BRS TRANSFORMATIONS AND SUPERFIELD FORMALISM IN GAUGE THEORIES*

By L. Bonora**

Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

(*Received November 20, 1981*)

The BRS and $\overline{\text{BRS}}$ transformations are reviewed and applied to the study of unitarity and gauge independence in gauge theories. A geometry of the BRS transformations is introduced through the superfield formalism and the relevance of it to the Adler-Bell-Jackiw anomalies is shown.

PACS numbers: 11.10.Np

In the last decade non-abelian gauge field theories, while gaining prominence as a fundamental tool of particle physics, have undergone a deep elaboration also from a formal point of view. Renormalization, unitarity and geometry of gauge theories have been the subject of a continuous investigation, which has improved the comprehension and considerably clarified and simplified the language of those theories.

The present article is a review of research done in this framework. To begin with, it illustrates the relevance of the Becchi-Rouet-Stora (BRS) and $\overline{\text{BRS}}$ (see Section 2) invariance to the unitarity and gauge independence of a renormalized gauge theory. Then the supersymmetric nature of BRS and $\overline{\text{BRS}}$ transformations is exploited to give a superfield formulation of a gauge theory. This reveals a new and larger geometric setting for gauge theories, which shows particularly useful in connection with Adler-Bell-Jackiw anomalies: they fit naturally in the larger geometric framework.

The article is organized as follows. In Section 1, the necessary notations are introduced. Section 2 is devoted to the illustration of the BRS and $\overline{\text{BRS}}$ transformations. In Section 3 a simple proof of unitarity and gauge independence for a theory with both BRS and $\overline{\text{BRS}}$ invariance is given. In the following Section the superfield formalism is introduced. Section 5

---

* Presented at the VI Autumn School of Theoretical Physics, Szczyrk, Poland, September 21–29, 1981, organized by the Silesian University, Katowice.

** Permanent address: Istituto di Fisica dell'Universitá di Padova and I.N.F.N., Sezione di Padova, Italy. Research supported by the U.S. DEPARTMENT OF ENERGY (DOE) under Contract DE-AC02--76ERO3069.

is devoted to some brief mathematical remarks and the final section to one of the applications of the superfield formalism, namely to the fact that the cohomology related to an Adler-Bell-Jackiw (ABJ) anomaly, is contained in the geometry on which this formalism is based.

1. Notations

The gauge group G is a compact non-abelian Lie group. Its Lie algebra will be denoted by $\mathfrak{G}$ and its generators by τ^α ($\alpha = 1, \ldots, N$). In matrix representation they are given by antihermitian matrices. They define the structure constants $f^{\alpha\beta\gamma}$ by the commutation relations $[\tau^\alpha, \tau^\beta]_- = f^{\alpha\beta\gamma}\tau^\gamma$. The gauge potentials $A^\alpha_\mu(x)$ will appear mostly in the matrix notation $A_\mu(x) = A^\alpha_\mu(x)\tau^\alpha$ (likewise for any N objects a^α, transforming according to the adjoint representation of the group G, let us write $a = a^\alpha\tau^\alpha$). Their curl $F_{\mu\nu}$ is given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]_-. \tag{1.1}$$

The form

$$A = A_\mu(x)dx^\mu, \tag{1.2}$$

defined locally in the space-time manifold M (which is supposed to be four-dimensional, unless otherwise specified) represents a connection in the principal fiber bundle $P(M, G)$, whose base space is M and structure group is G. The corresponding curvature form is

$$F = \sum_{\mu<\nu} F_{\mu\nu}(x)dx^\mu \wedge dx^\nu. \tag{1.3}$$

A gauge transformation is an automorphism of $P(M, G)$ and, on A, it takes the form

$$A \to A' = U^\dagger A U + U^\dagger dU, \tag{1.4}$$

where $U: M \to G$ can be written as:

$$U = \exp\{\lambda^\alpha(x)\tau^\alpha\} \tag{1.5}$$

and $\lambda^\alpha(x)$ are arbitrary functions. dU means $\dfrac{\partial U}{\partial x^\mu} dx^\mu$. As a consequence of Eq. (1.4) F transforms as

$$F \to F' = U^\dagger F U. \tag{1.6}$$

Now let us denote by $\phi^{(R)} = \{\phi_1, \ldots, \phi^{(R)}_k\}$ any matter field (either spinor or scalar) transforming according to the representation R of G and let us adopt the notation $a^{(R)} = a^\alpha\tau^{(R)\alpha}$, where $\tau^{(R)\alpha}$ are the generators of G in the representation R. Gauge transformations on $\phi^{(R)}$ operate as follows

$$\phi^{(R)} \to \phi^{(R)\prime} = U^{(R)}\phi^{(R)}, \tag{1.7}$$

where

$$U^{(R)} = \exp(\lambda^\alpha(x)\tau^{(R)\alpha}) = \exp\lambda^{(R)}(x).$$

2. *BRS and $\overline{BRS}$ invariance*

The classical gauge invariant Lagrangian involving gauge potentials and matter fields is

$$L_{\text{inv}} = +\frac{1}{4g^2}\,\text{Tr}\,(F_{\mu\nu}F^{\mu\nu})+L_{\text{matter}}(\phi;\,D_\mu\phi), \tag{2.1}$$

where ϕ is the generic symbol for all the $\phi^{(R)}$ involved and

$$D_\mu\phi^{(R)} = (\partial_\mu + A_\mu^{(R)})\phi^{(R)}. \tag{2.2}$$

L_{matter} is the most general Lorentz invariant functional of the fields ϕ and their covariant derivatives $D_\mu\phi$, invariant under global gauge transformations and with canonical dimension $\leqslant 4$ (renormalizability requirement).

As it stands the Lagrangian (2.1) is not quantizable, since the free propagator for the gauge fields is not defined. Indeed the free equations of motion are given by:

$$\Box P_{\mu\nu}A^\nu = 0 \tag{2.3}$$

where $\Box = \partial_\mu\partial^\mu$ and $P_{\mu\lambda} = g_{\mu\lambda} - \dfrac{\partial_\mu\partial_\lambda}{\Box}$. One easily sees that $P_{\mu\lambda}P^\lambda_\nu = P_{\mu\nu}$, so $P_{\mu\nu}$ is not invertible. In the Feynman path-integral language this can be ascribed to the fact that we sum over the same physical configuration an infinite number of times, as we sum over all copies of A_μ obtained through a gauge transformation. We need a prescription, or gauge choice, to avoid this multiple counting. If we want to preserve manifest Lorentz covariance, this can be achieved by adding to L_{inv} a gauge fixing term L_{GF}. In turn, this requires the introduction of the FP ghost fields c^α and $\bar{c}^\alpha$ ($\alpha = 1, \dots, N$), according to the Faddeev-Popov recipe. The ghost fields, which transform according to the adjoint representation of G, are scalar anticommuting fields (wrong statistics). Therefore, they introduce negative norm states into the theory. A way of phrasing it is to say that the covariant quantization induces an indefinite metric on the Fock space of the theory and the ghost fields are introduced to restore unitarity (see next section).

Therefore, the complete Lagrangian for a quantized gauge theory is:

$$L = L_{\text{inv}} + L_{\text{GF}} + L_{\text{FP}} \tag{2.4}$$

where L_{FP} takes into account the contribution from the ghost fields. To be definite let us stick to the standard gauge. Then

$$L_{\text{GF}} = \text{Tr}\left(+A_\mu\partial^\mu B - \frac{\xi}{2}BB\right), \tag{2.5}$$

where B_α are auxiliary fields and ξ is a gauge parameter. Consequently, the ghost term is given by:

$$L_{\text{FP}} = \text{Tr}\,(+\partial_\mu\bar{c}D^\mu c), \tag{2.6}$$

where $D_\mu = \partial_\mu + [A_\mu, \ ]_-$. The hermiticity convention for the ghost fields is:

$$c^{\alpha\dagger} = c^\alpha, \qquad \bar{c}^{\alpha\dagger} = -\bar{c}^\alpha. \tag{2.7}$$

Therefore, the complete Lagrangian in the standard gauge is:

$$L = \mathrm{Tr}\left(+ \frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu} + A_\mu \partial^\mu B - \frac{\xi}{2} BB + \partial_\mu \bar{c} D^\mu c \right) + L_{\mathrm{matter}}. \tag{2.8}$$

Now the theory can be quantized by defining the conjugate momenta and imposing the canonical commutation relations. We find three conserved charges, Q_B, $\bar{Q}_\mathrm{B}$ and Q_c:

$$\begin{aligned} Q_\mathrm{B} &= \int d^3x \, \mathrm{Tr}\,(B\partial_0 c - \partial_0 B c - \tfrac{1}{2}\, \partial_0 \bar{c}[c, c]_+) \\ \bar{Q}_\mathrm{B} &= \int d^3x \, \mathrm{Tr}\,(-BD_0\bar{c} + \partial_0 \bar{B}\bar{c} + \tfrac{1}{2}\, \partial_0 c[\bar{c}, \bar{c}]_+) \\ Q_\mathrm{c} &= \int d^3x \, \mathrm{Tr}\,(\partial_0 \bar{c} c - \bar{c} D_0 c), \end{aligned} \tag{2.9}$$

where

$$B + \bar{B} + [c, \bar{c}]_+ = 0. \tag{2.10}$$

iQ_c is the ghost number operator. Indeed,

$$\begin{aligned} &[iQ_\mathrm{c}, c]_- = c, \qquad [iQ_\mathrm{c}, \bar{c}]_- = -\bar{c}, \\ &[iQ_\mathrm{c}, A_\mu]_- = [iQ_\mathrm{c}, B]_- = [iQ_\mathrm{c}, \phi^{(\mathrm{R})}]_- = 0. \end{aligned} \tag{2.11}$$

Q_B and $\bar{Q}_\mathrm{B}$ are nilpotent charges and, together with iQ_c, form the following algebra

$$\begin{aligned} &Q_\mathrm{B}^2 = \bar{Q}_\mathrm{B}^2 = 0, \qquad [Q_\mathrm{B}, \bar{Q}_\mathrm{B}]_+ = 0, \\ &[iQ_\mathrm{c}, Q_\mathrm{B}]_- = Q_\mathrm{B}, \qquad [iQ_\mathrm{c}, \bar{Q}_\mathrm{B}]_- = -\bar{Q}_\mathrm{B}. \end{aligned} \tag{2.12}$$

As we see, Q_B carries ghost number $+1$ and $\bar{Q}$ carries ghost number -1. iQ_B is the generator of the BRS transformations [1]:

$$\begin{aligned} [iQ_\mathrm{B}, A_\mu]_- &= D_\mu c \\ [iQ_\mathrm{B}, c]_+ &= -\tfrac{1}{2}\,[c, c]_+ \\ [iQ_\mathrm{B}, \bar{c}]_+ &= B \\ [iQ_\mathrm{B}, B]_- &= 0 \\ [iQ_\mathrm{B}, \phi^{(\mathrm{R})}]_- &= -c^{(\mathrm{R})}\phi^{(\mathrm{R})}. \end{aligned} \tag{2.13}$$

$i\bar{Q}_\mathrm{B}$ is the generator of the $\overline{\mathrm{BRS}}$ transformations [2]:

$$\begin{aligned} [i\bar{Q}_\mathrm{B}, A_\mu]_- &= D_\mu \bar{c} \\ [i\bar{Q}_\mathrm{B}, \bar{c}]_+ &= -\tfrac{1}{2}\,[\bar{c}, \bar{c}]_+ \\ [i\bar{Q}_\mathrm{B}, c]_+ &= \bar{B} \\ [i\bar{Q}_\mathrm{B}, \bar{B}]_- &= 0 \\ [i\bar{Q}_\mathrm{B}, \phi^{(\mathrm{R})}]_- &= -\bar{c}^{(\mathrm{R})}\phi^{(\mathrm{R})}. \end{aligned} \tag{2.14}$$

It is extremely important that, though the gauge invariance was destroyed by adding L_{GF}, the Lagrangian (2.8) is invariant under the BRS transformations. This invariance, not only allows a much simpler proof of renormalization [1], but because of the nilpotency of Q_B, it also guarantees the unitarity of the physical S-matrix. Following Kugo and Ojima's scheme [3], we can define the physical space $\mathscr{V}_{ph}$ of the theory as the subspace formed by the vectors $|\psi\rangle$ such that $Q_B|\psi\rangle = 0$. Using the nilpotency of Q_B, they were able to show that $\mathscr{V}_{ph}$ has positive semidefinite norm, and so we can define a Hilbert space $\mathscr{H}_{ph}$ and a unitary S-matrix in it.

As the BRS invariance is exact at every order of the loop expansion (and the corresponding charge is nilpotent), we conclude that the S-matrix is unitary at every order of the loop expansion. As we see, the BRS invariance is the key for the success of the Faddeev--Popov recipe.

The Lagrangian (2.8) is invariant also under the $\overline{\text{BRS}}$ transformations (2.14) [2, 4]. In the next Section this further invariance will be exploited to give a simple proof of the unitarity and gauge independence of the S-matrix. It must be stressed that, although the $\overline{\text{BRS}}$ invariance is not strictly indispensable in this context, as unitarity and gauge independence can be proven even in theories where only BRS invariance is present, nonetheless, it simplifies the proof considerably.

3. *Unitarity and gauge independence*

In this section we will consider a theory invariant under both BRS and $\overline{\text{BRS}}$ transformations. The Lagrangian (2.8) is an example of it, but the explicit form of the Lagrangian will not be important here.

Unitarity. It is well known, since the electromagnetic field was quantized, that the Fock space of a covariantly quantized gauge theory contains negative and zero norm states. Therefore, among the postulates of QFT we have to give up the requirement that the Fock space of our theory is a Hilbert space. We simply require it to be a vector space $\mathscr{V}$ endowed with an inner product $\langle|\rangle$. We only add (without loss of generality) the condition that $\mathscr{V}$ be non-degenerate [3]: for every vector $|\psi\rangle \in \mathscr{V}$, there exists at least one vector $|\phi\rangle$, such that $\langle\psi|\phi\rangle \neq 0$. Now, slightly changing Kugo and Ojima's definition [3], let us define $\mathscr{V}_{ph}$ as the subspace of $\mathscr{V}$ annihilated by both Q_B and $\overline{Q}_B$:

$$\mathscr{V}_{ph} = \{|\psi\rangle \in \mathscr{V}: \quad Q_B|\psi\rangle = \overline{Q}_B|\psi\rangle = 0\}. \tag{3.1}$$

The two auxiliary conditions in Eq. (3.1) are quite plausible: only the states invariant under gauge transformations may be connected with physics. In the case of an abelian theory $Q_B|\psi\rangle = 0$ reduces to the well known auxiliary condition $\partial^\mu A_\mu^{(+)}|\psi\rangle = 0$.

Now we intend to show that, starting from $\mathscr{V}_{ph}$, it is possible to define a Hilbert space $\mathscr{H}_{ph}$ and an S-matrix operator in it, which is unitary. To this end let us consider the following theorem [3]:

Theorem 3.1. Let us suppose that in $\mathscr{V}$ an operator S is defined, such that:

(*i*) $SS^+ = S^+S = 1$ (pseudo-unitarity),

(*ii*) $S\mathscr{V}_{ph} \subset \mathscr{V}_{ph}$ and $S^{-1}\mathscr{V}_{ph} \subset \mathscr{V}_{ph}$.

Moreover, let us suppose that

(*iii*) $\mathscr{V}_{ph}$ is positive semi-definite.

Then we can define a physical Hilbert space $\mathscr{H}_{ph}$ and an operator S_{ph} in it such that $S_{ph}S_{ph}^{+} = S_{ph}^{+}S_{ph} = 1$.

The proof is simple. Let us call $\mathscr{V}_0$ the subset of 0 norm states in $\mathscr{V}_{ph}$, $\mathscr{V}_0 = \{\chi : \chi \in \mathscr{V}_{ph}, \langle\chi|\chi\rangle = 0\}$ as $\mathscr{V}_{ph}$ is positive semidefinite (i.e., no state has negative norm) the Schwartz inequality holds in it. So we can easily show that $\mathscr{V}_0$ is a subspace of $\mathscr{V}_{ph}$. Let us set $\mathscr{H}_{ph} = \mathscr{V}_{ph}/\mathscr{V}_0$, that is $\mathscr{H}_{ph} = \{\hat{\psi} : \hat{\psi} = \psi + \mathscr{V}_0, \psi \in \mathscr{V}_0\}$. It is easy to define a scalar product in $\mathscr{H}_{ph}$ by: $\langle\hat{\psi}|\hat{\phi}\rangle = \langle\psi|\phi\rangle$. That this is a good definition is ensured again by the Schwartz inequality. Now let us define $S_{ph}|\hat{\psi}\rangle = \widehat{S|\psi\rangle}$. Then $\langle\hat{\psi}|S_{ph}^{+}S_{ph}|\hat{\psi}\rangle = (\widehat{\langle\phi|S^{+}})\,(\widehat{S|\psi\rangle}) = \langle\phi|S^{+}S|\psi\rangle = \langle\phi|\psi\rangle = \langle\hat{\phi}|\hat{\psi}\rangle$, and the theorem is proven.

Going back to our theory, we see that condition (*i*) of theorem 3.1 is satisfied as a consequence of the (formal or pseudo-) hermiticity of the Lagrangian. (See (2.8) and the prescription (2.7).) Condition (*ii*) is satisfied because Q_B and $\bar{Q}_B$ are conserved charges. Therefore, in order to prove the unitarity of the theory we have to show that $\mathscr{V}_{ph}$ defined by Eq. (3.1) is positive semi-definite.

First of all, let us classify the states according to the eigenvalues of iQ_c:

$$iQ_c|\alpha, k\rangle = k\,|\alpha, k\rangle \tag{3.2}$$

where α represents all the other quantum numbers. As Q_c is (pseudo-) hermitian ($Q_c^{+} = Q_c$), $\langle\alpha, k|\alpha, k\rangle = 0$ for $k \neq 0$. Since $\mathscr{V}$ is non-degenerate there must exist a state $|\alpha', -k\rangle$ such that $\langle\alpha', -k|\alpha, k\rangle \neq 0$. This state will be called a conjugate state of $|\alpha, k\rangle$. It can be considered unique under the normalization condition

$$\langle\alpha', -k|\alpha, k\rangle = 1. \tag{3.3}$$

The proof of the positive semi-definiteness of $\mathscr{V}_{ph}$ is based on the classification [6] of the representations of the algebra (2.12). There are only three different kinds of representations:

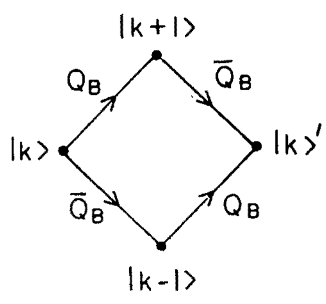

Fig. 1

1. *Singlet representations* ($k = 0$). They are formed by the states $|\alpha, 0\rangle$, such that $Q_B|\alpha, 0\rangle = \bar{Q}_B|\alpha, 0\rangle = 0$ and there exists no state $|\psi\rangle$ ($|\psi'\rangle$) such that $Q_B|\psi\rangle = |\alpha, 0\rangle$ ($\bar{Q}_B|\psi'\rangle = |\alpha, 0\rangle$).

2. *Quadruplet representations.* A quadruplet representation is given graphically in

Fig. 1, where for the sake of simplicity the labels α have been dropped. The arrows pointing upward represent the action of Q_B: $Q_B|k\rangle = |k+1\rangle$, $Q_B|k-1\rangle = |k\rangle'$ and the arrows pointing downward represent the action of $\bar{Q}_B$: $\bar{Q}_B|k+1\rangle = |k\rangle'$, $\bar{Q}_B|k\rangle = |k-1\rangle$. Due to the nilpotency of Q_B and $\bar{Q}_B$ there exists no other state (apart from the trivial one) connected with the four states of the diagram through the action of Q_B and $\bar{Q}_B$. To describe completely this representation we should add the conjugate states of each of the four states of the diagram (they are not represented in figure 1).

3. *Chain representations.* An example is given in Fig. 2 with the same conventions as for quadruplet representations. The chains can be finite or infinite. We should add that finite chains are ruled out by a remarkable theorem of Nakanishi [7].

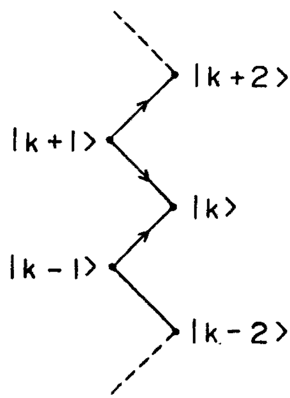

Fig. 2

Now, as far as quandruplet representations are concerned, we observe that the only states belonging to $\mathscr{V}_{ph}$ are the right most ones. They can be written as $Q_B\bar{Q}_B|k\rangle$ for some $|k\rangle$. Therefore, they are zero norm states and the relative conjugate states do not belong to $\mathscr{V}_{ph}$ (otherwise they would contradict condition (3.3)). This remark is very important because, if both $|\alpha, k\rangle$ and its conjugate $|\alpha', k\rangle$ belonged to $\mathscr{V}_{ph}$, then $\mathscr{V}_{ph}$ would contain negative norm states. Indeed, for instance, the combination $|\alpha, k\rangle - |\alpha', -k\rangle$ has norm -2 due to Eq. (3.3). As for chain representations, we remark that only the states on the right belong to $\mathscr{V}_{ph}$, and we can repeat the same argument as for quadruplet representations.

Apart from the states belonging to the singlet representations, we have shown that all the states in $\mathscr{V}_{ph}$ have non-negative norm. If we can draw the same conclusion for singlet representations, unitarity is proven. The positivity of the singlet sector can be proven for one particle states, but it must be assumed in general. One should add that this assumption must be made in any QFT, not only in the present case. Gauge theories are in the same conditions as any other QFT from this point of view.

Gauge independence [8]. Any gauge-fixing term invariant under both BRS and $\overline{\text{BRS}}$ transformations can be written as:

$$L - L_{inv} = \sum_i \xi_i [Q_B, L_{-1}^{(i)}]_+ \tag{3.4}$$

and

$$L - L_{\text{inv}} = \sum_i \xi_i [\bar{Q}_{\text{B}}, L^{(i)}_{+1}]_+, \tag{3.5}$$

where ξ_i are gauge parameters and $L^{(i)}_{-1}$ ($L^{(i)}_{+1}$) are local operators with ghost number -1 $(+1)$. For example the Lagrangian (4.17) below can be rewritten as

$$L = \text{Tr}\left\{+\frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}[\bar{Q}, [Q_{\text{B}}, A^\mu A_\mu]_-]_+ \right.$$

$$\left. -\xi_2[\bar{Q}_{\text{B}}, [Q_{\text{B}}, c\bar{c}]_-]_+ + \tfrac{1}{2}(\xi_2 - \xi_1)[\bar{Q}, \bar{B}c]_+\right\} + L_{\text{matter}}. \tag{3.6}$$

The physical S-matrix elements are defined by:

$$S_{\alpha\beta} = \langle \alpha_{\text{in}} | \beta_{\text{out}} \rangle, \qquad |\alpha_{\text{in}}\rangle, |\beta_{\text{out}}\rangle \in \mathscr{V}_{\text{ph}}. \tag{3.7}$$

From the Schwinger action principle we get

$$\frac{\delta S_{\alpha\beta}}{\delta \xi_i} = \int d^4x \langle \alpha_{\text{in}} | [Q_{\text{B}}, L^{(i)}_{-1}(x)]_+ | \beta_{\text{out}} \rangle = 0,$$

$$\frac{\delta S_{\alpha\beta}}{\delta \bar{\xi}_i} = \int d^4x \langle \alpha_{\text{in}} | [\bar{Q}_{\text{B}}, L^{(i)}_{+1}(x)]_+ | \beta_{\text{out}} \rangle = 0, \tag{3.8}$$

because Q_{B} and $\bar{Q}_{\text{B}}$ annihilate $\mathscr{V}_{\text{ph}}$. Therefore, the physical S-matrix elements are independent of the gauge parameters.

4. *The superfield formalism*

Due to the anticommutativity of the ghost fields the BRS transformations (2.13) and $\overline{\text{BRS}}$ transformations (2.14) have a supersymmetric nature which can be used to give a superfield formulation of gauge theories [5]. (See Ref. [9] for previous attempts in this sense.) To this end let us introduce a superspace Σ whose local coordinates are $\{x_\mu, \theta, \bar{\theta}\}$ where $\{x_\mu\}$ are the usual space-time coordinates and θ and $\bar{\theta}$ are anticommuting coordinates:

$$\theta^2 = \bar{\theta}^2 = \theta\bar{\theta} + \bar{\theta}\theta = 0. \tag{4.1}$$

Let us consider a 1-form ϱ in Σ. It can be written locally in the following way:

$$\varrho = \hat{A}_\mu (x, \theta, \bar{\theta}) dx^\mu + \eta(x, \theta, \bar{\theta}) d\bar{\theta} + \bar{\eta}(x, \theta, \bar{\theta}) d\theta. \tag{4.2}$$

Let us denote by $\psi(x, \theta, \bar{\theta})$ any of the superfields $\hat{A}_\mu$, η, $\bar{\eta}$. Then ψ can be expanded in the following way:

$$\psi(x, \theta, \bar{\theta}) = \psi_0(x) + \theta\psi_1(x) + \bar{\theta}\psi_2(x) + \theta\bar{\theta}\psi_3(x). \tag{4.3}$$

Now let us interpret ϱ as a connection form in Σ and define its curvature

$$R = \hat{d}\varrho + \tfrac{1}{2}[\varrho, \varrho] \tag{4.4}$$

where $\hat{d} = \frac{\partial}{\partial x^\mu} dx^\mu + \frac{\partial}{\partial \theta} d\theta + \frac{\partial}{\partial \bar{\theta}} d\bar{\theta}$ is the exterior differential in Σ, and $\frac{\partial}{\partial \theta}, \frac{\partial}{\partial \bar{\theta}}$ are left derivatives.

In local coordinates,

$$R = \sum_{\mu<\nu} R_{\mu\nu} dx^\mu \wedge dx^\nu + \sum_\mu (R_{\mu\theta} dx^\mu \wedge d\theta + R_{\mu\bar{\theta}} dx^\mu \wedge d\bar{\theta})$$

$$+ R_{\theta\theta} d\theta \wedge d\theta + R_{\bar{\theta}\bar{\theta}} d\bar{\theta} \wedge d\bar{\theta} + R_{\theta\bar{\theta}} d\theta \wedge d\bar{\theta}. \tag{4.5}$$

First we make the following ansatz: we require the connection ϱ to be flat in the θ and $\bar{\theta}$ directions. In other words, $R_{\mu\nu}$ is unconstrained, while

$$R_{\mu\theta} = R_{\mu\bar{\theta}} = R_{\theta\theta} = R_{\bar{\theta}\bar{\theta}} = R_{\theta\bar{\theta}} = 0. \tag{4.6}$$

These conditions determine all but four of the coefficients in the expansion (4.3) for the superfields $\hat{A}_\mu, \eta, \bar{\eta}$.

Now we go back to gauge theories, identifying these four free coefficients with fields relevant to gauge field theories:

$$\hat{A}_\mu(x, \theta, \bar{\theta}) = A_\mu(x) + \dots$$
$$\eta(x, \theta, \bar{\theta}) = c(x) + \dots$$
$$\bar{\eta}(x, \theta, \bar{\theta}) = \bar{c}(x) + \bar{\theta} B(x) + \dots \tag{4.7}$$

With these identifications we get:

$$\hat{A}_\mu(x, \theta, \bar{\theta}) = A_\mu + \theta D_\mu \bar{c} + \bar{\theta} D_\mu c + \theta\bar{\theta}(D_\mu B + [D_\mu c, \bar{c}]_+)$$
$$\eta(x, \theta, \bar{\theta}) = c + \theta \bar{B} - \bar{\theta}\, \tfrac{1}{2}\, [c, c]_+ + \theta\bar{\theta}[\bar{B}, c]_-$$
$$\bar{\eta}(x, \theta, \bar{\theta}) = \bar{c} - \theta\, \tfrac{1}{2}\, [\bar{c}, \bar{c}]_+ + \bar{\theta} B + \theta\bar{\theta}[\bar{c}, B]_-, \tag{4.8}$$

and

$$B + \bar{B} + [c, \bar{c}]_+ = 0$$

as in Eq. (2.10).

Similarly, starting with the superfield $\hat{\phi}^{(R)}(x, \theta, \bar{\theta})$, identifying the first coefficient of the $\theta - \bar{\theta}$ expansion with the matter field $\phi^{(R)}(x)$ and requiring the form $D\phi^{(R)} = \hat{d}\phi^{(R)} + \varrho^{(R)}\phi^{(R)}$ to vanish in the anticommuting directions, we get the expansion:

$$\hat{\phi}^{(R)}(x, \theta, \bar{\theta}) = \phi^{(R)} - \theta \bar{c}^{(R)} \phi^{(R)} - \bar{\theta} c^{(R)} \phi^{(R)} - \theta\bar{\theta}(B^{(R)} + \bar{c}^{(R)} c^{(R)})\phi^{(R)}. \tag{4.9}$$

If we denote by $\psi(x, \theta, \bar{\theta})$ any of the superfields $(\hat{A}_\mu, \eta, \bar{\eta}, \hat{\phi}^{(R)})$ and if we write

$$\psi(x, \theta, \bar{\theta}) = \psi_0 \theta \bar{s} \psi_0 + \bar{\theta} s \psi_0 + \theta\bar{\theta} s \bar{s} \psi_0, \tag{4.9'}$$

then, comparing (4.8) and (4.9) with Eqs. (2.13) and (2.14) we see that s and $\bar{s}$ represen the transformations induced by Q_B and $\bar{Q}_B$ respectively. In other words, BRS and $\overline{\text{BRS}}$ transformations are represented by translations in θ and $\bar{\theta}$ respectively. Speaking in term of the form ϱ we could say that the BRS ($\overline{\text{BRS}}$) transformations are represented by the Lie derivative in the direction $\frac{\partial}{\partial\bar{\theta}}\left(\frac{\partial}{\partial\theta}\right)$ respectively.

There is another (equivalent) way to arrive at the superfields (4.8) and (4.9). Let us start from the form A given by Eq. (1.2) and consider the supergauge transformation (that is, a gauge transformation in the superspace Σ)

$$A \to g^{-1}Ag+g^{-1}\hat{d}g, \tag{4.10}$$

where g is given by

$$g(x, \theta, \bar{\theta}) = \exp\{\theta\bar{c}+\bar{\theta}c+\theta\bar{\theta}(B+\tfrac{1}{2}[c, \bar{c}]_+)\} = 1+\theta\bar{c}+\bar{\theta}c+\theta\bar{\theta}(B+c\bar{c}). \tag{4.11}$$

The supergauge transformed of A is exactly the form ϱ with coefficients given by Eqs. (4.8). As a consequence of Eq. (4.10) the curvature R is given by

$$R = g^{-1}Fg. \tag{4.12}$$

Similarly for matter fields

$$\hat{\phi}^{(R)} = g^{(R)-1}\phi^{(R)}, \tag{4.13}$$

where $g^{(R)}$ has the same expression as g apart from the replacement of τ^α by $\tau^{(R)\alpha}$.

Eqs. (4.12) and (4.13) account for the identities:

$$\begin{aligned} \mathrm{Tr}\,(R_{\mu\nu}R^{\mu\nu}) &= \mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu}) \\ L_{\text{matter}}(\hat{\phi}, D_\mu\hat{\phi}) &= L_{\text{matter}}(\phi, D_\mu\phi) \end{aligned} \tag{4.14}$$

as a consequence L_{inv}, Eq. (2.1), is invariant under superlocal gauge transformations.

Therefore, L_{inv} is not quantizable (as we already saw in Section 2). We must break the superlocal gauge invariance, and we must comply with the requirements of unitarity (presence of a conserved nilpotent charge) and renormalizability. So the most general Lagrangian L will be built in such a way that $L-L_{\text{inv}}$:

(*i*) breaks completely the superlocal gauge invariance,
(*ii*) is invariant under BRS and $\overline{\text{BRS}}$ transformations,
(*iii*) has zero ghost number (the ghost number is conserved),
(*iv*) has canonical dimensions $\leqslant 4$ (renormalizability requirement),
(*v*) is hermitian.

If we want to recover the standard gauge breaking term, we have to add a further requirement, that $L-L_{\text{inv}}$ be invariant under gauge transformations of the first kind. The most general Lagrangian satisfying these conditions is:

$$L = \mathrm{Tr}\left(+\frac{1}{4g^2}R^{\mu\nu}R_{\mu\nu}-\tfrac{1}{2}\frac{\partial}{\partial\bar{\theta}}\frac{\theta}{\partial\theta}(\hat{A}_\mu\hat{A}^\mu)+\tfrac{1}{2}\xi_1\frac{\partial\bar{\eta}}{\partial\bar{\theta}}\frac{\partial\bar{\eta}}{\partial\bar{\theta}}-\tfrac{1}{2}\xi_2\frac{\partial\eta}{\partial\theta}\frac{\partial\eta}{\partial\theta}\right)$$
$$+L_{\text{matter}}(\hat{\phi}^{(R)}, D_\mu\hat{\phi}^{(R)}). \tag{4.15}$$

In order to satisfy condition (*v*) above we have introduced the hermiticity convention

$$\theta^+ = \theta, \quad \bar{\theta}^+ = -\bar{\theta}. \tag{4.16}$$

In terms of the component fields the Lagrangian (4.15) reduces to

$$L = \mathrm{Tr}\left(+\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu}+A_\mu\partial^\mu B+\partial_\mu\bar{c}D^\mu c-\tfrac{1}{2}\xi_1 BB+\tfrac{1}{2}\xi_2\bar{B}\bar{B}\right)$$
$$+L_{\mathrm{matter}}(\phi^{(\mathrm{R})}, D_\mu\phi^{(\mathrm{R})}). \tag{4.17}$$

This is the same as Eq. (2.8) apart from the term $\frac{\xi_2}{2}\bar{B}\bar{B}$. When $\xi_2 = 0$ the Lagrangian is invariant under the transformation $\bar{c} \to \bar{c}+\sigma$ where σ is constant. This invariance allows us to put $\xi_2 = 0$ consistently from the beginning, since renormalization cannot generate such a counterterm. If we want to recover the 't Hooft gauge we have to use the gauge breaking term:

$$L-L_{\mathrm{inv}} = \tfrac{1}{2}\frac{\partial^2}{\partial\theta\partial\bar{\theta}}(\hat{A}_\mu\hat{A}^\mu)+\xi\frac{\partial^2}{\partial\theta\partial\bar{\theta}}(\varepsilon, \hat{\phi})+\tfrac{1}{2}\xi\frac{\partial\bar{\eta}}{\partial\theta}\frac{\partial\bar{\eta}}{\partial\bar{\theta}}, \tag{4.18}$$

where $\varepsilon_r = \langle 0|\phi_r|0\rangle$ and (,) represents the scalar product in the representation space. In component fields it reduces to

$$L-L_{\mathrm{inv}} = -A_\mu\partial^\mu B-\partial_\mu\bar{c}D^\mu c+\xi(\phi, B^{(\mathrm{R})}\varepsilon)+\xi(\varepsilon, \bar{c}^{(\mathrm{R})}c^{(\mathrm{R})}\phi)+\frac{\xi}{2}B^2. \tag{4.19}$$

The background gauge can be dealt with in an analogous way. Other gauge choices, such as quadratic gauges and axial gauges, have only BRS invariance [6].

5. *A few mathematical remarks*

In the superfield formalism of Section 4 there are some mathematical points that deserve to be clarified. The supergauge transformation, Eq. (4.10), (4.11), works from an operational point of view, but it is quite unclear from a mathematical point of view. First of all, we should show that A is a connection form in Σ, therefore with reference to some principal bundle whose base space is Σ. Moreover, a difficulty arises when we try to figure out what is the gauge group implied in this super gauge transformation, $g:\Sigma \to$? This group cannot be G itself because, from the explicit form (4.11), we see that $\theta\bar{c}$, $\bar{\theta}c$, etc. are never real (complex) matrices. It is apparent that we must look for some sort of enlargement of the field of the real (complex) numbers in such a way as to allow a natural interpretation of such numbers as $\bar{\theta}c^\alpha(x)$, etc.

The clue for solving this difficulty is provided by the theory of supermanifolds developed in the last few years. All the details are explained in Ref. [10] and [11] and will not be repeated here. The main idea is to enlarge every geometrical structure by, roughly speaking, replacing commuting (anticommuting) variables with variables taking on their values in the even (odd) part of an infinite dimensional Grassmann algebra. In this way the superspace Σ is replaced by a corresponding supermanifold M_S, the group G by the corresponding Grassmann (enlarged) Lie group, etc. Moreover, Eqs. (4.10), (4.12) should be replaced rigorously by

$$\varrho = g^{-1}j^*Ag+g^{-1}\hat{d}g, \quad R = g^{-1}j^*Fg^{-1}, \tag{5.1}$$

where j^* means the pull-back through the projection map $j: M_S \to M$. However, the resulting structures work operationally in the same way as the rough picture given in the previous section does.

This is the reason why, once we have clarified this point, we forget about heavy mathematical details, following the naive attitude of the previous section (apart from keeping Eq. (5.1)). A rigorous procedure would give the same results.

6. *Superfields and ABJ anomalies*

Among the various applications of the superfield formalism, for example, to antisymmetric gauge tensors [12] or to gravitation theory [13], let us show here the connection between the geometry underlying the superfield formalism and the Adler-Bell-Jackiw (ABJ) anomalies in gauge field theories [14].

ABJ anomalies were discovered years ago [15] while studying the renormalizability properties of gauge theories. It was shown that renormalizability is destroyed by the presence of an anomaly. Precisely speaking, ABJ anomalies prevent the generalized WT identities from reproducing themselves recursively at orders higher than the zeroth in the loop expansion [1]; thus the renormalization procedure breaks down at the lowest perturbative order.

Let us consider a gauge field theory invariant under BRS transformations (for the sake of simplicity we will not consider $\overline{\text{BRS}}$ symmetry in this section; see, however, Ref. [14]). Let us call s the BRS transformation operator acting on the integrated or non-integrated polynomials of the fields involved. Then the Ward-Takahashi (WT) identity corresponding to the BRS symmetry breaks down at the first order if there exists a term $\Delta^{(1)} = \int \Delta(x)d^4x$, where $\Delta(x)$ is a polynomial of the fields with canonical dimension 5 and ghost number 1, such that

$$s\Delta^{(1)} = 0 \tag{6.1}$$

and there exists no $\Delta^{(0)}$ such that

$$s\Delta^{(0)} = \Delta^{(1)}. \tag{6.2}$$

Let us call $\Delta^{(1)}$ anomalous term. The aim of this section is to show that Eqs. (6.1), (6.2) fit naturally into the geometric structure of the principal fiber bundle whose base space is the superspace Σ. As we consider a theory invariant under BRS transformations only, the superfield formalism can be simplified. The superspace $\Sigma = (\{x_\mu\}, \theta)$ is obtained by adding to the usual space-time coordinates only one anticommuting coordinate θ. Therefore, the connections flat in the θ-direction have the form:

$$\varrho = \hat{A}_\mu(x, \theta)dx^\mu + \eta(x, \theta)d\theta \tag{6.3}$$

where

$$\hat{A}_\mu(x, \theta) = A_\mu + \theta D_\mu c$$
$$\eta(x, \theta) = c - \tfrac{1}{2}\theta[c, c]_+. \tag{6.4}$$

ϱ can be obtained by means of a supergauge transformation $g: \Sigma \to G$; starting from the 1-form A:

$$\varrho = g^{-1}j^*Ag + g^{-1}\hat{d}g, \qquad R = g^{-1}j^*Fg, \tag{6.5}$$

and

$$g(x, \theta) = \exp\theta c = 1 + \theta c, \tag{6.6}$$

where $\hat{d} = \dfrac{\partial}{\partial x^\mu}dx^\mu + \dfrac{\partial}{\partial\theta}d\theta.$

Now we need a few definitions. Let us introduce the set of symmetric multilinear mappings $f: \mathfrak{Z} \times \ldots \times \mathfrak{Z} \to R$, invariant under the adjoint action of the group G on $\mathfrak{Z}$, that is

$$\sum_{i=1}^{k} f(X_1, \ldots, [X_i, Y], \ldots, X_k) = 0 \tag{6.7}$$

for any $X_1, \ldots, X_k, Y \in \mathfrak{Z}$. If τ^α is a basis in $\mathfrak{Z}$,

$$f(\tau^{\alpha_1}, \ldots, \tau^{\alpha_k}) = d^{\alpha_1 \ldots \alpha_k} \tag{6.8}$$

is a completely symmetric tensor, and if $\omega_1, \ldots, \omega_k$ are $\mathfrak{Z}$-valued forms of order $i_1 \ldots, i_k$ respectively, we can construct the real-valued $i_1 + \ldots + i_k$ -form

$$F(\omega_1, \ldots, \omega_k) = \sum_{\alpha_1 \ldots \alpha_k} d^{\alpha_1 \ldots \alpha_k}\omega_1^{\alpha_1} \wedge \ldots \wedge \omega_k^{\alpha_k}. \tag{6.9}$$

Let ϱ_1 and ϱ_0 be any two connection forms in Σ. Then $\varrho_t = \varrho_0 + t(\varrho_1 - \varrho_0)$ is also a connection for t real, $0 \leqslant t \leqslant 1$. Let us call R_t the curvature corresponding to ϱ_t.

The following theorem is the main mathematical result we need:

Theorem: Let f be a k-linear symmetric invariant function as defined above and let us define

$$\hat{Q} = k\int_0^1 f(\varrho_1 - \varrho_0, R_t, \ldots, R_t)dt, \tag{6.10}$$

then

$$\hat{d}\hat{Q} = F(R_1, \ldots, R_1) - F(R_0, \ldots, R_0). \tag{6.11}$$

For classical manifolds, this theorem is the one on which the Weyl homomorphism is based. In the proof, an essential role is played by the geometric structure of the principal fiber bundle built over the base space [16]. In the case of the supermanifold M_s (corresponding to the superspace Σ) the proof is exactly parallel, provided that we take into account what was said in Section 5.

If the principal fiber bundle over Σ is trivial (this happens, for instance, when the space-time $M = R^4$), the 1-form equal to zero everywhere is a connection form in Σ. Therefore, in this case, we can choose $\varrho_0 = 0$. Then let us simplify the notations putting $\varrho_1 = \varrho$, so that

$$\varrho_t = t\varrho$$

$$R_t = t\hat{d}\varrho + \frac{t^2}{2}[\varrho, \varrho]$$

$$R_1 = R = \hat{d}\varrho + \tfrac{1}{2}[\varrho, \varrho]. \tag{6.12}$$

In this case, the above theorem says

$$\hat{d}\hat{Q} = f(R, \ldots, R), \tag{6.13}$$

where

$$\hat{Q} = k \int_0^1 f(\varrho, R_t, \ldots, R_t)dt. \tag{6.14}$$

From Eq. (6.5), Eq. (6.13) becomes

$$\hat{d}\hat{Q} = j^* f(F, \ldots, F), \tag{6.15}$$

due to the invariance property of f.

Let us take the particular case $k = 3$, then

$$\hat{d}\hat{Q} = 0 \tag{6.16}$$

where

$$\hat{Q} = 3 \int_0^1 f(g, R_t, R_t)dt. \tag{6.17}$$

Eq. (6.16) follows from the fact that $f(F, F, F)$ is a 6-form in a four-dimensional space.

Eq. (6.16) is the fundamental equation for the cohomology of the ABJ anomaly. Indeed let us decompose $\hat{Q}$ in the following way:

$$\hat{Q} = \sum_{i=0}^{5} Q^i_{5-i}(d\theta)^i, \tag{6.18}$$

where $(d\theta)^i = \underbrace{d\theta \wedge \ldots \wedge d\theta}_{i \text{ times}}$. Moreover, we decompose Q^i_{5-i} in the following way

$$\hat{Q}^i_{5-i} = Q^i_{5-i} + \theta s Q^i_{5-i} \tag{6.19}$$

where s is the BRS transformation operator as before, i is the ghost number and $5-i$ is the order of the form in the variables $\{x_\mu\}$. Eq. (6.16) implies a set of equations for the coefficients of Eq. (6.19) (remember that, trivially, $Q^0_5 = 0$):

$$\begin{aligned} dQ^1_4 &= 0 \\ dQ^2_3 + sQ^1_4 &= 0 \\ dQ^3_2 - sQ^2_3 &= 0 \\ dQ^4_1 + sQ^3_2 &= 0 \\ dQ^5_0 - sQ^4_1 &= 0 \\ sQ^5_0 &= 0. \end{aligned} \tag{6.20}$$

We see that Q_4^1 plays the role of Δ, introduced before Eq. (6.1). Indeed from Eqs. (6.20)

$$s\int Q_4^1 = 0. \tag{6.21}$$

Moreover, if there existed a Δ_4^0 such that $\int Q_4^1 = s\int \Delta_4^0$, then $Q_4^1 = s\Delta_4^0 + d\Delta_3^1$ for some Δ_3^1, then from the second of Eqs. (6.20), there would exist a Δ_2^2 such that $s\Delta_3^1 + Q_3^2 = d\Delta_2^2$, etc. Eventually, there would exist a Δ_0^4 such that

$$s\Delta_0^4 = -Q_0^5. \tag{6.22}$$

But from the explicit expression of Q_0^5, calculated by substituting Eqs. (6.3), (6.12) into Eq. (6.17) and given by

$$Q_0^5 = \tfrac{1}{40} f(c, [c, c]_+, [c, c]_+), \tag{6.23}$$

we see that there is no Δ_0^4 satisfying Eq. (6.22). Therefore, $\int Q_4^1$ is an anomalous term. The explicit form of Q_4^1 is given by:

$$Q_4^1 = f(c, dA, dA) + \tfrac{1}{4} f(c, dA, [A, A]) - \tfrac{1}{2} f(c, A, [A, A]). \tag{6.24}$$

We observe once more that the explicit form of Q_4^1 and the fact that it is an anomalous term are based on the information contained in the previously quoted theorem, and related to the geometry of the principal fiber bundle whose base space and structure group are, roughly speaking, the superspace Σ and the gauge group G. More information about the explicit form of the multilinear function f is not given by the geometry of the principal fiber bundle, but by the cohomology of the gauge group G.

The problem of the existence of the trilinear (and, in general, k-linear) invariant functions f was solved long ago by Chevalley and Borel [17, 18]. For completeness we quote their results. If we restrict ourselves to simple Lie groups, non-trivial k-linear invariant functions f (symmetric invariant tensors of k-th rank) exist only in the following cases:

Lie Algebra	k
A_N	2, 3, ..., $N+1$
B_N, C_N	2, 4, ..., $2N$
D_N	2, 4, ..., $2N-2$, N
G_2	2, 6
F_4	2, 6, 8, 12
E_6	2, 5, 6, 8, 9, 12
E_7	2, 6, 8, 10, 12, 14, 18
E_8	2, 8, 12, 14, 18, 20, 24, 30

As we see from this table, only SU(N) with $N \geq 3$ has a non-trivial third order invariant symmetric tensor, $d^{\alpha\beta\gamma}$, unique up to a constant factor. The anomalous term Q_4^1 may be cast into the form

$$Q_4^1 = c^\alpha(x) q^\alpha(x) d^4x \tag{6.25}$$

where $q^\alpha(x)$ is proportional to the familiar expression

$$q^\alpha(x) \sim \varepsilon_{\mu\nu\lambda\varrho}\partial^\mu[d^{\alpha\beta\gamma}\partial^\nu A_\beta^\lambda A_\gamma^\varrho + \tfrac{1}{12}(d^{\alpha\beta\varphi}f^{\varphi\gamma\delta} + d^{\alpha\gamma\varphi}f^{\varphi\delta\beta} + d^{\alpha\delta\varphi}f^{\varphi\beta\gamma})A_\beta^\nu A_\gamma^\lambda A_\delta^\varrho].$$

The constant of proportionality can be determined only on the basis of the explicit form of the Lagrangian. If in the Lagrangian there are only fermion fields belonging to real representations of G, or for every fermion field belonging to a nonreal representation there is also the counterpart belonging to the conjugate representation, then the constant is zero and the anomaly does not appear.

Summarizing, the possibility for anomalies to exist and their general form is contained in the geometry of the principal bundle over the superspace. Their detailed form is determined by the cohomology of the gauge group. Whether they appear or not, is based on the Lagrangian of the particular model considered. To conclude, we remark that the method illustrated in this section allows us to find the ABJ anomalies in any number d of dimensions. We must look for terms of the form Q_d^1 in the various $\hat{Q}$'s which can be obtained with different values of k. It is easy to realize that it must be $d = 2k-2$. Therefore, the present scheme can accomodate ABJ anomalies for even d only. For example, in two dimensions the anomalous term is $Q_2^1 = f(c, dA)$, where f in this case is given by the Killing form of $\mathfrak{J}$.

These lectures are based on work done in collaboration with P. Cotta-Ramunino, P. Pasti and M. Tonin, whose observations and advices have been essential in preparing them. It is a pleasure to thank the organizers of the School on Theoretical Physics at Katowice for the hospitality and for giving me the opportunity to lecture.

REFERENCES

[1] C. Becchi, A. Rouet, R. Stora, *Ann. Phys.* **98**, 287 (1976).
[2] G. Curci, R. Ferrari, *Nuovo Cimento* **35A**, 273 (1976).
[3] T. Kugo, I. Ojima, *Prog. Theor. Phys. Suppl.* **66**, 1 (1979).
[4] I. Ojima, *Prog. Theor. Phys.* **64**, 625 (1980); H. Quirós, F. J. de Urriés, H. Hoyes, M. L. Mazon, E. Rodriguez, *J. Math. Phys.* **22**, 1767 (1981).
[5] L. Bonora, M. Tonin, *Phys. Lett.* **B98**, 48 (1981).
[6] L. Bonora, P. Pasti, M. Tonin, *Nuovo Cimento* **64A**, 307 (1981); *Nuovo Cimento* **64A**, 378 (1981).
[7] N. Nakanishi, *Prog. Theor. Phys.* **62**, 1396 (1979).
[8] K. Fujikawa, *Prog. Theor. Phys.* **63**, 1364 (1980).
[9] S. Ferrara, O. Piguet, M. Schweda, *Nucl. Phys.* **B119**, 493 (1977); K. Fujikawa, *Prog. Theor. Phys.* **59**, 2045 (1977).
[10] L. Bonora, P. Pasti, M. Tonin, *Supermanifolds and BRS transformations*, IFPD 48/81, to appear in. *J. Math. Phys.*
[11] J. Hoyos, H. Quirós, J. Ramirez Mittelbrunn, F. J. de Urriés, *Superfiber bundle structure of quantized gauge theories* (Madrid preprint).
[12] P. A. Marchetti, M. Tonin, *Nuovo Cimento* 63A, 459 (1981).
[13] P. Pasti, M. Tonin, to appear.
[14] L. Bonora, P. Cotta-Ramunino, MIT preprint CTP#944 (1981), to appear in *Phys. Lett.* B.
[15] S. L. Adler, *Phys. Rev.* **177**, 2426 (1969); J. S. Bell, R. Jackiw, *Nuovo Cimento* **60A**, 47 (1969).
[16] S. Kobayashi, K. Nomizu, *Foundations of Differential Geometry*, vol. II, Interscience, New York 1969.
[17] C. Chevalley, A. Borel, *Am. Math. Soc. Mem.* **14**, 1 (1955).
[18] S. Okubo, *Phys. Rev.* **D16**, 3528 (1977).
[19] H. Georgi, S. L. Glashow, *Phys. Rev.* **D6**, 429 (1972).