THE SECOND CLASS CURRENTS IN HEAVY LEPTON DECAYS

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The second class current manifestations in the decays $\tau \to \nu\pi V$, $V = \rho, \omega, \varphi$ and $\tau \to \nu\pi\eta$ are studied. The energy distributions are obtained. The V meson density matrix elements are evaluated. We show that knowledge of these elements allows one to determine all hadron weak current formfactors.

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1. Introduction

The fraction of τ lepton hadron decays is more than 60% of its full width [1]. The investigation of hadron decays is useful for both weak and strong interactions. Owing to a large τ lepton mass one can investigate the hadron weak current at large momentum transfer squared (of the order $m_\tau^2 \simeq 3.2$ GeV). Therefore, we can consider ν-τ beams with varying energy. The variety of hadron decays is also an advantage. For example, the $\tau \to \nu\pi{-}\pi^0$ and $\tau \to \nu\pi\omega(\varphi)$ decays are important for the verification of the CVC hypothesis [2–4]. The $\tau \to \nu\varrho\pi$ decay [5, 6] is of interest for the investigation of the A_1 meson which is a classical object of current algebra [7, 8]. One can also study the locality [9, 10] of lepton weak interactions with hadrons in the unknown region of timelike momentum transfer [11, 12]. The T-invariance of charge weak current leads to specific predictions [13], which can be verified experimentally.

Finally the processes $\tau \to \nu$ +hadrons are helpful for the investigating second class currents [14]. The fact is that pion production is most favoured. However these hadron final states have certain G-parity [15, 16]. Therefore, the hadron production with positive G-parity ($\tau \to \nu\pi^-\pi^0$, $\tau \to \nu\pi\omega$) is determined by vector current (first class current), but the second class current ought to be axial. If the hadron state with negative G-parity is produced ($\tau \to \nu\varrho\pi$), the first class current ought to be an axial current, but the second class current ought to be a vector current.

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This comparison is only typical of τ lepton decays. The exception is the $\pi \to \pi^0 e\nu$ decay where the first class current is the vector current, but it is impossible to construct the axial current for this decay (even if the second class currents exist).

It was shown [17] that interference between the first and the second class currents distorts the hadron energy distributions in different τ lepton decays. But it was important to know the neutrino momentum.

In this paper we investigate the second class current manifestations in the decays $\tau \to \nu\pi V$, $V = \varrho, \omega, \varphi$. In the case of the unpolarized V meson the second class and first class current interference is determined by only two (of four) formfactors. The V meson density matrix is obtained.

The $\tau \to \nu\pi\eta$ decay which is determined by the second class current only is discussed as well.

2. *The vector meson production*

The matrix element of the $\tau \to \nu\pi V$, $V = \varrho, \omega, \varphi$ decay is given by

$$M = \frac{G}{\sqrt{2}} \bar{u}(p_2)\gamma_\mu(1+\gamma_5)u(p_1)J_\mu,$$

$$J_\mu = iF_1\varepsilon_{\mu\alpha\beta\gamma}U_\alpha^* K_\beta q_\gamma + (F_2 K_\mu + F_3 q_\mu) q \cdot U^* + F_4 U_\mu^*, \tag{1}$$

where G is the Fermi coupling, U and K are the polarization vector and momentum of the V meson, respectively, q, p_1 and p_2 are the momentum of π, τ and ν, respectively, $F(t)$, $t = (K+q)^2$ are the weak formfactors (complex in the region of the timelike momentum transfer).

In the case of ω and φ (ϱ) meson production the first class current is determined by the $F_1(F_2, F_3, F_4)$ formfactor, but the second class current is determined by the F_2, F_3 and F_4 (F_1) formfactors.

Owing to the different parity of the first and second class currents the contribution of their interference to the decay probability is zero. This result is independent of τ lepton polarization and the type of formfactors. Note, that the situation is not the same in the $\tau \to \nu K K^*$ decay. Moreover, this decay is suppressed due to the small phase space.

Therefore the contribution of this interference to the differential probability (for example, dW/dE_ν or dW/dE_π) is not zero.

After summing over the V meson and ν polarizations one can obtain the interference contribution for the matrix element squared the following formula

$$I_{12} = 2G^2 \operatorname{Re} F_1 F_4^* [q \cdot p_2 K \cdot p_1 - q \cdot p_1 K \cdot p_2 + m_\tau (K \cdot p_2 S \cdot q - q \cdot p_2 S \cdot K)]$$

where S is the τ lepton polarization vector. It is seen that even in the polarized τ lepton decay the interference contribution is determined by one combination only, viz. $\operatorname{Re} F_1 F_4^*$.

Note, that the sign of I_{12} is different for $\tau^+ \to \nu\pi^+ V^0$ and $\tau^- \to \nu\pi^- V^0$ decays.

For the numerical calculations, we have used the following formfactor parametrization ($V = \omega, \varphi$)

$$F_1 = F_1(0)\,[x_1 m_\varrho^2 (m_\varrho^2 - t - i m_\varrho \Gamma_\varrho)^{-1} + x_2 m_{\varrho'}^2 (m_{\varrho'}^2 - t - i m_{\varrho'} \Gamma_{\varrho'})^{-1}],$$

$$x_1 + x_2 = 1[19], \quad F_{1\omega}^2(0) = \frac{4.1}{m_\omega^2}, \quad F_{1\varphi}^2(0) = \frac{0.02}{m_\varphi^2},$$

$$F_4(t) = \frac{m_B^2}{m_B^2 - t - i m_B \Gamma_B},$$

where m_V and Γ_V are the mass and the width of the corresponding vector mesons.

For the $\tau \to \nu \varrho \pi$ decay we use axial formfactors, calculated using current algebra with "hard pions". In this approximation the axial formfactors are determined by the A_1 meson ($J^{PG} = 1^{+-}$) contribution [20]. These formfactors allowed all the experimental data to be explained. We suppose that the second class current formfactor (F_1) does not depend on t, because the hypothetical meson with $J^{PG} = 1^{--}$ (which can play the role of the second class current carrier) is, probably, the $q\bar{q}G$- or $qq\bar{q}\bar{q}$-system (q — quark and G — gluon). But the mass of such systems must be relatively large, say, several GeV.

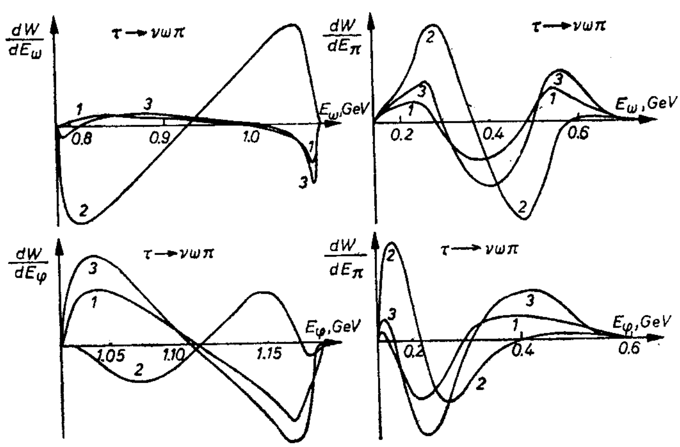

Fig. 1. The dW/dE_π and dW/dE_V distributions in $\tau \to \nu\pi\omega$, $\tau \to \nu\pi\varphi$ decays (arbitrary units). The curves correspond to $m_{\varrho'} = 0$, $\Gamma_{\varrho'} = 0$ (*1*), $m_{\varrho'} = 1.2$ GeV, $\Gamma_{\varrho'} = 0.13$ GeV (*2*), $m_{\varrho'} = 1.57$ GeV, $\Gamma_{\varrho'} = 0.34$ GeV (*3*)

It is seen from the figure that the interference contribution to the dW/dE_π and dW/dE_V distributions changes its sign (as expected).

The corresponding contribution to the V meson density matrix elements ($\varrho_{\lambda\lambda'} = \varrho^*_{\lambda'\lambda}$) is given in the Appendix. These elements are normalized by the condition $\sum_{\lambda=\lambda'} \varrho_{\lambda\lambda'} = I_{12}$, where λ and λ' are the V meson helicities. It is seen that knowing the density matrix elements one can, in principle, determine all the formfactor combinations.

3. The $\tau \to \nu\pi\eta$ decay

The matrix element of this decay is determined by only one formfactor

$$J_\mu = F(t)\,[K_\mu(m_\eta^2+K\cdot q)-q_\mu(m_\pi^2+K\cdot q)]. \tag{3}$$

We suppose that the vector current is conserved. Note, that generally one can not expect the conservation of second class current (even if it is a vector one). Yet, we used expression (3) to derive some information about this decay.

The Dalitz plot is given in the Appendix.

The hadron energy distribution is

$$\frac{dW}{dE_\eta} = \frac{G^2\sqrt{E_\eta^2-m_\eta^2}}{2^9\pi^3 m_\tau}\left(1+\frac{m_\pi^2}{2m_\tau E_\eta-m_\eta^2-m_\tau^2}\right)\int |M|^2 d\cos\theta,$$

$$|M|^2 = |F(t)|^2\,[(Q\cdot K)^2\,(2K\cdot p_1 K\cdot p_2-m_\eta^2 p_1\cdot p_2)+(Q\cdot q)^2\,(2q\cdot p_1 q\cdot p_2$$
$$-m_\pi^2 p_1\cdot p_2)+(Q\cdot K)\,(Q\cdot q)\,(K\cdot p_1 q\cdot p_2+K\cdot p_2 q\cdot p_1-K\cdot q p_1\cdot p_2)],$$

$$2p_1\cdot p_2 = m_\tau^2-t,\quad 2K\cdot p_1 = 2m_\tau E_\eta,\quad 2q\cdot K = i-m_\pi^2-m_\eta^2,\quad Q = q+K,$$

$$2K\cdot p_2 = 2m_\tau E_\eta-m_\eta^2-t,\quad 2q\cdot p_2 = m_\tau^2+m_\eta^2-m_\pi^2-2m_\tau E_\eta,$$

$$t = m_\tau^2+m_\tau\left(E_\eta-m_\tau+\sqrt{E_\eta^2-m_\eta^2}\cos\theta\right)\left(1+\frac{m_\pi^2}{2m_\tau E_\eta-m_\eta^2-m_\tau^2}\right).$$

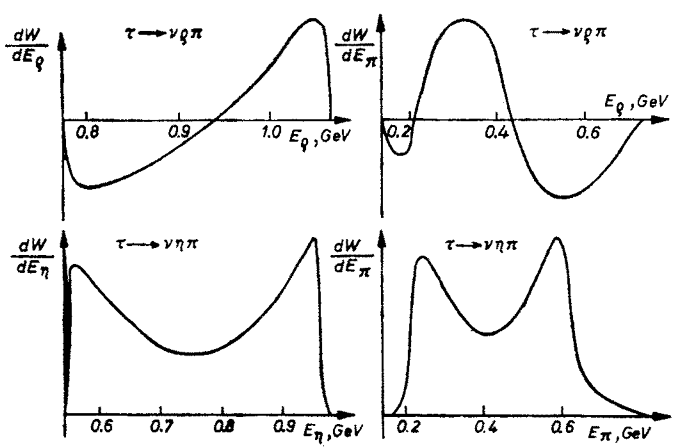

Fig. 2. The dW/dE_π, dW/dE_ϱ and dW/dE_η distributions in $\tau \to \nu\pi\varrho$ and $\tau \to \nu\pi\eta$ decays (arbitrary units)

For numerical computations we have used the formfactor

$$F(t) = m_\delta^2(m_\delta^2-t-im_\delta\Gamma_\delta)^{-1},$$

where m_δ and Γ_δ are the δ meson ($J^{PG} = 0^{+-}$) mass and width, respectively.

It is seen from the figure that the dW/dE_π and dW/dE_η distributions have the dip in the central part of the curve.

The total width is

$$\Gamma(\tau \to \nu\pi\eta) = 0.327 \times 10^{-11}\ \text{GeV}.$$

4. Conclusion

We have shown that τ lepton decays can be useful in second class current investigations. It is clear that the second class current problem cannot be avoided, for example, in the verification of the CVC hypotheses in $\tau \to \nu\pi\omega(\varphi)$ decays. However, there are some problems that can be solved without taking second class currents into account. Thus, the possibility of determining the ν_τ mass (using asymmetry in the $\tau \to \nu\tau$ + hadrons [13]) is independent of the existence of the second class current.

Note, finally, that the best estimates were obtained from nuclear β decay experiments [21]. But there are some difficulties in such experiments: the small momentum transfer, in accounting for electromagnetic and nucleon-nucleon interactions and lack of information of nucleon binding energy [22, 23].

Thus, the τ lepton decays seem to be most useful for the second class current search.

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APPENDIX

A. The Dalitz plot

$$\frac{dW}{dE_\pi dE_\eta} = \frac{G^2(1+\lambda^2)}{64\pi^3 m_\tau}|M|^2, \quad \lambda = \pm 1,$$

$$K \cdot p_1 = m_\tau E_\eta, \quad q \cdot p_1 = m_\tau E_\pi, \quad p_1 \cdot p_2 = m_\tau(m_\tau - E_\pi - E_\eta),$$

$$2q \cdot K = 2m_\tau(E_\pi + E_\eta) - m_\tau^2 - m_\eta^2 - m_\pi^2.$$

B. The V meson density matrix

$$\frac{1}{G^2}\varrho_{++} = \text{Im}\,(f_1 f_2^*)q_x(K^2 q \cdot p_2 B_y - K \cdot p_2 \kappa_2) + \text{Re}\,(f_1 f_2^*)$$

$$q_x(m_\pi\sigma_1 + K^2 u_x - q \cdot K r_x) + \text{Im}\,(f_1 f_3^*)q_x(q \cdot p_2\kappa_1 - q^2 K \cdot p_2 B_y)$$

$$+\text{Re}\,(f_1 f_3^*)q_x(m_\pi\sigma_2 + \delta q_x + q \cdot K u_x - q^2 r_x) + \text{Im}\,(f_1 f_4^*)$$

$$[m_\pi A_y(\Delta_1 + q_z p_{2x}) + q_x B_y K \cdot p_2] + \text{Re}\,(f_1 f_4^*)\,[-2\delta - q_x r_x$$

$$+ m_\pi(\Delta_1 A_x + \Delta_2 p_{2x} - 2A \cdot p_2 q_z)],$$

$$\frac{1}{G^2}\varrho_{--} = \mathrm{Im}\,(f_1 f_2^*) q_x (K\cdot p_2 \kappa_1 - K^2 q\cdot p_2 B_y) + \mathrm{Re}\,(f_1 f_2^*)$$

$$q_x(K^2 u_x - q\cdot K r_x - m_\pi \sigma_1) + \mathrm{Im}\,(f_1 f_3^*) q_x (q^2 K\cdot p_2 B_y$$

$$-q\cdot p_2 \kappa_2) + \mathrm{Re}\,(f_1 f_3^*) q_x (\delta q_x + q\cdot K u_x - q^2 r_x + m_\pi \sigma_2)$$

$$+\mathrm{Im}\,(f_1 f_4^*)\,(q_x B_y K\cdot p_2 - m_\pi \Delta_1 A_y - m_\pi A_y q_z p_{2x})$$

$$+\mathrm{Re}\,(f_1 f_4^*)\,[-2\delta - q_x r_x + m_\pi(2A\cdot p_2 q_z - A_x \Delta_1 - p_{2x}\Delta_2)],$$

$$\frac{1}{2G^2}\varrho_{00} = \mathrm{Re}\,(f_2 f_1^*) q_z (K^2 u_z - q\cdot K r_z) + \mathrm{Re}\,(f_3 f_1^*) q_z$$

$$(\delta q_z + q\cdot K u_z - q^2 r_z) - \mathrm{Re}\,(f_4 f_1^*)\,(\delta + q_z r_z) + m_\pi q_x A_y$$

$$\mathrm{Im}\, f_1\cdot (f_2 K\cdot p_2 q_z + f_3 q\cdot p_2 q_z + f_4 p_{2z})^*,$$

$$\frac{1}{\sqrt{2}G^2}\varrho_{0-} = \mathrm{Im}\,(f_1 f_2^*) q_z (K\cdot p_2 \kappa_1 - K^2 q\cdot p_2 B_y$$

$$+\mathrm{Re}\,(f_1 f_2^*) q_z (K^2 u_x - q\cdot K r_x - m_\pi \sigma_1) + \mathrm{Im}\,(f_3 f_1^*) q_z$$

$$(q\cdot p_2 \kappa_2 - q^2 K\cdot p_2 B_y) + \mathrm{Re}\,(f_3 f_1^*) q_z (\delta q_x - q^2 r_x$$

$$+q\cdot K u_x - m_\pi \sigma_2) + \mathrm{Im}\,(f_4 f_1^*) q_z (m_\pi A_y p_{2z} - K\cdot p_2 B_y)$$

$$-\mathrm{Re}\,(f_1 f_4^*)\,[q_z r_x + m_\pi(A\cdot p_2 q_x + \Delta_1 p_{1z} + \Delta_2 p_{2z})],$$

$$\frac{1}{G^2}\varrho_{+-} = \mathrm{Im}\,(f_1 f_2^*) q_x (K\cdot p_2 \kappa_1 - K^2 q\cdot p_2 B_y) + \mathrm{Re}\,(f_1 f_2^*)$$

$$q_x (K^2 u_x - q\cdot K r_x - m_\pi \sigma_1) + \mathrm{Im}\,(f_1 f_3^*) q_x (q^2 K\cdot p_2 B_y - q\cdot p_2 \kappa_2)$$

$$+\mathrm{Re}\,(f_1 f_3^*) q_x (\delta q_x + q\cdot K u_x - q^2 u_x - m_\pi \sigma_2) + \mathrm{Im}\,(f_1 f_4^*)$$

$$[m_\pi A_y(\Delta_1 - q_z p_{2x}) + q_x B_y K\cdot p_2] - \mathrm{Re}\,(f_1 f_4^*)\,[q_x r_x$$

$$+m_\pi(\Delta_1 A_x + \delta p_{2x})],$$

$$\frac{1}{\sqrt{2}G^2}\varrho_{+0} = \mathrm{Re}\,(f_1 f_4^*)\,(-r_z + m_\pi A\cdot p_2) q_x + \mathrm{Re}\,(f_1 f_3^*)$$

$$(\delta q_z + q\cdot K u_z - q^2 r_z) q_z + \mathrm{Re}\,(f_1 f_2^*) q_z (K^2 u_z - q\cdot K r_z)$$

$$+m_\pi A_y q_x\, \mathrm{Im}\, f_1 \cdot (f_2 K\cdot p_2 q_z + f_3 q\cdot p_2 q_z + f_4 p_{2x})^*,$$

$$\Delta_1 = q_x p_{1z}, \quad \Delta_2 = A_{1z} q_x - A_{1x} q_z,$$

$$\kappa_1 = q \cdot KB_y - m_\pi A_y q_z, \qquad \kappa_2 = q \cdot KB_y + m_\pi A_y q_z,$$

$$\sigma_1 = K \cdot A\Delta_1 + K \cdot p_2\delta, \qquad \sigma_2 = q \cdot A\Delta_1 + q \cdot p_2\delta,$$

$$\delta = q \cdot BK \cdot p_2 - q \cdot p_2 K \cdot B,$$

$$u_i = q \cdot p_2 B_i - q \cdot B p_{2i},$$

$$r_i = K \cdot p_2 B_i - K \cdot B p_{2i},$$

$$A = (1+\lambda^2)p_1 - 2\lambda m_\tau S,$$

$$B = 2\lambda p_1 - m_\tau(1+\lambda^2)S.$$

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