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THE NON-SYMMETRON

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The Generalised Field Theory GFT: (the nonsymmetric unified field theory of gravitation and electromagnetism with the metric hypothesis) is extended by subjecting it to the action of an U(1) gauge group. It is shown that it then implies a spherically symmetric, static model of an electron with a shell-like structure held by a magnetic force. It is claimed that this prediction represents a possible test of the theory which in principle could be carried out on a laboratory scale.

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1. Introduction

The nonsymmetric unified field theory of gravitation and electromagnetism first proposed by Einstein and Straus (Ref. [1]) has been developed in a series of articles (Refs. [2–4]; a complete list of relevant articles published to date is given in the review work Ref. [5]) to a point where it may be claimed that its overall structure and the more immediate physical implications are well understood. It has seemed hitherto that the possibility of an empirical verification of the theory to which we shall refer as GFT (generalised field theory, comprising the weak field equations of Einstein and Strauss together with the metric identification of Ref. [4]), lies exclusively with its cosmological implications (Ref. [6]). This was rather unfortunate since most of the data concerning distant objects necessitate interpretation in terms of an assumed theory and are therefore questionable as a means of testing its validity.

Hence it is the aim of the present article to show that GFT leads also to a classical model of a particle which we call a "nonsymmetron" relegating to the final conclusions any speculation as to its precise, physical nature. Construction of the nonsymmetron solution involves extension of GFT to include other fields than the purely macroscopic gravitation and electromagnetism. Indeed, it throws further light on the structure of the latter which is now known to be non-Maxwellian but rather of the nonlinear, Born-Infeld,

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type. The nonsymmetron will be seen to avoid some of the strange predictions of the Born-Infeld theory but because it has a well determined structure it may be susceptible to a laboratory investigation thus providing the required test of GFT.

In another article (Ref. [7]) we constructed spinor analysis of the nonsymmetric theory leading to Dirac equations. Here, the method of inclusion of new fields will be evolved in a more formal and precise way.

The formal part of the next section is described in detail, for examlpe in (Ref. [8]).

2. *Gauge fields in GFT*

Let us denote the space-time of GFT by M. The theory is fully determined through the field equations (together with the metric hypothesis or identification which is really another field law) by a sixteen component fundamental tensor g and the metric tensor a of M. Additional fields can be incorporated into its framework by erecting over M as a base manifold of a fibre structure. We avoid then in a natural way any need for an arbitrary extension of the original theory while enlarging its own content.

If G is a structural group, which can be freely chosen and P denotes the bundle manifold, a principal fibre bundle can be denoted by (P, M, G) and a projection

$$\pi: P \to M \tag{1}$$

defined if M is isomorphic to P/G. We wish to define the horizontal lift basis (Ref. [8]) in which we shall consider vector fields on P. In fact, if X is a tangent vector at a point $x \in M$, there is a unique horizontal vector $\hat{X}$ at $p \in \pi^{-1}(x)$ which satisfies

$$\pi_*(\hat{X}(p)) = X(x); \tag{2}$$

π_* denoting as usual the differential of π. Let us now denote by $\mathfrak{g}$ the Lie algebra of G, and let the linear homomorphism from $\mathfrak{g}$ to the algebra of vector fields on P be σ. Then, if $\mathfrak{a} \in \mathfrak{g}$, we have

$$\sigma: \mathfrak{a} \to \mathfrak{a}^* \tag{3}$$

say. Also, if ξ_i as a basis set of (vector fields on) $\mathfrak{g}$, ξ_μ a similar set on any open subspace of M then equations (2) and (3) define basis vector fields on P

$$(\hat{\xi}_\mu, \xi_i^*)$$

which are respectively horizontal and vertical: the horizontal lift basis on P. We shall retain Greek and Latin indices to refer to this splitting of the product space P. It can be shown easily that if f^i_{jk} are the structure constants of G

$$[\xi_i^*, \xi_j^*] = f^k_{ij}\xi_k^*, \tag{4}$$

and that there exist coefficients $F^i_{\mu\nu}$ such that (the bracket being vertical)

$$[\hat{\xi}_\mu, \hat{\xi}_\nu] = -F^r_{\mu\nu}\xi_r^*. \tag{5}$$

and that

$$[\xi_i^*, \hat{\xi}_\mu] = 0. \tag{6}$$

We can of course define in the usual way a connection and a connection one form (whose components with respect to the basis ξ_i are w^i), and cross-section Σ (over an open set $U \subseteq M$) of P as well as a cross-section dependent one form A_Σ on P.

If the components of A_Σ are $A_\mu{}^i$, $p \in P$ we define the trivial cross-section through p in terms of the isomorphism

$$\pi^{-1}(U) \sim U \times G \tag{7}$$

and the isomorphism ψ of the fibre over U to G. If

$$a = \psi(p) \tag{8}$$

and $\forall y \in U, \sigma(y)$ denotes the (unique) element of $\pi^{-1}(y)$ such that

$$\psi(\sigma(y)) = \text{a}$$

and we write

$$\xi_\mu = \sigma_* \xi_\mu, \tag{10}$$

then

$$w^i(\sigma_* \xi_\mu) = A_\mu{}^i, \tag{11}$$

as well as

$$\hat{\xi}_\mu = \xi_\mu - A_\mu{}^i \xi^*{}_i, \tag{12}$$

and

$$F^r_{\mu\nu} = \xi_\mu A_\nu{}^r - \xi_\nu A_\mu{}^r + f^r_{ij} A_\mu{}^i A_\nu{}^j, \tag{13}$$

formulae (12) and (13) being familiar from the gauge theory, with ξ_μ being just the partial derivatives (on an isomorph of at least part of M) and $\hat{\xi}_\mu$, the gauge covariant derivatives. Moreover, the coefficients $F^r_{\mu\nu}$ are now seen to be the nonzero components of (twice) the curvature two form Dw.

The above shows the usefulness of differential geometric language in formulating the field theory. We now turn to establishing the geometry of G and P remembering that on M we have standard generalised field theory (that is the weak field equations)

$$g_{\mu\nu,\lambda} - \tilde{\Gamma}^\sigma_{\mu\lambda} g_{\sigma\nu} - \tilde{\Gamma}^\sigma_{\lambda\nu} g_{\mu\sigma} = 0, \qquad \mathfrak{g}^{[\mu\nu]}, \nu = 0,$$
$$\tilde{R}_{(\mu\nu)} = 0, \qquad \tilde{R}_{[\mu\nu]} = \tfrac{2}{3}(\Gamma_{\mu,\nu} - \Gamma_{\nu,\mu}), \tag{14}$$

where the Ricci tensor $\tilde{R}_{\mu\nu}$ is constructed with the help of the affine connection $\tilde{\Gamma}^\lambda_{\mu\nu}$ whose contracted skew part $\tilde{\Gamma}_\mu = \tilde{\Gamma}^\sigma_{[\mu\sigma]}$ vanishes identically:

$$\tilde{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} + \tfrac{2}{3}\delta^\lambda{}_\mu \Gamma_\nu, \tag{15}$$

and in terms of which the symmetric metric $a_{\mu\nu}$ of M is defined by (metric hypothesis):

$$a_{\mu\nu,\lambda} - \tilde{\Gamma}^\sigma{}_{(\mu\lambda)} a_{\sigma\nu} - \tilde{\Gamma}^\sigma{}_{(\nu\lambda)} a_{\mu\sigma} = 0,$$

or

$$\begin{Bmatrix} & \lambda & \\ \mu & & \nu \end{Bmatrix}_a = \tilde{\Gamma}^{\lambda}{}_{\mu\nu}. \tag{16}$$

If ∇ os a Koszul connection on M and X, Y, Z are arbitrary vector fields there, the metric identification (16) can be written as

$$(\nabla_x a)(Y, Z)+\tfrac{1}{2}\, a(S_M(X, Y), Z)+\tfrac{1}{2}\, a(S_M(X, Z), Y) = 0, \tag{17}$$

where S_M is the torision on M, that is, a vector valued two form defined by

$$S_M(X, Y) = \nabla_X Y-\nabla_Y X-[X, Y]. \tag{18}$$

Before turning to the bundle space P we may note that on G we have a natural Riemannian geometry with the metric with components

$$g_{ij} = f^m_{in} f^n_{mj} \tag{19}$$

for which in particular $f_{ijk} = g_{im} f^m_{jk}$ is totally skew and $\frac{1}{2} f^i_{jk}$ is the affine connection while the corresponding Ricci tensor,

$$R_{ij} = R^m_{mij} = \tfrac{1}{4}\, g_{ij},$$

so that

$$R_G = \tfrac{1}{4}\,(\dim G) \text{ is a nonzero constant.} \tag{20}$$

3. The field theory on the bundle space

We can now define a metric α on P in terms of a on M and g_G on G. In fact α is fixed by the construction of P and because of the unique decomposition into horizontal and vertical parts of any vector on P

$$\alpha(X, Y) = \pi^* a(X, Y)+g_G(w(X), w(Y)). \tag{21}$$

It is less clear whether on P we should also have a tundamental field corresponding to g (or the nonsymmetric $g_{\mu\nu}$) on M. If however

$$P = M\times G$$

in a physical as well as geometrical sense, and since on G the metric and the fundamental fields coincide by our Riemannian hypothesis (in any case we want to associate with G, considered as a manifold, fields other than the GFT ones and there would be little point in including anything different), a γ field distinct from α should exist. We define then a fundamental tensor on P analogously to the definition (21) i.e.

$$\gamma(X, Y) = \pi^* g(X, Y), \qquad \text{if } X, Y \text{ are horizontal.}$$

$$\gamma(X, Y) = 0 \qquad \text{if one is vertical,}$$

$$\gamma(X, Y) = g_G(w(X), w(Y)) \qquad \text{if } X, Y \text{ are vertical.}$$

The torsion S_P on P can be defined similarly for the three cases, i.e. when both X, Y are horizontal (i), X is horizontal and Y vertical (ii), and both X, Y are vertical. We have:

(i) $\pi_* S_P(X, Y) = S_M(\pi_* X, \pi_* Y)$, but if p and q belong to the same fibre

$\pi_* S_P(P) = \pi_* S_P(q)$;

(ii) $S_P(X, Y) = 0$;

(iii) $w(S_P(X, Y)) = 0$. (22)

If for an arbitrary basis e_i on M

$$[e_i, e_j] = C^r_{ij} e_r. \tag{23}$$

then

$$S^i_{jk} = -C^i_{jk} + 2\tilde{\Gamma}_{[jk]}, \tag{24}$$

$\tilde{\Gamma}^i_{jk}$ representing the components w^i_j with respect to w^i of the one form entering the (Cartan) structural equations.

We observe also if n is the number of parameters of the gauge groups then

$$(\dim P = 4+n), \tag{25}$$

and

$$\tilde{\Gamma}^a_{bc} = \Gamma^a_{bc} - \frac{1}{3+n} S_b \delta^a_c \tag{26}$$

$\tilde{\Gamma}$, Γ and S referring to the bundle space.

From our definitions we see that

$$\alpha_{ab} = \left(\begin{array}{c|c} a_{\mu\nu} & 0 \\ \hline 0 & g_{ij} \end{array}\right) \quad \text{and} \quad \gamma_{ab} = \left(\begin{array}{c|c} g_{\mu\nu} & 0 \\ \hline 0 & g_{ij} \end{array}\right) \tag{27}$$

and that for the torsion on P

$$S_P{}^{\alpha}{}_{\mu\nu} = S_M{}^{\alpha}{}_{\mu\nu}, \quad S_P{}^{i}{}_{\mu j} = \tfrac{1}{3} S_{M\mu} \delta^i{}_j,$$

$$S_P{}^{\alpha}{}_{ij} \quad \text{and} \quad S_P{}^{i}{}_{\mu\nu} \quad \text{are indeterminate,}$$

while $S_P{}^{\alpha}{}_{\mu j}$ and $S_P{}^{i}{}_{jk}$ vanish identically. We may call these, the torsion conditions or rules.

We shall base our field theory in the bundle space on a Lagrangian

$$\sqrt{-\det \gamma}\, \gamma^{ab} \underset{P}{R}_{ab} \tag{28}$$

(principle of simplicity) and our aim is to express this quantity in terms of the GFT tensors $g_{\mu\nu}$, $a_{\mu\nu}$, the torsion and the (Yang-Mills) fields $F^r_{\mu\nu}$. In order to study the nonsymmetron of course, we shall have to specify the latter, that is the group G. In this sense the theory still remains arbitrary but we shall be guided by the interpretation of the electromagnetic field in GFT.

Meanwhile, we must settle the question of identification of the metric on the bundle space. A priori this seems to be entirely arbitrary unlike the GFT on M where the identification was forced by a consistency requirement with Einstein's principle of charge conjugation or of hermitian invariance (Ref. [9]). On the other hand, there is no reason why the same concept should not operate in P as well, and we accordingly assume that

$$\overset{P}{\nabla}_x \alpha(Y, Z) + \tfrac{1}{2}\,\alpha(S_P(X, Y), Z) + \tfrac{1}{2}\,\alpha(S_P(X, Z), Y) = 0. \tag{29}$$

Antisymmetry of f_{ijk} and repeated use of Jacobi identities gives

$$\gamma^{ab} \underset{P}{R}_{ab} = g^{\mu\nu} R_{\mu\nu} + R_G + \tfrac{1}{4}\, F^i_{\beta\alpha} F_i^{\alpha\beta} + \tfrac{1}{2}\, a^{\alpha\beta}(g^{(\mu\nu)} - a^{\mu\nu}) F_{r\mu\beta} F^r_{\alpha\nu} + \tfrac{1}{2}\, g^{[\mu\nu]}(\xi_i^* S^i_{\mu\nu} + S^i_{\alpha\mu} F_{i\nu}{}^{\alpha}). \tag{30}$$

The last term causes a difficulty in carrying out the variation because we do not know whether Gauss' theorem can be applied to the (pseudo) derivative ξ_i^*, but it can be omitted by imposing a permissible (though arbitrary) condition on P

$$\underset{P}{S}{}^i_{\mu\nu} = 0. \tag{31}$$

Instead of it we shall add to the Lagrangian a term which will take care automatically of the metric identification which hitherto has always been imposed after the field equations have been derived from a variational principle. The result will be (apart from an inclusion of the Yang-Mills fields) an apparently more comprehensive theory than GFT which nevertheless is completely equivalent to the latter.

4. U(1) Gauge

The Lagrangian is finally chosen to be

$$L = \sqrt{-g}\,[g^{\mu\nu} R_{\nu\mu} + R_G - \tfrac{1}{4}\, F^r_{\mu\nu} F_r^{\mu\nu} + \tfrac{1}{2}\, a^{\alpha\beta}(g^{(\mu\nu)} - a^{\mu\nu}) F_{r\mu\beta} F^r_{\alpha\nu}] + \lambda^{\alpha}_{\mu\nu} \tilde{\nabla}_{\alpha} a^{\mu\nu} \tag{32}$$

$\lambda^{\alpha}_{\mu\nu}$ being Lagrange multipliers, and $\tilde{\nabla}_{\alpha}$, the covariant derivative with respect to $\tilde{\Gamma}^{\lambda}_{(\mu\nu)}$. Since L can only be derived under the explicit assumption of the standard metric identification, inclusion of the last term appears necessary. Greek indices are raised and lowered with the M-metric $a_{\mu\nu}$ which, together with $g_{\mu\nu}$, $\Gamma^{\lambda}_{\mu\nu}$ and the gauge potentials $A_{\mu}{}^{i}$ (equation (13)) are the natural variational parameters. Defining

$$G_{\alpha}{}^{\mu\nu} = \partial_{\alpha} \mathfrak{g}^{\mu\nu} + \tilde{\Gamma}^{\mu}_{\sigma\alpha} \mathfrak{g}^{\alpha\nu} + \tilde{\Gamma}^{\nu}_{\alpha\sigma} \mathfrak{g}^{\mu\sigma} - \tilde{\Gamma}^{\sigma}_{\alpha\sigma} \mathfrak{g}^{\mu\nu},$$

$$W_r{}^{\mu\beta} = \sqrt{-g}\,(a^{\mu\nu} g^{(\alpha\beta)} + a^{\alpha\beta} g^{(\mu\nu)} - a^{\mu\nu} a^{\alpha\beta}) F_{r\alpha\nu}, \tag{33}$$

and

$$\Gamma_{\mu\nu} = \tilde{R}_{[\mu\nu]} + \tfrac{1}{2}\, R_G g_{[\mu\nu]} + \tfrac{1}{8}\, F^r_{\varrho\sigma} F_r^{\varrho\sigma} g_{[\mu\nu]},$$

the field equations are (on elimination of the multipliers $\lambda^{\alpha}{}_{\mu\nu}$).

$$\tilde{\nabla}_{\alpha}(G^{\mu(\nu\alpha)} + G^{\nu(\mu\alpha)} - G^{\alpha(\mu\nu)} - a^{\mu\nu} G_{\sigma}{}^{\sigma\alpha}) = (a^{\varrho\sigma} - g^{(\varrho\sigma)}) F_{r\varrho}{}^{\nu} F^{r\mu}{}_{\sigma} \sqrt{-g}\,, \tag{34}$$

$$G_{\alpha}{}^{[\mu\nu]} = 0, \quad (\tilde{\Gamma}_{\mu} = 0), \tag{35}$$

$$\tilde{R}_{(\mu\nu)} + \tfrac{1}{2}\, R_G g_{(\mu\nu)} = -\tfrac{1}{2}\, a^{\alpha\beta} F_{r\mu\beta} F^r{}_{\alpha\nu} - \tfrac{1}{8}\, g_{(\mu\nu)} F_r{}^{\alpha\beta} F^r{}_{\alpha\beta}, \tag{36}$$

$$\partial_{[\mu} \Gamma_{\nu\sigma]} = 0 \tag{37}$$

and

$$\partial_\mu W_r^{\mu\beta} + f^i{}_{rj} A_\mu{}^j W_i^{\mu\beta} = 0 \tag{38}$$

and describe the mutual interaction of the gauge and the GFT fields. Ignoring (dim P-4) and in effect the Yang-Mills potentials, there are now 54 equations for 54 unknowns ($a_{\mu\nu}$, $g_{\mu\nu}$, $\tilde{\Gamma}^\lambda_{[\mu\nu]}$, Γ_μ) so that the system is completely determined (as it must be in view of its variational derivation), the "unknows" having been chosen as variational parameters (and the multipliers $\lambda^\alpha{}_{\mu\nu}$ having been eliminated. In GFT, the metric hypothesis (16) was used to select from among the solutions of the field equations (14) those which happen to be compatible with it. Clearly, the differential equations (16) for the ten components $a_{\mu\nu}$ of the metric tensor cannot be integrable for an arbitrary set of functions $\tilde{\Gamma}^\lambda_{(\mu\nu)}$. This difficulty is now overcome by including the metric hypothesis into the variational principle from which the theory is derived. Let us now assume that the gauge group G is the abelian U(1). In the original formulation of GFT it has been assumed that $\tilde{R}_{[\mu\nu]}$ represented the electromagnetic intensity field tensor. There were two reasons for this. The first was that $\tilde{R}_{[\mu\nu]}$ necessarily appeared as a Maxwell tensor (the curl of a potential) and this was convenient for a preliminary and tentative interpretation of the theory. Secondly, and this was more important, the choice of $\tilde{R}_{[\mu\nu]}$ allowed the equations of motion of a charged test particle a quasi-Lorentz force to be derived from the field equations. On the other hand, it was already clear that the electromagnetism of GFT was most likely not to be Maxwellian but rather of the nonlinear ("Born-Infeld") type. The present, more elaborate structure, enables us to write down explicitely the proposed field equations of the electromagnetic theory. The structure constants $f^i{}_{jk}$ vanish for an abelian gauge group and we obtain

$$\partial_{[\lambda}\Gamma_{\mu\nu]} = 0, \tag{39}$$

$$\partial_\lambda W^{\lambda\mu} = 0 \tag{40}$$

with $W^{\lambda\mu} = -W^{\mu\lambda}$ being given in terms of the "second" electromagnetic field tensor $F_{\lambda\mu}$ by the second of the equations (33). Equation (39), of course, ensures that $\Gamma_{\mu\nu}$ is the curl of a "potential" just as in Maxwell's or the original GFT theory. Also, because equation (39) reduces to

$$\partial_{[\lambda}\tilde{R}_{[\mu\nu]]} = 0$$

in the limiting case

$$R_G \to 0, \quad F_{\lambda\mu} \to 0,$$

the basic results about the equations of motion are not disturbed. Just as in the nonlinear, Born-Infeld electrodynamics (eg. Ref. [10]) we can also define the electromagnetic charge--current vector J^μ by

$$J^\mu = \frac{1}{\sqrt{-a}}\,\partial_\lambda(\sqrt{-a}\,F^{\lambda\mu}) \tag{41}$$

the corresponding vector density

$$g^\mu = \sqrt{-a}\,J^\mu, \tag{42}$$

being conserved in the usual sense

$$\partial_\mu g^\mu = 0, \tag{43}$$

because of the skew symmetry of $F^{\lambda\mu}$. This definition appears to be much more natural then eiter the corresponding Born-Infeld one, or the definition originally employed by Einstein (eg. Ref. [10]).

Having now derived a complete description of the "nonsymmetric" or GFT electrodynamics we can proceed to the main objective of this article which is the construction of the model of a charged particle.

5. *Structure of the GFT charge*

We encounter a difficulty in attempting to obtain even a primitive, spherically symmetric static solution of the field equations (34)–(38). Indeed, the corresponding solution of GFT was only found by assuming the relevant symmetry of the field $g_{\mu\nu}$ which then forced the symmetry of space-time. In the current version of the theory a similar assumption is insufficient to reduce the system of the field equations to a manageable form. We know however that the GFT solutions (without gauge fields) remain as particular solutions of the new set of the (P, or bundle space) field equations.

Hence, we can take the exact GFT solutions as a fixed background and consider an exterior (that is gauge) field approximation, the result being a solution valid when the gauge fields are weak.

Let us consider first the electric GFT solution found by one of us (Ref. [4]) as the "background" field:

$$g_{00} = \sigma,\ g_{11} = -\alpha,\ g_{22} = g_{33}\operatorname{cosec}^2\theta = -\beta,\ g_{23} = -g_{32} = f\sin\theta, \tag{44}$$

with

$$\alpha = \left(1-\frac{r^2}{r_0^2}\right)^{-1}\left(1+c\sqrt{\frac{r_0^2}{r^2}-1}\right)^{-1}, \quad \sigma = w\left(1-\frac{r^2}{r_0^2}\right)^{-1}\alpha^{-1}, \quad r^2 = \sqrt{\beta^2+f^2} \tag{45}$$

c, w, r_0 constant (and where we can always put $w = 1$). Also, the diagonal metric tensor components are

$$d_{00} = \sigma, \quad d_{11} = -\alpha, \quad d_{22} = d_{33}\operatorname{cosec}^2\theta = -r^2.$$

In this spherically symmetric case, the only nonzero component of $F_{\lambda\mu}$ is F_{01} and from the (second of) equations (33)

$$W^{10} = -\sqrt{\frac{\beta^2+f^2}{\alpha\sigma}}\,F_{01}\sin\theta,$$

so that, from equation (40)

$$E_r = \frac{E_0/r^2}{\sqrt{1-r^2/r_0^2}}, \tag{46}$$

where E_0 is a constant and we have put $E_r = F_{01}$.

For a real field we must have

$$r < r_0 \tag{47}$$

which reinforces the cosmological (Ref. [6]) interpretation of r_0, and for

$$r \ll r_0$$

E_r behaves like an inverse square Coulomb field which is singular at the origin. However, that is precisely where the weak-field approximation breaks down. On the other hand, from the definition (41) the charge density

$$\sqrt{-a}\, J^0 = \frac{d}{dr}(\sqrt{-a}\, F^{10}) = 0 \tag{48}$$

in the present case which is what we would expect from a point particle solution. On the cosmological scale, the fall off of E_r is slower than for a Coulomb charge and in fact (for $E_0 > 0$), the field begins to increase beyond

$$r = \sqrt{\tfrac{2}{3}}\, r_0. \tag{49}$$

It may be therefore that this value should be interpreted as the radius of the universe. Alternatively, we could say that the weak field approximation fails also for

$$r \sim r_0$$

on the grounds of a Mach-like hypothesis (Ref. [4]) that globally charge manifests itself as mass and that "distant" masses are locally significant. The electric solution, however, does not tell us anything about the internal structure of the charge. Nevertheless, the non-linear electrodynamics we now have enables us to obtain information about the latter if the background field is assumed to correspond to the alternative magnetic solution $g_{23} = 0$, $g_{01} = -g_{10} = w$).

In GFT this solution is identical with Papapetrou's (Ref. [12]) solution of the strong field equations ($\tilde{R}_{[\mu\nu]} = 0$) and is rejected because it implies absence of the electromagnetic field (and of a Lorentz force on a charged test particle). It is nevertheless, a solution of the GFT field equations (with the metric hypothesis) and therefore also a solution of the modified field equations (34)–(38). Since there is now no reason for excluding any of the solutions and we are dealing in any case with a perturbation of a background field, we can assert that an electric charge still arises even if it is apparently excluded from the background. This indeed proves to be the case.

With $F_{01} = E_r$, we now have (in the notation of Ref. [4])

$$W^{10} = -\frac{K}{2} r^2 \left(y_0 \left(1 + \frac{k^2}{r^4} \right) + 1 + y_0 \right) E_r \sin\theta, \tag{50}$$

where K, k and Y_0 are constants, and therefore

$$E_r = \frac{E_0/r^2}{1 + q/r^4}, \tag{51}$$

where $q = y_0 k^2/1+2y_0 > 0$. Unlike the Born-Infeld field which is non-zero at the origin, E_r vanishes as $r \to 0$ and in fact increases to a maximum $(E_0 > 0)$ $E_0/2\sqrt{q}$ when $r = q^{1/4}$ and for $r > q^{1/4}$ behaves with increasing r more and more like a Coulomb field.

The charge density can be defined from equation (41) to be

$$e = J^0 = \frac{1}{\sqrt{-a}} \frac{d}{dr}(\sqrt{-a}\, a^{00} a^{11} E_r)$$

$$= -\frac{y_0 q K^2 E_0 r}{(r^4+q)^2}. \tag{52}$$

We may observe also that if we require that

$$a_{\mu\nu} = \text{symmetric limit } (q = 0) \text{ of } g_{(\mu\nu)} \tag{53}$$

then necessarily

$$y_0 = 1, \quad k^2 = 3q. \tag{54}$$

If we also assume that to the order to which the approximate solution of the field equations (34)–(38) is valid, the 3-space (r, θ, ψ) is Euclidean, we can define the charge of our particle to be

$$e = \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} e r^2 \sin\theta dr d\theta d\psi = -\pi K^2 E_0 \tag{55}$$

so that we can eliminate E_0 from equation (51):

$$E_r = \frac{-er^2}{\pi K^2 (r^4+q)}. \tag{56}$$

If $q = 0$ (that is, if the "magnetic" charge vanishes) the particle collapses to a Coulomb charge but otherwise the field is nonsingular. We may observe also, that K^{-2} appears to be proportional to the standard, Coulomb constant.

6. *Mass and spin of the GFT particle*

Let us next calculate the contribution to the total energy of the GFT particle due to the U(1) field. When

$$\tilde{\Gamma}^{\lambda}_{\mu\nu} = \left\{ {\lambda \atop \mu\ \ \nu} \right\}_a + \tfrac{1}{2} S^{\lambda}_{\mu\nu}, \quad S^{\lambda}_{\mu\nu} = -S^{\lambda}_{\nu\mu}, \quad S_\mu = 0,$$

we easily find that

$$\tilde{R}_{(\mu\nu)} = K_{\mu\nu} - \tfrac{1}{4} S^{\varrho}_{\mu\sigma} S^{\sigma}_{\nu\varrho},$$

where $K_{\mu\nu}$ is the Ricci tensor constructed from the Christoffel brackets. Hence equation (36) may be written as

$$K_{\mu\nu} = \tfrac{1}{4} S^{\varrho}_{\mu\sigma} S^{\sigma}_{\nu\varrho} - \tfrac{1}{2}(F_{\mu}{}^{\alpha} F_{\alpha\nu} - \tfrac{1}{4} F_{\varrho\sigma} F^{\sigma\varrho} g_{(\mu\nu)}) - \tfrac{1}{2} R_G g_{(\mu\nu)}, \tag{56a}$$

and therefore

$$K_\nu{}^\lambda - \tfrac{1}{2} S_\nu{}^\lambda K = \tfrac{1}{4} S^\varrho_{\mu\sigma} S^\sigma_{\nu\varrho} a^{\lambda\mu} - \tfrac{1}{8} S_\nu{}^\lambda a^{\alpha\beta} S^\varrho_{\alpha\sigma} S^\sigma_{\beta\varrho} - \tfrac{1}{2}(F^{\lambda\alpha} F_{\alpha\nu} - \tfrac{1}{4} F_{\varrho\sigma} F^{\sigma\varrho} a^{\lambda\mu} g_{(\mu\nu)})$$
$$+ \tfrac{1}{4} \delta_\nu{}^\lambda (1 - \tfrac{1}{4} a^{\alpha\beta} g_{\alpha\beta}) F_{\varrho\sigma} F^{\sigma\varrho} - \tfrac{1}{2} R_G (a^{\lambda\mu} g_{(\mu\nu)} - \tfrac{1}{2} \delta^\lambda_\nu a^{\alpha\beta} g_{(\alpha\beta)}), \tag{57}$$

where

$$K = a^{\alpha\beta} K_{\alpha\beta}.$$

In GFT, the tensor

$$K_\nu{}^\lambda - \tfrac{1}{2} \delta^\lambda_\nu K.$$

defines energy-momentum and satisfies identically the conservation law

$$\tilde{\nabla}_\lambda (K_\nu{}^\lambda - \tfrac{1}{2} \delta_\nu{}^\lambda K) = 0,$$

and, a fortiori, so does the tensor on the right-hand side of equation (57). Its first two terms evidently refer to the GFT fields, but as we know, the constant R_G cannot vanish in general. Let us consider the tensor

$$\tau_\nu{}^\lambda = a^{\lambda\mu} g_{(\mu\nu)} - \tfrac{1}{2} \delta_\nu{}^\lambda a^{\alpha\beta} g_{(\alpha\beta)}.$$

If $\tilde{\Gamma}^\lambda_{\mu\nu}$ is given in terms of $g_{\mu\nu}$ and its first derivatives (GFT) by

$$g_{\mu\nu,\lambda} - \tilde{\Gamma}^\sigma_{\mu\lambda} g_{\sigma\nu} - \tilde{\Gamma}^\sigma_{\lambda\nu} g_{\mu\sigma} = 0,$$

we can show easily that

$$\tilde{\nabla}_\lambda \tau^\lambda_\nu = \tfrac{1}{2} a^{\alpha\beta} [S^\sigma_{\nu\alpha} g_{[\sigma\beta]} + S^\sigma_{\beta\nu} g_{[\alpha\sigma]}]. \tag{58}$$

Then, for the magnetic solution (Ref. [3]), we find that

$$\tilde{\nabla}_\lambda \tau^\lambda_0 \equiv \tilde{\nabla}_\lambda \tau^\lambda_2 \equiv \tilde{\nabla}_\lambda \tau^\lambda_3 \equiv 0, \quad \tilde{\nabla}_\lambda \tau^\lambda_1 = a^{00} S^1_{01} g_{[01]}. \tag{58a}$$

However, we are interested only in the first of these equations, that is in the energy density. It now follows that for the energy component, the R_G term in (57) can be absorbed in the left-hand side so that we finally define the energy density of the U(1) field to be (with $F_{01} = E_r$ as the only component of the field)

$$\begin{aligned} W &= \tfrac{1}{2}(F^{0\alpha} F_{\alpha 0} - \tfrac{1}{4} F_{\varrho\sigma} F^{\sigma\varrho} a^{00} g_{00}) - \tfrac{1}{4}(1 - \tfrac{1}{4} a^{\alpha\beta} g_{\alpha\beta}) F_{\varrho\sigma} F^{\sigma\varrho} \\ &= \tfrac{1}{4} a^{00} a^{11} (a^{00} g_{00} - \tfrac{1}{2} a^{\alpha\beta} g_{\alpha\beta}) E_r^2 \\ &= \frac{K^2}{16}\left(1 - \frac{3q}{2r^4}\right) E_r^2. \end{aligned} \tag{59}$$

$\left(\text{for the magnetic solution } a^{00} = \frac{K^2}{4}\alpha,\ a^{ii} = g^{ii},\ g_{00} = \sigma \text{ and } \frac{K^2}{4}\alpha\sigma = 1 + \frac{3q}{r^4}\right).$

We notice that unless $q = 0$ (Coulomb case, giving here $g_{[01]} = 0$ so that there is no nonsymmetric field; this indicentally reconfirms the result discussed in (Ref. [11]), although from a different point of view, that we get "remnants" of electromagnetism when partial symmetry is imposed on the theory), energy density becomes negative when

$$r \leqslant \sqrt{\tfrac{3}{2}}\sqrt{q} = \sqrt{\tfrac{3}{2}}\, r_1 \tag{60}$$

say, then increases to a maximum at

$$r = 2r_1$$

and then decreases approaching rapidly the classical, r^{-4}, rate as r increases. For example, at $r = 4r_1$ (twice the radius of the particle) the energy is already some 98.64% of its classical value and at $r = 10r_1$, 99.97%.

The total energy of our particle is (again assuming a flat 3-space)

$$\frac{e^2}{4\pi K^2 r_1}\int_0^\infty \frac{x^2(x^4-\frac{3}{2})dx}{(1+x^4)^2} = \frac{9\sqrt{2}\, e^2}{128K^2 r_1}\,. \tag{61}$$

Thus it is finite and positive as required. Of course, it is very questionable whether this can be regarded as the mass of the particle since we have omitted all gravitational and electromagnetic (in the sense of GFT) contributions to the energy.

Finally, let us inquire whether the present theory can throw any light on the spin of the "non-symmetron". Since spin is a strictly quantum mechanical concept it can only be introduced into an essentially macroscopic theory by hypothesis and our considerations must be, at best, very tentative. The difficulty is compounded by us not having an exact solution of the generalised field equations (34)–(38) but only one obtained by assuming a GFT background.

We shall work by analogy with the Einstein-Cartan theory whose field equations are

$$R_{\mu\nu}-\tfrac{1}{2}\, a_{\mu\nu}a^{\alpha\beta}R_{\alpha\beta} = h\Sigma_{\mu\nu},$$

$$S^{\alpha}_{\mu\nu}+S^{\alpha}{}_{\mu}S_{\nu}-\delta^{\alpha}{}_{\nu}S_{\mu} = 2h\tau^{\alpha}_{\mu\nu}, \tag{62}$$

with $\Sigma_{\mu\nu}$ representing a generalised (nonsymmetric) energy momentum tensor and $\tau^{\alpha}_{\mu\nu}$, the spin density tensor. Conservation of angular momentum is expressed by

$$\nabla_{\alpha}\tau^{\alpha}{}_{\mu\nu} = \Sigma_{[\mu\nu]}. \tag{63}$$

The spin angular momentum tensor is then defined as

$$J_{\mu\nu} = \int \tau^{\alpha}_{\mu\nu}dS_{\alpha} = \int \tau^{0}_{\mu\nu}dS_0 \tag{64}$$

by the use of space-like hypersurfaces.

Since in GFT the tensor

$$\tilde{R}_{[\mu\nu]} = -\tfrac{1}{2}\,\tilde{\nabla}_{\alpha}\tilde{S}^{\alpha}_{\mu\nu}, \tag{65}$$

is identified with electromagnetic field tensor, we cannot use the "twiddled" connection to generate both electromagnetism and spin (if we do, the formula (64) gives zero $J_{\mu\nu}$ in any case) but we can use in this way the connection

$$\Gamma^{\lambda}_{\mu\nu} = \tilde{\Gamma}^{\lambda}_{\mu\nu}-\tfrac{2}{3}\,\delta^{\lambda}{}_{\mu}\Gamma_{\nu}$$

albeit that it is not well determined.

In the spherically symmetric, static case of GFT, the solution for Γ_μ is

$$(0, f(r), g(\theta), 3c\cos\theta)$$

where f and g are in general arbitrary functions of θ. It seems reasonable however to take the solution

$$(0, 0, 3c\sin\theta, 3c\cos\theta) \tag{66}$$

in our case. Then

$$J_{02} = \frac{1}{6h}\int_0^{2\pi}\int_0^{\pi} 3c\sin^2\theta d\theta d\psi = \tfrac{1}{2}\frac{\pi^2 c}{h} \tag{67}$$

all other components of $J_{\mu\nu}$ vanish. If the constant $\frac{\pi^2 c}{h}$ is disregarded, the above result at least does not exclude the possibility of interpreting the non-symmetron as an electron.

The GFT field equations also imply that

$$\tilde{\nabla}_\alpha \tau^\alpha_{\mu\nu} = -\tfrac{5}{4}\Sigma_{[\mu\nu]}\left(= -\frac{5}{4h}\tilde{R}_{[\mu\nu]}\right) \tag{68}$$

(the sign being due purely to the definition of the Ricci tensor employed here:

$$R_{\mu\nu} = -\Gamma^\sigma_{\mu\nu,\sigma} + \Gamma^\sigma_{\mu\sigma,\nu} + \Gamma^\sigma_{\mu\varrho}\Gamma^\varrho_{\sigma\nu} - \Gamma^\sigma_{\mu\nu}\Gamma^\varrho_{\sigma\varrho}).$$

7. *Discussion*

We have obtained above the macrophysical model of a relativistic particle tentatively identified as an electron. It consists essentially of a spherical shell with part of the interior being a region of negative energy. It is held together by a magnetic field, its own field being almost exactly Coulomb already at reasonably short (radial) distances from its surface, or rather from what we have interpreted as its surface. The field itself vanishes at the centre of the particle.

The model was derived from an ammended form of the field equations in which the electromagnetic induction is obtained from superimposing onto GFT of a U(1) gauge field, as well as in which the metric hypothesis of GFT is included in the variational principle. The resulting field equations (fewer in number than in GFT) are almost impossible to solve directly. The reason is that we do not know a priori on what to impose symmetry restrictions. It is clearly insufficient for mathematical tractability to impose these restriction only on the metric of space-time (as one presumably should). In GFT also, the static, spherically symmetric solution is obtained by assuming symmetry of the field and then deriving that of space-time (rather than the other way round). In the present case, the solution is found by an approximate procedure in which the GFT solution is taken as a fixed background.

It may be argued that because of this, the model proposed represents only a tentative prediction of the theory (and therefore also a test of GFT on a laboratory scale). This, however, is not the case. We have already pointed out that the present extension of GFT is a more comprehensive theory although we may have to rely on the former ror concrete solutions. The reason why our electron model is a definite prediction is that all solutions of GFT (unified field theory with the metric hypothesis) are also solutions of its generalised version and we have inquired here only as to what may be the possible consequences of adjoining to it a gauge group.

Perhaps the main theoretical result of this was resolution of the problem of electrodynamics whose structure will be explored further in a subsequent article.

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