CLUSTER MODEL FOR THE MULTIPLICITY DISTRIBUTIONS IN LEPTON-HADRON AND HADRON-HADRON COLLISIONS

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On the basis of a cluster model we demonstrate the significant difference of the charged multiplicity distributions between annihilation-like (νp, e^+e^- and $\bar{p}p$) and non-annihilation (pp, π^-p and K^-p) reactions. As for the number distribution of clusters we propose the Gaussian distribution in which a characteristic parameter is chosen according to these unlike reactions, and the asymptotic average numbers of clusters, which lead to KNO scaling, are attributed to the sorts of incident particles. The scaling functions of other reactions are predicted.

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1. Introduction

Recent experimental studies of the charged multiplicity distribution in νp interactions at the CERN SPS [1, 2] have presented an interesting dynamical view point concerning hadron production in contrast with ordinary hadron-hadron collisions. In this connection it is proposed [2, 3] that the charged multiplicity distributions are found to be devided into two types corresponding to two classes of reactions; i.e. (1) annihilation-like processes like $\bar{p}p$ (or e^+e^-) and νp to which diffraction dissociation of incident particles does not contribute, and (2) non-annihilation processes of normal hadron-hadron collisions or photoproduction where diffraction scattering is seemed to be present.

It should be stressed that indeed the charged multiplicity distributions of two types of reactions manifest the KNO scaling law [4], but the annihilation-like distributions are significantly much narrower than those of non-annihilation reactions. Furthermore we notice a slight difference between the scaling functions of $\bar{p}p$ and νp (or e^+e^-) reactions.

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Several authors [5–10] on the basis of the intermediate cluster formation have succeeded in explaining the adequate scaling function for pp scattering at high energy. However, only a few investigations [11, 12] have been proposed to demonstrate the charged multiplicity distribution of annihilation reactions in view of the KNO scaling law. Hence the analysis for interpretation on the relation between the multiplicity distributions of different reactions was very poor.

The purpose of this article is to demonstrate the significant difference of the KNO scaling functions of those unlike reactions by means of a cluster model. For the number distribution of clusters we propose the Gaussian distribution in which a characteristic parameter represents the features of the distribution functions for annihilation and non--annihilation reactions, whereas the asymptotic mean values of the cluster number are due to the sorts of incident particles. It follows from our cluster model that the narrow mutiplicity distributions of annihilation-like reactions are illustrated on account of production of nearly fixed number of clusters, which is compared with those of hadron-hadron collisions where the approximately independent emission of clusters seems probable. The result shows a remarkable agreement with the experimental data. The scaling functions of other reactions are predicted.

In Section 2 we present how to use the cluster model for the multiplicity distributions in general reactions with the help of the statistic thermodynamical method. We show in Section 3 comparison of our evaluations with the experimental data of the charged multiplicity distributions for various reactions making use of the Gaussian number distribution of clusters. Section 4 is devoted to summary and discussions.

2. *A cluster model for many particle production processes*

It has been emphasized in view of the particle correlation studies [13] that hadrons are produced via clusters in multiparticle production. As concerns the charged multiplicity distribution some authors [5,8–10] proposed interpretations of the emission of a fixed number of clusters in connection with the KNO scaling law. In this section we will show a general treatment of the cluster model concerning the multiplicity distribution for lepton--hadron and hadron-hadron collisions including annihilation reactions, and present the model calculations making use of the statistic thermodynamical method.

In terms of the probability for the production of m clusters A_m and the probability Q_n^m that n hadrons are finally emitted from m clusters, let us represent the n multiplicity distribution[1]

$$P_n = \sum_m A_m Q_n^m. \tag{1}$$

It is probable to consider that the hadron productions from respective clusters are the independent phenomena. Hence, the probability R_{n_i} that n_i hadrons are produced from i-th

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[1] As mentioned in the following, on the assumption of the production of clusters which emit at least one hadron, here we exclude the zero multiplicity events.

cluster yields, with the constraint of $n = \sum_{k=1}^{m} n_k$,

$$Q_n^m = \sum_{n_1} \cdots \sum_{n_m} R_{n_1} \cdots R_{n_m} \delta_n \sum_k n_k. \tag{2}$$

We suppose the energy of i-th cluster is given approximately by [5, 10]

$$M_i \simeq \langle E \rangle n_i, \tag{3}$$

where $\langle E \rangle$ is the average energy of emitted hadrons and M_i is the cluster mass, in the rest frame of cluster. It should be noted that at least one hadron is produced from each clusters. Assuming the statistic themodynamical energy distribution for rest clusters, we obtain in terms of the temperature T and the energy in the equilibrium state

$$R_{n_i} = (1-X)X^{n_i-1}, \tag{4}$$

where $X = \exp(-\langle E \rangle / T)$. Thus, it turns out from Eqs. (2) and (4)

$$Q_n^m = \frac{\Gamma(n)}{\Gamma(m)\Gamma(n-m+1)} (1-X)^m X^{n-m} \tag{5}$$

for $n \geqslant m \geqslant 1$.

It is convenient for the calculation of the correlation parameters to introduce the partition function [14] in the present investigation as follows;

$$\Phi(t) = \sum_n t^n P_n = \sum_m A_m t^m (1-X)^m (1-tX)^{-m}. \tag{6}$$

Eq. (6) gives

$$\left. \frac{\partial^k \Phi(t)}{\partial t^k} \right|_{t=1} = \sum_n n(n-1) \ldots (n-k+1) P_n = \langle n(n-1) \ldots (n-k+1) \rangle, \tag{7}$$

and hence we have

$$\langle n \rangle = (1-X)^{-1} \langle m \rangle,$$
$$\langle n^2 \rangle = (1-X)^{-2} \langle m^2 \rangle + X(1-X)^{-2} \langle m \rangle, \tag{8}$$

where $\langle m^k \rangle = \sum_m m^k A_m$. Therefore, substituting $X = 1 - (\langle m \rangle / \langle n \rangle)$ obtained from Eq. (8) into Eq. (5), we can rewrite Eq. (5) in terms of $\langle n \rangle$ and $\langle m \rangle$ as

$$Q_n^m = \frac{\Gamma(n)}{\Gamma(m)\Gamma(n-m+1)} \left(\frac{\langle m \rangle}{\langle n \rangle} \right)^m \left(1 - \frac{\langle m \rangle}{\langle n \rangle} \right)^{n-m}, \tag{9}$$

i.e. we get[2]

$$\langle n \rangle Q_n^m = \frac{\langle m \rangle^m}{(m-1)!} \left(z - \frac{1}{\langle n \rangle} \right) \ldots \left(z - \frac{(m-1)}{\langle n \rangle} \right) \left(1 - \frac{\langle m \rangle}{\langle n \rangle} \right)^{n-m}, \tag{9'}$$

where $z = n / \langle n \rangle$.

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[2] In comparison with the experimental data of the charged multiplicity distributions (see Section 3) we assume the charge invariance for emitted n_c charged pions, accordingly put $z = n_c / \langle n_c \rangle$.

3. Comparison with the experimental data of the charged multiplicity distribution

We want to show the explicit form of P_n to compare with the experimental data in connection with the KNO scaling law of the charged multiplicity distribution. It has been confirmed [5, 8–10] that KNO scaling is attributed to the emission of a fixed mean number of clusters in the scaling limit, i.e. $\langle m\rangle \to m_0$ (constant) in A_m and Q_n^m for the asymptotic energy. We shall follow up the previous investigations.

In the course of the actual calculations we assume the Gaussian form for A_m which reproduces the central plateau distribution [11, 15]. Now the asymptotic distribution[3] of A_m is

$$A_m = A\exp\{-\alpha^2(m-\langle m\rangle)^2\} \to A\exp\{-\alpha^2(m-m_0)^2\}, \tag{10}$$

where α is a parameter which ought to be chosen differently according to annihilation and non-annihilation reactions. Taking into consideration that in the scaling limit Eq. (9′) changes into the asymptotic form of the charged multiplicity distribution

$$\langle n_c\rangle Q_{nc}^m \to \frac{m_0^m}{(m-1)!}\, z^{m-1}e^{-m_0 z}, \tag{11}$$

where $z = n_c/\langle n_c\rangle$, we obtain from Eqs. (1), (10) and (11) the KNO scaling function[4]

$$\langle n_c\rangle P_{nc} = \Psi(z) = 2Ae^{-m_0 z}\sum_m \frac{m_0^m}{(m-1)!}\, z^{m-1}\exp\{-\alpha^2(m-m_0)^2\}. \tag{12}$$

As for ordinary hadron-hadron collisions our investigation chooses as follows; $m_0 = 6.5$ and $\alpha = 0.35$ for pp [16] (see Fig. 1), $m_0 = 7$ and $\alpha = 0.3$ for π^-p [17] (see Fig. 2) and $m_0 = 7$ and $\alpha = 0.35$ for K$^-$p [18] (see Fig. 3), while in the case of annihilation-like processes we have in the following; $m_0 = 8$ and $\alpha = 2$ for $\bar{\text{p}}$p [19] (see Fig. 4), $m_0 = 7$ and $\alpha = 1$ for νp [1, 2] (see Fig. 5) and $m_0 = 7$ and $\alpha = 2$ for e$^+$e$^-$ [20] (see Fig. 6). Fig. 7 manifests comparison of the theoretical values of $\langle n_c\rangle/D_c$ for annihilation-like reactions, where $D_c = \sqrt{\langle n_c^2\rangle - \langle n_c\rangle^2}$, and in Table I we show comparison of the higher moments of pp, π^-p and K$^-$p. Those suggest a good agreement between the theoretical calculations and the experimental data.

Since the parameter $\alpha = 1 \sim 2$ of Eq. (10) approximately leads to $A_m = \delta_{mm_0}$, we get [9] from Eq. (12)

$$\Psi(z) = \frac{2m_0^{m_0}}{(m_0-1)!}\, z^{m_0-1}e^{-m_0 z} \tag{13}$$

for annihilation-like processes. In this connection we dare say that the emission of the fixed number of clusters gives rise to annihilation reactions in contrast with the ordinary hadron-

[3] $A^{-1} = \sum_{m=1}^{\infty} \exp\{-\alpha^2(m-m_0)^2\}$ from the normalization condition for A_m.

[4] 2 comes from the normalization condition $\int_0^\infty dz\Psi(z) = 2$.

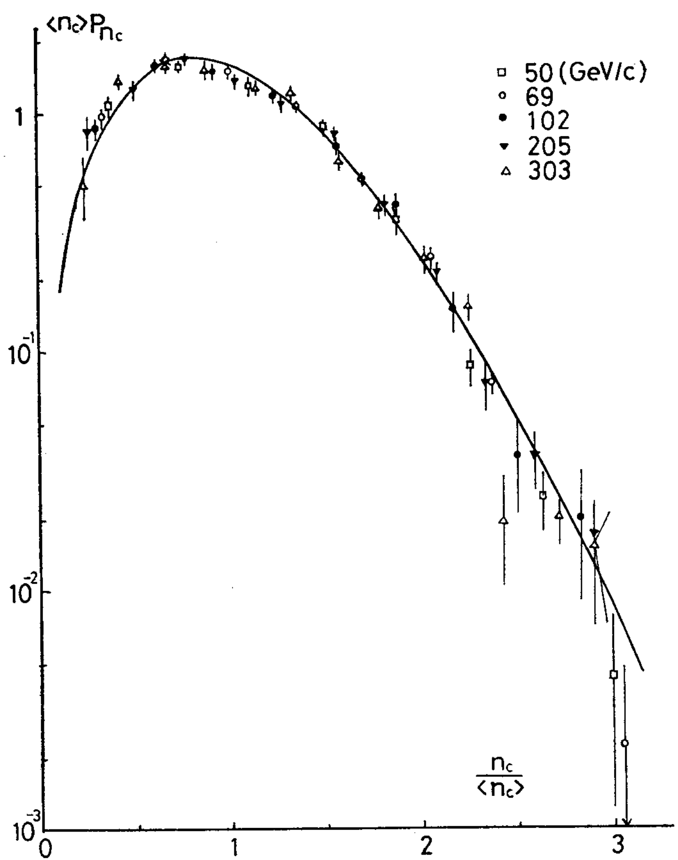

Fig. 1

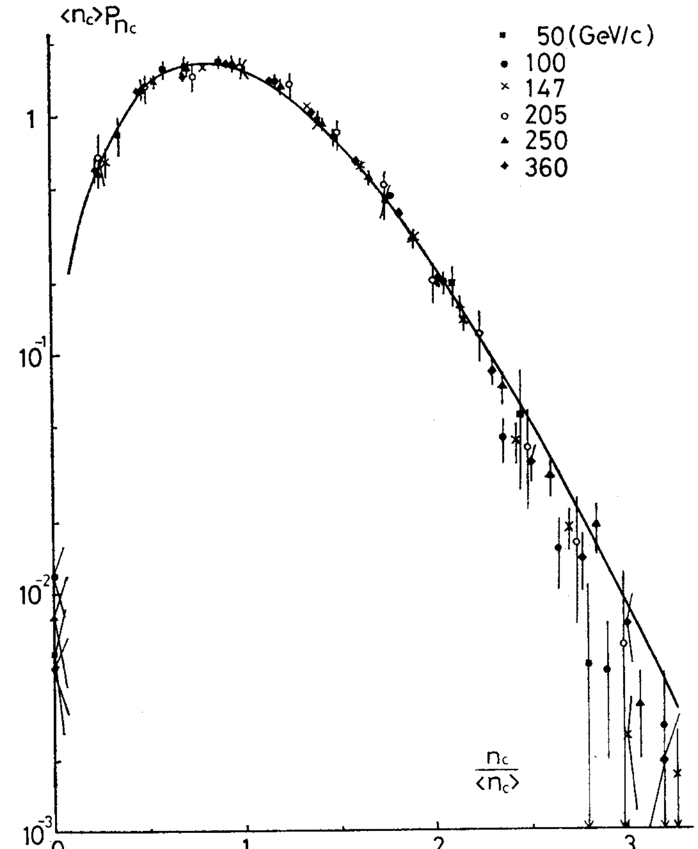

Fig. 2

Fig. 1. KNO scaling plot of $\langle n_c \rangle P_{n_c}$ versus $n_c/\langle n_c \rangle$ for pp scattering [16]. The solid line indicates Eq. (12) with $m_0 = 6.5$ and $\alpha = 0.35$

Fig. 2. KNO scaling plot of $\langle n_c \rangle P_{n_c}$ versus $n_c/\langle n_c \rangle$ for $\pi^- p$ scattering [17]. The solid line indicates Eq. (12) with $m_0 = 7$ and $\alpha = 0.30$

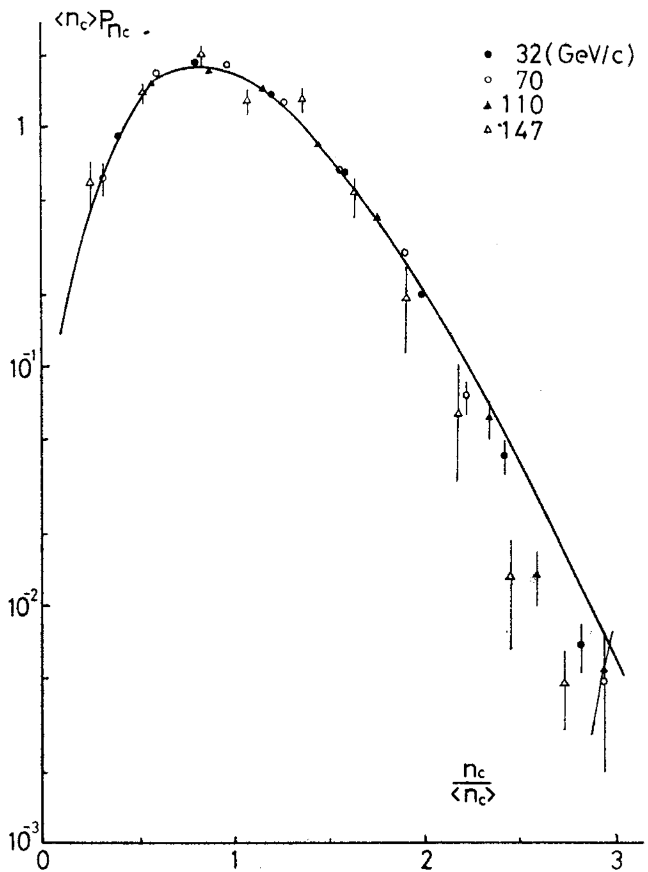

Fig. 3

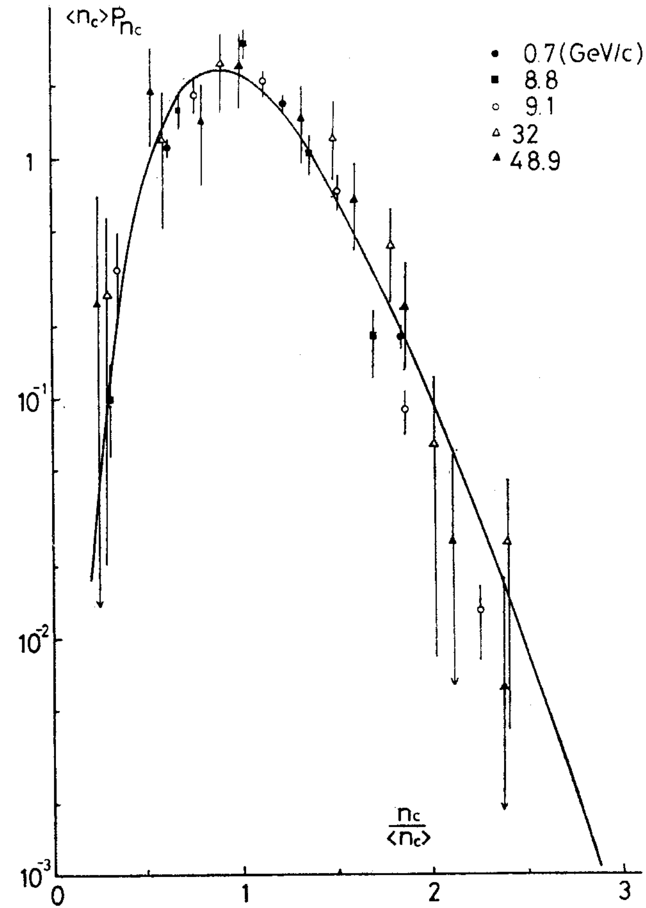

Fig. 4

Fig. 3. KNO scaling plot of $\langle n_c \rangle P_{n_c}$ versus $n_c / \langle n_c \rangle$ for $K^- p$ scattering [18]. The solid line indicates Eq. (12) with $m_0 = 7$ and $\alpha = 0.35$

Fig. 4. KNO scaling plot of $\langle n_c \rangle P_{n_c}$ versus $n_c / \langle n_c \rangle$ for $\bar{p}p$ annihilation [19]. The solid line indicates Eq. (12) with $m_0 = 8$ and $\alpha = 2$

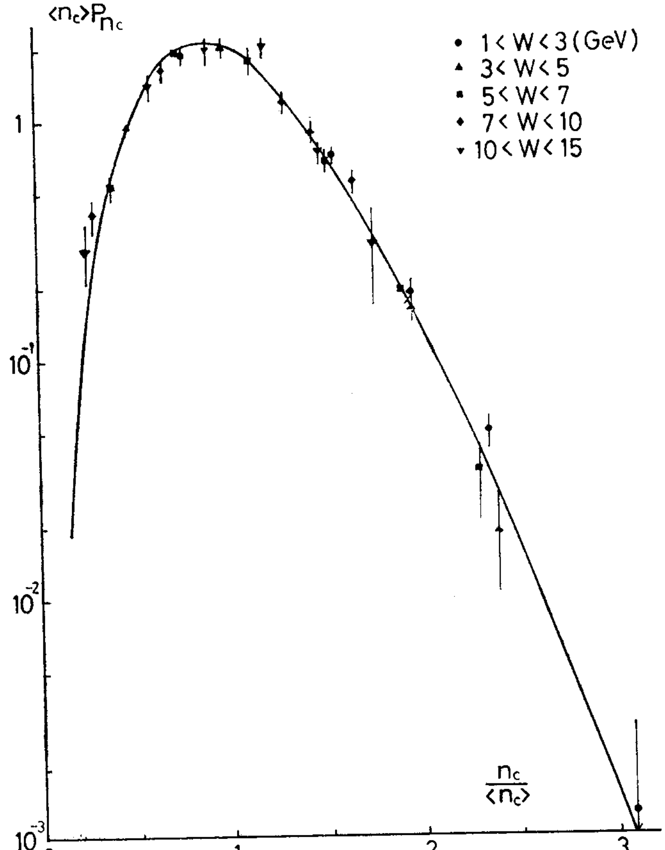

Fig. 5

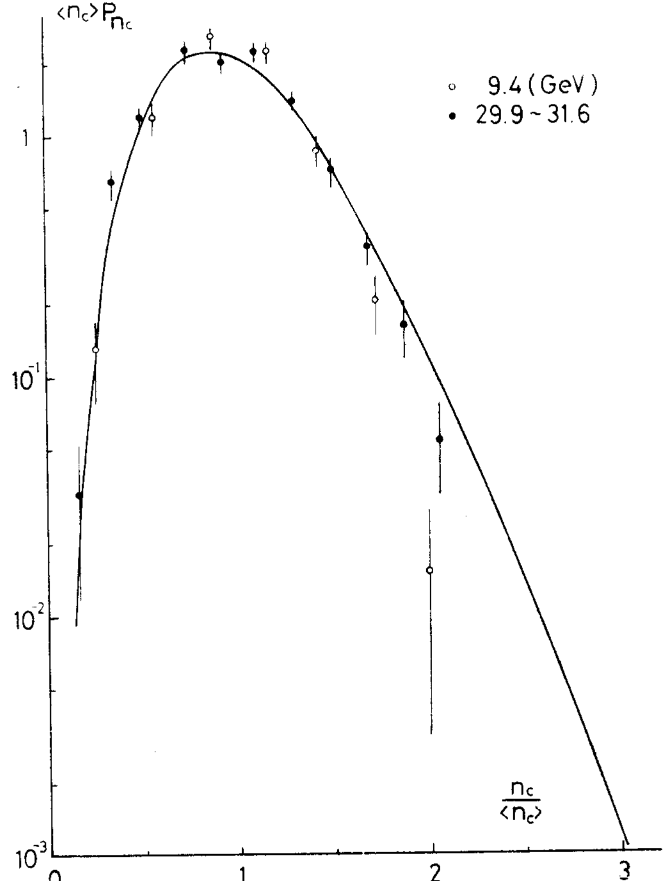

Fig. 6

Fig. 5. KNO scaling plot of $\langle n_c \rangle P_{n_c}$ versus $n_c/\langle n_c \rangle$ for νp scattering [1, 2]. The solid line indicates Eq. (12) with $m_0 = 7$ and $\alpha = 1$
Fig. 6. KNO scaling plot of $\langle n_c \rangle P_{n_c}$ versus $n_c/\langle n_c \rangle$ for e^+e^- annihilation [20]. The solid line indicates Eq. (12) with $m_0 = 7$ and $\alpha = 2$

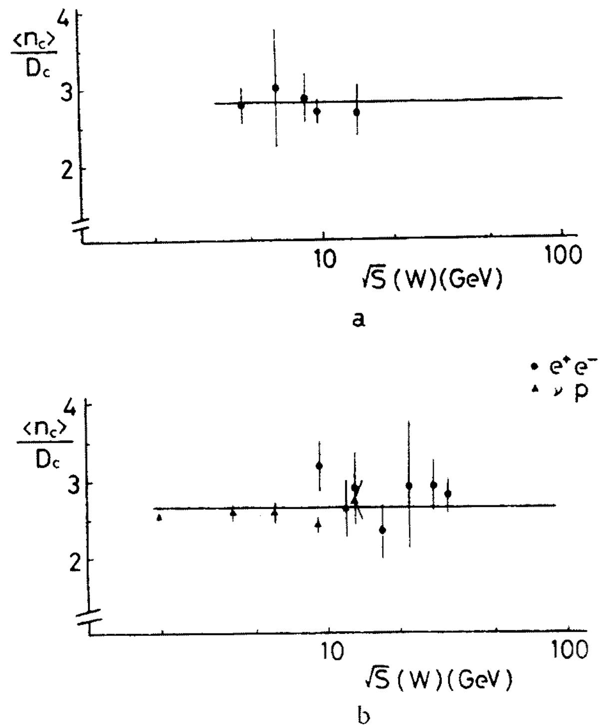

Fig. 7. Energy dependence of $\langle n_c \rangle / D_c$ for $\bar{p}p$ annihilation [19], νp scattering [1, 2] and e^+e^- annihilation [20]. (a) The solid line shows $\langle n_c \rangle / D_c = 2.83$ (cf. Eq. (16)) with $m_0 = 8$ for $\bar{p}p$ annihilation. (b) The solid line represents $\langle n_c \rangle / D_c = 2.65$ (cf. Eq. (16)) with $m_0 = 7$ for νp and e^+e^- reactions

-hadron collisions. For hadron-hadron interactions Eq. (10) with $\alpha \simeq 0.3$ (and $m_0 \simeq 7$) shows the similar behavior[5] with the Poisson distribution of the independent emission of clusters[6]

$$A_m = \frac{m_0^m}{m!} \cdot e^{-m_0}. \tag{14}$$

Eq. (14) leads to the KNO scaling function of hadron-hadron collisions [10]

$$\Psi(z) = \frac{2m_0}{\sqrt{z}} \exp\{-m_0(1+z)\} I_1(2m_0\sqrt{z}), \tag{15}$$

where I_1 is the first order modified Bessel function.

In the framework of our cluster model, where the number distribution of clusters is characterized with the parameter α independent of the primary energy in the scaling limit,

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[5] Provided $\alpha = \{1/(2m_0)\}^{1/2}$, the Gaussian distribution yields $\langle m^2 \rangle = m_0(m_0+1)$ which is equal to that of the Poisson distribution.

[6] In Eq. (1), if $Q_n^m = \delta_{mn/k}$ assuming that all clusters contain k particles, we obtain the scaling function of Levy [5], whereas it gives a poor fit for annihilation reactions with one parameter $\langle m \rangle$.

TABLE 1

Table of ten first normalized moments for pp, π^-p and K^-p scatterings, where the q-th normalized moment $c_q = \langle n^q \rangle / \langle n \rangle^q$. Parameters for m_0 and α are chosen to the same values as before. The experimental data for pp reaction are of 303 GeV/c [16]. For π^-p reaction, the experimental values are taken for 360 GeV/c by Firestone et al. [17]. Since it is probable for K^-p that the data have not reached the asymptotic value up to now, here we show only the theoretical estimates

q	pp		π^-p		K^-p
	c_q Theor.	c_q Exp.	c_q Theor.	c_q Exp.	c_q Theor.
2	1.239	1.245 ±0.015	1.246	1.243 ±0.059	1.219
3	1.805	1.816 ±0.051	1.822	1.802 ±0.093	1.732
4	2.97	2.99 ±0.14	3.01	2.94 ±0.19	2.77
5	5.42	5.43 ±0.39	5.49	5.27 ±0.45	4.91
6	10.7	10.7 ±1.0	10.9	10.3 ±1.1	9.4
7	22.6	22.4 ±2.8	22.9	21.5 ±2.9	19.4
8	50.3	49.6 ±7.9	50.9	48.1 ±8.1	42.1
9	117	115 ±22	118	114 ±23	95
10	282	278 ±63	284	283 ±67	226

the charged multiplicity distributions of all reactions, which have been observed in the available energy region, are well interpreted. The asymptotic mean value of the number of clusters comes from the information of the incident channels, while α is due to the type of reactions in which diffractive scattering contributes or not [3].

4. *Summary and discussions*

It has been long known that the particle correlations are accounted well by a cluster model with the intermediate formation of clusters and subsequent decay of hadrons from clusters. As regards the charged multiplicity distributions in hadron-hadron collisions several investigations denoted the cluster model to be an adequate picture to deduce the KNO scaling function. We have advanced those investigations in this study making use of the fruitful cluster distribution for lepton-hadron and hadron-hadron interactions including annihilation reactions.

With respect to indication that neutrino-nucleon scattering is annihilation-like process like $\bar{p}p$ (or e^+e^-) [3], we have proposed the well fitted KNO scaling functions for νp,

e^+e^- and $\bar{p}p$ reactions respectively on the basis of the cluster model in contrast with those of the ordinary hadron-hadron collisions. In terms ot the cluster model our results interprete well the feature that the multiplicity distributions of annihilation-like reactions are significantly narrower than those of non-annihilation reactions. The difference comes from how to make clusters, i.e. clusters are produced with the fixed number or not.

It should be noticed that one can see the characteristic feature of annihilation-like process for evaluation of the dispersion of the multiplicity distribution. It follows from Eqs. (8) and (10), provided n is proportional to n_c, that

$$D_c = \sqrt{\langle n_c^2 \rangle - \langle n_c \rangle^2} \simeq (\sqrt{1/m_0})\, \langle n_c \rangle, \tag{16}$$

since $1/(m_0\alpha)^2 \ll 1$ for annihilation-like processes (see Section 3). We see a good agreement of Eq. (16) with the relevant experimental data [1, 2, 19, 20] of νp, e^+e^- and $\bar{p}p$[7] which show the similar behavior (see Fig. 7).

Let us refer to two parameters m_0 and α in our cluster model. Though the asymptotic mean value of the cluster number varies from 6 to 8 according to the incident channels in this analysis, it is probable to expect them as an extension of the cluster number in cosmic-ray region [13]. We may recognize those as a prediction of the asymptotic number of quark and gluon jets. As concerns α of the cluster distribution it seems that $\alpha = 1 \sim 2$ for annihilation-like reactions like $\bar{p}p$, νp, $\mu^- p$, ep and e^+e^-, i.e. fixed number clusters are produced asymptotically in these reactions, while $\alpha = 0.3 \sim 0.4$ in ordinary hadron-hadron collisions appear to demand the distribution of the cluster number which is also expected for γp reaction. Our model shall be examined by the experiments for the charged multiplicity distribution of other reactions which have not been performed.

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