Section 5 that the relations of *class A* imply

$$\sin\theta_3/\sin\theta_2 = \sin\alpha'/\sin\alpha, \tag{1.2}$$

$$\cos(\alpha+\alpha') \leqslant \cos\theta_1 \leqslant \cos(\alpha-\alpha') \tag{1.3}$$

and

$$\sin\theta_2 \leqslant \sin\alpha/\sin\theta_1, \quad \sin\theta_3 \leqslant \sin\alpha'/\sin\theta_1, \tag{1.4}$$

where θ_1, θ_2 and θ_3 are the weak mixing angles [60] and the angles α, α' are defined by

$$\mathrm{tg}^2\,\alpha = m_d/m_s, \tag{1.5}$$

$$\mathrm{tg}^2\,\alpha' = m_u/m_c. \tag{1.6}$$

Eq. (1.2) was derived in Ref. [26], [33] and the inequality (1.3) was considered by many authors. Our main point is not a new derivation, but the observation that from the point of view of sequential mass generation these relations are more fundamental than others, also disscussed in Ref. [12], [26] and [33].

Since, as seen from Eq. (1.5), (1.6), we have $\alpha' \ll \alpha$, inequality (1.3) implies $\sin^2\theta_1 \approx \frac{m_d}{m_s}$ and thus one recovers the well-known relation between the Cabbibo angle and the ratio m_d/m_s [1–4].

Relations of *class B* allow separate determination of the mixing angles θ_2 and θ_3. One obtains

$$\sin\theta_2 = \sin\alpha\sin\gamma/\sin\theta_1, \qquad \sin\theta_3 = \sin\alpha'\sin\gamma/\sin\theta_1, \tag{1.7}$$

where $\gamma = \theta - \theta'$ and the angles θ and θ' are defined by

$$\mathrm{tg}^2\,\theta = \tfrac{1}{2}\,\frac{m_s+m_d}{m_b}, \qquad \mathrm{tg}^2\,\theta' = \tfrac{1}{2}\,\frac{m_u+m_c}{m_t}. \tag{1.8}$$

Finally, the specific example of the relation of *class C* which we consider in this paper (symmetry between up- and down-families in the first order approximation) implies $\theta = -\theta'$ and thus

$$\frac{m_u+m_c}{m_t} = \frac{m_s+m_d}{m_b}. \tag{1.9}$$

Consequently, the mass of the top quark is fixed in terms of other masses.

We show in Section 6 that all these relations are compatible with present experimental limits.

In the next section we describe our construction of the quark mass matrices. In Section 3, the connection of this construction to the idea of sequential mass generation is explained. The classification of the relations between the mass matrices of up- and down-quark families is presented in Section 4. The consequences for the quark masses and mixing angles are derived in Section 5. In Section 6 the numerical values of these parameters are disscussed. Comparison with the results of other authors is presented in Section 7. In the last section we speculate on possible consequences and interpretation of our results.

2. *Construction of the fermion mass matrix*

Let us consider first the down (d, s, b) quarks. We shall assume that the mass matrix is hermitian with non-negative eigenvalues and we write it in the form

$$M_{ik} = \sum_{\mu=1}^{3} a_i(\mu)a_k^*(\mu) \tag{2.1}$$

with $i, k = 1, 2, 3$.

The form (2.1) does not restrict generality of the hermitian matrix M. On the contrary, for any matrix M satisfying our assumptions the choice of the vectors $\mathbf{a}(1)$, $\mathbf{a}(2)$ and $\mathbf{a}(3)$ is not unique. To avoid this ambiguity we require that the following conditions are satisfied by the vectors $\mathbf{a}(\mu)$, $\mu = 1, 2, 3$:

$$|\mathbf{a}(3)|^2 = |\mathbf{a}(2)|^2, \tag{2.2}$$

$$(\mathbf{a}(3)+\mathbf{a}(2))\mathbf{a}^*(1) = 0, \tag{2.3}$$

$$|\mathbf{a}(1)|^2 = \tfrac{1}{2}\,|\mathbf{a}(2)-\mathbf{a}(3)|^2. \tag{2.4}$$

Conditions (2.2)–(2.4) are invariant with respect to unitary transformations in the families space. In particular, they are invariant with respect to arbitrary phase-changing transformations,

$$P(\delta_1, \delta_2, \delta_3) = \begin{pmatrix} e^{i\delta_1} & & \\ & e^{i\delta_2} & \\ & & e^{i\delta_3} \end{pmatrix}, \tag{2.5}$$

so they are valid for arbitrary phase factors of the quark states. Furthermore, conditions (2.2)–(2.4) guarantee that there is one-to-one correspondence between the matrix M and the set of vectors $\mathbf{a}(\mu)$, $\mu = 1, 2, 3$ apart from arbitrary phases.

In the reference frame where the matrix M is diagonal, it is not difficult to find a parametrization of vectors $\mathbf{a}(\mu)$ which satisfies conditions (2.2)–(2.4). We have (up to an arbitrary transformation (2.5))

$$\mathbf{a}_0(3) = \Omega \begin{bmatrix} -\sin\theta\sin\alpha \\ \sin\theta\cos\alpha \\ \cos\theta \end{bmatrix}, \quad \mathbf{a}_0(2) = \Omega \begin{bmatrix} \sin\theta\sin\alpha \\ -\sin\theta\cos\alpha \\ \cos\theta \end{bmatrix}, \quad \mathbf{a}_0(1) = \sqrt{2}\,\Omega \begin{bmatrix} \sin\theta\sin\alpha \\ \sin\theta\cos\alpha \\ 0 \end{bmatrix}, \tag{2.6}$$

where Ω, θ and α are arbitrary real parameters. The matrix M itself takes the form

$$M_{ik}^{(0)} = \Omega^2 \begin{bmatrix} 4\sin^2\theta\sin^2\alpha & 0 & 0 \\ 0 & 4\sin^2\theta\cos^2 x & 0 \\ 0 & 0 & 2\cos^2\theta \end{bmatrix}. \tag{2.7}$$

Finally, the three eigenvectors $\mathbf{e}(\nu)$, $\nu = 1, 2, 3$ of the matrix M, normalized by the conditions

$$|\mathbf{e}(\nu)|^2 = \lambda_\nu,$$

where λ_ν are the eigenvalues of M given by Eq. (2.7) are related to the vectors $\mathbf{a}(\mu)$ as follows:

$$\mathbf{e}(\nu) = \sum_\mu G(\nu; \mu)\mathbf{a}(\mu), \tag{2.8}$$

where

$$G(\nu;\mu) = \begin{pmatrix} 1/\sqrt{2} & \frac{1}{2} & -\frac{1}{2} \\ -1/\sqrt{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}. \tag{2.9}$$

The geometrical relations between vectors $\mathbf{a}(\mu)$ and $\mathbf{e}(\mu)$ are illustrated in Fig. 1.

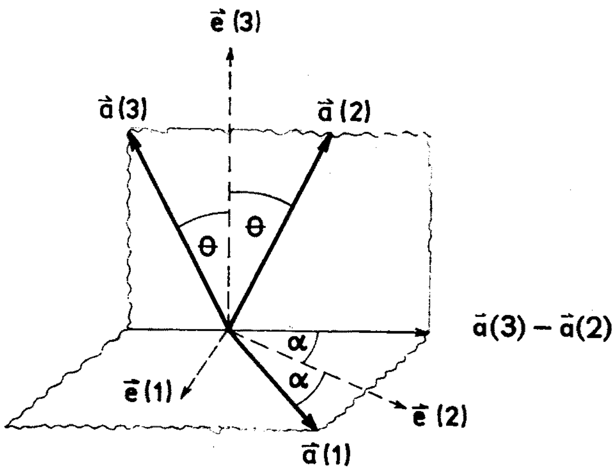

Fig. 1. Geometry of the construction of the mass matrix

Using Eq. (2.7) we can express the angles α and θ in terms of the quark mass ratios

$$\mathrm{tg}^2\,\alpha = m_d/m_s, \tag{2.10}$$

$$\mathrm{tg}^2\,\theta = \frac{1}{2}\,\frac{m_d+m_s}{m_b}. \tag{2.11}$$

For the up-quark families (u, c, t) the construction is identical, but the values of parameters are, in general, different. Denoting the relevant quantities by the same letters as in former case, but primed, we have

$$\mathrm{tg}^2\,\alpha' = m_u/m_c, \tag{2.12}$$

$$\mathrm{tg}^2\,\theta' = \frac{1}{2}\,\frac{m_u+m_c}{m_t}, \tag{2.13}$$

and all other formulas of this section remain valid for primed quantities.

3. *Relation to the idea of sequential mass generation*

In this section we show that the parametrization of mass matrices presented in Section 2 is convenient for implementing the idea of sequential generation of the quark masses [12, 26, 33, 49] by perturbative corrections to interaction described by a Lagrangian which gives vanishing u, d, s and c masses at the tree level.

To fix attention, let us consider b, s and d quarks. At the tree level we have $m_d = m_u = m_s = m_c = 0$. This situation can be realized by the choice

$$\mathbf{a}(2) = \mathbf{a}(3), \qquad \mathbf{a}(1) = 0 \tag{3.1}$$

satisfying conditions (2.2)–(2.4). Thus in this case the Lagrangian singles out one particular direction $\mathbf{a}(3)$ in the space of the quark families.

Following the idea of sequential mass generation we now assume that the first order corrections define another direction, different from $\mathbf{a}(3)$. As a result vector $\mathbf{a}(2)$ may become different from $\mathbf{a}(3)$. If we choose $\mathbf{a}(2)$ in such a way as to satisfy condition (2.2), the vector $\mathbf{a}(1)$, of the form $\mathbf{a}(1) = \frac{1}{\sqrt{2}}(\mathbf{a}(2)-\mathbf{a}(3))$ satisfies conditions (2.3) and (2.4). Furthermore, at this stage $\mathbf{a}(1)$ lies in the $\mathbf{a}(2)\,\mathbf{a}(3)$ plane, so that the eigenvalue $\lambda_1 = m_d$ vanishes.

The second order corrections define a new direction which is used to rotate $\mathbf{a}(1)$ off the $\mathbf{a}(2)\,\mathbf{a}(3)$ plane, thus generating non-vanishing m_d.

4. *Relations between the mass matrices of up-and down-quark families*

In this section we discuss relations between the matrices M and M' (defined in Section 2) which follow from the restrictions imposed on vectors $\mathbf{a}(2)$, $\mathbf{a}(3)$ and $\mathbf{a}(2)'$, $\mathbf{a}'(3)$ defining the first approximation in sequential mass generation. We propose to classify these relations as follows.

Relations of class A follow from the assumption that only third and second quark families are mixed in the first order approximation. This implies that all vectors $\mathbf{a}(2)$, $\mathbf{a}(3)$ $\mathbf{a}'(2)$ and $\mathbf{a}'(3)$ lie in the same plane. We write this condition in the form

$$\mathbf{a}(2)+\mathbf{a}(3) = \eta'_2\mathbf{a}'(2)+\eta'_3\mathbf{a}'(3), \tag{4.1}$$

$$\mathbf{a}'(2)+\mathbf{a}'(3) = \eta_2\mathbf{a}(2)+\eta_3\mathbf{a}(3). \tag{4.2}$$

Relations of class B require additional assumption that only third generation couples to the mass matrix at the tree level. This implies that the vectors $\mathbf{a}(3)$ and $\mathbf{a}'(3)$ are parallel:

$$\mathbf{a}(3)/\Omega = \mathbf{a}'(3)/\Omega'. \tag{4.3}$$

Relations of class C are obtained from any additional constraint. In particular, we considered the possibility that there is a symmetry between the up- and down-families in the first approximation. This condition implies that

$$\frac{\eta_3}{\eta_2} = \frac{\eta'_3}{\eta'_2}. \tag{4.4}$$

5. *Quark masses and mixing angles*

We present now the consequences of the relations discussed in the previous section for the quark masses and for the weak mixing angles.

Relations of class A

It follows from Eq. (4.2) that in the reference frame in which the matrix M' is diagonal we have

$$\eta_2 \mathbf{a}(2)+\eta_3 \mathbf{a}(3) = 2\Omega' \cos\theta' \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{5.1}$$

If U is the unitary matrix which transforms the diagonal frame of $\boldsymbol{M}$ into the diagonal frame of M', we obtain

$$\eta_2 U \mathbf{a}_0(2)+\eta_3 U \mathbf{a}_0(3) = 2\Omega' \cos\theta' \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \tag{5.2}$$

where $\mathbf{a}_0(\mu)$ are given by Eq. (2.6). Repeating the same argument for $\mathbf{a}'_0(\mu)$ we obtain

$$\eta'_2 U^{-1} \mathbf{a}'_0(2)+\eta'_3 U^{-1} \mathbf{a}'_0(3) = 2\Omega \cos\theta \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{5.3}$$

Let us take U in the form

$$U = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\phi} & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} \cos\chi & \sin\chi & 0 \\ -\sin\chi & \cos\chi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{5.4}$$

Using Eqs (2.6) and (5.2)–(5.4) we have

$$(\eta_3-\eta_2)\cos\psi \sin\theta \sin(\chi-\alpha)e^{i\phi}+(\eta_2-\eta_3)\sin\theta \sin\psi \cos\gamma \cos(\chi-\alpha)$$
$$+(\eta_2+\eta_3)\cos\theta \sin\psi \sin\gamma = 0,$$
$$(\eta_3-\eta_2)\sin\psi \sin\theta \sin(\chi-\alpha)e^{i\phi}+(\eta_3-\eta_2)\sin\theta \cos\psi \cos\gamma \cos(\chi-\alpha)$$
$$-(\eta_2+\eta_3)\cos\theta \cos\psi \sin\gamma = 0, \tag{5.5}$$

and two analogous equations obtained from (5.5) by substitutions

$$\chi \to \psi, \quad \psi \to \chi, \quad \gamma \to -\gamma, \quad \phi \to -\phi,$$
$$\alpha \to \alpha', \quad \theta \to \theta', \quad \eta_2 \to \eta'_2, \quad \eta_3 \to \eta'_3. \tag{5.6}$$

From (5.5) we deduce, assuming $\eta_2 \neq \eta_3$,

$$\alpha = \chi, \tag{5.7}$$

$$\operatorname{tg} \theta = \frac{y+1}{y-1} \operatorname{tg} \gamma, \tag{5.8}$$

where

$$y = \eta_3/\eta_2 \tag{5.9}$$

and from the other two equations we thus have

$$\alpha' = \psi, \tag{5.10}$$

$$\operatorname{tg} \theta' = -\frac{y'+1}{y'-1} \operatorname{tg} \gamma, \tag{5.11}$$

where

$$y' = \eta_3'/\eta_2'. \tag{5.12}$$

No restrictions are obtained on ϕ.

To obtain the values of the Kobayashi-Maskawa mixing angles we observe that the matrix U given by Eq. (5.4) must be related to the Kobayashi-Maskawa matrix K by the phase transformations of the type

$$K = P(\delta_1, \delta_2, \delta_3)UP(\delta_1', \delta_2', \delta_3'), \tag{5.13}$$

where δ_i and δ_i' are arbitrary. Writing the matrix K in the standard form [60]

$$K = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & -c_1s_2c_3 - c_2s_3e^{i\delta} & -c_1s_2s_3 + c_2c_3e^{i\delta} \end{pmatrix}, \tag{5.14}$$

where

$$c_i = \cos \theta_i \quad \text{and} \quad s_i = \sin \theta_i,$$

and using Eqs (5.4), (5.7), (5.10), (5.13) we obtain the following relations for the weak mixing angles

$$|\cos \theta_1| = |\cos \alpha \cos \alpha' e^{i\phi} + \sin \alpha \sin \alpha' \cos \gamma|, \tag{5.15}$$

$$\sin \theta_1 \sin \theta_2 = \sin \alpha \sin \gamma, \tag{5.16}$$

$$\sin \theta_1 \sin \theta_3 = \sin \alpha' \sin \gamma, \tag{5.17}$$

$$\operatorname{Arg}(e^{i\phi} \cos \alpha \cos \alpha' + \sin \alpha \sin \alpha' \cos \gamma) = \operatorname{Arg}(\cos \theta_2 \cos \theta_3 e^{i\delta} - \cos \theta_1 \sin \theta_2 \sin \theta_3). \tag{5.18}$$

Since γ, ϕ, y and y' are arbitrary parameters, we obtain only one equality between the measurable quantities

$$\frac{\sin\theta_3}{\sin\theta_2} = \frac{\sin\alpha'}{\sin\alpha} \approx \left(\frac{m_u m_s}{m_c m_d}\right)^{1/2}. \tag{5.19}$$

This relation was derived in Ref. [26] and [33]. It is also implicit in other models, e.g. [45].

Eq. (5.15) implies the inequality

$$\cos(|\alpha|+|\alpha'|) \leqslant \cos\theta_1 \leqslant \cos(|\alpha|-|\alpha'|) \tag{5.20}$$

which is quite restrictive, because $\alpha' \ll \alpha$ (we disscuss the numerical values of the parameters in the next section). In particular, it implies the approximate relation

$$\mathrm{tg}^2\,\theta_1 \approx \mathrm{tg}^2\,\alpha = m_d/m_s \tag{5.21}$$

which is known to be phenomenologically successful [1–4]. The inequality (5.20) was obtained and studied by many of the authors of Refs. [1–57].

Finally, Eqs (5.16) and (5.17) imply the inequalities

$$|\sin\theta_1\sin\theta_2| \leqslant \sin\alpha, \qquad |\sin\theta_1\sin\theta_3| \leqslant \sin\alpha'. \tag{5.22}$$

Relations of class B

Eq. (4.3) implies

$$\eta_2\eta_2' = 1. \tag{5.23}$$

Multiplying Eq. (4.1) by (4.2) and using conditions (5.23) and (2.2) we obtain

$$\cos 2\theta+\cos 2\theta'+1 = y'\cos 2\theta+y\cos 2\theta'+yy'. \tag{5.24}$$

This equality, together with Eqs (5.8) and (5.11) implies

$$\mathrm{tg}\,\gamma = \mathrm{tg}\,(\theta-\theta'). \tag{5.25}$$

Using this formula and Eqs (5.16), (5.17) and (2.11), (2.13) one can determine separately mixing angles θ_2 and θ_3 from the quark mass ratios. The determination is not unique, however, because the signs of the angles θ and θ' are not determined by Eqs (2.11) and (2.13).

Relations of class C

Eq. (4.4) implies

$$y = y' \tag{5.26}$$

and thus, using Eqs (5.8) and (5.11) we obtain $\theta = -\theta'$. This condition fixes the mass of the top quark in terms of other masses. Indeed, from Eqs (2.11) and (2.13) we obtain

$$\frac{m_u+m_c}{m_t} = \frac{m_d+m_s}{m_b}. \tag{5.27}$$

Furthermore, Eq. (5.25) then implies

$$\mathrm{tg}\,\gamma = \mathrm{tg}\,2\theta$$

and thus θ_2 and θ_3 are determined up to a sign.

6. *Analysis of experimental data*

The relations derived in the previous section connect various parameters of the Weinberg-Salam model. We shall discuss now the numerical values of the weak mixing angles and of the top quark mass predicted by these relations.

We shall use the following values of the input parameters:

$$\begin{aligned} m_d/m_s &= 0.045 \pm 0.011 && \text{Ref. [61]},\\ m_u/m_s &= 0.017 \pm 0.008 && \text{Ref. [61]},\\ m_c &= 1260 \pm 10 \text{ MeV} && \text{Ref. [62]},\\ m_b &= 4790 \pm 30 \text{ MeV} && \text{Ref. [63]}. \end{aligned} \tag{6.1}$$

For m_s we take $m_s = 150$ MeV, following Ref. [4]. However, we also investigate the m_s dependence of the results, varying m_s in the range

$$100 \text{ MeV} \leqslant m_s \leqslant 200 \text{ MeV} \tag{6.2}$$

Using these values we obtain for angles α, α' and θ (assuming $m_s = 150$ MeV)

$$\sin\alpha = 0.208 \pm 0.024, \quad \sin\alpha' = 0.045 \pm 0.011, \tag{6.3}$$

$$\sin\theta = 0.1269 \pm 0.0006. \tag{6.4}$$

The m_s dependence of $\sin\alpha'$ and of $\sin\theta$ is displayed in Fig. 2 (α does not depend on m_s).

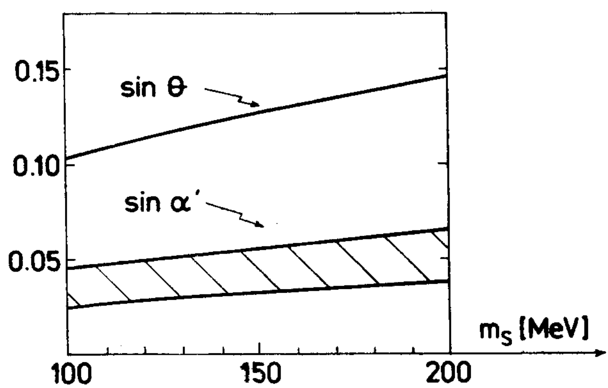

Fig. 2. m_s dependence of $\sin\alpha'$ and of $\sin\theta$

Let us first disscuss the *class A* relation (5.19). Using Eq. (6.3) we find

$$\frac{\sin\theta_3}{\sin\theta_2} = 0.216 \pm 0.059. \tag{6.5}$$

This result is compatible with the existing experimental bounds [64], [65]. The m_s dependence of Eq. (6.5) is shown in Fig. 3.

Turning to the Cabbibo angle θ_1 we observe that since all angles α, α' and θ are rather small and, furthermore $\alpha' \ll \alpha$, inequality Eq. (5.15) can be approximated by $\theta_1 \approx \alpha$, and thus we obtain the approximate relation (5.21). However, to calculate more precisely the mixing angle θ_1 from Eq. (5.15) we would have to know the value of the phase φ, which is related to the *CP* violating phase δ by Eq. (5.18), and of the angle γ. Nevertheless,

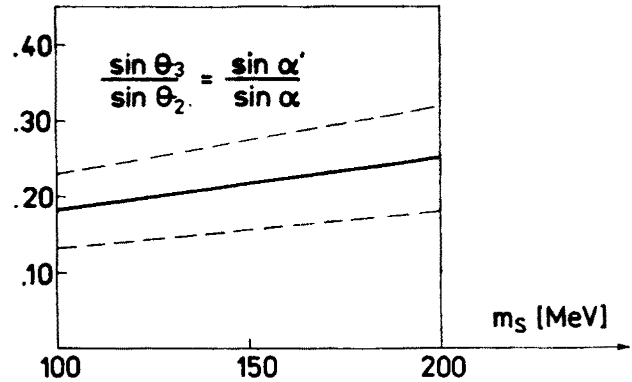

Fig. 3. m_s dependence of the ratio $\sin\theta_3/\sin\theta_2 = \sin\alpha'/\sin\alpha$

since the angles α and α' are rather small, the value of the $\cos\theta_1$ given by Eq. (5.15) is insensitive to ϕ and γ. The lower and upper limits on $\cos\theta_1$ from Eq. (5.15) are

$$\cos(|\alpha|+|\alpha'|) = 0.9679\pm.0056,$$

$$\cos(|\alpha|-|\alpha'|) = 0.9865\pm.0058, \tag{6.6}$$

again assuming $m_s = 150$ MeV. Thus one can see that the uncertainty due to ϕ and γ is comparable to other errors. In Fig. 4 we show the dependence of the limits (6.6) on the adopted value of m_s. One sees that for m_s in the interval (6.2) the bounds are consistent with the present experimental estimates of $\cos\theta_1$ [66].

Let us now turn to the *class B* relation (5.25). To obtain the values of $\sin\theta_2$ and of $\sin\theta_3$ we employed Eqs (5.16) and (5.17) using $\sin\theta_1$ as determined from experiment [66],

$$\sin\theta_1 = 0.228\pm0.011. \tag{6.7}$$

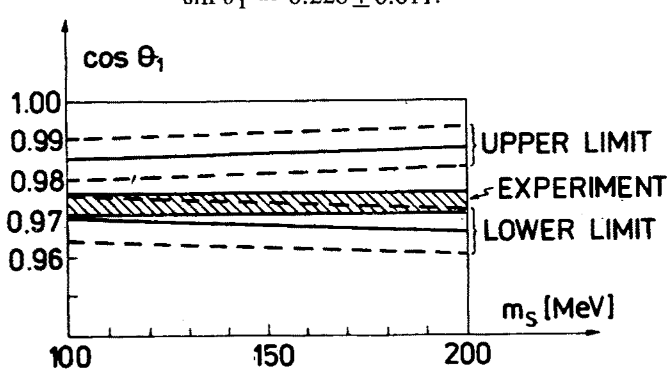

Fig. 4. m_s dependence of the upper and lower limits on $\cos\theta_1$

As seen from Eq. (5.25), it is not possible to determine separately $\sin \theta_2$ and $\sin \theta_3$ without information on θ', i.e. on the value of the top quark mass. Therefore, we used the value of the top quark mass obtained from the *class C* relation (5.27). This relation gives, assuming (6.1) and $m_s = 150$ GeV,

$$m_t = 38.6 \pm .8 \text{ GeV} \tag{6.8}$$

a rather large value. The dependence of m_t on the assumed value of m_s is plotted in Fig. 5. One sees that m_t is quite sensitive to the choice of m_s. Nevertheless, we can conclude from Fig. 5 that m_t turns out to be comfortably above the present experimental limit [67] even

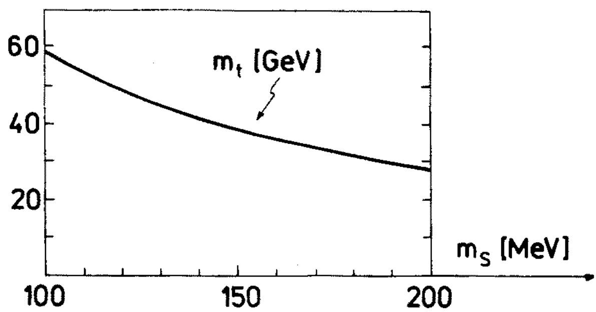

Fig. 5. m_s dependence of the mass of the top quark

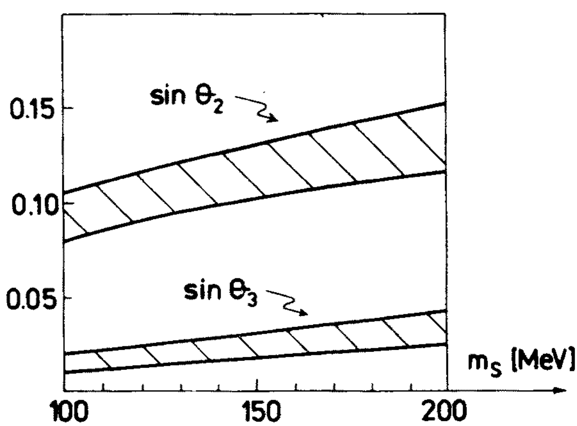

Fig. 6. m_s dependence of the mixing angles θ_2 and θ_3

for quite large values of m_s. For m_s above 140 MeV, m_t is also consistent with the upper bound derived recently by Buras [68].

Using the value (6.8) and assuming that θ and θ' have opposite signs we obtain for $\sin \theta_2$ and $\sin \theta_3$

$$\sin \theta_2 = 0.230 \pm 0.030, \qquad \sin \theta_3 = 0.050 \pm 0.012. \tag{6.9}$$

The m_s dependence of this result is shown in Fig. 6. The obtained values are consistent with the limits given in Ref. [64], [65] and [69].

Thus we conclude that the relations considered in this paper give the value of the Cabbibo angle in agreement with experiment and predict the values of two other mixing angles and of the mass of the top quark in agreement with the present experimental limits.

7. *Comparison with other models*

We compare here some of the results of other authors to ours.

(a) *Mass of the top quark*. The estimates of the top quark mass were reviewed in Ref. [70]. Most of them gave masses in the range or below 20 GeV. However, also the formulae similar to our Eq. (5.27) were obtained. Pakwasa and Sugawara derived $m_t/m_c = m_b/m_s$ in the $SU(2) \times U(1)$ model with S_4 horizontal symmetry [9]. The same relation was found by Ebrahim [11] in $SU_R(2) \times SU_L(2) \times U(1)$ model. Crombrugghe [19] obtained the relation $(m_s - m_d)/m_b = (m_c - m_u)/m_t$ in the $SU_R(2) \times SU_L(2) \times U(1)$ model with left-right symmetry.

b) *Cabbibo angle*. All models (including ours) were constructed in such a way, as to recover the phenomenologically successful relation (5.21). In most models this relation is an approximation following from the condition $m_u/m_c \ll m_d/m_s$. With improving data it may be possible to discriminate between these different approximations. For the moment, all models are in reasonable agreement with the data.

(c) *Mixing angles θ_2 and θ_3*. Here the predictions vary substantially, but most models give rather small values for these angles. The *class A* relation (5.19) (implying $\theta_3 \ll \theta_2$) was explicitly or implicitly found in several models [26], [33], [45]. There are also other predictions giving $\theta_3 \ll \theta_2$ [16], [48]. However, in some models $\theta_3 \approx \theta_2$ [11], [15], [24], and there is also one prediction $\theta_3 \gg \theta_2$ [29].

8. *Conclusions and outloook*

We have proposed a classification of the relations between the mass matrices of up- and down-quark families. The classification is related to the idea of sequential mass generation and can be useful in testing this general idea: Relations of *class A* are particularly fundamental in this respect because they practically follow from the very assumption of sequential mass generation and can thus be treated as necessary conditions for this mechanism to be valid. Relations of *class B* are connected to our specific proposal of the construction of the mass matrices and thus provide a check for the physical relevance of this construction. Finally, the relations of *class C* are somewhat ad hoc conditions which are chosen by requirements of symmetry.

We have shown in Section 6 that all relations (*class A, B* and *C*) give phenomenologically acceptable predictions for the weak mixing angles and for the mass of the top quark. It seems therefore of interest to disscuss their possible origin and physical significance. We would like to make 3 remarks.

(*i*) Relations of *class A* follow from all theories which incorporate the sequential mass generation — e.g. the one considered in Ref. [26].

(*ii*) All relations we considered are invariant with respect to "horizontal" unitary transformations in the space of 3 quark families. Thus they might be relevant in models which assume such a symmetry [38], [57].

(*iii*) If one interprets the mass matrix M as a matrix of Yukawa couplings between Higgs bosons and left- and right-handed quarks, the form (1.1) is natural in models where Higgs bosons are composite particles. For example, in the technicolor scheme [58] the vectors $\mathbf{a}(\mu)$ might be those particular combinations of techniquarks which form the Higgs bound states. (This would require 3 families of techniquarks). The relations connecting $\mathbf{a}(\mu)$ with the quark eigenstates $\mathbf{e}(\nu)$ can then be regarded as following from extended technicolor interactions between the quarks and techniquarks. It remains an open and interesting question whether it is possible to find a group structure of technicolor and extended technicolor interactions consistent with relations (2.2)–(2.4) and (2.8), (2.9).

I would like to thank M. Namiki and I. Ohba for their kind hospitality at the Department of Physics of the Waseda University where a large part of this work was done. Thanks are also due to N. Sakai for illuminating discussions.

REFERENCES

[1] S. Gatto, G. Sartori, M. Tonin, *Phys. Lett.* **28B**, 128 (1968).
[2] N. Cabbibo, L. Maiani, *Phys. Lett.* **28B**, 131 (1968).
[3] R. Jackiw, H. J. Schnitzer, *Phys. Rev.* **D5**, 2008 (1972).
[4] S. Weinberg, *The Problem of Mass, Festschrift for I. I. Rabi*, transactions of the Academy of Sciences, N.Y. (1977) p. 185.
[5] De Rújula, H. Georgi, S. L. Glashow, *Ann. Phys.* (*USA*) **109**, 258 (1977).
[6] F. Wilczek, A. Zee, *Phys. Lett.* **70B**, 418 (1977).
[7] H. Fritzsch, *Phys. Lett.* **70B**, 436 (1977).
[8] T. Kitazoe, K. Tanaka, *Phys. Rev.* **D18**, 3476 (1978).
[9] S. Pakvasa, H. Sugawara, *Phys. Lett.* **73B**, 69 (1978).
[10] R. N. Mohapatra, G. Senjanovic, *Phys. Lett.* **73B**, 176 (1978).
[11] A Ebrahim, *Phys. Lett.* **73B**, 181 (1978).
[12] H. Fritzsch, *Phys. Lett.* **73B**, 317 (1978).
[13] G. Branco, *Phys. Lett.* **76B**, 70 (1978).
[14] T. Hagiwara, T. Kitazoe, G. B. Mainland, K. Tanaka, *Phys. Lett.* **76B**, 602 (1978).
[15] A. Ebrahim, *Phys. Lett.* **76B**, 605 (1978).
[16] H. Harari, H. Haut, J. Weyers, *Phys. Lett.* **78B**, 459 (1978).
[17] H. Harari, *Phys. Rep.* **42C**, 238 (1978).
[18] F. Gürsey, M. Sergaloglu, *Nuovo Cimento Lett.* **21**, 28 (1978).
[19] M. A. Crombrugghe, *Phys. Lett.* **80B**, 365 (1979).
[20] E. Derman, *Phys. Rev.* **D19**, 317 (1979).
[21] D Wyler, *Phys. Rev.* **D19**, 3369 (1979).
[22] E. Derman, H.-S. Tsao, *Phys. Rev.* **D20**, 1207 (1979).
[23] H. Sato, *Nucl. Phys.* **B148**, 433 (1979).
[24] E. Ma, *Phys. Rev.* **D20**, 2408 (1979).
[25] S. Pakvasa, H. Sugawara, *Phys. Lett.* **82B**, 105 (1979).
[26] H. Georgi, D. V. Nanopoulos, *Nucl. Phys.* **B155**, 52 (1979).
[27] H. Georgi, D. V. Nanopoulos, *Phys. Lett.* **82B**, 392 (1979).
[28] C. D. Froggatt, H. B. Nielsen, *Nucl. Phys.* **B147**, 227 (1979).
[29] G. Preparata, *Phys. Lett.* **82B**, 398 (1979).

[30] K. Higashima, T. K. Kuo, P. G. Lauwers, *Phys. Lett.* **83B**, 213 (1979).
[31] G. Sègre, H. A. Weldon, J. Weyers, *Phys. Lett.* **83B**, 351 (1979).
[32] T. Hagiwara, *Phys. Lett.* **84B**, 465 (1979).
[33] H. Fritzsch, *Phys. Lett.* **85B**, 81 (1979); *Nucl. Phys.* **B155**, 189 (1979).
[34] K. T. Manhanthappa, M. A. Sher, *Phys. Lett.* **85B**, 294 (1979).
[35] Wu Dan-Di, *Phys. Lett.* **85B**, 364 (1979).
[36] H. Georgi, C. Jarlskog, *Phys. Lett.* **86B**, 279 (1979).
[37] D. Grosser, *Phys. Lett.* **86B**, 301 (1979).
[38] F. Wilczek, A. Zee, *Phys. Rev. Lett.* **42**, 421 (1979).
[39] J. Ellis, M. K. Gaillard, *Phys. Lett.* **88B**, 315 (1979).
[40] G. Ecker, W. Konetschny, *Z. Phys.* **C3**, 155 (1979); *Phys. Lett.* **91B**, 225 (1980).
[41] W. Kummer, *Acta Phys. Austriaca* **51**, 247 (1979).
[42] M. Sokolovsky, *Phys. Lett.* **90B**, 116 (1980).
[43] S. Melianac, S. Pallus, *Phys. Lett.* **90B**, 138 (1980).
[44] M. D. Scadron, *Phys. Lett.* **91B**, 265 (1980).
[45] S. Nandi, K. Tanaka, *Phys Lett.* **92B**, 107 (1980).
[46] C. W. Kim, J. Kim, A. Sato, *Phys. Lett.* **93B**, 286 (1980).
[47] M. Krammer, M. Voloshin, V. Zakharov, *Phys. Lett.* **93B**, 447 (1980).
[48] G. Ecker, W. Grismus, W. Konetschny, *Phys. Lett.* **94B**, 381 (1980); *Nucl. Phys.* **B177**, 489 (1981).
[49] R. Barbieri, D. V. Nanopoulos, *Phys. Lett.* **95B**, 43 (1980).
[50] E. Ma, *Phys. Lett.* **96B**, 115 (1980).
[51] S. K. Bose, A. Paschos, *Nucl. Phys.* **B169**, 384 (1980).
[52] K. Hikasa, K. Igi, *Z. Phys.* **C6**, 47 (1980).
[53] H. Terazawa, K. Akama, *INS-Rep.* 396 (1980)
[54] S. L. Glashow, *Phys. Rev. Lett.* **45**, 1914 (1980).
[55] H. Sato, *Phys. Rev. Lett.* **45**, 1997 (1980).
[56] M. Gronau, R. Yahalom, *Phys. Lett.* **98B**, 441 (1981).
[57] O. Shanker, *Phys. Rev.* **D23**, 1555 (1981).
[58] L. Susskind, *Phys. Rev.* **D19**, 1277 (1979).
[59] S. Weinberg, *Phys. Rev.* **D20**, 2619 (1979).
[60] M. Kobayashi, K. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
[61] P. Langacker, H. Pagels, *Phys. Rev.* **D19**, 2070 (1979).
[62] M. A. Shifman et al., *Nucl. Phys.* **B147**, 448 (1979).
[63] M. B. Voloshin, *Yad. Fiz.* **29**, 1368 (1979).
[64] V. Barger, W. F. Long, S. Pakvasa, *Phys. Rev. Lett.* **42**, 1585 (1979).
[65] R. E. Schrock S. B. Treiman, L. L. Wang, *Phys. Rev. Lett.* **42**, 1589 (1979).
[66] R. E. Schrock, L. L. Wang, *Phys. Rev. Lett.* **41**, 1692 (1978).
[67] For review see B. Wiik, Report on Madison Conference 1980.
[68] A. Buras, *Phys. Rev. Lett.* **46**, 1354 (1981).
[69] R. E. Schrock, M. B. Voloshin, *Phys. Lett.* **87B**, 375 (1979).
[70] M. K. Gaillard, Proc. Lepton-Photon Symposium — FERMILAB 1978, p. 397.