ON SPIN AND ISOSPIN STABILITY OF DENSE NEUTRON AND NUCLEAR MATTER WITH HARD CORE INTERACTION. II*

By M. Hen, W. Piechocki†, J. Rożynek and J. Dąbrowski

Institute of Nuclear Research, Warsaw**

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Spin and isospin stability of dense neutron and nuclear matter with hard core interaction of radius c is investigated with the Iwamoto-Yamada-Jastrow method. Different correlation functions for two nucleons with identical and nonidentical third components of spin-isospin are used. Neutron matter shows spin stability. Onset of spin and isospin instability of nuclear matter is predicted at $k_F c \cong 1.1$.

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1. Introduction

In our recent work [1] (hereafter to be referred as I) the problem of spin stability of neutron matter ($\widetilde{\mathrm{NM}}$), and of spin and isospin stability of nuclear matter (NM) was investigated. In particular, it was found out that for NM with hard core interaction there is a possibility of an instability at high densities. The conclusion, however, was not definite because at these densities the applicability of the methods applied in I was doubtful.

One of the conditions for the occurrence of spin instability of $\widetilde{\mathrm{NM}}$, considered in I, was

$$\tilde{\Delta} = \tilde{E}(\alpha_\sigma = 1)/N - \tilde{E}(\alpha_\sigma = 0)/N < 0, \tag{1.1}$$

where $\tilde{E}(\alpha_\sigma)$ is the ground state energy of NM with spin excess $\alpha_\sigma = (N_\uparrow - N_\downarrow)/N$.

In case of NM, there are three types of polarization: spin (σ) polarization, isospin (τ) polarization, and spin-isospin ($\sigma\tau$) polarization, measured respectively by the parameters: $\alpha_\sigma = (A_\uparrow - A_\downarrow)/A$, $\alpha_\tau = (N-Z)/A$, and $\alpha_{\sigma\tau} = (N_\uparrow + Z_\downarrow - N_\downarrow - Z_\uparrow)/A$. The condition for the occurrence of κ-instability ($\kappa = \sigma, \tau, \sigma\tau$), analogous to (1.1), is:

$$\Delta_\kappa = E_\kappa(\alpha_\kappa = 1)/A - E_\kappa(\alpha_\kappa = 0)/A < 0, \tag{1.2}$$

where $E_\kappa(\alpha_\kappa)$ is the ground state energy of NM with a given value of α_κ.

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** Address: Instytut Badań Jądrowych, Hoża 69, 00-681 Warszawa, Poland.

† Present address: UMIST, Department of Mathematics, P.O.Box 88, Manchester M60 1QD, England.

In the Jastrow method of calculating the ground state energies in (1.1) and (1.2), we used in I the Iwamoto-Yamada (IY) [2] cluster expansion with state independent correlation functions (cf's) $f(r_{ij})$. However, even the simplest state dependence of the cf's — different cf's in different spin-isospin states of the two nucleons — could change the results obtained for $\tilde{\Delta}$ and Δ_κ, and the conclusions reached in I. Notice that both $\tilde{\Delta}$ and Δ_κ are differences of two big numbers, the ground state energies of polarized and unpolarized states, and an accurate calculation of these energies is essential for obtaining reliable results for $\tilde{\Delta}$ and Δ_κ.

In the present paper, the σ stability of $\widetilde{\text{NM}}$ and the κ stability of NM for hard core interaction is reinvestigated with spin and isospin dependent cf's. In the case ot $\widetilde{\text{NM}}$, the two-body correlation operator is assumed to be of the form:

$$\hat{f}_{ij} = f(r_{ij})\,(1+\sigma_{iz}\sigma_{jz})/2+\bar{f}(r_{ij})\,(1-\sigma_{iz}\sigma_{jz})/2, \tag{1.3}$$

with two cf's: $f(r_{ij})$ and $\bar{f}(r_{ij})$ for parallel and anti-parallel spins of the two correlated neutrons respectively. In case of NM, we assume for $\hat{f}_{ij}$ a similar form which involves not only spin projection operators ot Eq. (1.3) but also analogous isospin projection operators. This simplified treatment of the spin and isospin dependence of the cf's (applied before, e.g., in [4] in the problem of liquid ^{3}He) has the advantage that all our cf's commute. Consequently, the IY calculation of $\tilde{\Delta}$ and Δ_κ requires only minor modification of the procedure presented in I.

In Section 2, we outline the IY method of calculating ground state energies of polarized and unpolarized $\widetilde{\text{NM}}$ and NM for Wigner type two-body interaction, in the case of spin--isospin dependent cf's of the type given in Eq. (1.3). In Section 3, we present and discuss results obtained for $\tilde{\Delta}$ and Δ_κ in case of a pure hard core interaction. Expressions for the energy expectation values, obtained in the IY approximation with our spin-isospin dependent cf's, are listed in the Appendix.

Our main result is the predicted occurrence of κ-instability of hard core NM at high densities. Thus the suggestion of I is confirmed by the present improved analysis.

2. *Calculation of energy*

We apply the Jastrow method, and approximate the ground state energies in instability conditions (1.1), (1.2) by expectation values of the hamiltonian H,

$$\mathscr{E}_\nu = \langle \Psi_\nu | H | \Psi_\nu \rangle / \langle \Psi_\nu | \Psi_\nu \rangle, \tag{2.1}$$

calculated with Jastrow type wave functions Ψ_ν. By ν we denote the spin-isospin degeneracy.

All our equations are written for the case of a Wigner type two-body interaction $v(r_{ij})$ with a hard core radius c.

(a) Neutron matter

In the case of $\widetilde{\text{NM}}$ we have

$$\tilde{E}(\alpha_\sigma = 1) \cong \mathscr{E}_{\nu=1}, \quad \tilde{E}(\alpha_\sigma = 0) \cong \mathscr{E}_{\nu=2}. \tag{2.2}$$

To calculate $\mathscr{E}_{\nu=2}$, we could start with the Jastrow Ansatz for Ψ_ν with the two body correlation operators given in (1.3). However, in the case of spin independent v, we may consider the $N/2$ spin up neutrons and the $N/2$ spin down neutrons as different particles, and write the Jastrow Ansatz in the form:

$$\Psi_{\nu=2}(1 \dots N) = \prod_{i,j} \bar{f}(r_{ij}) \prod_{i<i'} f(r_{ii'}) \prod_{j<j'} f(r_{jj'}) \Phi_\uparrow(1 \dots N/2) \Phi_\downarrow(1 \dots N/2), \tag{2.3}$$

where i, i' (j, j') denote spin up (spin down) neutrons, and $\Phi_{\uparrow(\downarrow)}(1 \dots N/2)$ are Slater determinants of $N/2$ spin up (spin down) neutrons. Each of the indices $i, \dots, j'$ assumes values from 1 to $N/2$.

The energy $\mathscr{E}_{\nu=2}$ consists of three parts: the energy of spin up neutrons, the energy of spin down neutrons (these two parts are equal), and the interaction energy between the two types of neutrons. By applying to each of them the IY cluster expansion, we obtain:

$$\mathscr{E}_\nu \cong \tfrac{3}{5}\, \varepsilon_{F\nu} \mathscr{N} + \mathscr{E}_\nu^{(2)} + \mathscr{E}_\nu^{(3)}, \tag{2.4}$$

where $\mathscr{N} = N$ for $\widetilde{\text{NM}}$ ($\mathscr{N} = A$ for NM). The Fermi energy

$$\varepsilon_{F\nu} = \hbar^2 k_{F\nu}^2 / 2M, \tag{2.5}$$

where the Fermi momentum $k_{F\nu}$ is connected with the density ϱ (number of particles per unit volume) by

$$k_{F\nu}^3 = 6\pi^2 \varrho / \nu. \tag{2.6}$$

Expressions for the two-body cluster energy $\mathscr{E}_\nu^{(2)}$, as well as for $\mathscr{E}_\nu^{(3)}$, are given in Appendix.

To determine the cf's, f and $\bar{f}$, we follow the procedure of I. First, we find f and $\bar{f}$ which minimize $\mathscr{E}_{\nu=2}^{(2)}/N$ with subsidiary healing conditions (see Appendix for definitions of γ_ν and $\bar{\gamma}_\nu$),

$$\varrho \int d\boldsymbol{r}(1-f^2)\gamma_\nu = \text{const}, \tag{2.7}$$

$$\varrho \int d\boldsymbol{r}(1-\bar{f}^2)\bar{\gamma}_\nu = \text{const}, \tag{2.8}$$

imposed on f and $\bar{f}$. Second, we determine the two Lagrange multipliers connected with conditions (2.7) and (2.8), β^2 and $\bar{\beta}^2$, by minimizing $\mathscr{E}_\nu^{(2)} + \mathscr{E}_\nu^{(3)}$.

The minimization of $\mathscr{E}_\nu^{(2)}$ with the subsidiary healing conditions leads to the following Euler equations:

$$\frac{d^2 f}{d\zeta^2} + \left(\frac{2}{\zeta} + \frac{d\gamma_\nu}{d\zeta}\Big/\gamma_\nu\right)\frac{df}{d\zeta} - \tfrac{1}{2}\, v f/\varepsilon_{F\nu} - \beta^2(f-1) = 0, \tag{2.9}$$

$$\frac{d^2 \bar{f}}{d\zeta^2} + \frac{2}{\zeta}\frac{d\bar{f}}{d\zeta} - \tfrac{1}{2}\, v \bar{f}/\varepsilon_{F\nu} - \bar{\beta}^2(\bar{f}-1) = 0, \tag{2.10}$$

where $\zeta = k_{F\nu} r$. For spin unpolarized $\widetilde{\text{NM}}$ $\nu = 2$ in Eqs. (2.7)–(2.10).

The energy of spin polarized $\widetilde{\text{NM}}$, $\mathscr{E}_{\nu=1}$, depends only on one cf, f, and the calculation of $\mathscr{E}_{\nu=1}$ presented in I is not affected by the spin dependence of the cf's, Eq. (1.3). Consequently, in calculating $\tilde{\Delta}$, we use for $\mathscr{E}_{\nu=1}$ the results of I.

(b) Nuclear matter

In the case of NM, we have

$$E_\kappa(\alpha_\kappa = 1) \cong \mathscr{E}_{\nu=2}, \quad E_\kappa(\alpha_\kappa = 0) \cong \mathscr{E}_{\nu=4}. \tag{2.11}$$

The calculation of $\mathscr{E}_{\nu=2}$ has been described in Section 2(a). To calculate $\mathscr{E}_{\nu=4}$ we closely follow the method of calculating $\mathscr{E}_{\nu=2}$. Obviously, we could generalize the two--body correlation operator, Eq. (1.3), by introducing isospin projection operators. However, in the case of spin-isospin independent v, we may consider the $A/4$ spin up neutrons, the $A/4$ spin down neutrons, the $A/4$ spin up protons, and the $A/4$ spin down protons as different particles, and write $\Psi_\nu(1 \ldots A)$ in a form obtained by generalizing Eq. (2.3) to four different kinds of particles. Although we now have four types of particles, we have only two distinct cf's: the cf of a pair of identical particles f, and the cf of a pair of different particles $\bar{f}$. The energy $\mathscr{E}_{\nu=4}$ is now four times the energy of $A/4$ identical particles plus six times the interaction energy between $A/4$ particles of one kind with $A/4$ particles of a different kind. By applying the IY cluster expansion to each of the two parts of $\mathscr{E}_{\nu=4}$, we obtain for $\mathscr{E}_{\nu=4}$ Eq. (2.4) with expressions for $\mathscr{E}^{(2)}_{\nu=4}$ and $\mathscr{E}^{(3)}_{\nu=4}$ given in Appendix. The cf's f and $\bar{f}$ are determined as in case of $\nu = 2$. The healing conditions, Eqs (2.7)–(2.8), and the Euler equations, Eqs (2.9)–(2.10) remain unchanged (except that now $\nu = 4$).

3. *Results and discussion*

In the case of pure hard core interaction of radius c, considered here, the solution of Eq. (2.10) (with $v = 0$), which vanishes at $\zeta = x = k_{F\nu}c$ and approaches unity at large distances, is:

$$\bar{f}(\zeta) = -\exp[-\bar{\beta}(\zeta - x)]/(\zeta/x). \tag{3.1.}$$

On the other hand, Eq. (2.9) for f has to be solved numerically for a few values of β. With these functions f and $\bar{f}$, $\mathscr{E}^{(2)}_\nu + \mathscr{E}^{(3)}_\nu$ was calculated, and the values of β and $\bar{\beta}$ were

TABLE I

Results for energy compared with those obtained in I and in the x^3-approximation

ν	x	β	$\bar{\beta}$	$\mathscr{E}^{(2)}/\mathscr{N}$	$\mathscr{E}^{(3)}/\mathscr{N}$	$\hat{\mathscr{E}}/\mathscr{N}$	$\hat{\mathscr{E}}(\mathrm{I})/\mathscr{N}$	$\hat{\mathscr{E}}(x^3)/\mathscr{N}$	$\mathscr{E}^{(3)}/\mathscr{E}^{(2)}$
2	1.01	1.1	0.6	0.39	0.19	1.18	1.23	1.16	0.50
	1.26	1.1	0.7	0.59	0.38	1.57	1.64	1.50	0.64
	1.39	1.1	0.7	0.71	0.52	1.84	1.91	1.72	0.74
	1.51	1.1	0.8	0.85	0.71	2.16	2.24	1.96	0.84
	1.76	1.1	0.8	1.15	1.27	3.03	3.11	2.57	1.11
4	0.8	1.5	0.9	0.77	0.41	1.77	1.81	1.72	0.54
	1.0	1.5	1.0	1.07	0.79	2.46	2.52	2.30	0.74
	1.1	1.5	1.0	1.24	1.07	2.91	2.98	2.75	0.86
	1.2	1.5	1.0	1.44	1.41	3.45	3.53	3.20	0.98
	1.4	1.5	1.1	2.87	2.37	4.84	4.94	4.30	1.10

determined for which $\mathscr{E}_v^{(2)} + \mathscr{E}_v^{(3)}$ attains its minimum. The whole procedure was repeated at each density considered.

Results obtained for $\tilde{\mathscr{E}}_v = \mathscr{E}_v/\varepsilon_{Fv}$, together with the optimal values of β and $\bar{\beta}$, are shown in Table I which also contains results obtained in I, $\tilde{\mathscr{E}}_v(\mathrm{I})$, and in the x^3-approximation, $\tilde{\mathscr{E}}_v(x^3)$. (The x^3-approximation is the known cubic approximation of the expansion of $\mathscr{E}_v$ in powers of $x = k_F c$ — see, e.g., I.) The resulting values of $\tilde{\Delta}/\varepsilon_{F2}$ and $\Delta_\kappa/\varepsilon_{F4}$ are shown in Figs 1 and 2 in the interesting ranges of $x_v = k_{Fv}c$. (Values of $\mathscr{E}_{v=1}$ for calculating $\tilde{\Delta}$ were taken from I).

We settle the question of the range of reliability of our results similarly as in I. We consider our results as unreliable in the range $x_v > \bar{x}_v$ in which $\mathscr{E}_v^{(3)}/\mathscr{E}_v^{(2)} > 1$ (we have

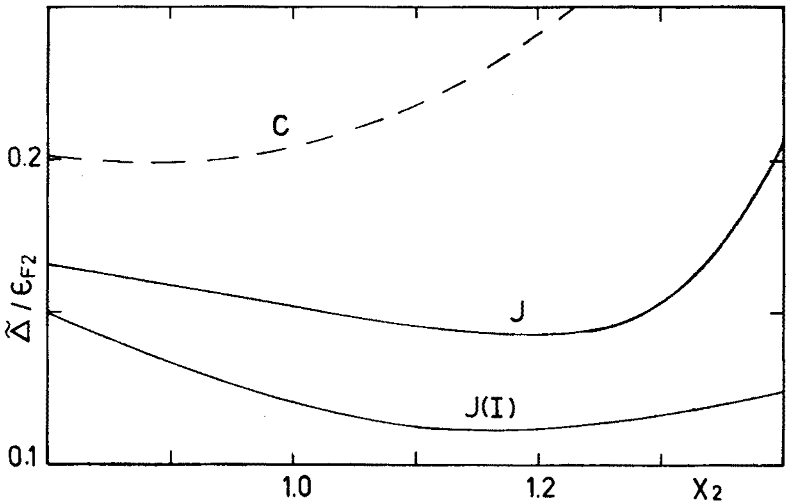

Fig. 1. The function $\tilde{\Delta}/\varepsilon_{F2}$ obtained in the present work (J), in I (J(I)), and in the x^3-approximation (C)

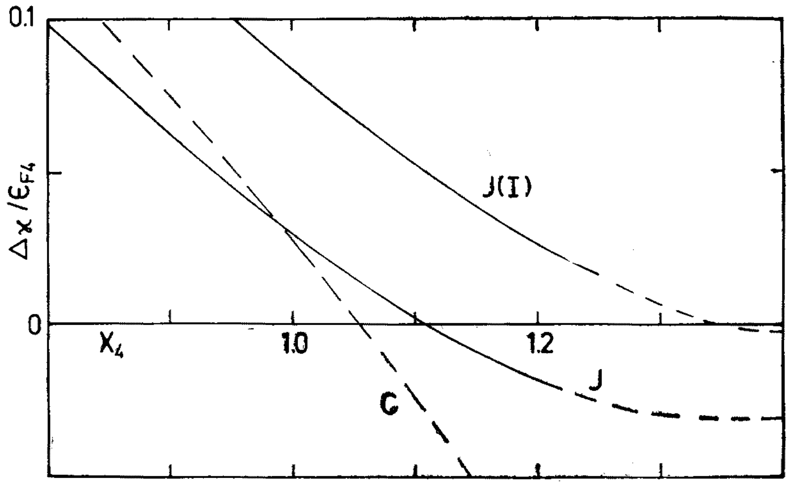

Fig. 2. The function $\Delta_\kappa/\varepsilon_{F4}$ (notation as in Fig. 1)

$\bar{x}_1 = 2.3$, $\bar{x}_2 = 1.67$, and $\bar{x}_4 = 1.22$). As far as the x^3-approximation is concerned, we expect it to be meaningful only for $x_\nu < 1$. The broken parts of the curves in Figs 1 and 2 lie in the regions in which we consider the results to be unreliable.

Our present results confirm the spin stability of dense $\widetilde{\mathrm{NM}}$ treated as nonrelativistic hard core gas. Actually, Fig. 1 reveals a greater spin stability of $\widetilde{\mathrm{NM}}$ according to our present results (curve J) compared to the results of I (curve J(I)). The reason for it is that our present more flexible correlations, Eq. (1.3), lower the expectation value of the energy of unpolarized $\widetilde{\mathrm{NM}}$, whereas this flexibility is ineffective in case of spin polarized $\widetilde{\mathrm{NM}}$.

In case of NM the flexibility of our present correlations is effective in both polarized and unpolarized states. Notice that in, e.g., spin polarized NM ($\nu = 2$) there are two possible intrinsic states of each nucleon: the proton (p) and the neutron (n) state, and accordingly there are four possible intrinsic states of a pair of nucleons: pp, nn, pn, np. In the first two of them the two nucleons are correlated by f, and in the remaining two by $\bar{f}$. Consequently, we expect f and $\bar{f}$ to appeare in our calculation of $\mathscr{E}_{\nu=2}$ with the same weight 1/2. The same argument applied to unpolarized NM ($\nu = 4$) suggests that here f and $\bar{f}$ should appeare with weights 1/4 and 3/4 respectively. This, in fact, is revealed in expression (A.1) for $\mathscr{E}_\nu^{(2)}$. Obviously, introducing two different cf's, f and $\bar{f}$, should be most important when the two cf's appear with the same weights, i.e., in polarized NM. This explains why introducing two cf's lowers $\mathscr{E}_{\nu=2}$ more than $\mathscr{E}_{\nu=4}$ (compare $\mathscr{E}_\nu$ and $\mathscr{E}_\nu$(I) in Table 1). Consequently, the resulting values of Δ_κ, curve J in Fig. 2, lie below those obtained in I, curve J(I) in Fig. 2.

Our present results confirm the suggestion of I concerning κ-instability of NM. At $x_4 = 1.1$ Δ_κ changes sign. Since it happens at the point where $\mathscr{E}_\nu^{(3)}/\mathscr{E}_\nu^{(2)} \cong 0.86$ (0.74) for $\nu = 4(2)$, we hope that our calculation of Δ_κ at this point is still reliable. Consequently, we expect at $x_4 = 1.1$ the onset of σ, τ and $\sigma\tau$ instability of NM treated as nonrelativistic hard core gas. (For $c = 0.4(0.5)$ fm, $x_4 = 1.1$ corresponds to about eight (four) times the equilibrium density of NM).

APPENDIX

Expressions for $\mathscr{E}_\nu^{(2)}$ and $\mathscr{E}_\nu^{(3)}$

We apply the Clark-Westhaus [5] form of the kinetic energy. We denote the number of particles by $\mathscr{N}$ ($\mathscr{N} = N$ for $\widetilde{\mathrm{NM}}$, and $\mathscr{N} = A$ for NM).

For the two-body cluster term, $\mathscr{E}_\nu^{(2)}$, we have:

$$\mathscr{E}_\nu^{(2)}/\mathscr{N} = \tfrac{1}{2}\,\varrho \int d\boldsymbol{r}_{12}\{w_{12}\gamma_\nu(k_{F\nu}r_{12})+(\nu-1)\bar{w}_{12}\bar{\gamma}_\nu\}, \tag{A.1}$$

where γ_ν and $\bar{\gamma}_\nu$ are the radial distribution functions in the absence of interaction for identical nucleons (i.e., with the same third components of spin and isospin) and nonidentical nucleons respectively,

$$\gamma_\nu(k_{F\nu}r_{12}) = [1-l(k_{F\nu}r_{12})^2]/\nu, \qquad \bar{\gamma}_\nu = 1/\nu, \tag{A.2}$$

where

$$l(k_{F\nu}r_{12}) = 3j_1(k_{F\nu}r_{12})/(k_{F\nu}r_{12}) = l_{12}. \tag{A.3}$$

The effective two-body potentials between identical and nonidentical nucleons, w_{12} and $\bar{w}_{12}$, are:

$$w_{12} = f(r_{12})^2 v(r_{12}) + (\hbar^2/M) f'(r_{12})^2,$$
$$\bar{w}_{12} = \bar{f}(r_{12})^2 v(r_{12}) + (\hbar^2/M) \bar{f}'(r_{12})^2. \tag{A.4}$$

In presenting the results for $\mathscr{E}_v^{(3)}$, we apply Clark's notation (see, e.g., [6]):

$$\mathscr{E}_v^{(3)} = (1-17/105)\mathscr{E}_{hv} + \mathscr{E}_{hhv} + \mathscr{E}_{tv}, \tag{A.5}$$

where the four-body term linear in $h = f^2-1$ and $\bar{h} = \bar{f}^2-1$ (approximated, as suggested in [3]) is included into $\mathscr{E}_{hv}$, and

$$\mathscr{E}_{tv}/\mathscr{N} = (\hbar^2/2M)(\varrho/v)^2 \int d\boldsymbol{r}_{12} d\boldsymbol{r}_{13} \hat{\boldsymbol{r}}_{12} \hat{\boldsymbol{r}}_{13} \{ f_{23}^2 f_{12} f_{12}' f_{13} f_{13}' (1+2l_{12}l_{23}l_{31} - 2l_{12}^2 - l_{23}^2)$$
$$+ 2(v-1)\bar{f}_{23}^2 f_{12} f_{12}' \bar{f}_{13} \bar{f}_{13}' (1-l_{12}^2) + (v-1) f_{23}^3 \bar{f}_{12} \bar{f}_{12}' \bar{f}_{13} \bar{f}_{13}' (1-l_{23}^2)$$
$$+ (v-1)(v-2)\bar{f}_{23}^2 \bar{f}_{12} \bar{f}_{12}' \bar{f}_{13} \bar{f}_{13}' \}, \tag{A.6}$$

$$\mathscr{E}_{hv}/\mathscr{N} = (\varrho/v)^2 \int d\boldsymbol{r}_{12} d\boldsymbol{r}_{13} \{ w_{12} h_{13} (l_{12}l_{23}l_{31} - l_{23}^2) - (v-1)\bar{w}_{12} \bar{h}_{13} l_{23}^2 \}, \tag{A.7}$$

$$\mathscr{E}_{hhv}/\mathscr{N} = \tfrac{1}{2}(\varrho/v)^2 \int d\boldsymbol{r}_{12} d\boldsymbol{r}_{13} \{ w_{12} [h_{13} h_{23} (1 - l_{12}^2$$
$$+ 2l_{12}l_{23}l_{31} - 2l_{23}^2) + (v-1)\bar{h}_{13}\bar{h}_{23}(1-l_{12}^2)]$$
$$+ \bar{w}_{12}[2(v-1)\bar{h}_{13} h_{23}(1-l_{23}^2) + (v-1)(v-2)\bar{h}_{13}\bar{h}_{23}] \}. \tag{A.8}$$

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