SPIN SYMMETRY ENERGY OF A HARD CORE NUCLEON MATTER*

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The spin symmetry energy, ε_σ, of a hard core neutron and nuclear matter is expanded in powers of $x = k_Fc$ (k_F = Fermi momentum, c = hard core radius). Coefficients of the expansion are calculated for all terms up to those $\sim x^3$.

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1. Introduction

The ground state energy E_{NM} of nuclear matter (NM), composed of $A_\uparrow$ nucleons with spin up and $A_\downarrow$ nucleons with spin down, depends on the spin excess:

$$E_{\mathrm{NM}}/A = \varepsilon_{\mathrm{vol}} + \tfrac{1}{2}\,\varepsilon_\sigma[(A_\uparrow - A_\downarrow)/A]^2. \tag{1.1}$$

The spin symmetry energy, ε_σ, plays a crucial role in the problem of spin stability of NM [1–3], and is related to the energy of the σ-mode of the giant resonance [4], and to the spin dependent part of the single particle potential [5]. Similarly, we define the spin symmetry energy of neutron matter ($\widetilde{\mathrm{NM}}$). It is directly related to the magnetic susceptibility of $\widetilde{\mathrm{NM}}$ (see, e.g., [6]) and is crucial in discussing spin stability of $\widetilde{\mathrm{NM}}$ [1–3, 7].

In the present paper, we calculate the spin symmetry energy for the model of pure hard core (h.c.) two-body interaction of radius c. We apply the expansion in powers of the gas parameter $x = k_Fc$ (k_F is the Fermi momentum in units of $\hbar$). Whereas the ground state energy of a spin saturated system of fermions with h.c. interaction has been investi-

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gated for a long time [8–21], the calculation of the spin symmetry energy, so far, was carried out in the quadratic approximation (x^2-approximation) only [22]. In the present paper, we calculate the next, cubic, term in the expansion of ε_σ in powers of x. As it turns out, the cubic term plays a decisive role in the discussion of spin stability of NM and $\widetilde{\text{NM}}$.

Let us notice that in the h.c. model, the spin symmetry energy of NM is equal to the isospin symmetry energy, ε_τ, and to the spin-isospin symmetry energy, $\varepsilon_{\tau\sigma}$. (The symmetry energies ε_τ and $\varepsilon_{\sigma\tau}$ are defined similarly as ε_σ, except that instead of spin excess we consider isospin (or neutron) excess $(N-Z)/A$, and the spin-isospin excess $(N_\uparrow+Z_\downarrow-N_\downarrow-Z_\uparrow)/A$ respectively.)

At sufficiently high densities, where the short range repulsion is the decisive part of nuclear forces, our h.c. model should approximately describe real NM and $\widetilde{\text{NM}}$. Also at lower densities, the h.c. model may be used as a starting point of an approximation procedure in which the attractive interaction is treated as a perturbation (see, e.g., [23–26]).

Needless to say that the h.c. model has been applied to other physical systems, notably to liquid ^{3}He (see, e.g., [27]), and also electrons in metals [28].

The paper is organized as follows: In Section 2, we describe the calculation of ε_σ in the x^3-approximation in the case of a general system of h.c. fermions with a degeneracy number $\nu = 2\mu$. In Section 3, we present and discuss the results for NM and $\widetilde{\text{NM}}$. Formulas for integrations, which appear in Section 2, are collected in Appendix.

2. *Calculation of* ε_σ

To obtain results for ε_σ valid for NM and $\widetilde{\text{NM}}$, as well as for any other h.c. fermion system, we consider the general system of $\mathcal{N}$ fermions ($\mathcal{N}_\uparrow$ with spin up, and $\mathcal{N}_\downarrow$ with spin down), in which a single particle state of momentum $\boldsymbol{k}$ (measured in units of $\hbar$) may be occupied by ν particles. This means that a single particle state of momentum $\boldsymbol{k}$ and with a given spin direction (up or down) may be occupied by $\mu = \nu/2$ particles. For NM the spin-isospin degeneracy $\nu = 4$, and the isospin degeneracy $\mu = 2$. For $\widetilde{\text{NM}}$ the spin degeneracy $\nu = 2$, and $\mu = 1$.

At a fixed density $\varrho = \mathcal{N}/\Omega$ (Ω is the volume), the ground state energy E of our system depends on the spin excess parameter

$$\alpha = (\mathcal{N}_\uparrow - \mathcal{N}_\downarrow)/\mathcal{N}, \tag{2.1}$$

and we define the spin symmetry energy by

$$\varepsilon_\sigma = \{d^2\,[E(\alpha)/\mathcal{N}]/d\alpha^2\}_0, \tag{2.2}$$

where the zero at the bracket indicates that the derivative is calculated at $\alpha = 0$.

If we write $E(\alpha)$ in the form

$$E(\alpha) = T(\alpha) + \Delta E(\alpha), \tag{2.3}$$

where $T(\alpha)$ is the (kinetic) energy of the noninteracting system, and $\Delta E(\alpha)$ is the change in $E(\alpha)$ due to the h.c. interaction, we obtain:

$$\varepsilon_\sigma = \tfrac{2}{3}\,\varepsilon(k_\text{F}) + \Delta\varepsilon_\sigma, \tag{2.4}$$

where $\varepsilon(k_F)$ is the Fermi energy, and

$$\Delta\varepsilon_\sigma = \{d^2\,[\Delta E(\alpha)/\mathscr{N}]/d\alpha^2\}_0 \tag{2.5}$$

is the contribution of the h.c. interaction to ε_σ.

We use the notation $\varepsilon(k) = k^2/2\mathscr{M}$ for the single particle kinetic energy ($\mathscr{M}$ is the mass of the particle divided by $\hbar^2$), and k_F for the Fermi momentum of our system (at $\alpha = 0$). It is connected with the density ϱ by:

$$k_F^3 = 6\pi^2\varrho/\nu = 3\pi^2\varrho/\mu. \tag{2.6}$$

To achieve our final goal — the expansion of $\Delta\varepsilon_\sigma$ in powers of $x = k_F c$ — we will expand $\Delta E(\alpha)$ in powers of x. We start with the expression for $\Delta E(\alpha)$ in terms of the Brueckner reaction matrix $\mathscr{K}$:

$$\Delta E(\alpha) = \Delta E(\alpha)^{(1)} + \Delta E(\alpha)^{(3)} + \ldots \tag{2.7}$$

where $\Delta E(\alpha)^{(n)}$ denotes the part of $\Delta E(\alpha)$, which is of the n-th order in $\mathscr{K}$. Obviously, there are no second order terms.

(*i*) Terms of first order in $\mathscr{K}$

The first order part of $\Delta E(\alpha)$ is:

$$\Delta E(\alpha)^{(1)} = \tfrac{1}{2}\mu^2$$

$$\times \Bigg\{ \sum_{m_1}^{<\kappa} \sum_{m_2}^{<\kappa} \left[(m_1m_2|\mathscr{K}_{\kappa\kappa}|m_1m_2) - \frac{1}{\mu}(m_1m_2|\mathscr{K}_{\kappa\kappa}|m_2m_1) \right]$$

$$+ \sum_{m_1}^{<\lambda} \sum_{m_2}^{<\lambda} \left[(m_1m_2|\mathscr{K}_{\lambda\lambda}|m_1m_2) - \frac{1}{\mu}(m_1m_2|\mathscr{K}_{\lambda\lambda}|m_1m_2) \right],$$

$$+2 \sum_{m_1}^{<\kappa} \sum_{m_2}^{<\lambda} (m_1m_2|\mathscr{K}_{\kappa\lambda}|m_1m_2) \Bigg\}, \tag{2.8}$$

where κ and λ are Fermi momenta of the spin up and spin down particles:

$$\kappa = k_F(1+\alpha)^{1/3}, \qquad \lambda = k_F(1-\alpha)^{1/3}. \tag{2.9}$$

We write the reaction matrix equations for the on-energy-shell $\mathscr{K}$ matrices, which appear in expression (2.8) in the form [22]:

$$(p_1p_2|\mathscr{K}_{\kappa\lambda}|m_1m_2) = (p_1p_2|\mathscr{K}^0|m_1m_2)$$

$$+ \sum_{k_1} \sum_{k_2} (p_1p_2|\mathscr{K}^0|k_1k_2) \frac{Q_{\kappa\lambda}(k_1, k_2) - P}{\varepsilon(m_1)+\varepsilon(m_2)-\varepsilon(k_1)-\varepsilon(k_2)}$$

$$\times (k_1k_2|\mathscr{K}_{\kappa\lambda}|m_1m_2), \tag{2.10}$$

where $\mathscr{P}$ indicates the principal value, and the exclusion principle operator

$$Q_{\kappa\lambda}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \begin{cases} 1 \text{ for } k_1 > \kappa \text{ and } k_2 > \lambda, \\ 0 \text{ otherwise.} \end{cases} \tag{2.11}$$

By $\mathscr{K}^0$, we denote the reaction matrix for an isolated pair of particles. The expansion of $\mathscr{K}^0$ into partial waves is [20]:

$$(\boldsymbol{p}_1\boldsymbol{p}_2|\mathscr{K}^0|\boldsymbol{m}_1\boldsymbol{m}_2) = \frac{1}{\Omega}\,\delta_{\boldsymbol{PM}} \sum_l (2l+1)\mathscr{K}^0_l(p, m)P_l(\hat{p}\hat{m}), \tag{2.12}$$

where

$$\mathscr{K}^0_l(p, m) = -\frac{4\pi c}{\mathscr{M}}\frac{1}{mc}j_l(pc)/n_l(mc). \tag{2.13}$$

Here, $\boldsymbol{M}$ and $\boldsymbol{m}$ (and similarly $\boldsymbol{P}$ and $\boldsymbol{p}$) denote the total and relative momenta:

$$\boldsymbol{M} = \boldsymbol{m}_1+\boldsymbol{m}_2, \qquad \boldsymbol{m} = (\boldsymbol{m}_1-\boldsymbol{m}_2)/2. \tag{2.14}$$

By expanding $\mathscr{K}^0_l$ in powers of c, and neglecting terms of higher order than c^3, we obtain:

$$\mathscr{K}^0_l(p, m) = \frac{4\pi c}{\mathscr{M}} \times \begin{cases} 1+\frac{1}{2}(mc)^2-\frac{1}{6}(pc)^2 \\ \frac{1}{3}\,pmc^2 \\ 0 \end{cases} \quad \text{for} \quad l \begin{cases} = 0, \\ = 1, \\ > 1. \end{cases} \tag{2.15}$$

All the contributions to $\Delta E(\alpha)$ which are linear and quadratic in c, are contained in $\Delta E(\alpha)^{(1)}$. They are produced by the S wave part ($l = 0$) of $\mathscr{K}^0_l$, approximated by $4\pi c/\mathscr{M}$, Eq. (2.15). The approximation $\mathscr{K} \simeq \mathscr{K}^0$ in Eq. (2.8) gives the linear contribution. To get the quadratic contribution, we insert into Eq. (2.8) the $\mathscr{K}$ matrices obtained by the first iteration of Eq. (2.10) for $\mathscr{K}$ in terms of $\mathscr{K}^0$. The corresponding contributions to $\Delta\varepsilon_\sigma$ are then obtained by inserting the linear and quadratic parts of $\Delta E(\alpha)$ into Eq. (2.5). The final result for ε_σ in the x^2-approximation is [22]:

$$\varepsilon_\sigma = \tfrac{2}{3}\,\varepsilon(k_F)\left\{1-\frac{2}{\pi}\,x+\frac{8}{\pi^2}\,[(2\mu-1)\,(11-2\ln 2)/15-\mu]x^2\right\}. \tag{2.16}$$

Now let us consider the three contributions to $\Delta E(\alpha)^{(1)}$ cubic in c, which give rise to the following three parts of $\Delta\varepsilon_\sigma$ proportional to x^3.

($i1$) The S wave part $(\Delta\varepsilon_\sigma)_{S3}$

This part is produced by the c^3-part of $\mathscr{K}^0_{l=0}$, Eq. (2.15),

$$\mathscr{K}^0(m, m)_{S3} = \frac{4\pi c^3}{\mathscr{M}}\,\tfrac{1}{3}\,m^2, \tag{2.17}$$

which in the approximation $\mathscr{K} \simeq \mathscr{K}^0$ gives:

$$(\boldsymbol{m}_1\boldsymbol{m}_2|\mathscr{K}|\boldsymbol{m}_1\boldsymbol{m}_2)_{S3} = (\boldsymbol{m}_1\boldsymbol{m}_2|\mathscr{K}|\boldsymbol{m}_2\boldsymbol{m}_1)_{S3} = \frac{1}{\Omega}\,\frac{4\pi c^3}{\mathscr{M}}\,\tfrac{1}{3}\,\mathrm{m}^2. \tag{2.18}$$

By inserting expression (2.18) into Eq. (2.8), we easily obtain:

$$[\Delta E(\alpha)^{(1)}/\mathcal{N}]_{S3} = \frac{1}{15\pi}\varepsilon(k_F)x^3[(\mu-1)f(\alpha)+\mu g(\alpha)], \tag{2.19}$$

where

$$f(\alpha) = \tfrac{1}{2}[(1+\alpha)^{8/3}+(1-\alpha)^{8/3}],$$

$$g(\alpha) = \tfrac{1}{2}[(1-\alpha)(1+\alpha)^{5/3}+(1+\alpha)(1-\alpha)^{5/3}], \tag{2.20}$$

and Eq. (2.5) gives:

$$(\Delta\varepsilon_\sigma)_{S3} = \tfrac{2}{3}\varepsilon(k_F)x^3(\mu-2)\frac{2}{9\pi}. \tag{2.21}$$

(*i*2) The P wave part $(\Delta\varepsilon_\sigma)_P$

This part is produced by $\mathcal{K}^0_{l=1}$, Eq. (2.15). Proceeding similarly as in the case (*i*1), we get:

$$[\Delta E(\alpha)^{(1)}/\mathcal{N}]_P = \frac{1}{5\pi}\varepsilon(k_F)x^3[(\mu+1)f(\alpha)+\mu g(\alpha)], \tag{2.22}$$

and

$$(\Delta\varepsilon_\sigma)_P = \tfrac{2}{3}\varepsilon(k_F)x^3(\mu+2)\frac{2}{3\pi}. \tag{2.23}$$

(*i*3) The second order exclusion principle correction $(\Delta\varepsilon_\sigma)_{Ex2}$

This part arises from the second iteration of Eq. (2.10), in which $\mathcal{K}^0$ is approximated by its part linear in c:

$$(\boldsymbol{p}_1\boldsymbol{p}_2|\mathcal{K}^0|\boldsymbol{m}_1\boldsymbol{m}_2) \cong \frac{1}{\Omega}\delta_{\boldsymbol{PM}}\frac{4\pi c}{\mathcal{M}}. \tag{2.24}$$

Notice that approximation (2.24) is valid for both on- and off-energy shell. The off-energy shell effects appear first in the next term (cubic in c), [19], [29].

The second iteration of Eq. (2.10), with approximation (2.24) for $\mathcal{K}^0$, leads to the following expression for the part of $\mathcal{K}_{\kappa\lambda}$ cubic in c.

$$(\boldsymbol{m}_1\boldsymbol{m}_2|\mathcal{K}_{\kappa\lambda}|\boldsymbol{m}_1\boldsymbol{m}_2)_{Ex2} = \frac{16c^3}{\Omega\pi\mathcal{M}}I_{\kappa\lambda}(\boldsymbol{m}_1, \boldsymbol{m}_2)^2, \tag{2.25}$$

where

$$I_{\kappa\lambda}(\boldsymbol{m}_1, \boldsymbol{m}_2) = \frac{1}{4\pi}\int d\boldsymbol{k}_1\int d\boldsymbol{k}_2\,\frac{Q_{\kappa\lambda}(\boldsymbol{k}_1, \boldsymbol{k}_2)-\mathcal{P}}{m^2-k^2}\,\delta(\boldsymbol{M}-\boldsymbol{K}). \tag{2.26}$$

Expression (2.25) for $\mathscr{H}_{\kappa\lambda}$ inserted into (2.8) leads to the following contribution to $\Delta E(\alpha)^{(1)}/\mathscr{N}$:

$$[\Delta E(\alpha)^{(1)}/\mathscr{N}]_{\mathrm{Ex}2} = \varepsilon(k_{\mathrm{F}})x^3 \frac{12}{\pi^2 k_{\mathrm{F}}^8}$$

$$\{(\mu-1)\,[X(\kappa,\kappa)+X(\lambda,\lambda)]+\mu[X(\kappa,\lambda)+X(\lambda,\kappa)]\}, \tag{2.27}$$

where

$$X(\kappa,\lambda) = \left(\frac{1}{4\pi}\right)^2 \int^{\kappa} d\boldsymbol{m}_1 \int^{\lambda} d\boldsymbol{m}_2 I_{\kappa\lambda}(\boldsymbol{m}_1,\boldsymbol{m}_2)^2, \tag{2.28}$$

where we use the notation:

$$\int^{\kappa} d\boldsymbol{m}_1 = \int_0^{\kappa} dm_1 m_1^2 \int d\hat{m}_1. \tag{2.29}$$

To obtain $(\Delta\varepsilon_\sigma)_{\mathrm{Ex}2}$, we insert expression (2.27) into Eq. (2.5). Taking into account the dependence of the X's on α through κ, and λ, Eq. (2.9), we get:

$$(\Delta\varepsilon_\sigma)_{\mathrm{Ex}2} = \tfrac{2}{3}\,\varepsilon(k_{\mathrm{F}})x^3 \frac{4}{\pi^3 k_{\mathrm{F}}^8}$$

$$\times \left\{(2\mu-1)\left[k_{\mathrm{F}}^2 \frac{d^2X(k_{\mathrm{F}},k_{\mathrm{F}})}{dk_{\mathrm{F}}} - 2k_{\mathrm{F}}\frac{dX(k_{\mathrm{F}},k_{\mathrm{F}})}{dk_{\mathrm{F}}}\right]\right.$$

$$\left. - 4\mu k_{\mathrm{F}}^2 \left[\frac{\partial^2 X(\kappa,\lambda)}{\partial\kappa\partial\lambda}\right]_0\right\}, \tag{2.30}$$

where the zero at the last term indicates that the derivative is taken at $\alpha = 0$, i.e., at $\kappa = \lambda = k_{\mathrm{F}}$.

Since $X(k_{\mathrm{F}}, k_{\mathrm{F}})$ is equal to $k_{\mathrm{F}}{}^8$ times a constant, we have

$$k_{\mathrm{F}} \frac{dX(k_{\mathrm{F}},k_{\mathrm{F}})}{dk_{\mathrm{F}}} = 8X(k_{\mathrm{F}},k_{\mathrm{F}}),$$

$$k_{\mathrm{F}}^2 \frac{d^2X(k_{\mathrm{F}},k_{\mathrm{F}})}{dk_{\mathrm{F}}^2} = 56X(k_{\mathrm{F}},k_{\mathrm{F}}), \tag{2.31}$$

and Eq. (2.30) takes the form:

$$(\Delta\varepsilon_\sigma)_{\mathrm{Ex}2} = \tfrac{2}{3}\,\varepsilon(k_{\mathrm{F}})x^3 \frac{16}{\pi^3 k_{\mathrm{F}}^8}$$

$$\times \left\{10(2\mu-1)X(k_{\mathrm{F}},k_{\mathrm{F}}) - \mu k_{\mathrm{F}}^2 \left[\frac{\partial^2 X(\kappa,\lambda)}{\partial\kappa\partial\lambda}\right]_0\right\}. \tag{2.32}$$

From the definition (2.28) of $X(\kappa, \lambda)$, we easily obtain:

$$(4\pi)^2 \left[\frac{\partial^2 X(\kappa, \lambda)}{\partial\kappa\partial\lambda}\right]_0 = k_F^4 \int d\hat{k}_{F1} \int d\hat{k}_{F2} I_{k_F k_F}(\boldsymbol{k}_{F1}, \boldsymbol{k}_{F2})^2$$

$$+k_F^2 \int d\hat{k}_{F1} \int^{k_F} d\boldsymbol{m}_2 \frac{d}{dk_F} I_{k_F k_F}(\boldsymbol{k}_{F1}, \boldsymbol{m}_2)^2$$

$$+ \int^{k_F} d\boldsymbol{m}_1 \int^{k_F} d\boldsymbol{m}_2 \Big\{ \tfrac{1}{2} \left[dI_{k_F k_F}(\boldsymbol{m}_1, \boldsymbol{m}_2)/dk_F \right]^2$$

$$+2 I_{k_F k_F}(\boldsymbol{m}_1, \boldsymbol{m}_2) \left[\partial^2 I_{\kappa\lambda}(\boldsymbol{m}_1, \boldsymbol{m}_2)/\partial\kappa\partial\lambda\right]_0 \Big\}, \tag{2.33}$$

where $\boldsymbol{k}_{F1}$ and $\boldsymbol{k}_{F2}$ are vectors of length k_F.

An elementary integration gives (for $M \leqslant 2k_F$, and $m \leqslant k_F$) for $I_{\kappa\lambda}$, Eq. (2.26):

$$I_{k_F k_F}(\boldsymbol{m}_1, \boldsymbol{m}_2) = \tfrac{1}{2} \Big\{ k_F + M/2 + [(k_F^2 - M^2/4 - m^2)/M]$$

$$\times \ln \frac{(k_F + M/2)^2 - m^2}{k_F^2 - M^2/4 - m^2} + m \ln \frac{M/2 + k_F - m}{M/2 + k_F + m} \Big\}. \tag{2.34}$$

To calculate $\partial^2 I_{\kappa\lambda}/\partial\kappa\partial\lambda$, we use the definition (2.26):

$$\frac{\partial^2 I_{\kappa\lambda}(\boldsymbol{m}_1, \boldsymbol{m}_2)}{\partial\kappa\partial\lambda} = \frac{1}{4\pi} \frac{\partial^2}{\partial\kappa\partial\lambda} \int_\kappa d\boldsymbol{k}_1 \int_\lambda d\boldsymbol{k}_2 \frac{\delta(\boldsymbol{M}-\boldsymbol{K})}{m^2 - k^2}, \tag{2.35}$$

where we use the notation:

$$\int_\kappa d\boldsymbol{k}_1 = \int_\kappa^\infty dk_1 k_1^2 \int d\hat{k}_1. \tag{2.36}$$

From the expression (2.35), we get (for $M \leqslant 2k_F$, and $m \leqslant k_F$):

$$\left[\frac{\partial^2 I_{\kappa\lambda}(\boldsymbol{m}_1, \boldsymbol{m}_2)}{\partial\kappa\partial\lambda}\right]_0 = \frac{k_F^2}{2M} \Big/ (M^2/4 + m^2 - k_F^2). \tag{2.37}$$

After inserting expressions (2.34) and (2.37) into Eq. (2.33), we perform the integrations in Eq. (2.33) numerically (by Gauss quadrature), applying the formulas given in Appendix. In this way, we obtain the value of $[\partial^2 X(\kappa, \lambda)/\partial\kappa\partial\lambda]_0$. A calculation of $X(k_F, k_F)$ is not necessary since its value is known from previous calculations of $\Delta E(\alpha = 0)^{(3)}$ [18], [19].

Our result for $(\Delta\varepsilon_\sigma)_{Ex2}$, Eq. (2.32), is:

$$(\Delta\varepsilon_\sigma)_{Ex2} = \tfrac{2}{3} \varepsilon(k_F) x^3 [0.4271(2\mu - 1) - 0.6038\mu]. \tag{2.38}$$

(*ii*) Terms of third order in $\mathscr{K}$

As we are interested in the x^3-approximation of ε_σ, we approximate $\mathscr{K}$ by $\mathscr{K}^0$ given in Eq. (2.24).

We shall consider two types of third order terms: (1) hole-hole (hh) interaction terms, and (2) particle-hole (ph) interaction terms.

There are two more types of third order terms: hole self-energy terms, and particle self-energy terms, but their contributions to $\Delta E(\alpha)^{(3)}$ cancel if $\mathscr{K}$ is approximated by (2.24). (Notice that in the so called "standard" low order Brueckner theory hole self-energy terms are included while particle self-energy terms are neglected.)

(*ii*1) The hh interaction part $(\Delta\varepsilon_\sigma)_{\mathrm{hh}}$

The contribution of the hh interaction to $\Delta E(\alpha)^{(3)}/\mathscr{N}$, $[\Delta E(\alpha)^{(3)}/\mathscr{N}]_{\mathrm{hh}}$, may be easily obtained by properly modifying the expression for $[\Delta E(\alpha=0)^{(3)}/\mathscr{N}]_{\mathrm{hh}}$ given in [19] (and denoted there by ε_4). We simply have to split this expression into three parts representing respectively interaction between two spin-up holes, between two spin-down holes, and between a spin-up hole and a spin-down hole. In this way, we get

$$[\Delta E(\alpha)^{(3)}/\mathscr{N}]_{\mathrm{hh}} = \varepsilon(k_{\mathrm{F}})x^3\,\frac{12}{\pi^3 k_{\mathrm{F}}^8}$$

$$\times\{(\mu-1)\,[Y(\kappa,\kappa)+Y(\lambda,\lambda)]+\mu[Y(\kappa,\lambda)+Y(\lambda,\kappa)]\},\tag{2.39}$$

where

$$Y(\kappa,\lambda) = \left(\frac{1}{4\pi}\right)^2 \int\limits_{\kappa} d\boldsymbol{k}_1 \int\limits_{\lambda} d\boldsymbol{k}_2 J_{\kappa\lambda}(\boldsymbol{k}_1,\boldsymbol{k}_2)^2,\tag{2.40}$$

$$J_{\kappa\lambda}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{1}{4\pi}\int\limits^{\kappa} d\boldsymbol{m}_1 \int\limits^{\lambda} d\boldsymbol{m}_2\,\frac{\delta(\boldsymbol{K}-\boldsymbol{M})}{m^2-k^2}\,.\tag{2.41}$$

Proceeding with expression (2.39) similarly as in (*i*3), we get

$$(\Delta\varepsilon_\sigma)_{\mathrm{hh}} = \tfrac{2}{3}\,\varepsilon(k_{\mathrm{F}})x^3\,\frac{16}{\pi^3 k_{\mathrm{F}}^8}$$

$$\times\{10(2\mu-1)Y(k_{\mathrm{F}},k_{\mathrm{F}})-\mu k_{\mathrm{F}}^2[\partial^2 Y(\kappa,\lambda)/\partial\kappa\partial\lambda]_0\},\tag{2.42}$$

and from (2.40) we obtain:

$$(4\pi)^2\,[\partial^2 Y(\kappa,\lambda)/\partial\kappa\partial\lambda]_0$$

$$= k_{\mathrm{F}}^4 \int d\hat{\boldsymbol{k}}_{\mathrm{F}1} \int d\hat{\boldsymbol{k}}_{\mathrm{F}2} J_{k_{\mathrm{F}}k_{\mathrm{F}}}(\boldsymbol{k}_{\mathrm{F}1},\boldsymbol{k}_{\mathrm{F}2})^2$$

$$-k_{\mathrm{F}}^2 \int d\hat{\boldsymbol{k}}_{\mathrm{F}1} \int\limits_{k_{\mathrm{F}}} d\boldsymbol{k}_2 dJ_{k_{\mathrm{F}}k_{\mathrm{F}}}(\boldsymbol{k}_{\mathrm{F}1},\boldsymbol{k}_2)^2/dk_{\mathrm{F}}$$

$$+\int\limits_{k_{\mathrm{F}}} d\boldsymbol{k}_1 \int\limits_{k_{\mathrm{F}}} d\boldsymbol{k}_2[dJ_{k_{\mathrm{F}}k_{\mathrm{F}}}(\boldsymbol{k}_1,\boldsymbol{k}_2)/dk_{\mathrm{F}}]^2/2+2J_{k_{\mathrm{F}}k_{\mathrm{F}}}(\boldsymbol{k}_1,\boldsymbol{k}_2)\,[\partial^2 J_{\kappa\lambda}(\boldsymbol{k}_1,\boldsymbol{k}_2)/\partial\kappa\partial\lambda]_0.\tag{2.43}$$

From Eq. (2.41), we obtain:

$$J_{k_F k_F}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \tfrac{1}{2}\Big\{k_F - K/2 - [(k_F^2 - K^2/4 - k^2)/K]$$

$$\times \ln\frac{(k_F - K/2)^2 - k^2}{k_F^2 - K^2/4 - k^2} + k\ln\frac{k + K/2 - k_F}{k - K/2 + k_F}\Big\}\,\theta(2k_F - K), \tag{2.44}$$

$$[\partial^2 J_{\kappa\lambda}(\boldsymbol{k}_1, \boldsymbol{k}_2)/\partial\kappa\partial\lambda]_0 = k_F^2\theta(2k_F - K)/[2K(k_F^2 - K^2/4 - k^2)]. \tag{2.45}$$

Integrations in Eq. (2.43) were done numerically, by applying formulas given in Appendix. The value of $Y(k_F, k_F)$ was taken from previous calculations of $\Delta E(\alpha = 0)^{(3)}$ [18], [19].

Our result for $(\Delta\varepsilon_\sigma)_{hh}$, Eq. (2.42), is:

$$(\Delta\varepsilon_\sigma)_{hh} = \tfrac{2}{3}\,\varepsilon(k_F)x^3[0.0765(2\mu - 1) - 0.1562\mu]. \tag{2.46}$$

(*ii*2) The ph interaction part $(\Delta\varepsilon_\sigma)_{ph}$

We obtain the contribution of the ph interaction to $\Delta E(\alpha)^{(3)}/\mathcal{N}$, $[\Delta E(\alpha)^{(3)}/\mathcal{N}]_{ph}$, by modifying the known expression for $[\Delta E(\alpha = 0)^{(3)}/\mathcal{N}]_{ph}$ given in [19] (and denoted there by ε_5), which may be represented as a sum of eight diagrams shown in Fig. 9 of Ref. [19] as diagrams (a)–(h). Three of them (diagrams (f)–(h)) involve only one particle, and each of them, in our case of $\alpha \neq 0$, gives rise to two distinct diagrams involving either a spin up or a spin down particle. Four of them (diagrams (b)–(e)) involve two particles, and here the modification in the case of $\alpha \neq 0$ is essentially the same as in (*ii*1). One of them, the three body cluster diagram (a) involves three particles, and in the case of $\alpha \neq 0$ gives rise to eight distinct diagrams with specified spin direction of each particle. In this way, we obtain

$$[\Delta E(\alpha)^{(3)}/\mathcal{N}]_{ph} = \varepsilon(k_F)x^3\,\frac{24}{\pi^3 k_F^8}$$

$$\times\{(\mu - 1)(\mu - 3)\,[Z_1(\kappa, \kappa) + Z_1(\lambda, \lambda)]$$

$$+\mu(\mu - 1)\,[Z_1(\kappa, \lambda) + 2Z_2(\kappa, \lambda) + Z_1(\lambda, \kappa) + 2Z_2(\lambda, \kappa)]$$

$$-\mu[Z_3(\kappa, \lambda) + Z_3(\lambda, \kappa)]\}, \tag{2.47}$$

where

$$Z_1(\kappa, \lambda) = \left(\frac{1}{4\pi}\right)^2 \int^{\kappa} d\boldsymbol{m}_1 \int_{\kappa} d\boldsymbol{k}_1 L_{\lambda\lambda}(\boldsymbol{m}_1, \boldsymbol{k}_1)^2, \tag{2.48}$$

$$Z_2(\kappa, \lambda) = \left(\frac{1}{4\pi}\right)^2 \int^{\kappa} d\boldsymbol{m}_1 \int_{\kappa} d\boldsymbol{k}_1 L_{\lambda\lambda}(\boldsymbol{m}_1, \boldsymbol{k}_1) L_{\kappa\kappa}(\boldsymbol{m}_1, \boldsymbol{k}_1), \tag{2.49}$$

$$Z_3(\kappa,\lambda) = \left(\frac{1}{4\pi}\right)^2 \int^{\kappa} d\boldsymbol{m}_1 \int_{\lambda} d\boldsymbol{k}_1 L_{\lambda\kappa}(\boldsymbol{m}_1,\boldsymbol{k}_1)^2, \tag{2.50}$$

$$L_{\lambda\kappa}(\boldsymbol{m}_1,\boldsymbol{k}_1) = \frac{1}{4\pi}\int^{\lambda} d\boldsymbol{m}_2 \int_{\kappa} d\boldsymbol{k}_2 \frac{\delta(\boldsymbol{m}_1+\boldsymbol{m}_2-\boldsymbol{k}_1-\boldsymbol{k}_2)}{\left(\frac{\boldsymbol{m}_1-\boldsymbol{m}_2}{2}\right)^2 - \left(\frac{\boldsymbol{k}_1-\boldsymbol{k}_2}{2}\right)^2}. \tag{2.51}$$

Proceeding with expression (2.51) similarly as in ($i3$) and ($ii1$), we get:

$$(\Delta\varepsilon_\sigma)_{\text{ph}} = \tfrac{2}{3}\,\varepsilon(k_{\text{F}})x^3 \frac{32}{\pi^3 k_{\text{F}}^8}\{10(2\mu-1)(2\mu-3)Z(k_{\text{F}},k_{\text{F}})$$

$$-\mu(\mu-1)k_{\text{F}}^2[\partial^2(Z_1(\kappa,\lambda)+2Z_2(\kappa,\lambda))/\partial\kappa\partial\lambda]_0$$

$$+\mu k_{\text{F}}^2[\partial^2 Z_3(\kappa,\lambda)/\partial\kappa\partial\lambda]_0\}. \tag{2.52}$$

(At $\kappa = \lambda = k_{\text{F}}$ the three functions Z_i become identical, and are denoted by Z.)

From (2.48)–(2.50), we easily obtain:

$$(4\pi)^2\{\partial^2[Z_1(\kappa,\lambda)+2Z_2(\kappa,\lambda)]/\partial\kappa\partial\lambda\}_0$$

$$= 2\{k_{\text{F}}^2 \int d\hat{k}_{\text{F}1}[\int_{k_{\text{F}}} d\boldsymbol{p}_1 - \int^{k_{\text{F}}} d\boldsymbol{p}_1] dL_{k_{\text{F}}k_{\text{F}}}(\boldsymbol{p}_1,\boldsymbol{k}_{\text{F}1})^2/dk_{\text{F}}$$

$$+ \int^{k_{\text{F}}} d\boldsymbol{m}_1 \int_{k_{\text{F}}} d\boldsymbol{k}_1[dL_{k_{\text{F}}k_{\text{F}}}(\boldsymbol{m}_1,\boldsymbol{k}_1)/dk_{\text{F}}]^2\}, \tag{2.53}$$

$$(4\pi)^2[\partial^2 Z_3(\kappa,\lambda)/\partial\kappa\partial\lambda]_0$$

$$= -k_{\text{F}}^4 \int d\hat{k}_{\text{F}1}\int d\hat{k}_{\text{F}2} L_{k_{\text{F}}k_{\text{F}}}(\boldsymbol{k}_{\text{F}1},\boldsymbol{k}_{\text{F}2})^2$$

$$+2k_{\text{F}}^2\int d\hat{k}_{\text{F}1}\{\int_{k_{\text{F}}} d\boldsymbol{p}_1[\partial L_{\lambda\kappa}(\boldsymbol{p}_1,\boldsymbol{k}_{\text{F}1})/\partial\lambda]_0$$

$$-\int^{k_{\text{F}}} d\boldsymbol{p}_1[\partial L_{\lambda\kappa}(\boldsymbol{p}_1,\boldsymbol{k}_{\text{F}1})/\partial\kappa]_0\} L_{k_{\text{F}}k_{\text{F}}}(\boldsymbol{p}_1,\boldsymbol{k}_{\text{F}1})$$

$$+2\int^{k_{\text{F}}} d\boldsymbol{m}_1 \int_{k_{\text{F}}} d\boldsymbol{k}_1[\partial L_{\lambda\kappa}(\boldsymbol{m}_1,\boldsymbol{k}_1)/\partial\lambda]_0\,[\partial L_{\lambda\kappa}(\boldsymbol{m}_1,\boldsymbol{k}_1)/\partial\kappa]_0$$

$$+L_{k_{\text{F}}k_{\text{F}}}(\boldsymbol{m}_1,\boldsymbol{k}_1)\,[\partial^2 L_{\lambda\kappa}(\boldsymbol{m}_1,\boldsymbol{k}_1)/\partial\kappa\partial\lambda]_0. \tag{2.54}$$

From Eq. (2.51), we obtain:

$$L_{k_{\text{F}}k_{\text{F}}}(\boldsymbol{m}_1,\boldsymbol{k}_1) = -\frac{1}{4p_1}\Big\{2p_1k_{\text{F}}+k_{\text{F}}P_1x_1+[k_{\text{F}}^2-(p_1+\tfrac{1}{2}P_1x_1)^2]$$

$$\times \ln\frac{p_1+k_{\text{F}}+P_1x_1/2}{p_1-k_{\text{F}}+P_1x_1/2}+P_1x_1\left[p_1-k_{\text{F}}+2p_1\ln\frac{P_1x_1/2}{k_{\text{F}}-p_1+P_1x_1/2}\right]\theta(k_{\text{F}}-p_1)\Big\}, \tag{2.55}$$

$$[\partial L_{\lambda\kappa}(\boldsymbol{m}_1, \boldsymbol{k}_1)/\partial\lambda]_0 = -\frac{k_F}{4p_1}\left[\theta(k_F-p_1)\ln\frac{p_1+k_F+P_1x_1/2}{P_1x_1/2} + \theta(p_1-k_F)\ln\frac{p_1+k_F+P_1x_1/2}{p_1-k_F+P_1x_1/2}\right], \quad (2.56)$$

$$[\partial L_{\lambda\kappa}(\boldsymbol{m}_1, \boldsymbol{k}_1)/\partial\kappa]_0 = -\frac{k_F}{4p_1}\theta(k_F-p_1)\ln\frac{P_1x_1/2}{k_F-p_1+P_1x_1/2},$$

$$[\partial^2 L_{\lambda\kappa}(\boldsymbol{m}_1, \boldsymbol{k}_1)/\partial\kappa\partial\lambda]_0 = \left(\frac{k_F}{p_1}\right)^2\frac{\theta(k_F-p_1)}{4p_1x_1} \quad (2.57)$$

where

$$\boldsymbol{P}_1 = \boldsymbol{k}_1+\boldsymbol{m}_1, \quad \boldsymbol{p}_1 = (\boldsymbol{k}_1-\boldsymbol{m}_1)/2, \quad x_1 = \hat{P}_1\hat{p}_1. \quad (2.58)$$

Integrations in Eqs (2.53) and (2.54) were done numerically, with the help of formulas given in Appendix. The value of $Z(k_F, k_F)$ was taken from the previous calculations of $\Delta E(\alpha = 0)^{(3)}$ [18], [19].

Our result for $(\Delta\varepsilon_\sigma)_{ph}$, Eq. (2.52), is:

$$(\Delta\varepsilon_\sigma)_{ph} = \tfrac{2}{3}\varepsilon(k_F)x^3[0.3817(2\mu-1)(2\mu-3)-1.7289\mu(\mu-1)+0.5500\mu]. \quad (2.59)$$

3. Results and discussion

Adding all the contributions to ε_σ, calculated in the preceding section, we obtain in the x^3-approximation:

$$\varepsilon_\sigma = \tfrac{2}{3}\varepsilon(k_F)\left\{1-\frac{2}{\pi}x+\frac{8}{\pi^2}[(2\mu-1)(11-2\ln 2)/15-\mu]x^2 + (0.9245-0.2447\mu-0.2020\mu^2)x^3\right\}. \quad (3.1)$$

In the x^3-approximation, only the interaction in S and P states contributes to ε_σ. The contribution of the P state interaction, $(\Delta\varepsilon_\sigma)_P$, appears only in the term cubic in x, and is given in Eq. (2.23).

For $\widetilde{\mathrm{NM}}$ ($\mu = 1$), and NM ($\mu = 2$), Eq. (3.1) gives:

$$\varepsilon_\sigma(\widetilde{\mathrm{NM}}) = \tfrac{2}{3}\varepsilon(k_F)(1-0.6366x-0.2911x^2+0.4778x^3), \quad (3.2)$$

$$\varepsilon_\sigma(\mathrm{NM}) = \tfrac{2}{3}\varepsilon(k_F)(1-0.6366x-0.0626x^2-0.3728x^3). \quad (3.3)$$

Eqs (3.2) and (3.3) agree quite well with Eqs (3.13) and (3.21) of [1], where the coefficients of the cubic terms were estimated only.[1]

[1] The coefficient 0.5583 in Eq. (3.21) of [1] should be replaced by 1.1875. Furthermore, the number 0.6 in column C of Table 1 of [1] should be replaced by 0.96. Also the factor 1/2 in Eqs (2.3) and (2.11) of [1] is superfluous.

The cubic terms in $\varepsilon_\sigma(\widetilde{\mathrm{NM}})$, and $\varepsilon_\sigma(\mathrm{NM})$, calculated in the present paper; are essential for discussing spin stability. The positive cubic term makes $\varepsilon_\sigma(\widetilde{\mathrm{NM}})$ positive at all values of x, and consequently $\widetilde{\mathrm{NM}}$ is stable against spin polarization. In the case of NM, Eq. (3.3) shows that $\varepsilon_\sigma(\mathrm{NM}) = 0$ at $x = x(\varepsilon_\sigma) = 0.95$, and is negative for $x > x(\varepsilon_\sigma)$. This means that in our h.c. model, the x^3-approximation predicts the onset of instability of NM against spin polarization at $x = 0.95$, i.e. for $c = 0.4$ fm, at about six times the equilibrium density of NM. Although the validity of the x^3-approximation at $x = 0.95$ may be questioned, nevertheless the predicted outset of instability of h.c. NM at $x \simeq 1$ is supported by variational estimates [1], [3].

In the case of NM, the most important contributions to the cubic term in ε_σ are $(\Delta\varepsilon_\sigma)_{\mathrm{ph}}$ which is negative and $(\Delta\varepsilon_\sigma)_{\mathrm{P}}$ which is positive. The other contributions are of an order of magnitude smaller. Consequently, the value of the cubic term in the symmetry energy of h.c. NM obtained in [30] in a calculation, in which the ph interaction is neglected, is misleading.

We might mention, that the cubic term in ε_σ, Eq. (3.1) becomes more negative with increasing μ, i.e., with increasing number of internal degrees of freedom of particles with a fixed spin direction. As a consequence, a system with $\mu = 4$ would become spin unstable at $x(\varepsilon_\sigma) = 0.61$.

APPENDIX

Formulas for integrals

Here, we list the formulas for integrals which appear in the present paper. We use the notation: $\boldsymbol{P} = \boldsymbol{p}_1+\boldsymbol{p}_2$, $\boldsymbol{p} = (\boldsymbol{p}_2-\boldsymbol{p}_1)/2$, $x = \hat{\boldsymbol{P}}\hat{\boldsymbol{p}}$, and $\beta = (p^2+P^2/4-k_F^2)/Pp$. For the functions $F(\boldsymbol{p}_1, \boldsymbol{p}_2) = f(P, p)$, and $G(\boldsymbol{p}_1, \boldsymbol{p}_2) = g(P, p, x)$, we have:

$$(4\pi)^{-2}\int^{k_F} d\boldsymbol{p}_1 \int^{k_F} d\boldsymbol{p}_2 F(\boldsymbol{p}_1, \boldsymbol{p}_2)$$

$$= \int_0^{k_F} dp\,p^2\Big\{\int_0^{2(k_F-p)} dP\,P^2 + \int_{2(k_F-p)}^{2\sqrt{k_F^2-p^2}} dP\,P^2(-\beta)\Big\} f(P, p), \tag{A.1}$$

$$(4\pi)^{-2}\int_{k_F} d\boldsymbol{p}_1 \int_{k_F} d\boldsymbol{p}_2\, \theta(2k_F-|\boldsymbol{p}_1+\boldsymbol{p}_2|) F(\boldsymbol{p}_1, \boldsymbol{p}_2)$$

$$= \int_0^{2k_F} dP\,P^2\Big\{\int_{\sqrt{k_F^2-P^2/4}}^{k_F+P/2} dp\,p^2\beta + \int_{k_F+P/2}^{\infty} dp\,p_2^2\Big\} f(P, p), \tag{A.2}$$

$$(4\pi)^{-2}\int^{k_F} d\boldsymbol{p}_1 \int_{k_F} d\boldsymbol{p}_2 G(\boldsymbol{p}_1, \boldsymbol{p}_2)$$

$$= \tfrac{1}{2}\Big\{\int_0^{2k_F} dP\,P^2\Big[\int_{k_F-P/2}^{\sqrt{k_F^2-P^2/4}} dp\,p^2 \int_{-\beta}^{1} dx + \int_{\sqrt{k_F^2-P^2/4}}^{k_F+P/2} dp\,p^2 \int_{\beta}^{1} dx\Big]$$

$$+ \int_{2k_F}^{\infty} dP\,P^2 \int_{P/2-k_F}^{P/2+k_F} dp\,p^2 \int_{\beta}^{1} dx\Big\} g(P, p, x), \tag{A.3}$$

$$(4\pi)^{-2}\int d\hat{k}_{F1}\int^{k_F} d\boldsymbol{p}_2 G(\boldsymbol{k}_{F1},\boldsymbol{p}_2)$$

$$= k_F^{-1}\int_0^{k_F} dpp \int_{2(k_F-p)}^{2\sqrt{k_F^2-p^2}} dPPg(P,p,x=\beta), \tag{A.4}$$

$$(4\pi)^{-2}\int d\hat{k}_{F1}\int_{k_F} d\boldsymbol{p}_2 G(\boldsymbol{k}_{F1},\boldsymbol{p}_2)$$

$$= k_F^{-1}\{\int_0^{k_F} dpp \int_{2\sqrt{k_F-p^2}}^{2(k_F+p)} dPP + \int_{k_F}^{\infty} dpp \int_{2(p-k_F)}^{2(p+k_F)} dPP\} g(P,p,x=\beta), \tag{A.5}$$

$$(4\pi)^{-2}\int d\hat{k}_{F1}\int d\hat{k}_{F2} G(\boldsymbol{k}_{F1},\boldsymbol{k}_{F2})$$

$$= 2k_F^{-2}\int_0^{k_F} dppg(P=2\sqrt{k_F^2-p^2}, p, x=0), \tag{A.6}$$

where $k_{F1} = k_{F2} = k_F$.

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