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IS CABIBBO MIXING INDUCED BY HIGGS BOSONS OF THE SECOND GENERATION?

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It is shown that Cabibbo-like mixing is related to a non-diagonal fermion mass which may be generated by the Higgs mechanism involving scalars of higher generations. Then a spontaneously broken symmetry group is given by SU(2)×U(1)×U′(1), the factor U′(1) being generated by the properly defined particle-generation number.

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First, we point out that the standard model of electroweak and strong interactions [1], involving the diagonal mass and Cabibbo-like mixing, is equivalent to another version of the standard model with a *non-diagonal* mass and *no* Cabibbo-like mixing.

To this end let us consider the former model, where the mass and electroweak-interaction fermion lagrangians are

$$\mathscr{L}_{\rm M} = -\bar{\nu} M_\nu \nu - \bar{e} M_{\rm e} e - \bar{u} M_{\rm u} u - \bar{d} M_{\rm d} d \tag{1}$$

and

$$\mathscr{L}_{\rm EW} = -e\left[J^\lambda_{\rm EM} A_\lambda + \frac{1}{\sin\theta_{\rm W}\cos\theta_{\rm W}} J^\lambda_{\rm N} Z_\lambda + \frac{1}{\sqrt{2}\sin\theta_{\rm W}} (J^\lambda_{\rm C} W^+_\lambda + J^{\lambda *}_{\rm C} W^-_\lambda)\right], \tag{2}$$

respectively. Here, M_ν, $M_{\rm e}$, $M_{\rm u}$, $M_{\rm d}$ are diagonal fermion-mass matrices acting on

$$\nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \vdots \end{pmatrix}, \quad e = \begin{pmatrix} e^- \\ \mu^- \\ \tau^- \\ \vdots \end{pmatrix}, \quad u = \begin{pmatrix} u \\ c \\ t \\ \vdots \end{pmatrix}, \quad d = \begin{pmatrix} d \\ s \\ b \\ \vdots \end{pmatrix}, \tag{3}$$

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and the electroweak fermion currents have the form

$$J^{\lambda}_{\mathrm{EM}} = -\bar{e}\gamma^{\lambda}e+\tfrac{2}{3}\,\bar{u}\gamma^{\lambda}u-\tfrac{1}{3}\,\bar{d}\gamma^{\lambda}d, \tag{4}$$

$$J^{\lambda}_{\mathrm{N}} = \bar{\nu}\,\tfrac{1}{2}\gamma^{\lambda}\,\frac{1-\gamma_5}{2}\,\nu+\bar{e}\left(-\tfrac{1}{2}\gamma^{\lambda}\,\frac{1-\gamma_5}{2}+\sin^2\theta_{\mathrm{W}}\gamma^{\lambda}\right)e$$
$$+\bar{u}\left(\tfrac{1}{2}\gamma^{\lambda}\,\frac{1-\gamma_5}{2}-\tfrac{2}{3}\sin^2\theta_{\mathrm{W}}\gamma^{\lambda}\right)u+\bar{d}\left(-\tfrac{1}{2}\gamma^{\lambda}\,\frac{1-\gamma_5}{2}+\tfrac{1}{3}\sin^2\theta_{\mathrm{W}}\gamma^{\lambda}\right)d, \tag{5}$$

$$J^{\lambda}_{\mathrm{C}} = \overline{\nu^{(\mathrm{C})}}\gamma^{\lambda}\,\frac{1-\gamma_5}{2}\,e+\bar{u}\gamma^{\lambda}\,\frac{1-\gamma_5}{2}\,d^{(\mathrm{C})}. \tag{6}$$

In Eq. (6), $d^{(\mathrm{C})}$ denotes the Cabibbo-like-rotated d [2], while $\nu^{(\mathrm{C})}$ describes the *usual* neutrinos as they appear in weak interactions. In Eq. (1), for the sake of lucidity, we take into account the possibility that neutrinos may have some non-zero Dirac masses and denote the corresponding mass eigenstates by ν (which is in general different from $\nu^{(\mathrm{C})}$). If there are no right-handed neutrinos, we put $M_\nu \equiv 0$. In the case of $M_\nu \equiv 0$, or more generally if M_ν has all eigenvalues degenerate, there is no reason to *distinguish* ν from $\nu^{(\mathrm{C})}$, because then the unitary transformation $\nu \to \nu^{(\mathrm{C})}$ (expressing ν, whereever it appears, by $\nu^{(\mathrm{C})}$) does not change the *form* of the lagrangian. Thus, in this case, we can *choose* $\nu = \nu^{(\mathrm{C})}$. If $M_\nu \neq 0$ with non-degenerate eigenvalues, ν cannot be expressed everywhere by $\nu^{(\mathrm{C})}$ without changing the form of the lagrangian. So, in this case, Cabibbo-like mixing of neutrinos is not trivial (and neutrino oscillations of the Dirac type appear). Generally, we shall write the unitary transformations $\nu^{(\mathrm{C})} \to \nu$ and $\mathrm{d}^{(\mathrm{C})} \to \mathrm{d}$ in the form

$$\nu^{(\mathrm{C})} = U^{(\mathrm{C})}_{\nu}\nu, \qquad d^{(\mathrm{C})} = U^{(\mathrm{C})}_{\mathrm{d}}d. \tag{7}$$

Now, let us introduce the arbitrary unitary transformations

$$\nu^{(0)} = U_\nu \nu, \qquad e^{(0)} = U_e e, \qquad u^{(0)} = U_u u, \qquad d^{(0)} = U_d d, \tag{8}$$

subject to the two conditions:

$$U_e^{-1}U_\nu = U^{(\mathrm{C})}_{\nu}, \qquad U_u^{-1}U_d = U^{(\mathrm{C})}_{\mathrm{d}}. \tag{9}$$

Here, $U_\nu = U_{\nu_R}(1+\gamma_5)/2+U_{\nu_L}(1-\gamma_5)/2$, etc., with U_{ν_R} and U_{ν_L} being unitary matrices. Then it can be easily seen from Eqs. (1) and (4), (5), (6) that

$$\mathscr{L}_{\mathrm{M}} = -\overline{\nu^{(0)}}M^{(0)}_{\nu}\nu^{(0)}-\overline{e^{(0)}}M^{(0)}_{e}e^{(0)}-\overline{u^{(0)}}M^{(0)}_{u}u^{(0)}-\overline{d^{(0)}}M^{(0)}_{d}d^{(0)} \tag{10}$$

and

$$J^{\lambda}_{\mathrm{EM}} = -\overline{e^{(0)}}\gamma^{\lambda}e^{0}+\tfrac{2}{3}\,\overline{u^{(0)}}\gamma^{\lambda}u^{(0)}-\tfrac{1}{3}\,\overline{d^{(0)}}\gamma^{\lambda}d^{(0)} \tag{11}$$

$$J^{\lambda}_{\mathrm{N}} = \overline{\nu^{(0)}}\,\tfrac{1}{2}\gamma^{\lambda}\,\frac{1-\gamma_5}{2}\,\nu^{(0)}+\overline{e^{(0)}}\left(-\tfrac{1}{2}\gamma^{\lambda}\,\frac{1-\gamma_5}{2}+\sin^2\theta_{\mathrm{W}}\gamma^{\lambda}\right)e^{(0)}$$
$$+\overline{u^{(0)}}\left(\tfrac{1}{2}\gamma^{\lambda}\,\frac{1-\gamma_5}{2}-\tfrac{2}{3}\sin^2\theta_{\mathrm{W}}\right)u^{(0)}+\overline{d^{(0)}}\left(-\tfrac{1}{2}\gamma^{\lambda}\,\frac{1-\gamma_5}{2}+\tfrac{1}{3}\sin^2\theta_{\mathrm{W}}\gamma^{\lambda}\right)d^{(0)}, \tag{12}$$

$$J^{\lambda}_{\mathrm{C}} = \overline{\nu^{(0)}}\gamma^{\lambda}\,\frac{1-\gamma_5}{2}\,e^{(0)}+\overline{u^{(0)}}\gamma^{\lambda}\,\frac{1-\gamma_5}{2}\,d^{(0)}, \tag{13}$$

where there are *non-diagonal* fermion masses,

$$\beta M_{\nu}^{(0)} = U_{\nu}\beta M_{\nu}U_{\nu}^{-1}, \quad \beta M_{e}^{(0)} = U_{e}\beta M_{e}U_{e}^{-1}, \quad \beta M_{u}^{(0)} = U_{u}\beta M_{u}U_{u}^{-1},$$
$$\beta M_{d}^{(0)} = U_{d}\beta M_{d}U_{d}^{-1}, \tag{14}$$

but *no* Cabibbo-like mixing. That completes our proof of the equivalence of both versions of the standard model, as the rest of the fermion lagrangian (consisting of the kinetic and strong-interaction fermion lagrangians) has the same form before and after the transformations (8). Let us remark that the latter version of the standard model may be considered as a more *fundamental* if a physical mechanism generating the non-diagonal fermion masses exists.

In fact, it is very natural to expect that the non-diagonal fermion masses are generated by the Higgs mechanism. In such a case, however, beside the usual Higgs bosons of the first generation, there should exist Higgs bosons of higher generations in order to couple pairs of fermions of different generations in the way conserving some diagonal charges (whose eigenvalues characterize particle generations) and possibly also some non-diagonal charges (closing together with the former the algebra of a new symmetry group called sometimes the "horizontal group" [1]). Such a coupling generates in the lagrangian off-diagonal fermion-mass terms, after the symmetry of this coupling has been spontaneously broken. The respective symmetry group which at the lower mass scale is spontaneously broken to $U_{EM}(1)\times SU(3)$, is evidently given by the direct product of three factors: the usual electroweak group $SU(2)\times U(1)$, the "horizontal group" or its certain diagonal subgroup, and the colour group SU(3). At some higher mass scale the two former factors of this direct product may be unified into a large electroweak group, before the grand unification with the colour group SU(3) will take place at some still higher mass scale (what is in contrast to the usual scheme [3]). Let us note that the existence of Higgs bosons of higher generations would imply the existence of electroweaki ntermediate bosons of higher generations [4] which, however, should be very heavy in order to avoid their interference with the action of the familiar $W^{\pm}$, Z and γ belonging to the first generation. If the "horizontal group" works in the standard model, these familiar $W^{\pm}$, Z and γ, in view of Eqs. (2)–(6), must constitute its scalar representations.

A diagonal subgroup of the "horizontal group" corresponds to the group U′(1) generated by the overall particle-generation number N [4] which takes the eigenvalues $N = 0, 1, 2, \ldots$ (or $N = 0, -1, -2, \ldots$) for one-fermion (or one-antifermion) states belonging to the successive generations[1]. If there are three fermion generations, the minimal "horizontal group" is provided by the group SO′(3) [5] with $I_3' = \mathrm{Tr}\, N/\mathrm{Tr}\, 1 - N$ and the next-to-minimal one by the group SU′(3) with $I_3'+\frac{3}{2}Y' = \mathrm{Tr}\, N/\mathrm{Tr}\, 1-N$ (in the former case, the group SU′(2) with $I_3' = \mathrm{Tr}\, N/\mathrm{Tr}\, 1-N$ instead of SO′(3) might be considered for preons which are possible subelementary constituents of leptons and quarks). In the present paper we will not specify further the hypothetic "horizontal group", restricting

[1] If the "horizontal group" is unimodular, its subgroup corresponding to U′(1) is the group SU′(1) generated by $N - \mathrm{Tr}\, N/\mathrm{Tr}\, 1$. The relation between U′(1) and SU′(1) resembles the correspondence between the familiar groups generated by strangeness and hypercharge (as $Y = S - \mathrm{Tr}\, S/\mathrm{Tr}\, 1$).

ourselves to the more modest group U′(1). Let us stress that this group may be applicable even if the "horizontal group" does not work.

In order to illustrate the action of the possible Higgs bosons of higher generations let us consider quarks of the first and second generations, u, d and c, s, and assume, besides the familiar Higgs of the first generation

$$\begin{pmatrix} \Phi_0^+ \\ \Phi_0^0 \end{pmatrix} \tag{15}$$

with $N = 0$, the simplest Higgs of the second generation

$$\begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix}, \quad \begin{pmatrix} \Phi_{-1}^+ \\ \Phi_{-1}^0 \end{pmatrix} \tag{16}$$

corresponding to $N = 1$ and $N = -1$, respectively. Then there appear the SU(2)×U(1) ×U′(1)-invariant couplings

$$g_1 \overline{\begin{pmatrix} c^{(0)} \\ s^{(0)} \end{pmatrix}}_L u_R^{(0)} \begin{pmatrix} \Phi_{-1}^{0*} \\ -\Phi_{-1}^{+*} \end{pmatrix} + g_{-1} \overline{\begin{pmatrix} u^{(0)} \\ d^{(0)} \end{pmatrix}}_L c_R^{(0)} \begin{pmatrix} \Phi_1^{0*} \\ -\Phi_1^{+*} \end{pmatrix} + \text{h.c.} \tag{17}$$

and

$$h_1 \overline{\begin{pmatrix} c^{(0)} \\ s^{(0)} \end{pmatrix}}_L d_R^{(0)} \begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix} + h_{-1} \overline{\begin{pmatrix} u^{(0)} \\ d^{(0)} \end{pmatrix}}_L s_R^{(0)} \begin{pmatrix} \Phi_{-1}^+ \\ \Phi_{-1}^0 \end{pmatrix} + \text{h.c.} \tag{18}$$

If their symmetry is spontaneously broken, they lead to the off-diagonal mass terms

$$\overline{c^{(0)}} m_{cu}^{(0)} u^{(0)} + \text{h.c.}, \qquad \overline{s^{(0)}} m_{sd}^{(0)} d^{(0)} + \text{h.c.}, \tag{19}$$

where

$$m_{cu}^{(0)} = \frac{1+\gamma_5}{2} g_1 \langle \Phi_{-1}^0 \rangle_0 + \frac{1-\gamma_5}{2} g_{-1} \langle \Phi_1^0 \rangle_0,$$

$$m_{sd}^{(0)} = \frac{1+\gamma_5}{2} h_1 \langle \Phi_1^0 \rangle_0 + \frac{1-\gamma_5}{2} h_{-1} \langle \Phi_{-1}^0 \rangle_0 \tag{20}$$

and $\langle \Phi_{\pm 1}^0 \rangle_0$ are chosen real. It is very natural to expect that the "physical" Higgs of the second generation described by $\Phi_{\pm 1}^0 - \langle \Phi_{\pm 1}^0 \rangle_0$ are very heavy so that the flavour-changing corrections to the neutral current (5), mediated by these Higgs, are negligible [6].

The larger "horizontal group" containing a subgroup corresponding to U′(1) may provide

$$g_1 = g_{-1}, \qquad h_1 = h_{-1} \tag{21}$$

and also

$$\langle \Phi_1^0 \rangle_0 = \langle \Phi_{-1}^0 \rangle_0, \tag{22}$$

the latter if the subgroup generated by the non-diagonal charge responsible for the transformation $\Phi_1^0 \to \Phi_{-1}^0$ is not spontaneously broken (though the U′(1) generated by N is). If Eqs. (21) and (22) hold, the off-diagonal masses (20) do not contain γ_5-parts:

$m_{cu}^{(0)} = g_1\langle\Phi_1^0\rangle_0$ and $m_{sd}^{(0)} = h_1\langle\Phi_1^0\rangle_0$. In order to give an example, let us consider in the case of three fermion generations the next-to-minimal "horizontal group" SU'(3) with $I'_3+\frac{3}{2}Y' = \mathrm{Tr}\,N/\mathrm{Tr}\,1-N$. Then, denoting by $\Phi_0^{+'}$ and $\Phi_0^{0'}$ some Higgs with $N=0$ [2], we may consider Φ_{-1}^+, $\Phi_0^{+'}$, Φ_1^+ and Φ_{-1}^0, $\Phi_0^{0'}$, Φ_1^0 as two triplets with $Y'=0$ of an SU'(2) ⊂ SU'(3), allowing easily the expressions (17) and (18) to be parts of two SU'(2)-invariant couplings (if Eq. (21) holds)[3]. In this case, the non-diagonal charge, possibly responsible for Eq. (22), is $I'_1 = F'_1$, while the charges $I'_2 = F'_2$ and $I'_3 = F'_3$ as well as $Y' = \frac{2}{\sqrt{3}}F'_8$ generate spontaneously broken subgroups of SU'(3) (in consistency with the spontaneous breakdown of the U'(1) generated by N).

So, if neglecting for simplicity the influence of quarks of the third and possible higher generations, we get in the case of Higgs (15) and (16) the quark-mass matrices

$$\beta M_u^{(0)} = \beta\begin{pmatrix} m_u^{(0)}, & m_{uc}^{(0)} \\ m_{cu}^{(0)}, & m_c^{(0)} \end{pmatrix} = U_u\beta M_u U_u^{-1}, \qquad \beta M_d^{(0)} = (u\to d,\ c\to s), \tag{23}$$

where $\beta m_{uc}^{(0)} = m_{cu}^{(0)}\beta$ and $\beta m_{ds}^{(0)} = m_{ds}^{(0)}\beta$, whilst $m_u^{(0)}$, $m_c^{(0)}$ and $m_d^{(0)}$, $m_s^{(0)}$ do not contain γ_5-parts. Here, M_u and M_d are diagonal, and their diagonal elements are

$$\begin{matrix} m_u \\ m_c \end{matrix} = \frac{m_u^{(0)}+m_c^{(0)}}{2} \mp \frac{m_c^{(0)}-m_u^{(0)}}{2}\left[1+\frac{4m_{uc}^{(0)}m_{cu}^{(0)}}{(m_c^{(0)}-m_u^{(0)})^2}\right]^{1/2} \simeq \begin{matrix} m_u^{(0)} \\ m_c^{(0)} \end{matrix} \mp \frac{m_{uc}^{(0)}m_{cu}^{(0)}}{m_c^{(0)}-m_u^{(0)}} \tag{24}$$

and

$$\begin{matrix} m_d \\ m_s \end{matrix} = (u\to d,\ c\to s), \tag{25}$$

containing no γ_5-parts. Hence, if Eqs. (21) and (22) hold,

$$U_d^{(C)} = U_u^{-1}U_d = \begin{pmatrix} \cos\theta_C, & \sin\theta_C \\ -\sin\theta_C, & \cos\theta_C \end{pmatrix}, \tag{26}$$

where

$$\sin\theta_C = \cos\theta_u \sin\theta_d - \sin\theta_u\cos\theta_d \tag{27}$$

with[4]

$$\sin^2\theta_u = \tfrac{1}{2}\left\{1-\left[1+\frac{4m_{uc}^{(0)}m_{cu}^{(0)}}{(m_c^{(0)}-m_u^{(0)})^2}\right]^{-1/2}\right\} \simeq \frac{m_{uc}^{(0)}m_{cu}^{(0)}}{(m_c^{(0)}-m_u^{(0)})^2} \tag{28}$$

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[2] Very probably, $\Phi_0^{+'}$ and $\Phi_0^{0'}$ are different from the familiar Φ_0^+ and Φ_0^0 which, like $W^\pm$, W^0 and B, are presumably singlets of SU'(3).

[3] If the "horizontal group" SU'(3) works, there may exist the following extra Higgs multiplets of SU'(2) (forming together with the two former triplets two octets of SU'(3)): two singlets $\Phi_0^{+''}$ and $\Phi_0^{0''}$ with $Y'=0$, two doublets Φ_{-2}^+, $\Phi_{-1}^{+'}$ and Φ_{-2}^0, $\Phi_{-1}^{0'}$ with $Y'=1$, and two doublets $\Phi_1^{+'}$, Φ_2^+ and $\Phi_1^{0'}$, Φ_2^0 with $Y'=-1$, the lower index denoting N (here Tr $N=0$). There may also exist the higher-generation analogues of $W^\pm$, W^0 and B, belonging to four octets of SU'(3).

[4] Assuming $m_u \simeq 0$, $m_u^{(0)} \ll m_c^{(0)}$ and $m_d \simeq 0$, $m_d^{(0)} \ll m_s^{(0)}$ wet get from Eqs (24) and (25) $m_{uc}^{(0)}m_{cu}^{(0)} \simeq m_u^{(0)}m_c^{(0)}$, and $m_{ds}^{(0)}m_{sd}^{(0)} \simeq m_d^{(0)}m_s^{(0)}$, and then from Eqs (28) and (29) $\sin^2\theta_u \simeq m_u^{(0)}/m_c^{(0)}$ and $\sin^2\theta_d \simeq m_d^{(0)}/m_s^{(0)}$. In this case Eq. (27) gives $\sin\theta_c \simeq \sqrt{m_d^{(0)}/m_s^{(0)}} - \sqrt{m_u^{(0)}/m_c^{(0)}}$ [7].

and

$$\sin^2\theta_d = (u \to d,\ c \to s). \tag{29}$$

Thus we obtain in this case Cabibbo mixing

$$d^{(C)} = \cos\theta_C d + \sin\theta_C s,$$
$$s^{(C)} = -\sin\theta_C d + \cos\theta_C s. \tag{30}$$

Finally, let us note that the conservation of the overall particle-generation number N, described by the unbroken U′(1), implies the *separate* conservations of the particular fermionic numbers

$$n_{\nu_e}+n_{e^-}, \quad n_{\nu_\mu}+n_{\mu^-}, \quad n_{\nu_\tau}+n_{\tau^-}, \ldots \tag{31}$$

and

$$n_u+n_d, \quad n_c+n_s, \quad n_t+n_b, \ldots \tag{32}$$

if there are no electroweak intermediate bosons of higher generations or *if* their action can be neglected due to their very large masses [4]. When U′(1) is spontaneously broken, Cabibbo-like mixing appears, perturbing the strict conservations of quark numbers (32) but not those of leptonic numbers (31) if $M_\nu \equiv 0$ (if $M_\nu \neq 0$ with nondegenerate eigenvalues, the latter conservations are also perturbed). If electroweak intermediate bosons of higher generations exist, they also cause the strict conservations of leptonic and quark numbers (31) and (32) to be perturbed, but presumably to a very small extent.

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