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STUDY OF ELASTIC $\pi^-\pi^-$ SCATTERING IN $\pi^-n \to p\pi^-\pi^-$ REACTION AT 21 GeV/c

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Data for reaction $\pi^-n \to p\pi^-\pi^-$ at 21 GeV/c are analysed in order to extract information on elastic $\pi^-\pi^-$ scattering. $\sigma_{el}^{\pi^-\pi^-}$ is found to decrease with energy from 5 mb at $s_{\pi\pi} \simeq 2$ GeV^2 to value below 1 mb for $s_{\pi\pi} > 8$ GeV^2. The data are also compared with predictions of the modified Drell-Hiida-Deck models and reasonable agreement is found.

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1. *Introduction*

We present an analysis of the exclusive reaction

$$\pi^-n \to p\pi^-\pi^- \tag{1}$$

at the incident momentum of 21 GeV/c. Diagram representing reaction (1) is shown in Fig. 1a.

Reactions of type $\pi N \to N\pi\pi$, involving isospin one exchange at the nucleon vertex are studied either to extract information on elastic $\pi\pi$ scattering or to analyze nucleon dissociation into $N\pi$ system. In this paper we shall address both problems. However the emphasis in the analysis will be put on the elastic $\pi^-\pi^-$ scattering.

The one-pion-exchange amplitude (OPE) is believed to be a dominant process in the kinematic region of small momentum transfer between nucleons, t_{np}, and low $\pi\pi$ c.m. energy squared, $s_{\pi\pi}$. Reaction (1) and its isospin conjugate partner $\pi^+p \to n\pi^+\pi^+$ have been used as a main source of information on isospin $I = 2$ elastic $\pi\pi$ scattering. The $s_{\pi\pi}$ range explored for such an analysis has been usually limited to $s_{\pi\pi}/s < 0.2$, where s is the c.m. energy squared of the initial πN system. Due to the above cut and low beam momenta of previous experiments the $I = 2$ elastic $\pi\pi$ scattering has been studied only for $s_{\pi\pi}$ less than 4 GeV^2. It was found that $\sigma_{el}^{\pi^-\pi^-}$ decreases from the value of 5 mb at $s_{\pi\pi} \approx 1.5$ GeV^2 to 2 mb at $s_{\pi\pi} \approx 4$ GeV^2 and that $\pi^-\pi^-$ elastic amplitude is predominantly real at these energies [1–3]. Above $s_{\pi\pi} = 4$ GeV^2 the entire experimental information on elastic $I = 2\pi\pi$ cross sections comes from a single analysis of the $\pi p \to \pi\pi\Delta$ reaction [3].

The production of large $\pi\pi$ masses ($s_{\pi\pi}/s > 0.2$) corresponds roughly to the production of low invariant masses of the Nπ system, thus reflecting nucleon diffractive dissociation (DD) into Nπ. The gross features of DD are reasonably reproduced by OPE amplitude with purely imaginary $\pi\pi$ elastic off-shell scattering at one of the vertices (Fig. 1b).

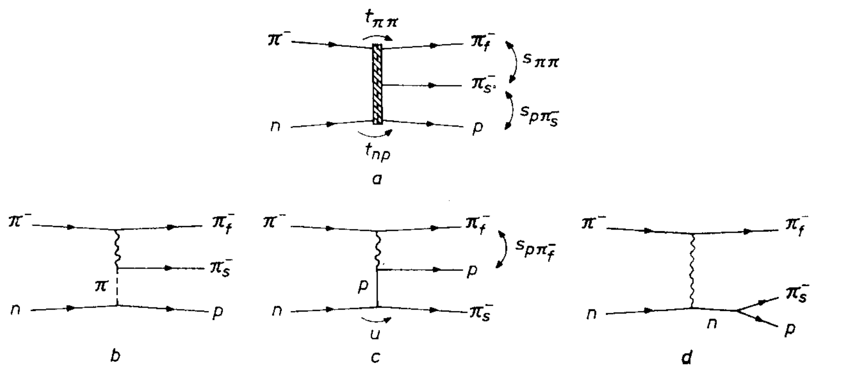

Fig. 1. a. Diagram representing reaction $\pi^-n \to p\pi^-\pi^-$; b-d. diagrams of the exchange processes expected to contribute to cross section for reaction $\pi^-n \to p\pi^-\pi^-$: pion-exchange (b), baryon exchange (c), baryon exchange-direct nucleon pole. Kinematic variables are indicated in (a) and (c)

This fact is known as a Drell–Hiida–Deck (DHD) effect [4]. However any theoretically correct and phenomenologically adequate description of diffractive dissociation in terms of exchange amplitudes must include in addition to DHD diagrams representing baryon exchange (Fig. 1c), baryon exchange direct nucleon pole (Fig. 1d) and absorptive corrections [5]. The baryon exchange graphs tend to cancel each other in the relevant region of phase space, but nevertheless effects of their presence are clearly visible in the data and obscure analysis of $\pi\pi$ scattering at high $\pi\pi$ energies.

The data used in this analysis cover the whole region of phase space available for reaction (1). However we shall concentrate on interactions with four momentum transfer between nucleons smaller than 1 GeV^2 and $s_{\pi\pi} < 20$ GeV^2 ($s_{\pi\pi}/s < 0.5$). The same kinematic cuts were applied in our recent analyses of the inclusive production of protons off neutrons: $\pi^-n \to p+X$ [6, 7], where reggeised pion exchange amplitude was found to be dominant even in such extended region of phase space. Consequently most of the experimental distributions for reaction (1) are presented in terms of variables natural for the OPE diagram (Fig. 1b). We find that this way of displaying data has also some advantages in studying DD phenomena. Namely, it demonstrates the effects of various DD diagrams on the results of standard Chew–Low extrapolation techniques and provides new sensitive tests for DD models.

The paper is organized as follows. In Section 2 we discuss the event selection and describe kinematic variables used in the analysis. The values of elastic $\pi^-\pi^-$ cross sections obtained by standard Chew–Low extrapolation techniques are presented in Section 3. We also show that production of low mass $\pi^-\pi^-$ systems is correctly described by a modified poor man's absorption model and use a procedure proposed by Ochs and Wagner to determine independently $\pi\pi$ elastic cross sections. The experimental effects of nucleon diffractive dissociation are demonstrated in Section 4. In Section 5 we compare predictions of various models for diffractive dissociation with the data. Our conclusions are given in Section 6.

2. *Event selection. Kinematic variables*

The data for reaction (1) come from a 458000 picture exposure of π^-d interactions at 21 GeV/*c* in the CERN 2m bubble chamber filled with deuterium. The experiment was performed by the Cambridge-Cracow-Warsaw Collaboration and many results have been already published [6–11].

The reaction actually studied experimentally is

$$\pi^- d \rightarrow p_s p \pi^- \pi^- , \tag{2}$$

where p_s stands for proton spectator. Events of reaction (2) have been obtained simultaneously with events of the coherent reaction $\pi d \rightarrow \pi^+\pi^-\pi^- d$ [11]. The candidates for both reactions appear in the deuterium bubble chamber as 4 prong interactions with at least one heavily ionizing track or as 3 prong events with the proton spectator (or deuteron) so slow as to be unseen. A total sample of 87000 such candidates was measured on SWEEPNIK automatic machines and processed through the standard geometrical reconstruction and kinematic fit programs. Full details of the experimental procedure and cuts applied to the data may be found in Refs. [12, 13].

The spectator proton in reaction (2) was chosen to be the slower of the two protons. Events with spectator momenta greater than 250 MeV/*c* were eliminated from the final data sample. The ionization of the fast positive track with momentum less than 1.2 GeV/*c* was checked to agree with the proton hypothesis. We were left with a total of 1735 events for physical analysis.

The measured cross section σ for reaction (2) at 21 GeV/*c* is equal to (198 ± 11) μb [12, 13]. To deduce corresponding cross section for reaction (1) on unbound neutron targets model dependent corrections for Glauber shielding and rescattering must be made [8–10]. In order to avoid these corrections we have taken as $\sigma(\pi^- n \rightarrow p\pi^-\pi^-)$ the extrapolated value of the cross section for the isospin conjugated reaction $\pi^+ p \rightarrow n\pi^+\pi^+$ [14]. By this method one obtains $\sigma(\pi^- n \rightarrow p\pi^-\pi^-)$ at 21 GeV/*c* equal to (231 ± 20) μb [12], roughly 15% larger than the value of σ for reaction (2) found in this experiment.

The diagram representing reaction (1) is shown in Fig. 1a. There is an ambiguity in deciding which of the two negative pions is the fast one (π_f^-) and which the slow one (π_s^-). The pion with the higher laboratory momentum was taken as π_f^-. Let us note that the number of possible ambiguous events is very small. There are only 4% of events with both π^-'s having momenta in a wide range of (5–15) GeV/c.

Events of reaction (1) are described by five independent kinematic variables. They are (p_i — four momenta of particles involved):

$$s = (p_{\pi^-} + p_n)^2 \text{ — c.m. energy squared of the initial } \pi n \text{ system}^1$$

$$s_{\pi\pi} = M_{\pi\pi}^2 = (p_{\pi_s^-} + p_{\pi_f^-})^2 \text{ — } \pi_s^- \pi_f^- \text{ c.m. energy squared}$$

$$t_{np} = (p_p - p_n) \text{ — four momentum transfer to the nucleon}$$

$\cos\theta, \phi$ — decay angles of the π_f^- in the $\pi^-\pi^-$ rest system in the t-channel helicity frame.

In addition we shall use Lorentz invariant variables:

$$s_{p\pi_s^-} = (p_p + p_{\pi_s^-})^2;$$

$$s_{p\pi_f^-} = (p_p + p_{\pi_f^-}),$$

$$u = (p_n - p_{\pi_s^-}),$$

$$t_{\pi\pi} = (p_{\pi^-} - p_{\pi_f^-})^2$$

and $t'_{\pi\pi} = t_{\pi\pi} - t_{min}$, where t_{min} stands for the minimum absolute value of $t_{\pi\pi}$ at a given value of $s_{p\pi_s^-}$.

3. *Elastic $\pi^-\pi^-$ cross sections*

In order to extract information on elastic $\pi^-\pi^-$ scattering from the measured differential cross section for reaction (1) we have used both standard Chew-Low extrapolation procedure [15] as well as a method proposed by Ochs and Wagner [16] and derived from a generalization of the poor man's absorption model (PMA) [17].

The one pion exchange formula we use is given by

$$s\frac{d^4\sigma}{ds_{\pi\pi}dt_{np}d\Omega} = \frac{g_{\pi np}^2}{16\pi^2}\frac{|t_{np}|}{(t_{np}-\mu^2)^2}\cdot\frac{s_{\pi\pi}}{s}\cdot F(t_{np})\,(1-1/2\cdot H(t_{np}))\cdot\left(\frac{d\sigma_{el}^{\pi\pi}}{d\Omega}\right)_{on}, \tag{3}$$

where $g_{\pi np}^2/4\pi = 29.2$ is the on-mass-shell coupling, $F(t_{np})$ is a formfactor to account for off mass shell corrections and the term $1-1/2\ H(t_{np})$ is a correction for losses due to the Pauli exclusion principle [2, 18]. To extract $\pi^-\pi^-$ elastic cross section, we extrapolated Eq. (3) to the pion pole:

$$\sigma_{el}^{\pi^-\pi^-} = \lim_{t_{np}\to\mu^2} s\frac{d^2\sigma}{ds_{\pi\pi}dt_{np}}\frac{g_{\pi np}^2}{16\pi^2}\frac{|t_{np}|}{(t_{np}-u^2)^2}\cdot\frac{s_{\pi\pi}}{s}F(t_{np})\,(1-\tfrac{1}{2}H(t_{np})). \tag{4}$$

The extrapolation was performed for two types of form factors $F(t_{np})$: (a) the Dürr–Pilkuhn factor [19] with the values of parameters taken from Ref. [20] and (b) Regge type form

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[1] $p_n = p_d - p_s$, where $p_d = (m_d, 0, 0, 0)$ and p_s are 4-momenta of deuteron and proton spectator respectively.

factors $(s_{\pi\pi}/s)^{-2\alpha_\pi(t_{np})}$, where $\alpha_\pi(t_{np}) = t_{np} - \mu^2$ is the pion Regge trajectory. The t_{np} region from which extrapolation was done depended on the $s_{\pi\pi}$ bin and varied from $0.025 < |t_{np}| < 0.35$ GeV^2 for $1 < s_{\pi\pi} < 2$ GeV^2 to $0.15 < |t_{np}| < 1.0$ GeV^2 for $12 < s_{\pi\pi} < 16$ GeV^2. The extrapolated function in Eq. (4) exhibits only weak dependence on t_{np} and linear fits were sufficient to follow the data. The extracted values of $\sigma_{el}^{\pi\pi}$ are given in Table I.

TABLE I

Estimates for elastic $\pi^-\pi^-$ cross sections

$s_{\pi\pi}$, GeV^2	$-t_{np}$, GeV^2	Events	Dürr–Pilkuhn	$\sigma_{el}^{\pi^-\pi^-}$, mb [a] OPER	MLHF	Optical theorem[b]
1-2	0.025–0.45	87	5.2±1.0	4.2±1.1	5.0±1.1	—
2–4	0.025-0.55	198	3.7±0.4	3.1±0.4	2.8±0.4	—
4–6	0.025–0.55	182	1.7±0.2	1.35±0.15	1.2±0.2	0.60–0.80
6–8	0.025-0.55	198	1.4±0.4	1.25±0.15	1.2±0.2	0.63-0.84
8–12	0.05–1.0	315	1.15±0.15	0.72±0.08	0.60±0.12	0.67–0.89
12–16	0.15–1.0	224	0.8±0.2	0.45±0.10	0.63±0.17	0.70–0.93
16–20	0.25–1.0	139	1.4±0.4	0.6 ±0.2	—	0.73–0.97

[a] The errors are statistical only. We estimate systematic error due to the normalization of the total cross section for reaction (1) to equal 15%.

[b] These predictions were calculated assuming $\sigma_t^{\pi^-\pi^-} = (8.7\ \text{mb}) s_{\pi\pi}^{0.0755}$ and values of slopes for $\pi^-\pi^-$ differential cross sections equal to 6 and 8 GeV^{-2} respectively.

It is well known that Eq. (3) does not describe adequately angular distributions of π_f^- in the $\pi\pi$ c.m. system. Experimental ϕ distributions are not flat (see Fig. 7) even in the region of small $s_{\pi\pi}$ and t_{np}, where pion exchange should dominate, contrary to the predictions of Eq. (3). According to PMA model [17] the observed deviations are due to absorptive corrections to evasive π-exchange amplitudes. The generalized PMA model developed by Ochs and Wagner gives [16]:

$$s\frac{d^4\sigma}{ds_{\pi\pi}dt_{np}d\Omega} = I_0(s_{\pi\pi}, t_{np}, \theta) + I_1(s_{\pi\pi}, t_{np}, \theta)\cos\phi, \tag{5}$$

where

$$I_0 = N\left\{\frac{-t_{np}}{(t_{np}-\mu^2)^2} F_0^2(t_{np})\,|T_0|^2 + \frac{|C_A|^2 s_{\pi\pi}}{(s_{\pi\pi}-\mu^2)^2} F_1(t_{np}) \left|\frac{\partial}{\partial\theta} T_0\right|^2\right\},$$

$$I_1 = N\left\{\frac{\sqrt{-t_{np}}}{\mu^2 - t_{np}} F_0(t_{np})F_1(t_{np})\,\mathrm{Re}\,C_A \frac{\sqrt{s_{\pi\pi}}}{s_{\pi\pi}-\mu^2}\left(-\frac{\partial}{\partial\theta}\right)|T_0|^2\right\},$$

$$N = \frac{g_{\pi np}^2}{16\pi^2}\frac{s_{\pi\pi}}{s}\left[1-\tfrac{1}{2}H(t_{np})\right],$$

and

$$\left(\frac{d\sigma_{\text{el}}^{\pi\pi}}{d\Omega}\right)_{\text{on}} = |T_0(s_{\pi\pi}, \theta)|^2$$

$F_i(t_{\text{np}})$ are form factors, $C_A(s_{\pi\pi})$ is the absorption strength parameter.

These formulae predict [16] that:

— spherical harmonics moments $\langle Y_l^m \rangle$ with $m \geqslant 2$ are negligible. Our data (not shown) are in agreement with this prediction;

— the ratio $R = -\sqrt{l(l+1)}\,\langle Y_l^0 \rangle / \langle \text{Re}\, Y_l^1 \rangle$ is related to the absorption parameter C_A through

$$-\sqrt{l(l+1)}\,\frac{\langle Y_l^0 \rangle}{\langle \text{Re}\, Y_l^1 \rangle} = \frac{s_{\pi\pi}^{1/2}}{\text{Re}\, C_A}(s_{\pi\pi}), \tag{6}$$

and should not depend on value of l.

The experimental values of R displayed in Fig. 2 are consisten t with the model predictions. They are also in agreement with the corresponding values of the $\pi^+\pi^-$ data [21, 22] shown in the same figure for comparison. The approximately linear increase of R with $s_{\pi\pi}$ suggests that the parameter C_A decreases with $s_{\pi\pi}$ like $s_{\pi\pi}^{-1/2}$.

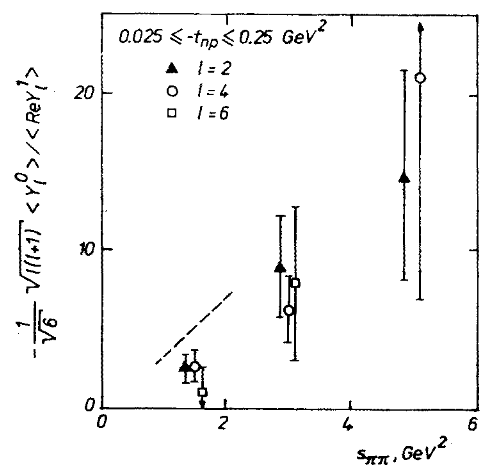

Fig. 2. Ratios of the t-channel moments: $-\frac{1}{\sqrt{6}}\sqrt{l(l+1)}\,\langle Y_l^0 \rangle / \langle \text{Re}\, Y_l^1 \rangle$ integrated over $0.025 < -t_{\text{np}} < 0.25$ GeV^2 as a function of $s_{\pi\pi}$. Dotted line represents trend of $\pi^+\pi^-$ data of Ref. [21]

We conclude that the predictions of the Ochs and Wagner model are consistent with our data. Therefore we use a maximum likelihood technique (MLHF) to fit formulae (5) to the data and thus extract $\pi^-\pi^-$ elastic cross sections. Following Ref. [23] we assume C_A to be real, take $F_0 = F_1 = \exp\,(b(t_{\text{np}} - \mu^2))$ and introduce parameters $T_0 = |T|$ and $T_1 = \frac{2C_A}{M_{\pi\pi}}\left|\frac{dT}{d\theta}\right|$. The fit is performed for various $s_{\pi\pi}$ regions in t_{np} interval $0.025 \leqslant -t_{\text{np}} < 0.25$ GeV^2 and for one to three $\cos\theta$ bins.

We neglect a relative phase change between T and $\frac{dT}{d\theta}$ and moreover we assume $\left\langle \frac{dT}{d\theta} \Big/ T \right\rangle^2 = \left\langle \left|\frac{dT}{d\theta}\right|^2 \Big/ T^2 \right\rangle$ to hold in a given $\cos\theta$ bin. These simplifications are equivalent to taking into account only the minimal contribution of the absorptive amplitude. The two independent parameters of the fit are b, and the ratio T_1/T_0. $\sigma_{el}^{\pi\pi}$ is then calculated

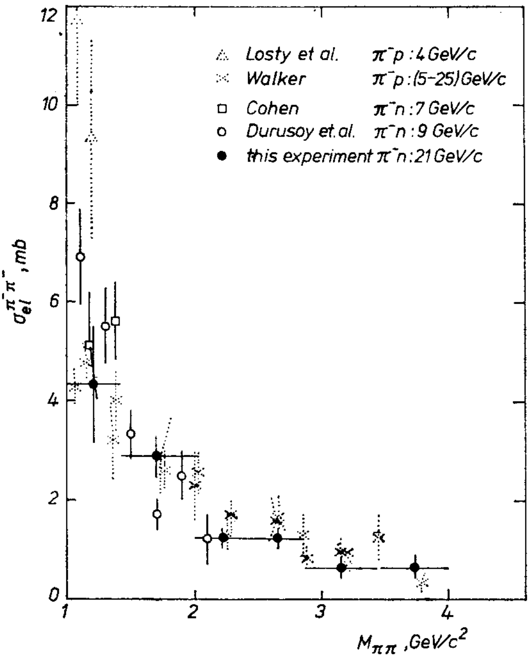

Fig. 3. $\sigma_{el}^{\pi^-\pi^-}$ as a function of $M_{\pi\pi} = \sqrt{s_{\pi\pi}}$. The results of various experiments are compiled [1–3]. The values of $\sigma_{el}^{\pi^-\pi^-}$ extracted in this experiment (MLHF method) are given in Table 1

from $\sigma_{el}^{\pi\pi} = \int T_0^2 d\cos\theta$. The values of $\sigma_{el}^{\pi\pi}$ obtained using this method are given in Table I and compared with results of previous experiments [1–3] in Fig. 3.

The values of $\sigma_{el}^{\pi\pi}$ are obtained by the MLHF procedure and by extrapolation method with the Regge type form factors are equal within errors but are systematically lower than results from the Dürr–Pilkuhn parametrization. We note that values of $\sigma_{el}^{\pi\pi}$ obtained by any of the methods are subject to 15% systematic error.

Our results confirm the trend of $\sigma_{el}^{\pi\pi}$ found by previous experiments. $\sigma_{el}^{\pi\pi}$ for $\pi\pi$ system in isospin $I = 2$ state decreases monotonically from a value of $\sim$ 5 mb at $s_{\pi\pi} \approx 1.5$ GeV^2 to below 1 mb at $s_{\pi\pi}$ above 8 GeV^2. Similar small values of $\sigma_{el}^{\pi\pi}$ around 1 mb have been recently reported for elastic $\pi^+\pi^-$ scattering at these energies [23].

In a previous publication by our collaboration [6] we have shown that extracted values of total $\pi^-\pi^-$ cross sections, $\sigma_t^{\pi^-\pi^-}$, are consistent with the parametrisation $\sigma_t^{\pi^-\pi^-} = (8.7\,\text{mb})s_{\pi\pi}^{0.0755}$. We have used this parametrisation to estimate $\sigma_{el}^{\pi\pi}$ from the optical theorem: $\sigma_{el}^{\pi\pi} = \sigma_t^{\pi\pi^2}/16\pi b_{\pi\pi}$, where $b_{\pi\pi}$ is a slope of elastic $\pi^-\pi^-$ scattering. Assuming $6 < b_{\pi\pi} < 8$ GeV^{-2} one gets values of $\sigma_{el}^{\pi^-\pi^-}$ given in Table I. Predicted $\sigma_{el}^{\pi\pi}$ are below 1 mb and for $s_{\pi\pi} > 8$ GeV^2 they lie between values of $\sigma_{el}^{\pi\pi}$ obtained by the Dürr–Pilkuhn procedure and the two other methods. The large excess of the experimental $\sigma_{el}^{\pi\pi}$ for $s_{\pi\pi} < 8$ GeV^2 suggests that elastic $\pi^-\pi^-$ amplitude at these energies is predominantly real.

4. *Differential cross sections $d\sigma/dt'_{\pi\pi}$*

The differential cross sections $d\sigma/dt'_{\pi\pi}$ studied for various $s_{\pi\pi}$ regions and for $|t_{np}| < 1$ GeV^2 are presented in Fig. 4. The distributions are sharply peaked at $t'_{\pi\pi} \approx 0$ and flatten off with increasing $t'_{\pi\pi}$. The observed structure is similar for all $s_{\pi\pi}$ except for extreme regions of $s_{\pi\pi} > 16$ GeV^2. We have checked that $d\sigma/dt'$ exhibit little if any dependence on t_{np} cut as long as $-t_{np}$ is less than 1 GeV^2.

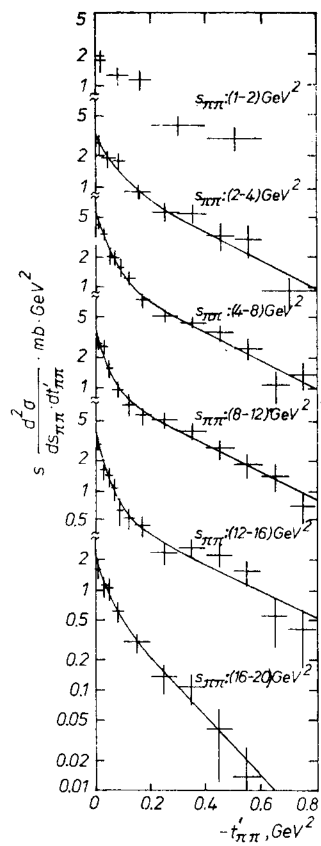

Fig. 4. Invariant differential cross section $s\,\dfrac{d^2\sigma}{ds_{\pi\pi}dt'_{\pi\pi}}$ as a function of $t'_{\pi\pi}$ integrated over $-t_{np} < 1$ GeV^2 for various indicated $s_{\pi\pi}$ regions. Curves represent fit to the data with formula (7). The values of fitted parameters are given in Table II

The fit to the data with formula

$$s\frac{d^2\sigma}{ds_{\pi\pi}dt'_{\pi\pi}} = A(e^{Bt_{\pi\pi}'}+C\cdot e^{Dt_{\pi\pi}'}), \tag{7}$$

gives for slopes B and D values of $\geqslant 20$ GeV^2 and 3 GeV^{-2} respectively (see Table II). The values of parameter B are much larger than expected for elastic $\pi\pi$ slope and must reflect the well known structures observed for nucleon diffractive dissociation and usually studied as a function of the $p\pi_s^-$ invariant mass [24–26].

In Fig. 5 we present invariant $p\pi_s^-$ mass, $m(p\pi_s^-)$, distributions for two categories of events with: (a) $|t'_{\pi\pi}| < 0.1$ GeV^2 and (b) $|t'_{\pi\pi}| > 0.1$ GeV^2. Within given category the data

TABLE II

Results of a fit to $s\frac{d^2\sigma}{ds_{\pi\pi}dt'_{\pi\pi}}$ with formula (7)

$s_{\pi\pi}$ GeV^2	A mb GeV^{-2}	B GeV^{-2}	C	D GeV^{-2}
2–4	2.0	17	0.62	3.2
4–8	3.6	21.3	0.32	3.15
8–12	2.7	29.0	0.36	3.15
12–16	3.0	29.3	0.24	3.35
16–20	1.2	22.8	0.67	6.56

Values of parameters A, B, C, and D are strongly correlated. Typical errors are: $\Delta A = 0.6$ mb GeV^{-2}; $\Delta B = 8$ GeV^{-2}; $\Delta C = 0.10$; $\Delta D = 0.45$ GeV^{-2}.

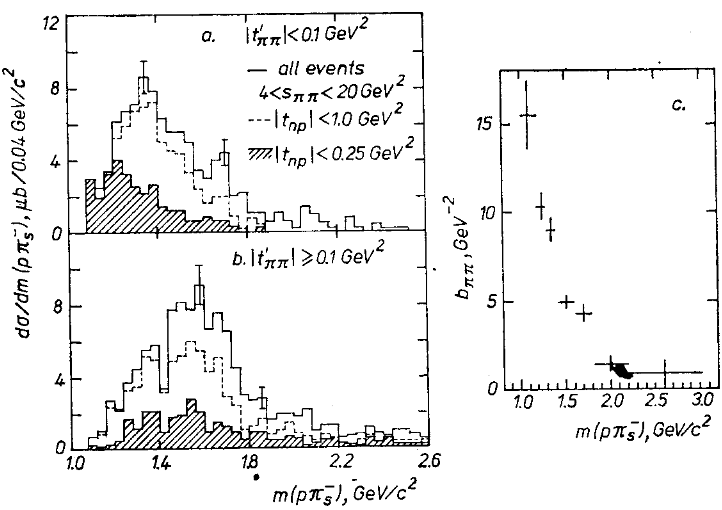

Fig. 5. a-b. Invariant $p\pi_s^-$ mass distributions for: $|t'_{\pi\pi}| < 0.1$ GeV^2 (a) and $|t'_{\pi\pi}| \geqslant 0.1$ GeV^2 (b). Data are shown without additional cuts and with $s_{\pi\pi}$ and t_{np} cuts indicated in the figures; c. fitted values of the slope $b_{\pi\pi}$ of $d\sigma/dt'_{\pi\pi}$ distributions as a function of $m(p\pi_s^-)$. The fit with formula $d\sigma/dt'_{\pi\pi} \sim e^{-b_{\pi\pi}t_{\pi\pi}'}$ was performed for $|t'_{\pi\pi}| < 0.3$ GeV^2

are shown: (*i*) without additional cuts and for sub-samples of events selected by requiring: $4 \leqslant s_{\pi\pi} \leqslant 20$ GeV^2 and (*ii*) $0.025 < |t_{np}| < 1.0$ GeV^2 or (*iii*) $0.025 < |t_{np}| < 0.25$ GeV^2. The broad maximum at $m(p\pi_s^-) \approx (1.2\text{–}1.5)$ GeV/c^2 observed for events with $|t'_{\pi\pi}| < 0.1$ GeV^2 shifts dramatically towards the region of small $p\pi_s^-$ masses when cut (*iii*) is imposed on the data. On the other hand the maximum at $m(p\pi_s^-) \approx (1.4\text{–}1.7)$ GeV/c^2 observed for events with $|t'_{\pi\pi}| > 0.1$ GeV^{-2}, whose resonance character is very controversial [27], almost disappears for interactions with small values of t_{np}.

The data exhibit strong dependence of a $d\sigma/dt'_{\pi\pi}$ slope, $b_{\pi\pi}$, on $m(p\pi_s^-)$ (Fig. 5c) found in many NN → NNπ experiments [24, 25] and confirmed for πN → πNN reactions [26].

It is well known that DHD diagram alone (Fig. 1b) fails to explain large values of $b_{\pi\pi}$ and structures observed in $d\sigma/dt'_{\pi\pi}$ distributions. The predictions of modified DHD models are compared with the data in the following section.

5. *Model descriptions for reaction* $\pi^- n \to p\pi^-\pi^-$

In this section we discuss models of diffractive disociation formulated in terms of exchange amplitudes. The experimental features presented in Figs. 4 and 5 have been interpreted as:

— a manifestation of the importance of baryon exchange diagrams (Figs. 1cd) [28–30] and/or

— evidence for strong absorptive corrections to the DHD amplitude [31–33] due to e.g. final state rescattering of the $\pi_f^- p$ system.

The predictions of models based on these ideas have been compared with the experimental data for the NN → NNπ reaction. The best agreement was found for Cutler and Berger (C–B) parametrisation of diagrams of the type 1b-d [29], however only after strong absorption effects had been taken into account [33]. The successful description of the data was also achieved with the dual parametrisation of pion and baryon exchange amplitudes [30, 25].

We consider it interesting to test the predictions of both models against our data. The relative contributions of pion and baryon exchange amplitudes are expected to be different for πN → πNπ and NN → NNπ reactions. The strength of absorptive corrections is proportional to the total cross section for beam type particle-nucleon interactions. Therefore absorption should play less important role for pion induced reactions than for those induced by nucleons.

Following Cutler and Berger [29] we parametrise the differential cross section for reaction (1) by:

$$s \cdot \frac{d^4\sigma}{ds_{\pi\pi}dt_{np}d\Omega} = \frac{1}{(4\Pi)^4}\{-t_{np}|\Pi|^2+|B+D|^2\cdot[2(\mu^2(t_{\pi\pi}-4m^2)$$

$$-(s_{p\pi_s^-}-m^2)(u-m^2))]+\Pi(B+D)[4m(t_{\pi\pi}-\mu^2-t_{np})]\}, \tag{8}$$

where μ and m are pion and nucleon masses and Π, B and D stand for the pion exchange baryon exchange and direct baryon production amplitudes corresponding to diagrams 1b, c and respectively. These amplitudes are assumed to be purely imaginary and are given by:

$$\Pi = ig_{\pi np}\frac{F_\pi(t_{np})}{t_{np}-\mu^2}\, s_{\pi\pi}\sigma_t^{\pi^-\pi^-}(s_{\pi\pi})e^{b_{\pi\pi}\cdot t_{\pi\pi}/2}; \qquad F_\pi(t_{np}) = e^{b_\pi(t_{np}-\mu^2)},$$

$$B = ig_{\pi np}\frac{F_B(u)}{u-m^2}\cdot\frac{s_{p\pi_f^-}}{2m}\cdot\sigma_t^{\pi^-p}(s_{p\pi_f^-})\cdot e^{b_{\pi N}t_{\pi\pi}/2}; \qquad F_B(u) = e^{b_u(u-m^2)}$$

$$D = ig_{\pi np}\frac{F_D(s_{p\pi_s^-})}{s_{p\pi_s^-}-m^2}\cdot\frac{s}{2m}\cdot\sigma_t^{\pi^-n}(s)\cdot e^{b_{\pi N}t_{\pi\pi}/2}; \qquad F_D(s_{p\pi_2^-}) = e^{b_s(m^2-s_{p\pi_s^-})}, \tag{9}$$

where $\sigma_t^{ij}(s_{ij})$ are total ij cross sections at ij c.m. energy squared s_{ij}, $b_{\pi\pi}$ and $b_{\pi N}$ are slopes at differential elastic $\pi\pi$ and πN cross sections and b_π, b_u and b_s are parameters of form factors. For the total πN cross sections we take experimental values and $\sigma_t^{\pi^-\pi^-}$ is parametrised as $\sigma_t^{\pi^-\pi^-} = (8.7\,\text{mb})\cdot s_{\pi\pi}^{0.0755}$ (see Sect. 3 and Ref. [6]).

We put $b_{\pi\pi} = 6$ GeV^{-2}, $b_\pi = 2$ GeV^{-2}, $b_u = 1$ GeV^{-2} and $b_s = 1.5$ GeV^{-2}. These values are the same as used in Ref. [33] for NN → NNπ channel and we have checked that they also optimize an agreement of the model predictions with the data for reaction (1).

The absorption corrections are introduced by substituting amplitude Π in Eq. (8) by $\Pi_{abs} = \Pi + \Pi_{corr}$. Absorptive correction Π_{corr} is proportional to both Π and $\sigma_t^{\pi^-p}(s_{p\pi_f^-})$ and is calculated following prescription given in Ref. [32]. Contrary to Ref. [33] we do not introduce the phenomenological factor $\lambda \geqslant 1$ to increase the strength of the correction. We note that C–B parametrisation predicts both the shape and the absolute normalisation for $d^4\sigma/ds_{\pi\pi}dt_{np}d\Omega$.

We also calculate predictions for reggeised dual parametrisation of amplitudes (9) as given by formulae (12–14) and (17) in Ref. [30]. The values of total πN and $\pi\pi$ cross sections and slopes $b_{\pi\pi}$ and $b_{\pi N}$ are taken the same as for the C–B parametrisation. We have found that introduction of asymptotic values for $\sigma_t^{\pi N}$ and large values of $b_{\pi\pi}$, as suggested in Refs. [25, 30], worsen the agreement with the data. The predictions of the dual model are normalized to the total number of events in the kinematic region: $4 < s_{\pi\pi} < 20$ GeV^2 and $0.025 < -t_{np} < 1.0$ GeV^2.

We compare predictions of three models with the data for:

(*i*) $s\dfrac{d^2\sigma}{ds_{\pi\pi}dt'_{\pi\pi}}$ in three $s_{\pi\pi}$ regions between $4 < s_{\pi\pi} < 12$ GeV^2 and $0.025 < -t_{np} < 0.25$ GeV^2, (Fig. 6).

(*ii*) $\dfrac{d\sigma}{d\phi}$ in five $s_{\pi\pi}$ regions between $4 < s_{\pi\pi} < 20$ GeV^2 and two t_{np} bins: $0.025 < -t_{np} < 0.25$ GeV^2 and $0.25 < -t_{np} < 1.0$ GeV^2 (Fig. 7). ϕ is the azimuthal angle of π_f^- calculated in $\pi^-\pi^-$ rest frame (t-channel helicity). We find:

(a) The C–B parametrization fairly reproduces the overall normalisation and the $s_{\pi\pi}$ dependence of the data in the low t_{np} region. Too small values of cross section predicted for $s_{\pi\pi} < 8$ GeV^2 probably reflect negligence of the real part of the elastic $\pi^-\pi^-$ amplitude in the calculations. In the low t_{np} region the Π amplitude dominates because contributions from the B and D amplitudes cancel each other (see upper right inlet in Fig. 7). Similar deviations in the predicted $s_{\pi\pi}$ dependence are observed for the dual parametrisation.

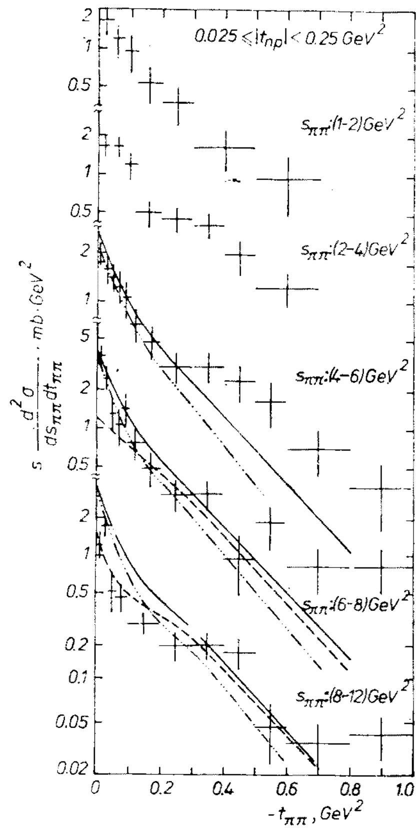

Fig. 6. Invariant differential cross section $s\dfrac{d^2\sigma}{ds_{\pi\pi}dt_{\pi\pi}}$ as a function of $t_{\pi\pi}$ integrated over $0.025 < |t_{np}| < 0.25$ GeV^2. Curves represent predictions of modified DHD models: Cutler–Berger without (full) and with (dash-dotted) absorption correction (Eq. (8)) and dual (dashed)

(b) The C–B model fails to reproduce the data for larger t_{np} region, where mutual cancellations of the B and D amplitudes are less efficient. This failure of the model can be probably cured by introducing more sophisticated parametrization of baryonic amplitudes.

(c) Both C–B and dual parametrisations qualitatively reproduce the main deviations of the data from behaviour predicted by the pion-exchange amplitude Π, namely

— strong and steep maximum in the small $t'_{\pi\pi}$ region (Fig. 6),

— increase in planarity of events with increasing t_{np} (maximum of $d\sigma/d\phi$ distribution at $\phi = 0$ in Fig. 7).

Above deviations are explained in the C–B model as a manifestation of the contributions

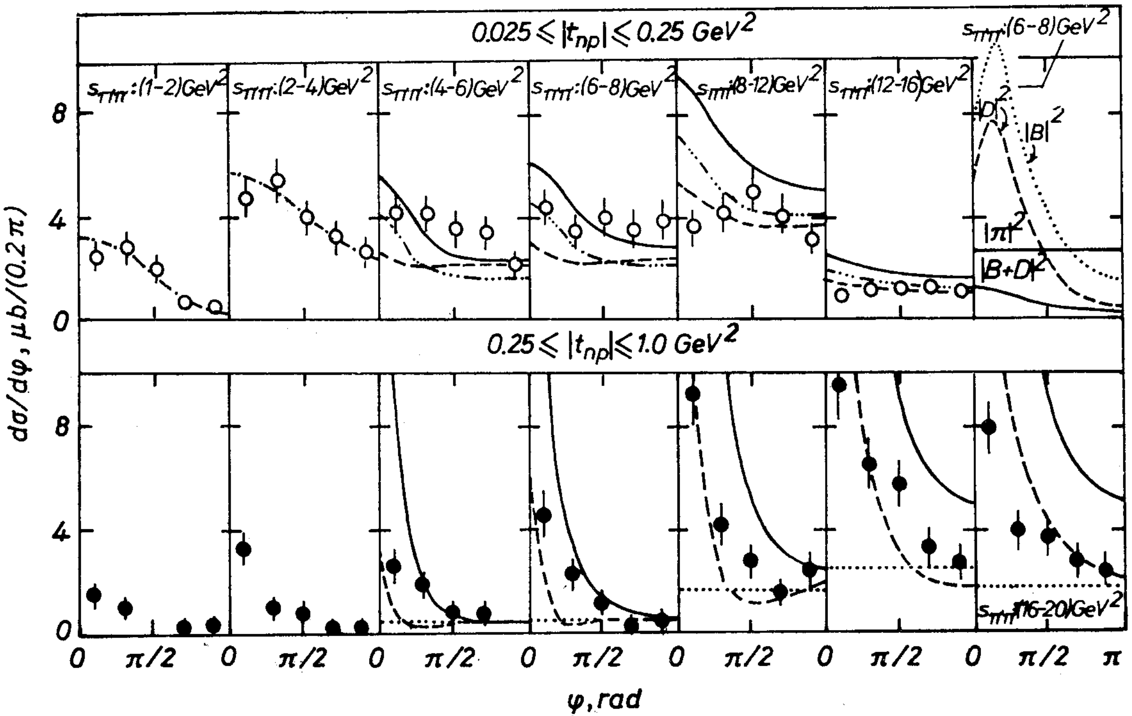

Fig. 7. Distributions of ϕ-azimuthal angle of π_f^- in $\pi\pi$ t-channel rest frame, for various t_{np} and $s_{\pi\pi}$ regions. Curves in figures corresponding to $s_{\pi\pi} > 4\,\mathrm{GeV}^2$ represent predictions of the same models as in Fig. 6. An inlet in the upper-right corner shows contributions of various terms in Eq. (8) to the data in the indicated region of $s_{\pi\pi}$ and t_{np}. The curves for $s_{\pi\pi} < 4\,\mathrm{GeV}^2$ represent results of the fit to the data with Ochs-Wagner parametrisation (Eq. (5)). The predictions of the CB parametrisation without and with absorption corrections for $0.25 < -t_{np} < 1.0\,\mathrm{GeV}^2$ region are almost identical. Therefore the latter have been omitted in the lower part of the figure. We show there instead contribution of the $|\Pi|^2$ term (Eq. (8)) to the cross section (dotted lines)

from the baryonic amplitudes and their interference with the Π amplitude. The absorptive effects are weak and are predicted to show up in $d\sigma/dt_{\pi\pi}$ only for $|t'_{\pi\pi}| > 0.8$ GeV^2. The present data do not give clear indication for the presence and strength of absorptive corrections.

(d) Both models fail to account for flattening off of $d\sigma/dt'_{\pi\pi}$ for $|t_{\pi\pi}| > 0.4$ GeV^2. Similar effect observed for NN → NNπ reaction was explained in Ref. [25] as due to the resonance production in $p\pi_s^-$ system. However in the kinematic region studied in this experiment we find very little evidence for resonance production (see Fig. 5b).

In summary, we consider the observed agreement between the theory and the data as satisfactory if one takes into account simplicity of the model parametrisations. Our findings have also some implications for the analysis of the $\pi^-\pi^-$ scattering.

(a) The assumed values of $\sigma_t^{\pi^-\pi^-}$ give reasonable normalization of the DHD amplitude in the low t_{np} region.

(b) The absorptive corrections and contributions from baryon exchange amplitudes tend to flatten off the experimental t_{np} distributions for reaction (1) (Regge form factor in Eq. (4) works only on the average) and as a consequence may diminish the values of $\sigma_{el}^{\pi^-\pi^-}$ obtained from the Chew–Low extrapolation.

(c) The contribution of baryonic amplitudes at $t'_{\pi^-\pi} = 0$ does not allow meaningful application of the optical theorem to determine total $\pi\pi$ cross section from elastic $\pi\pi$ cross sections extracted in the analysis of reactions of type (1) (see Ref. [23]).

6. *Summary and conclusions*

We have analysed the data for reaction $\pi^-n \to p\pi^-\pi^-$ at 21 GeV/c in order to extract information on elastic $\pi^-\pi^-$ scattering.

The total elastic $\pi^-\pi^-$ cross section decreases with energy from 5 mb at $s_{\pi\pi} = 2$ GeV^2 to values below 1 mb for $s_{\pi\pi} > 6$ GeV^2.

The analysis of elastic $\pi\pi$ scattering is obscured by strong contributions to reaction $\pi^-n \to p\pi^-\pi^-$ from baryon exchange amplitudes even in the kinematic region of $s_{\pi\pi}/s < 0.2$ and $|t_{np}| < 0.25$ GeV^2. They manifest themselves mainly in the observed structures of $t_{\pi\pi}$ distributions and are reasonably reproduced by the modified Drell–Hiida–Deck models.

The absorption effects present in low $s_{\pi\pi}$ region and related to the virtual pion are correctly reproduced by phenomenological model of Ochs and Wagner. On the other hand data do not provide information on the strength of the absorptive corrections due to the final state interactions.

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