Vol. B13 (1982) ACTA PHYSICA POLONICA No 4

SUPERUNIFICATION*

BY J. ELLIS

CERN, Geneva**

MARY K. GAILLARD***

LAPP, Annecy-le-Vieux, France

AND B. ZUMINO***

CERN, Geneva

(Received October 6, 1981)

We explore scenarios for exploiting supersymmetry in unified field theories. Supersymmetry must be broken at some level, and we discuss theories with different supergaps. In particular we review the possibility that the hierarchy problem may be alleviated in simply supersymmetric grand unified theories, thanks to certain non-renormalization theorems. Supersymmetric grand unified theories are not truly unified since the supersymmetry generators do not carry internal quantum numbers and gravity is not included. We therefore favour unification in extended supergravity theories, and analyze possible schemes of this type, including the possibility that one supersymmetry is retained down to low energies in the form of a supersymmetric GUT. A major problem of unification in extended supergravity theories is the disposal of unwanted helicity states, and we propose some new ideas for achieving this. Our favoured approach postulates the existence in the physical spectrum of infinite-dimensional unitary representations of the noncompact symmetry groups of extended supergravities. Such representations could enable the unwanted helicity states to be paired off, leaving a desirable spectrum of states with masses much less than the Planck mass.

PACS numbers: 12.20.Hx, 12.40.-y

---

* Presented at the XXI Cracow School of Theoretical Physics, Paszkówka, Poland, May 28 — June 9, 1981.

** Address: CERN, 1211 Genève 23, Switzerland.

*** Present address: Department of Physics, University of California, Berkeley, CA 94720, USA.

I. INTRODUCTION

1. *Preliminary remarks*

Many theorists take the point of view that supersymmetry [1], particularly in its local realization through supergravity, is so elegant that it must be true. The only remaining questions are how, when and where? In this paper we review [2] various different approaches to combining supersymmetry with gauge theories of the strong, weak and electromagnetic interactions. We emphasize in particular our own approach [3] which seeks to identify all the known "elementary" particles as composite states, bound on the Planck scale, of "preons" which are the elementary quanta of the $N = 8$ supergravity theory [4]. We mention in this connection some new ideas we have been pursuing, which may shed some light on the fate of the $N = 8$ supergravity composite states which are not apparently observed at energies much less than the Planck mass. Perhaps these composite fields never bind to form well-defined particle states? Or perhaps they do, but these states have unobservable hyperweak interactions characterized by inverse powers of the Planck mass? Or perhaps all these bound states acquire masses of order the Planck mass? This last possibility is allowed group-theoretically if the physical spectrum of the theory contains a unitary representation of the non-compact E_7 group underlying the $N = 8$ supergravity theory.

The outline of this paper is as follows. First we review the framework of the unified gauge theories [5–7] which one may want to supersymmetrize, high-lighting outstanding problems which such a supersymmetrization might solve. Then we discuss in sections II and III various alternatives for the energy scale at which supersymmetry may become apparent. These include the scale of weak-electromagnetic unification of order 10^2 to 10^3 GeV [8–11], the conjectured grand unification scale of about 10^{15} GeV [12, 13], and the Planck mass of 10^{19} GeV [3, 14–18]. We emphasize in particular recent ideas about the dynamical breaking of supersymmetry on an energy scale of order 1 TeV (the so-called supercolour or supersymmetric technicolour scenarios [9–11]), and the possibility of constructing a supersymmetric GUT [12, 13]. We finish in section III with an extended discussion of attempts to use composite fields from extended supergravity theories, starting from reminders of the work of Cremmer and Julia [4] and of our own [3], continuing through more recent work by Frampton [16], by Derendinger, Ferrara and Savoy [17] and by Kim and Song [18], and culminating in our own recent ideas mentioned above. At the end of the paper we summarize open problems in the pursuit of superunification.

2. *Framework in which supersymmetry may be included*

In Fig. 1 we sketch the generally accepted picture of fundamental particle interactions, ordered by the hierarchy of decreasing distance scales. The first stage of $SU(3) \times U(1)$ for the strong and electromagnetic interactions is uncontroversial, but one may harbour some lingering doubts whether $SU(2) \times U(1)$ is the full group [5] of the electroweak unification encountered at energies between 10^2 and 10^3 GeV. Then there is the great desert which may extend to the conjectured scale of grand unification estimated to be around 10^{15} GeV. This desert is as yet uncharted, and may contain oases as well as mirages —

which category fits technicolour [19]? In fact, even the existence of a desert is conjectural, since we have no experimental evidence for or against any interaction scales between those of the weak interactions and gravitation. The best evidence for GUTs would be the detection of proton decay. Even if the principle of grand unification is valid, it could be

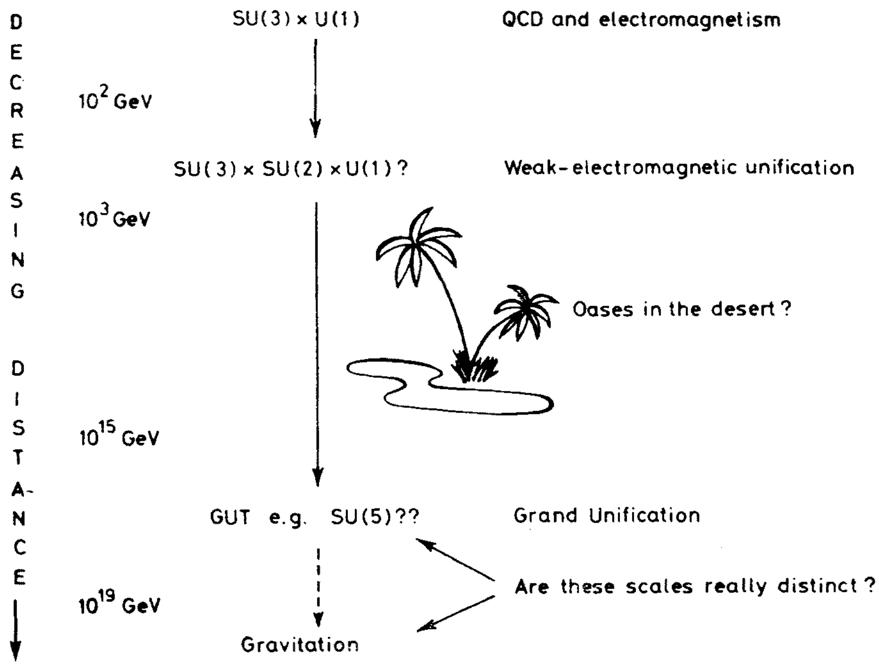


Fig. 1. An impressionistic sketch of fundamental particle interactions, arranged by order of decreasing distance scales

that its energy scale is rather closer to that of gravitation, and less distinct from it than is the case in simple models such as SU(5) [6]. In this case the proton lifetime could well be unobservably long, and then a conclusive test of the grand unification idea would be hard to come by.

Contemplation of Fig. 1 reveals no shortage of problems that supersymmetry may be called upon to solve, while one should always recall that supersymmetrization may trail new hazards in its wake, owing to the fact that no supersymmetric partner of any elementary particle has ever been detected. For example: if one supersymmetrizes QCD, where are the partners — gluinos and squarks — of the conventional gluons and quarks? Among the problems which supersymmetry might be able to solve are the choices of gauge group at various different energy scales — which may be restricted if one is to be able to include the gauge theory in an extended supergravity theory [2, 4]. One may also hope that the choice of fermion representations is also restricted by supersymmetry, and there is in fact a severe danger that it will be over-restricted. It seems highly desirable, both phenomenologically and theoretically in the framework of GUTs, that the "fundamental" fermions lie in a chiral representation of the intermediate energy $SU(3)\times SU(2)\times U(1)$ group. This is possible in the context of simple $N = 1$ supersymmetry, but not in gauge theories with $N > 1$. It seems that if extended supersymmetries appear anywhere in physics they can only appear in the context of an extended supergravity theory at an energy scale $\simeq 10^{19}$ GeV. Furthermore this extended supergravity theory must find some way of acquiring a spectrum of fermions which is complex with respect to the internal symmetry group: "breaking reality" so as to be left with some fermions at low energies which are chiral, but not excessively so.

Returning to the recital of problems which supersymmetry may be called upon to solve, one is the origin of the hierarchies of mass scales in Fig. 1. Why is the weak scale $m_W \simeq 10^2$ GeV $\ll$ the grand unification scale $m_X \approx 10^{15}$ GeV (perhaps in turn $\ll$ the Planck mass $\approx 10^{19}$ GeV)? Two aspects of the hierarchy problem [20] should be distinguished: one is how to fix $m_W \ll m_X$ without specifying some bizarre relation between coupling constants to dozens of decimal places, while the other aspect is how to maintain this relation despite the depradations of radiative corrections. Dynamical symmetry breaking [19] may in principle provide answers to both these problems, but the existing models are cumbersome, unaesthetic, and susceptible to phenomenological difficulties with flavour-changing neutral currents [21] and the proliferation of light pseudo-Goldstone bosons [22]. Supersymmetry enables [23] one to maintain a hierarchy if it is imposed at the tree level [12], but no model exists yet where the imposition of the hierarchy condition emerges in an elegant way as a consequence of some higher symmetry [24]. In addition, one must eventually break supersymmetry, as it is not immediately apparent in the low energy spectrum. If this is done explicitly [12, 13, 25], one is left to wonder why this is done at a scale much less than the Planck mass, even though it may be technically "naturally" small in the sense of not acquiring destructively large radiative corrections. An alternative to the explicit breaking of supersymmetry is dynamical breaking [9, 10], although it is not yet clear [11] that this is possible in four-dimensional models of interest.

The problems of naturalness arise when one has scalar bosons in the theory, and one might question whether there exist any "fundamental" scalars, in the sense of not being composite on a distance scale much larger than the scale of compositeness of fermions and gauge bosons. The proliferation of quark and lepton flavours strongly suggests that they are composite, but the theory of electroweak unification [5] assumes this is not manifest at energies < 100 GeV. Many theorists explore schemes wherein quarks and leptons are composite on a scale of order 10^3 GeV or so [26], whereas grand unification [6, 7] postpones the scale of compositeness beyond 10^{15} GeV. Supersymmetry may help specify the scale of compositeness, and we will see for example that it is necessary [14, 15, 3, 27, 28] to assume that the known quarks, leptons and gauge bosons are composite on a scale of 10^{19} GeV, if not before, if one is to be able to embed them in an extended supergravity theory.

We abstract from this section's recital of problems the message that supersymmetry may be relevant to physics on any of a number of different energy scales, and of course we also know that it is not an exact symmetry all the way down to zero energy. We are therefore faced with the basic question: What is the supergap? Subsequent sections of this paper comment on the possibility [8] that it is $\lesssim 10^2$ GeV (the region we now probe in accelerator experiments), is of order 10^3 to 10^4 GeV (the so-called supercolour or supersymmetric technicolour scenarios [9, 10]), the possibility that GUTs are supersymmetric [12, 13], and the idea we favour [3] that supersymmetry plays a rôle in combining particle physics with gravitation on a scale of order 10^{19} GeV.

II. UNIFICATION AND SIMPLE SUPERSYMMETRY

1. Supergap $\lesssim 10^2$ GeV?

In this section we discuss the possibility pursued energetically by Fayet [8], frequently in collaboration with Farrar [29], that simple supersymmetry is spontaneously broken along with the electroweak $SU(2) \times U(1)$ ($\times$?) symmetry. Such a theory abounds in unobserved low mass superpartners of known particles which we denote by wiggles: e.g. $g \to \tilde{g}$, $\nu \to \tilde{\nu}$. For example, it is known [8, 29] that the gluinos $\tilde{g}$ must have a mass $\gtrsim 2$ or 3 GeV because of upper limits in beam dump experiments on events of the type $p+N \to \tilde{g}+X$, followed by $\tilde{g} \to \tilde{\nu}+X$ decay and a subsequent secondary interaction of the $\tilde{\nu}$ in a detector. Gluinos could also show up in heavy quarkonium decays, but the limits from charmonium decays (e.g. of ${}^3\tilde{g}_1 \to g+(\tilde{g}+\tilde{g})$) are not as stringent as from the beam dump experiments, and studies of bottomonium decays are not yet sufficiently precise to rule on the gluinos' existence. Much more stringent limits on sleptons (possible supersymmetric partners of leptons) come from PETRA: for example one knows from the absence of $e^+e^- \to \tilde{\mu}+\bar{\tilde{\mu}}$, $\tilde{\mu} \to \tilde{\nu}+\mu$ decays that any smuon $\tilde{\mu}$ must have a mass $\gtrsim 15$ GeV, while any selectron $\tilde{e}$ must have a mass $\gtrsim 16$ GeV [8].

Such limits conflict with general sum rules in $SU(2) \times U(1)$ weak gauge theories for the masses of scalar partners $\tilde{f}$ of known fermions f.

$$m_{\tilde{f}}^2 - m_f^2 = \alpha Q_f^\gamma + \beta Q_f^Z \tag{1}$$

where Q_f^γ and Q_f^Z are the electric and weak charges of the fermion in question and the α and β are model-dependent constants of proportionality. To avoid this conflict, it has been proposed [30] that the electroweak gauge group be extended to $SU(2) \times U(1) \times U(1)$ by the inclusion of a new neutral gauge boson with axial vector couplings to quarks and leptons. In doing so, care must be taken to avoid anomalies involving the new U(1) current. In the simplest versions of such a theory one has the general bounds

$$\left(\frac{m_{\tilde{f}_1}^2 + m_{\tilde{f}_2}^2}{2}\right)^{1/2} \lesssim \tfrac{1}{2}\, m_{W^\pm} \approx 40 \text{ GeV} \tag{2}$$

for the two scalar partners of a light fermion f, which is uncomfortable but cannot be excluded by experiments before the next generation of e^+e^- storage rings. Furthermore, the bound (2) can be relaxed if need be, by extending the Higgs structure of the theory.

There are strong experimental constraints on the form of neutral currents which prevent the existence of a new neutral gauge boson with large axial vector couplings and a mass

$O(m_Z)$. A possible way [9, 30] to realize the SU(2) × U(1) × U(1) model is to make the new gauge boson U very light and with a very small gauge coupling constant g':

$$m_U \sim g'/g m_W \tag{3}$$

if there is no extra SU(2) $\underline{1}$ Higgs field. If the U boson is very light then its gauge interactions are small compared with its residual pseudoscalar interaction through the divergence of the axial current, and its phenomenology is similar to that of a traditional axion. The beam dump experiments then restrict its mass to

$$m_U < 1 \text{ MeV or between } (7{,}300) \text{ MeV} \tag{4}$$

and future searches in K, J/ψ, Υ and toponium decays are indicated [9, 30, 31].

While the existence either of light supersymmetric partners of known fermions or of a very light gauge boson cannot be rigorously excluded on the basis of present experiments, the class of supersymmetric theories discussed in this section looks rather unhealthy, even if it is not dead yet.

2. *Supergap* $\sim 10^3$ *or* 10^4 *GeV*

In this section we discuss various attempts [9–11] that have been made to introduce supersymmetry on a scale $\lesssim$ 1 to 10 TeV as a solution to the "naturalness" problems associated with elementary scalar fields. Putting aside the question whether some higher symmetry exists, elementary scalars such as those in the Weinberg-Salam model in general acquire quadratically divergent contributions to their masses from radiative corrections:

$$\delta m_s^2 \sim \Lambda^2, \tag{5}$$

where the cut-off Λ could range up to $m_{PL} \sim 10^{19}$ GeV. The radiative correction (5) would be larger than the physical m_s^2, and hence "unnatural", if $\Lambda > O(1)$ TeV.

An empirical approach to this problem has been taken by Veltman [32], who has examined explicitly the one-loop quadratic divergences in δm_H^2 and δm_W^2 for the Weinberg-Salam model [5]. He finds that these two quadratic divergences are cancelled if a simple sum rule is obeyed:

$$\sum_{\text{fermions f}} \frac{m_f^2}{m_W^2} = \tfrac{3}{2} + \frac{3}{2\cos 2\theta_W} + \tfrac{3}{4}\frac{m_H^2}{m_W^2}. \tag{6}$$

He makes the point that the quadratic divergences in higher loop diagrams also involve possible superheavy ($m \sim m_X$? m_{PL}?) particles so that it is not obviously crazy to use (6) to solve the one-loop problem and leave the superheavies to sort out the higher loops. If one assumes that there are only 3 generations of fermions and that $m_H \lesssim m_W$, one finds from (6) that

$$m_t \approx (69 \text{ to } 78) \text{ GeV}. \tag{7}$$

This prediction is sufficiently precise to be testable, though it would require LEP to see such a heavy top quark. Veltman [32] feels that the relation (7) would be "suggestive of supersymmetry if true".

The problem of cancelling the quadratic divergences (5) has been studied systematically in several recent papers [9, 11, 33]. Quadratic divergences are not automatically avoided even in supersymmetric theories. If one has a U(1) gauge group factor with an associated gauge supermultiplet $V = (A_\mu, \chi, D)$, then one can encounter a quadratic divergence in the coefficient of the D term in the Lagrangian if its dimension 4 terms do not exhibit a parity invariance: $V \leftrightarrow -V$ [9]. Of course one could avoid this problem by constructing a theory with no U(1) factor at an energy scale $\Lambda \sim 1$ TeV. No-one has constructed a particularly satisfactory weak interaction model where the low energy $SU(2)_L \times U(1)$ group is absorbed into a simple group at such a low energy scale, and such a theory would contain monopoles of mass $\lesssim 10$ TeV which might prove to be cosmologically embarrassing. An alternative is to specify the conventional Weinberg-Salam U(1) hypercharges of light particles so as to make the theory "safe" from quadratic divergences. An obvious way to do this is by introducing right-handed fermion doublets to "mirror" the known fermions, but this makes it difficult to understand the fermion matrix, since the left- and right-handed fermions could combine into an $SU(2) \times U(1)$ invariant mass term. Actually, such "mirroring" is not necessary: Dimopoulos and Raby [9] noticed that the conventional fermions with their usual U(1) hypercharge assignments did not give a quadratically divergent D term in low orders of perturbation theory, and Witten [11] observed that this was true to all orders if the fermions were "grand unifiable", i.e. could be embedded in representations of a simple group. Even this requirement is not necessary: it turns out [33] that all one needs to eliminate the divergent D term is the sum rule

$$\sum_f Y_f = 0 \tag{8}$$

a condition of which the above examples are special cases.

In a realistic model supersymmetry must be broken, and the scale should not be much larger than 1 TeV if the scalars in the theory are to be protected from acquiring naturally large masses. Two scenarios for supersymmetry breaking have recently been investigated: one is dynamical and the other is explicit.

In the former case one supposes [9, 10] that there is a new set of asymptotically free exact gauge interactions which become strong on a scale Λ_{sc} of order 1 to 10 TeV. These are supposed to form vacuum condensates of vector, spinor and scalar fields:

$$\langle 0|F_{\mu\nu}F^{\mu\nu}|0\rangle = O(\Lambda_{sc}^4), \quad \langle 0|\psi\psi|0\rangle = O(\Lambda_{sc}^3), \quad \langle 0|ss|0\rangle = O(\Lambda_{sc}^2), \tag{9}$$

which violate supersymmetry. It is not necessary, and may even be desirable, that these condensates do not break the weak interaction $SU(2)_L \times U(1)$ symmetry. This can be done by still another set of technicolour interactions which are strong on a scale of order 1 TeV. An obvious question is why not economize by abolishing one of these two new sets of strong interactions or telescoping them into the same scale. If one brings supercolour interactions down to the technicolour scale one runs a more serious risk that particles with masses $O(\Lambda_{sc})$ will mediate flavour-changing interactions at an unacceptably high level [10], and one tends to get an axion with characteristics similar to those of the original Peccei-Quinn-Weinberg-Wilczek [34] axion which seems to be experimentally excluded. On the other hand, it is claimed [10] that a theory without a lower energy technicolour scale, i.e. so that $SU(2)_L \times U(1)$ breaking comes about through Higgs vacuum expectation values, runs into problems with unwanted extra U(1)'s, real Goldstone bosons and axions.

In the explicit model of Dine, Fischler and Srednicki [10], the elementary scalar fields are protected from acquiring masses or vacuum expectation values until supersymmetry is broken by the supercolour interactions. One then has

$$\langle 0|H|0\rangle \simeq g_{Y_s}\Lambda_{sc}^3/m_{\tilde{H}}^2, \tag{10}$$

where g_{Y_s} is a Yukawa coupling and

$$m_{\tilde{H}}^2 = O(\alpha)\,(m_{\tilde{W}}^2, m_{\tilde{B}}^2), \tag{11}$$

and the $\tilde{W}$ and $\tilde{B}$ are supersymmetric partners of the $SU(2)_L \times U(1)$ gauge fields, with masses

$$m_{\tilde{W},\tilde{B}} \sim O(g_2^2)\Lambda_{sc}. \tag{12}$$

The Higgs vacuum expectation values (10) give masses to the ordinary fermions:

$$m_f \approx g_{Y_f}\langle 0|H|0\rangle \approx g_{Y_f}g_{Y_s}\Lambda_{sc}^3/m_{\tilde{H}}^2, \tag{13}$$

but do not give all the masses of the $W^{\pm}$ and Z^0:

$$m_W^2 = \frac{g_2^2}{4}\,[F_T^2+\langle 0|H|0\rangle^2], \tag{14}$$

where $F_T \approx 250$ GeV is the conventional [19] technipion decay constant. Putting in plausible numbers: $g_{Y_s} \approx g_{Y_f} \approx e = 0.3$, and $m_t \approx 30$ GeV, one finds from (13) that $\langle 0|H|0\rangle \approx 100$ GeV and hence from (10, 11 and 12) that

$$\Lambda_{sc} \sim 10 \text{ to } 100\,\text{TeV}, \tag{15}$$

with $m_{\tilde{W}}$ and $m_{\tilde{B}}$ each a few TeV and $m_{\tilde{H}}$ a few hundred GeV. However, since one cannot do reliable numerical calculations in the strong coupling regime, and the Yukawa couplings g_{Y_s} and g_{Y_f} are largely undetermined, the absolute scale of Λ_{sc} is difficult to estimate precisely.

There are some interesting superpartners in this theory which are expected to have relatively low masses. For example, it is expected [10] that the mass of the gluino $\tilde{g}$ is of order:

$$m_{\tilde{g}} \approx O(5)\,\text{GeV}, \tag{16}$$

while the mass of the fermionic partner of the Higgs $\tilde{H}$ is expected to be:

$$m_{\tilde{H}} \approx O(30)\,\text{GeV}. \tag{17}$$

The gluino lifetime is expected to be $O(10^{-10})$ seconds, so it should be quite detectable at present accelerators.

Many questions about these super/technicolour models remain unanswered. For example: is dynamical breaking of supersymmetry possible at all? (Witten [11] has raised questions about this). Is it really necessary to have technicolour interactions on a separate scale from supercolour? What is the scale of supercolour? Can one avoid the appearance of unacceptable axions? (Axions with masses much less than the original Paccei-Quinn-Weinberg-Wilczek [34] variant are excluded by astrophysical arguments [35].) However, at least these models meet all criteria of naturalness for scalar fields, while probably avoiding the worst of the flavour-changing neutral current problems that plagued [21] the original extended technicolour models. At the moment one's primary objection to such models may be aesthetic: they look rather cumbersome with an exact gauge group containing 5 factors (supercolour, technicolour, colour, $SU(2)_L$ and $U(1)$) and many supermultiplets (gauge, supermatter, technimatter, matter and Higgs), and it is not clear how the whole structure can be unified.

3. *Supersymmetric GUTs?*

The problem of "naturalness" for scalar fields becomes particularly acute in the context of GUTs [6, 7], whose usual incarnations contain at least two sets of Higgs fields with vacuum expectation values differing by 10^{12} or 10^{13}. It has been emphasized that this hierarchy of scales can only be realized at the expense of fine-tuning one or more combinations of parameters of the theory with amazing precision [20], and that this fine-tuning must be adjusted in each order of perturbation theory [20, 36]. Several authors [12, 13, 24] have recently investigated whether supersymmetry [23] can alleviate these hierarchy problems, with at least some success in dealing with radiative corrections to the fine-tuning.

The philosophy is to postulate a grand unified gauge group with simple symmetry, break it down at $O(10^{15}$ to $10^{18})$ GeV to the intermediate energy $SU(3)\times SU(2)_L\times U(1)$ group while keeping simple supersymmetry, and then finally break supersymmetry (susy) at an energy scale comparable with that where $SU(2)_L\times U(1)$ breaks down to the $U(1)$ of electromagnetism:

$$G_{GUT}\times(N=1\ \text{susy}) \underset{10^{15}\text{ to }10^{18}\text{ GeV}}{\rightarrow} SU(3)\times SU(2)_L\times U(1)\times(N=1\ \text{susy})$$
$$\underset{10^{2}\text{ to }10^{3}\text{ GeV}}{\rightarrow} SU(3)\times U(1). \tag{18}$$

The breaking of supersymmetry on a scale of 10^3 GeV or so may be either explicit or dynamical: the specific models proposed [12, 13] to date break supersymmetry explicitly because of the difficulties encountered when one tries to unify the dynamical models which were mentioned in the previous subsection.

The explicit breaking of supersymmetry is constructed to be soft (from terms of dimension $d < 4$ in the Lagrangian) and protects [12, 13, 25] the SU(2) doublet Higgs boson of the Weinberg-Salam model from acquiring a large mass. It is sometimes said rather lazily that supersymmetry can be used to protect the masses of scalars by putting them in multiplets where their fermionic partners are protected by chiral symmetry from acquiring large masses. This is not in fact the way things work in the supersymmetric GUTs proposed: instead these models use another remarkable property [23] of supersymmetric theories, namely that the only renormalization of coupling constants comes from wave function renormalization. The strategy is the following: one fine-tunes the parameters of the supersymmetric Higgs potential in such a way that the SU(2) doublet Higgs has very small (or zero) mass, while baryon-number violating Higgses have masses $\gtrsim O(10^{14}$ GeV). There is no improvement yet on standard GUTs but the peculiar renormalization properties [23] of supersymmetric theories ensure that this finely-tuned relationship can be maintained in all orders of perturbation theory. Thus the problem of radiative corrections is avoided, although the initial necessity to fine-tune is still present, so that the progress is rather technical. An obvious question is whether a suitably sophisticated G_{GUT} would in fact impose masslessness on the SU(2) doublet Higgs [24], in which case the initial fine-tuning problem would also be solved.

As long as the supersymmetry is eventually broken explicitly, there are also a number of dimensionful parameters specifying the mass differences between different superpartners which must also be fixed at values which are presumably much smaller than the Planck mass. (Note however that the allowable upper bounds on these dimensionful parameters have not all been determined.) Once again, supersymmetry generally ensures that these parameters only acquire logarithmically divergent radiative corrections, and they are therefore technically "naturally" small in the sense of 't Hooft. The corrections are not finite, but are proportional to the bare supersymmetry breaking masses, just as conventional radiative corrections to fermion masses are proportional to the bare fermion masses which break chiral symmetry. These properties of the radiative corrections apply as long as supersymmetry is not broken by explicit fermion masses or other terms which do not respect supersymmetric relations between dimension 3 coupling terms: such terms would induce quadratic divergences because they are less "soft". One is uneasy about having so many finely-tuned "small" mass parameters to break supersymmetry, even if they are in some sense natural, and this is a major motivation for the dynamical models [9–11] of the previous subsection.

There is one dramatic consequence of the supersymmetric GUTs discussed, namely that the grand unification mass can be substantially altered [13, 37, 38] with the nucleon lifetime perhaps becoming unobservably long. If one just includes in the evolution equations for the $SU(3)\times SU(2)\times U(1)$ intermediate energy gauge couplings the contributions of supersymmetric partners of the gauge bosons (gluinos and the like), one finds [37] that the rate of approach of the coupling constants is significantly reduced:

$$\frac{1}{\alpha_3(Q^2)} - \frac{1}{\alpha_2(Q^2)} = -\frac{11}{12\pi}\ln(m_X^2/Q^2) \rightarrow \frac{-9}{12\pi}\ln(m_X^2/Q^2). \tag{19}$$

This means that the estimate of $\ln(m_X/m_W)$ is increased by about 20% and m_X becomes $O(10^{17})$ GeV. In a realistic theory one must supplement equation (19) by the contributions of Higgs multiplets which have the effect [38] of somewhat reducing m_X again, but the proton lifetime is nevertheless estimated to be considerably longer than the usual 10^{31} years or so, unless one has several light Higgs supermultiplets. This circumstance at least provides us with a possible experimental test of supersymmetric GUTs; if nucleon decays are detected in the forthcoming generation of experiments, the simplest versions of these theories are excluded. However, observation of nucleon decay does not necessarily mean that supersymmetric GUTs are wrong. Indeed they make slightly less implausible an alternative mode of baryon decay via intermediate scalars. In a conventional GUT it is necessary that scalars have masses $O(10^{10}$ to $10^{11})$ GeV if their contribution is to be competitive with gauge boson exchange. In conventional GUTs it is unnatural to have scalars with masses $\ll m_X$, but this is technically natural in a supersymmetric GUT. Supersymmetrists need not despair if nucleon decays are observed soon!

III. UNIFICATION IN SUPERGRAVITY

1. *Models, and motivations for compositeness*

While the models of the previous section have several encouraging features, they are by no means completely satisfactory. One can also object that none of those models is truly superunified, in that each of them has simple supersymmetry grafted on to a more or less unified gauge group, and none of them combine gauge and supersymmetry transformations into a more comprehensive algebraic structure. Furthermore, none of the previous models gave a thought to gravity, whose understanding at the quantized level was one of the initial motivations for introducing supersymmetry [2]. Supergravity theories are candidates to fulfill all these objectives: the difficulty is that they are relatively few in number and their internal symmetry properties bear little obvious correspondence to our phenomenological requirements.

Supergravity theories [2] are characterized by $N = 1, 2, 3, 4, 5, 6$ or 8 local supersymmetry generators and have corresponding SU(N) global internal symmetries, with vector boson contents appropriate for gauging SO(N). It is believed that all these internal SO(N) symmetries may in fact be gauged. The constructions are completed [39] for $2 \leqslant N \leqslant 4$, the $N = 5$ case is almost completed and no problems are anticipated [40], while there are no known obstacles to the extension to $N = 8$. Gauging the internal symmetry is necessarily accompanied by the introduction of a cosmological constant:

$$\Lambda_{\text{cosmo}} \propto g^2 m^4_{\text{Planck}}. \tag{20}$$

It is not clear whether this is a virtue or an embarrassment: we want the presently observed value of Λ_{cosmo} to be zero, or at least microscopic on any particle physics scale, and this means that Λ_{cosmo} must have been non-zero before the spontaneous breakdown of gauge symmetries. Hawking and collaborators [27, 28] have argued in the context of quantum gravity that the apparent Λ_{cosmo} must be of order m^4_{PL} when it is measured over Planck volume scales. If one wants to introduce $\Lambda_{\text{cosmo}} \neq 0$ on such small scales, presumably one must either gauge supergravity or else violate local supersymmetry already at the

Planck scale. However, it is not obvious that one can do the latter while retaining even a simple $N = 1$ supersymmetry at larger distance scales, necessary for making some contact with the models of the previous section.

There is still another problem, though, namely that the internal symmetry of the maximal extended supergravity theory is insufficient [14] to contain all the "observed" low energy group, let alone a GUT:

$$SO(8) \not\supset SU(3)\times SU(2)\times U(1); \quad SO(8) \supset SU(3)\times U(1)\times U(1). \tag{21}$$

We see from (21) that even $N = 8$ supergravity does not contain fields which are candidates to be the $W^{\pm}$, and it is also impossible to accommodate all the observed fermion spectrum. Furthermore, the fermions are real with respect to SO(8), and it is not clear why they do not all acquire large masses, as happens in GUTs which do not have chiral fermion representations. These observations suggest that if supergravity is to be of any relevance, some [15, 4] at least of the known "elementary" particles must in fact be composites of preon fields taken from an underlying extended supergravity theory. And if some of the "elementary" particles are to be composite, why not all [3] of them?

This point of view gets some indirect support from recent work by Hawking and collaborators [27, 28] on the propagation of elementary particles through space-time foam. They claim [27] that any elementary scalar field would acquire a mass of order m_{PL} from propagating through the foam of pure gravity, while any elementary spin 1/2 field would acquire [28] a similar mass from propagating through the foam of a supergravity theory, at least if it were not protected by an exact gauge symmetry. If these arguments are correct, it might be impossible to imagine the original fields of an extended supergravity theory surviving as essentially massless down to energies $\ll m_{PL}$, and one may be forced to describe the known particles using composite fields.

The remainder of this section describes attempts to follow this line of thought, exploiting some concealed symmetries of supergravity theories.

2. Concealed symmetries of supergravities

It has been realized [4] that one can formulate SO(N) supergravity theories in such a way that they are invariant under a larger global "parent" group, e.g. non-compact E_7 in the case of $N = 8$ supergravity, with a local gauge symmetry e.g. SU(8) in the case of $N = 8$ supergravity. This gauge symmetry enables one to reduce to the physical degrees of freedom, e.g. from 133 scalars in the basic representation of E_7 via 63 SU(8) gauge transformations down to the 70 scalars in the $N = 8$ supergravity theory. The connections for these gauge transformations are not elementary, but composite fields containing for example combinations $(\partial_\mu v)v^{-1}$ where v is a matrix of scalar fields. They therefore do not correspond to any physical states in the original Lagrangian — they have no poles in their propagators. Cremmer and Julia [4] however conjectured that they might become physical, propagating states as a result of the supergravitational dynamics becoming strong on an energy scale $E \sim m_{PL}$. They emphasized a possible analogy with CP^{N-1} models in 2 dimensions [41], which start off with a "parent" SU(N) symmetry and have a gauge

U(1) symmetry. Quantum corrections generate a long range U(1) gauge potential which eventually confines the constituents in the two-dimensional model. A similar phenomenon has been found in a supersymmetric three-dimensional model by Nissimov and Pacheva [42], with the dynamics generating a physical long-range gauge particle. There are even arguments [43] that a nonlinear spinor theory in 4 dimensions will give rise to non-Abelian gauge interactions at energies $E \ll$ the cutoff Λ, presumably to be identified with m_{PL}. Notice that all these examples start with a local symmetry in the Lagrangian, in contrast with some other subconstituent models which hope to get a (possibly approximate) gauge theory from a symmetry which is initially global.

3. *"Fundamental" particles as composites*

If one is willing to suggest that gauge bosons may be composite, it is natural to go further and suggest [3] that all the observed "elementary" particles — quarks, leptons and Higgs scalars — may also be bound states made out of preons appearing in the basic supermultiplet of some extended supergravity. The only low-mass particle which might be left as truly "elementary" is the graviton which is a singlet of all the symmetries in the extended supergravity theories. All other "fundamental" particles would be composite with a binding scale of order m_{PL}.

The first problem which then arises is the choice of composite supermultiplet(s) with which to identify the known particles. Since the gauge connection $\supset (\partial_\mu \mathrm{v})\mathrm{v}^{-1} + \ldots$ is similar to an SU(8) current, it is natural to start [3] with the massless supercurrent multiplet shown in Table I.

TABLE I

Helicity λ	$-3/2$	-1	$-1/2$	0	$+1/2$	$+1$	$+3/2$	$+2$	$+5/2$
Representation R	$\overline{8}$	63 $+1$	216 $+8$	420 $+28$	504 $+56$	$\overline{378}$ $+70$	$\overline{168}$ $+\overline{56}$	$\overline{36}$ $+\overline{28}$	$\overline{8}$

The first row of representations have one upper and several antisymmetrized lower indices $R^{\mathrm{A}}_{[\mathrm{BC}\ldots\mathrm{F}]}$, while the second row contains trace representations $R^{\mathrm{A}}_{[\mathrm{AC}\ldots\mathrm{F}]}$ which have been removed from the top representations to make them irreducible. There is then a question what to do with the trace representations: originally we discarded [3] them by analogy with the SU(8) singlet vector field which cannot be identified with a gauge field in the analysis of Cremmer and Julia [4], though this immediately implies the breaking of supersymmetry [44].

The next problem is how to embed the observed "low-energy" symmetries into this representation structure. Even if one does not worry about embedding a GUT such as SU(5), there is a difficulty with embedding $SU(3)_{\mathrm{colour}}$ in a vector-like way so as to conserve parity in the strong interactions. It is in fact impossible [3] if one retains all the composite fields in the massless supercurrent multiplet, and it is confidently conjectured that this problem remains if one uses any finite set of supermultiplets which is not completely vector-like with respect to all the internal symmetry group, as in the case of a set of massless supermultiplets making up a complete massive supermultiplet. This result has been proved by Frampton [16] for the restricted case where the maximal helicity in each supermultiplet is in an n-fold completely antisymmetric representation, and each supermultiplet appears with multiplicity 0 or 1.

It therefore seems that in order to make contact with the real world, one must either select judiciously among the supermultiplet states to be retained *and/or* use an infinite set of supermultiplets.

4. *A possible superGUT*

We propose [3] that one proceed by demanding that the only particles left with masses $\ll m_{PL}$ should be a subset with renormalizable interactions. This means that there should be no light particles with $|\lambda| > 1$, and that the only particles with $|\lambda| = 1$ should be gauge bosons. It means further that the residual fermions should be free of anomalies with respect to the low-energy gauge group. It is also possible that there may be small non-renormalizable interactions at low energies scaled by inverse powers of the Planck mass: $(E/m_{PL})^{n\geq 1}$. In our search for candidate superGUTs obeying these criteria, and starting off from the massless supercurrent multiplet shown in Table I for the case $N = 8$, we established [3] the two following theorems.

Theorem 1: The only extended supergravity theory whose massless supercurrent multiplet is large enough to include enough particles for a GUT is the largest $N = 8$ supergravity with its local SU(8) symmetry.

Theorem 2: The only plausible GUT group which can be included in the $N = 8$ supergravity theory is SU(5).

Clearly other frequently discussed [7] GUT groups such as SO(10) and E_6 are not subgroups of SU(8). We also found two indications that SU(8) breaks down directly to SU(5), and that SU(6) and SU(7) are not likely to be useful intermediate symmetry groups. The first reason is that the only SU(6) or SU(7) anomaly-free representations of fermions in the massless supercurrent multiplet which are vector-like with respect to $SU(3)_{colour}$ are also vector-like with respect to the full SU(6) or SU(7) group, giving rise to the familiar problems in understanding the fermion mass spectrum. The second reason is that if any component of the $\underline{420}$ of scalar fields in Table I acquires a vacuum expectation value, then SU(6) and SU(7) get broken. The maximal simple group which has a chiral fermion representation that is vector-like with respect to colour SU(3) is SU(5). It is possible [17] to combine this with an SU(2) generation (or family) group which is automatically anomaly--free.

It is natural to ask what is the maximal (in the sense of the largest number of helicity states) SU(5) anomaly-free representation of left-handed fermions vector-like with respect to colour SU(3) which one can extract from the massless supermultiplet of Table I. It turns out [3] to be

$$(\underline{45}+\overline{\underline{45}})+4(\underline{24})+9(\underline{10}+\overline{\underline{10}})+3(\overline{\underline{5}}+\underline{5})+9(\underline{1})+3(\overline{\underline{5}}+\underline{10}). \qquad (22)$$

We are encouraged to see that the representation (22) contains three generations of chiral SU(5) fermions, which agrees with our phenomenological prejudices. To fit the observed particles requires at least 3 generations, the SU(5) etc. GUT calculation of the b quark mass fails [36, 45] if there are more than 3 low-mass generations, and cosmological nucleosynthesis is uncomfortable [46] with more than 3 or perhaps 4 almost massless neutrinos. Another amusing observation is that the representation (22) contains large numbers of fermions which are vector-like with respect to SU(5) and can hence acquire masses of order m_X to m_{PL} (10^{15} to 10^{19} GeV). In fact there is so much vector-like stuff with masses $\geqslant m_X$ that above all thresholds [3]

$$\beta_{SU(5)} = 147\tfrac{1}{2} \tag{23}$$

in a normalization convention where the usual scenario with N_G generations and ignoring Higgses and superheavy particles would yield $-55+4N_G$. With such a large positive β-function it is possible [47] that the gauge coupling constant becomes of order unity at the Planck mass, as working this trick only requires

$$\langle\beta_{SU(5)}\rangle \approx 70 \tag{24}$$

in the energy range between m_X and m_{PL}, as indicated in Fig. 2. There is therefore no need for non-perturbative supergravity to manufacture somehow a small coupling α_{GUT} at the

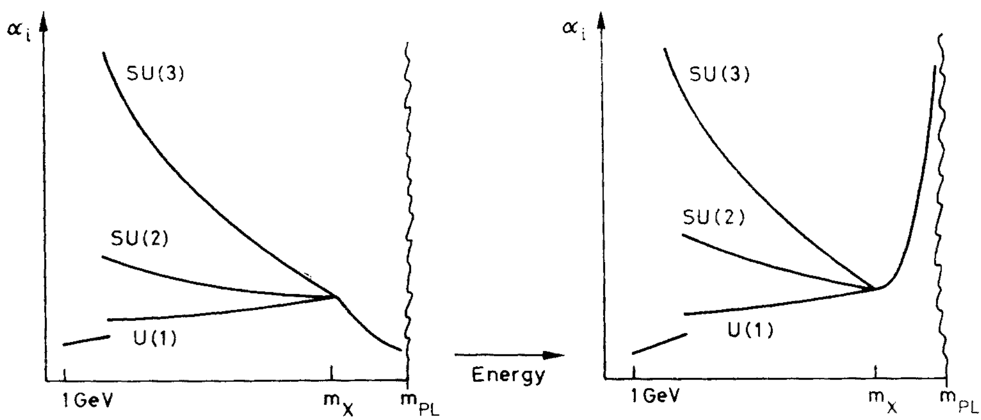


Fig. 2. The conventional picture (a) of the renormalization of gauge coupling constants is modified (b) above the grand unification mass in theories with a large number of superheavy particles of mass $O(m_X)$. The wriggly vertical lines in this and the following figure indicate the onset of quantum gravity effects of $O(1)$

Planck mass. The smallness of $\alpha_{GUT} \approx 1/42$ at $m_X \simeq 10^{15}$ GeV — and hence the smallness of the fine structure constant also — find a natural explanation through the existence of very many massive particles: $m_X \lesssim m \lesssim m_{PL}$.

It is natural to ask how the value of $m_X \approx O(10^{-4}$ to $10^{-5})m_{PL}$ could be generated. Some time ago a possible solution was proposed [48] in the context of conventional GUTs, which is illustrated in Fig. 3. The Higgs self-couplings are supposed to be of order g^2 (the gauge coupling squared) at the Planck mass. Then because they are not asymptotically free, they decrease as the energy decreases. When a suitable combination of them decreases to $O(g^4)$, radiative corrections to the Higgs potential become important and can cause symmetry breaking [49]. Calculations [48] show that they naturally generate $m_X = O(10^{-4}$ or $10^{-5})m_{PL}$. It is possible that this scenario may be adapted to our superGUT picture as also shown in Fig. 3. Although we have not made any explicit calculations in this case,

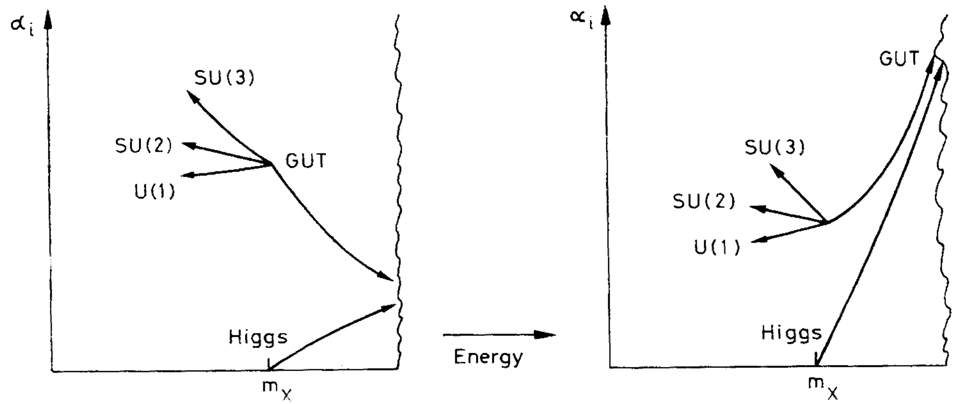


Fig. 3. A possible scenario (a) for the generation of the grand unification mass-scale by radiative corrections when an appropriate grand unified Higgs coupling becomes $O(g^4)$, which may be carried over (b) into a theory where the gauge coupling increases at energies $\gtrsim m_X$, as in Fig. 2b

it seems quite possible that a relic of the previously unique asymptotic unfreedom of the multi-Higgs couplings will still drive them to zero at an energy scale of order m_X, triggering spontaneous symmetry breaking in the same way as before.

It is clear that the elimination of unwanted helicity states does violence to the supersymmetry of the underlying theory: hitherto we have presumed that all supersymmetries are dynamically broken in the process of generating an effective renormalizable "low energy" gauge theory. An alternative possibility is that one of the original eight supersymmetries survives, so that one can play all the supersymmetric GUT games of section II. 3. Imposing a simple supersymmetry in addition to anomaly freedom restricts us to a smaller fermion spectrum than that of Eq. (22), and we find that (with or without traces) the maximal allowed spectra do not lead naturally to a three generation theory, though non-maximal spectra may contain enough states to embed any of the models of section II. 3. For example, including traces the maximal three generation spectrum is:

$$6(\underline{1})+4(\underline{24})+(\underline{45}+\overline{\underline{45}})+3(\underline{10}+\overline{\underline{10}})+3(\underline{5}+\overline{\underline{5}})+3(\underline{10}+\overline{\underline{5}}); \tag{25}$$

and without traces we can keep:

$$3(\underline{1})+4(\underline{24})+(\underline{45}+\overline{\underline{45}})+2(\underline{10}+\overline{\underline{10}})+(\underline{5}+\overline{\underline{5}})+3(\underline{10}+\overline{\underline{5}}). \tag{26}$$

The spectra increase by one $(\underline{10}+\overline{\underline{10}})$ and two $(\underline{5}+\overline{\underline{5}})$ if we identify the basic 8 as right-handed instead of left-handed as was done above.

Another remark concerns the admissibility of a global Peccei-Quinn [34] U(1) chiral symmetry in the superGUT framework [50]. There are certainly sufficiently many degrees of freedom to accomodate such a $U(1)_{PQ}$ in the originally proposed superGUT (22), or even in the smaller susy GUT versions (25) or (26). However, it is not clear how such a $U(1)_{PQ}$ can be generated dynamically in a plausible way. It should be noticed that the candidate superGUT (22) has a sufficiently complex structure to generate enough (n_B/n_γ), thereby avoiding a pitfall [51] of the minimal SU(5) axion model [50].

These examples indicate that the superGUT framework is sufficiently rich to incorporate many features desired in supersymmetric and other GUTs.

5. *Objections and alternatives*

The philosophy [3] that all "elementary" particles are bound states of preons from supergravity is by no means compelling, and even if one accepts it, there are many objections that one can make to the specific realization described in the previous subsection. Why choose only the massless supercurrent multiplet? Why throw away the trace representations? What happens to all the unwanted helicity states in the supermultiplet? How do the dynamics discard them and select a subset of states with renormalizable interactions? In view of these questions it is to be expected that alternatives to the superGUT of the previous subsection have been proposed [17, 18].

One alternative philosophy has been adopted by Derendinger, Ferrara and Savoy (DFS) [17]. As was done before, they discard all states with $|\lambda| > 1$ and all states of $|\lambda| = 1$ which are not adjoints of the SU(N) group. They demand that the residual set of $|\lambda| = 1/2$ fermions be real under SU(5) with the exception of some number of conventional chiral SU(5) $\bar{5}+\underline{10}$ generations. Finally, they demand that theory be free of anomalies for the full SU(N) gauge group. They construct such theories out of supermultiplets whose maximum helicity state is in an n-fold totally antisymmetric representation, and demand that the multiplicity of each supermultiplet be 0 or 1. Starting with the different SU(N) theories they find the following numbers of generations:

$$\begin{array}{ccccl} N=5 & N=6 & N=7 & N=8 & \\ 2 & 1 & 1 & 3 & \text{generations,} \end{array} \tag{27}$$

which seems to favour the $N = 8$ theory. Unfortunately, not all the supermultiplets they need in their models can be constructed as bound states of finite numbers of preons. It is easy to see from the work of Cremmer and Julia [4] that states of integer (half-integer) spin can only have an even (odd) number of SU(8) indices if they are to be interpretable as bound states of preons, and this is not true for the supermultiplets originally used by DFS. They have also considered models in which the maximum helicity state has a two--column Young tableau representation, getting in this way 5 generations from the $N = 8$ theory, but again at the expense of using multiplets which cannot be bound states of supergravity preons. If they impose this restriction, they [17] have to be prepared to use supermultiplets with multiplicities greater than one, and it turns out that SU(5) $\bar{5}+\underline{10}$ generations can only be obtained in pairs.

Kim and Song [18] propose a model in which several supermultiplets are used, and the only trace representations retained in the effective gauge theory are those of fermions. Optimistically, they also discard high helicity composite fields, and disregard the constraint that the SU(8) supermultiplets be obtainable as composites of $N = 8$ preons. A distinctive feature of their model is that they break SU(8) down to SU(5)×SU(3) and use the latter factor as a technicolour group. In fact, in their model, some of the observed low-mass fermions are composites of hyperfermions, and hence composite on a scale $\ll m_{PL}$.

We do not feel that any of these schemes has strong arguments for preferring it over the original superGUT scheme [3]. Also, many of the previously unanswered questions still remain unanswered: what happens to the unwanted helicity states with $|\lambda| \geq 1$? (though none of the $|\lambda| = 1/2$ states are discarded in the DFS scheme) how do the dynamics select which renormalizable low-energy subtheory? The remainder of this paper is devoted to three different possible fates for the unwanted helicity states:

— They were never bound in the first place;
– Some of them are present in the theory as massless states with very weak couplings;
– They all exist, and have acquired masses by combining with "partner" helicity states.

6. *Possible fates of unwanted helicity states*

A. They were never bound?

Is it possible that the dynamics of the extended supergravity theory could avoid forming bound states in unwanted helicity channels, while forming the bound states that are desirable? Earlier we have taken the point of view that the binding dynamics could implement Veltman's theorem [32] by forming poles corresponding to a renormalizable subset of composite fields – gauge spin 1, anomaly-free spin 1/2 and spin 0 — while singularities associated with nascent unrenormalizability could preclude binding in other composite channels.

Soluble supersymmetric non-linear σ models in 2 and 3 dimensions do not shed much light on this problem. In the two-dimensional case [41] anomalies give a mass to the composite gauge field and its superpartner acquires the same mass. This is an interesting example how a potentially anomalous gauge interaction can be removed from the low energy theory [52]. The three-dimensional model [42] exhibits a strong coupling phase in which both the gauge fields and their superpartners exhibit zero mass poles. In neither case are there any high helicity fields that one might fear could become physical and mess up renormalizability. In both cases simple supersymmetry is preserved by the dynamics.

In our case the breaking of some supersymmetries is necessary, since as noted previously low energy ($\ll m_X$) phenomenology can tolerate at most $N = 1$ supersymmetry, and at least some high helicity states from the supermultiplet of Table I must disappear since there is no known consistent interacting field theory for particles of helicity > 2. As far as we know, there is no theorem or principle requiring the existence of a bound-state pole in all composite channels. Indeed, there are even arguments to the opposite effect. Weinberg and Witten [53] have argued that there can be no massless (composite or elementary) state with helicity $|\lambda| > 1/2$ in a theory with a Lorentz-covariant conserved current, and no massless state with helicity $|\lambda| > 1$ in a theory with a Lorentz-covariant energy-momentum tensor. However, these technical assumptions of Lorentz covariance exclude most cases of physical interest. Thus gauge theories possess vector currents which are Lorentz--covariant only to within a gauge transformation, and theories of gravitation — such as extended supergravities — do not possess a Lorentz-covariant energy-momentum tensor either. In the case of $N = 8$ extended supergravity there are currents associated with rigid SU(8) transformations, and explicit construction [54] shows that they are covariant only up to an Abelian gauge transformation. Therefore the Weinberg-Witten [53] theorem is inapplicable to our problem — fortunately, since it would have been disastrous if it had been relevant.

We conclude this discussion in the belief that our knowledge of strong coupling dynamics is not sufficient either to exclude or to sanction the hypothesis that the unwanted helicity states in our massless supercurrent multiplet just were never bound in the first place.

B. Are the unwanted helicity states in fact present?

What if the unwanted helicity states do bind, and show up in the physical spectrum? Unless one can find mechanisms to marry them off with partners so as to acquire large masses (see discussion C), they may be around as zero (or low) mass particles. This is for the most part phenomenologically disastrous, but not universally so. Massless states of high helicity are tolerable because their couplings are characterized by inverse powers of an intrinsic mass parameter, in our case the Planck mass [55]. We will not prove this here in all generality, but give a simple illustrative example. It suffices for our purposes to consider helicity amplitudes for $2 \leftrightarrow 2$ scattering of massless particles. Ader, Capdeville and Navelet (ACN) [56] have shown that dimensionless amplitudes F have the following kinematic singularities:

$$F(1+2 \to 3+4) = (\sqrt{s})^{\lambda_s} (\sqrt{-t})^{\lambda_t} (\sqrt{-u})^{\lambda_u} \tilde{F}, \tag{28}$$

where s, t and u are the conventional Mandelstam variables and

$$\begin{aligned} \lambda_s &\equiv |\lambda_1+\lambda_2+\lambda_3+\lambda_4|, \\ \lambda_t &\equiv |\lambda_1-\lambda_2-\lambda_3+\lambda_4|, \\ \lambda_u &\equiv |\lambda_1-\lambda_2+\lambda_3-\lambda_4|, \end{aligned} \tag{29}$$

and the dynamical structure is manifested in the kinematic singularity-free amplitude $\tilde{F}$. As a warm-up, consider spin 1/2 + spin 1/2 → spin 1/2 + spin 1/2:

$$\lambda_1 = 1/2, \quad \lambda_2 = -1/2, \quad \lambda_3 = \pm 1/2, \quad \lambda_4 = \mp 1/2. \tag{30}$$

In this case

$$\lambda_s = 0, \quad \lambda_t = 0 \text{ or } 2, \quad \lambda_u = 2 \text{ or } 0 \tag{31}$$

so that

$$F \sim (u \text{ or } t)\tilde{F}. \tag{32}$$

The amplitude $\tilde{F}$ can contain a direct channel photon (or gluon) pole $\sim 1/s$ so that the final dimensionless amplitude

$$F \sim \left(\frac{u \text{ or } t}{s}\right) \tag{33}$$

need contain no dimensionful parameter (cf. the well-known amplitudes and cross-sections for $e^+e^- \to \mu^+\mu^-$, $q\bar{q}$ or $q\bar{q} \to \mu^+\mu^-$, $q'\bar{q}'$). Now consider the possible production of a massless helicity 3/2 particle in e^+e^- or $q\bar{q}$ collisions via spin 1/2 + spin 1/2 $\to$ helicity 3/2 + spin 1/2:

$$\lambda_1 = 1/2, \quad \lambda_2 = -1/2, \quad \lambda_3 = 3/2, \quad \lambda_4 = 1/2. \tag{34}$$

In this case

$$\lambda_s = 2, \quad \lambda_t = 0, \quad \lambda_u = 2,$$

and if we put in a direct channel photon (or gluon) pole we get

$$F \sim \left(\frac{us}{s}\right)\tilde{F}, \tag{35}$$

and for F to be dimensionless we need a factor in $\tilde{F}$ with the dimension of $(\text{mass})^{-2}$, which can in our case only be $(m_{PL})^{-2}$. The cross-sections for singly producing such a massless helicity 3/2 particle at present energies would therefore be completely negligible. A similar analysis can be made of the process

$$\lambda_1 = 1/2, \quad \lambda_2 = -1/2, \quad \lambda_3 = 3/2, \quad \lambda_4 = -3/2, \tag{36}$$

with the conclusion that the dimensionless amplitude

$$F \sim (tu^2)\tilde{F}. \tag{37}$$

If we look for a direct channel photon (or gluon) pole in $\tilde{F}$, this time we get a coefficient $O(m_{PL})^{-4}$. This may seem surprising since one might imagine giving the helicity 3/2 state a conventional dimensionless minimal gauge coupling. However, the only known [57] field-theoretically consistent way of doing this involves giving a non-zero mass to the spin 3/2 particle, which we have excluded by hypothesis.

One can make a similar analysis of scattering amplitudes involving massless spin 1 particles. There it is well-known that individual Feynman diagrams give amplitudes which would lead to cross-sections blowing up at large s, but that one can get cancellations between different Feynman diagrams with direct and crossed-channel exchanges, which make the theory renormalizable, if and only if the couplings are those of a gauge theory. Since the unwanted states of helicity ± 1 in the supermultiplet of Table I do not lie in an adjoint representation, they cannot have gauge couplings. Hence they must run into power-counting problems analogous to equation (35) (which can easily be checked using the ACN rules) and hence have low-energy production cross-sections suppressed by powers of the Planck mass. Similar arguments work *a fortiori* for states of higher helicity.

There is one possible way in which one might anticipate a possible very weak low energy coupling to show up: if it can interact coherently with large amounts of matter as does the graviton. Clearly any other massless spin 2 field which is a singlet of the low energy exact gauge symmetry $SU(3)_{colour} \times U(1)$ could have such an interaction, and clearly such states abound in the supermultiplet of Table I. We have no excuse why their exchanges should not show up with a magnitude comparable to conventional graviton exchange: perhaps they do? Although the exchanges of bosons with non-trivial $SU(3) \times U(1)$ transformation properties of fermions cannot act coherently on large bodies of matter, one might worry whether multiple exchanges could, e.g. the simultaneous exchange of two

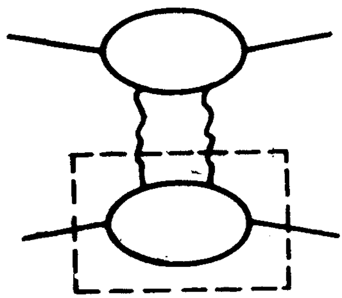


Fig. 4. An attempt to generate a long-range potential by the exchange of a pair of states of helicity 3/2

helicity 3/2 fermions shown in Fig. 4. If these two fermions were massless there would be no mass gap and a long-range potential could result. However, the helicity rules (28,29) applied to the matter-matter-helicity 3/2 — helicity 3/2 amplitude F of Fig. 4 ensure a strong suppression at the $t \to 0$ tip of the crossed-channel branch cut which dominates the long--range potential, making it unobservably small.

It therefore seems that certain of the unwanted helicity states might indeed be present and have interactions with known particles which are too weak to be detected directly experimentally. This does not necessarily mean however that they can have no observable consequences. As has been emphasized by Steigman, Olive and Schramm [58], their gravitational interactions cause the Universe to expand faster than would be expected on the basis of the observed degrees of freedom, and this could affect primordial nucleosynthesis. From the successful comparison of the cosmological ^{4}He abundance and calculations assuming a conventional expansion rate with three species of light neutrinos, one can infer [46, 58] that the Universe could only have been expanding a factor ξ faster than the conventionally assumed expansion rate with two low-mass neutrinos, where

$$\xi^2 - 1 < 0.15 \tag{38}$$

at the time of nucleosynthesis. In general one has

$$\xi^2 = 1 + \tfrac{4}{43}\, g_d \left(\frac{T_d}{T}\right)^4, \quad g_d = \tfrac{7}{8}\, g_f + g_b, \tag{39}$$

where g_f and g_b are the numbers of fermions f and bosons b which decouple from the rest of matter at a temperature T_D, and T_d is the (time-dependent) effective temperatures of f and b, which we assume to be universal. At the time of nucleosynthesis we have

$$\left(\frac{T_d}{T}\right) = \left(\frac{43}{4g(T_D)}\right)^{1/3}, \tag{40}$$

where $g(T_D)$ is the total number of helicity states which were thermally active at the decoupling temperature T_D, including observable fermions F and bosons B, as well as the ones which decouple. The condition (38) requires

$$g_b + \tfrac{7}{8} g_f < 0.07[g(T_D)]^{4/3}. \tag{41}$$

Equation (41) is satisfied for the desired values of g_b, g_f since if we include all the helicity states in the original supermultiplet of Table I

$$g(T_D) = 1920. \tag{42}$$

While it is unlikely that all of the unwanted states in the original supermultiplet of Table I could be massless with very weak interactions characterized by inverse powers of the Planck mass, this possibility cannot yet be ruled out by the nucleosynthesis constraint [46, 58]. However this constraint could perhaps be useful if it were tightened up in the future.

C. Have the unwanted states combined with "partner" helicity states to acquire large masses?

It was recalled [3] in subsection II.3 that it is probably impossible to embed a vector-like $SU(3)_{colour}$ group in a theory with any plausible finite set of complete massless supermultiplets, the only known exceptions being superpositions of massive supermultiplets in which case there is no obvious reason why all the states do not have masses $O(m_{PL})$. Thus one must either take only some judicious subsets of complete supermultiplets, as was done in subsection II.4, or else one must postulate an infinite set of supermultiplets [16]. It is interesting to note in this connection that Grisaru and Schnitzer [59] have recently advanced arguments that scattering amplitudes in extended supergravity theories may Reggeize. They have deduced the quantum numbers of the physical zero mass states which should appear on some of these Regge trajectories, and have shown that they correspond to a set of supermultiplets which include the one in Table I on which we have been focussing our attention. However, they also find other supermultiplets, and it seems likely that a full study of all scattering amplitudes will reveal an infinite set of supermultiplets. In fact, previous to their work, we had already been led to imagine a scenario in which such an infinite set might actually be present in the physical spectrum.

It is a striking fact that the physical spectra in both 2- and 3-dimensional non-linear σ-models [41, 42, 60] contain unitary representations of the "parent" global symmetry, e.g. $SU(N)$ in the case of 2-dimensional CP^{N-1} models [60] and $SO(N)$ in the 3-dimensional model of Nissimov and Pacheva [42]. In unpublished work, Rabinovici [61] has studied 2-dimensional non-linear σ-models with a "parent" non-compact SO(N, 1) symmetry. He found an apparent infinity of solitons in the physical spectrum: could these form a unitary representation of the "parent" non-compact symmetry? If so, one would be led to hypothesize that the physical (composite) spectrum of an extended supergravity theory may not only contain unitary representations of the local symmetry (SU(8) in the case of the $N = 8$ theory) but in fact full unitary representations of the non-compact global "parent" symmetry (non-compact E_7 in the case of the $N = 8$ theory). Unitary representations of these non-compact groups would of course contain infinite sets of unitary representations of their compact local subgroups. These then reopen the possibility of "partner" helicity states combining with all the unwanted helicity states in the original massless supermultiplet of Table I.

The structure of unitary irreducible representations of U(1, 1) is well-known [62], and the analogous Barut force bosonic constructions of unitary irreducible representations of some non-compact groups appearing in supergravity theories have also been made recently [63]. However, these methods have not yet been extended to the case of the non-compact E_7 group encountered in $N = 8$ supergravity. We would like to know not only what are the appropriate unitary representations of non-compact E_7 which can be obtained using the $\underline{70}$-dimensional scalar fields of the $N = 8$ theory, but also how one can represent the supersymmetry transformations on them and whether this requires the use of reducible E_7 representations at each spin. For orientation we consider first the simpler case of $N = 4$ supergravity, which is mathematically more tractable.

The $N = 4$ theory is invariant under SU(4)$\times$SU(1, 1). The scalar fields of the theory can be described by a complex scalar z which is an SU(4) singlet. Under infinitesimal SU(1, 1) transformations:

$$L_+ z = i, \quad L_- z = iz^2, \quad L_0 z = -iz,$$
$$L_+ \bar{z} = -i\bar{z}^2, \quad L_- \bar{z} = -i, \quad L_0 \bar{z} = i\bar{z}, \tag{43}$$

$$[L_+, L_-] = -2iL_0,$$
$$[L_0, L_+] = iL_+,$$
$$[L_0, L_-] = -iL_-, \tag{44}$$

here L_+ is the hermitean conjugate of L_-, and L_0 is the generator of the compact U(1) subgroup. The group SU(1, 1) acts non-linearly on the fundamental supergravity multiplet (preons) in the way described explicitly in Ref. [54]) for the more general case of Sp($2n$, R), where here $n = 1$ [Sp(2, R) = SU(1, 1)].

Multiple commutators of SU(1, 1) generators with a supersymmetry generator produce new fermionic generators, which in turn, when anticommuted with each other give rise to generalized momentum operators. In this way, starting from the SU(1, 1) algebra together with the original supersymmetry algebra, one generates an infinite dimensional superalgebra. It is easy to see that the commutator of an SU(1, 1) transformation with a local supersymmetry transformation of parameter $\varepsilon(x)$ gives a supersymmetry transformation of parameter $\varepsilon'(x)$, where

$$L_+: \quad \varepsilon' = \frac{i}{4}\,\bar{z}\gamma_5\varepsilon,$$

$$L_-: \quad \varepsilon' = \frac{i}{4}\,z\gamma_5\varepsilon,$$

$$L_0: \quad \varepsilon' = -\frac{i}{4}\,\gamma_5\varepsilon, \tag{45}$$

plus field-dependent transformations. Since we wish to find the rigid (global) algebra generated by supersymmetry and SU(1, 1), we have to consider the asymptotic limit of large x. In this limit all fields vanish except for the scalars which tend to some constant, and therefore we drop all field-dependent terms except the supersymmetry transformations with parameters (45), in which asymptotically both ε and z are constant. This result appears plausible since a generator of supersymmetry transformations is expected to transform under SU(1, 1) like the spin 3/2 field which is a gauge field for supersymmetry. By repeating the process we see that multiple commutators give rise to the fermionic generators

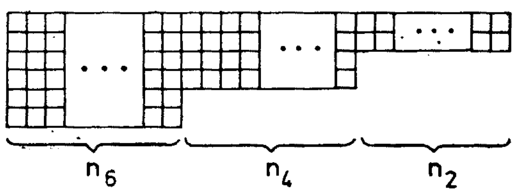
$$Q_{\mathrm{A},n,m} = \bar{z}^n z^m Q_{\mathrm{A}}$$

$$\bar{Q}^{\mathrm{A}}{}_{,n,m} = \bar{z}^n z^m \bar{Q}^{\mathrm{A}} = \overline{Q_{\mathrm{A},m,n}}, \tag{46}$$

where z and $\bar{z}$ are the asymptotic constant values of the scalars and Q_{A} and $\bar{Q}^{\mathrm{A}}$ carry opposite helicities. Without writing here the complete infinite dimensional superalgebra we note that the above construction bears a strong similarity to that by which infinite Kac--Moody [64] algebras are obtained from ordinary Lie algebras. While in the Kac-Moody case one makes the generators of a Lie algebra functions of the points on the unit circle (the group U(1)) in our case we make the generators of the original supersymmetry algebra functions of points in the quotient space SU(1, 1)/U(1). Clearly the infinite dimensional superalgebra admits SU(1, 1) as a group of automorphisms since this group can be represented on the quotient space. Observe that in the Kac-Moody case the infinite algebra has c-number terms similar to Schwinger terms, which are necessary for the existence of certain unitary representations. It is possible that such c-number terms are also needed in our case. This type of algebraic structure is also reminiscent of the Virasoro algebras encountered in dual models [65]: it would indeed be remarkable if the supergravities eventually turned out to have a deeper connection with dual theories.

Some of the above results can be generalized to the more complicated case of $N = 8$ supergravity where the group is non-compact E_7. The quotient space E_7/SU(8) can be parametrized by a matrix Z constructed from the scalar fields which are in the $\underline{70}$ of SU(8). The non-compact generators of E_7 operating on the eight supersymmetry charges will generate an infinite number of supersymmetry charges which transform under SU(8) like the product of an 8 with a symmetrized power of 70's. Similarly they will generate generalized momenta and these together with the supersymmetry charges will form an infinite dimensional supersymmetry algebra. We do not have to write it down to guess at the form of a representation. If we start with a supermultiplet of $N = 8$ supergravity and apply to it the non-compact generators of E_7 we obtain an infinite set of states. A basis for this set consists of the original supermultiplet plus supermultiplets whose SU(8) content for each helicity is a product of the original SU(8) representation for the same helicity with a symmetric product of 70's. This infinite multiplet can be thought of as obtained from the original multiplet of $N = 8$ supergravity by multiplying it by an arbitrary number of scalar fields, but we consider it as a linear set of states on which one can obviously represent both the generators of E_7 and the original supersymmetry charges. Consequently the full infinite algebra obtained by commuting these quantities an arbitrary number of times can also be represented. In this infinite multiplet every representation which appears for a given helicity recurs an infinite number of times and therefore the E_7 representations are not irreducible. However it could be that the infinite supermultiplet is irreducible as a representation of the infinite supersymmetry algebra.

It is easy to satisfy oneself that the symmetric products of 70's of scalar fields include an infinite number of times each and every SU(8) representation with the Young tableau representation

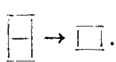


(47)

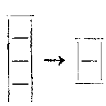
where

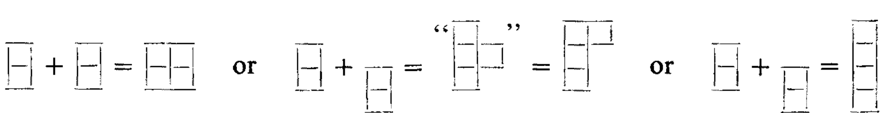
$$n_6 + n_2 = \text{even}. \tag{48}$$

To see this, it is convenient to lump together the two boxes representing a pair of antisymmetrized SU(8) indices:



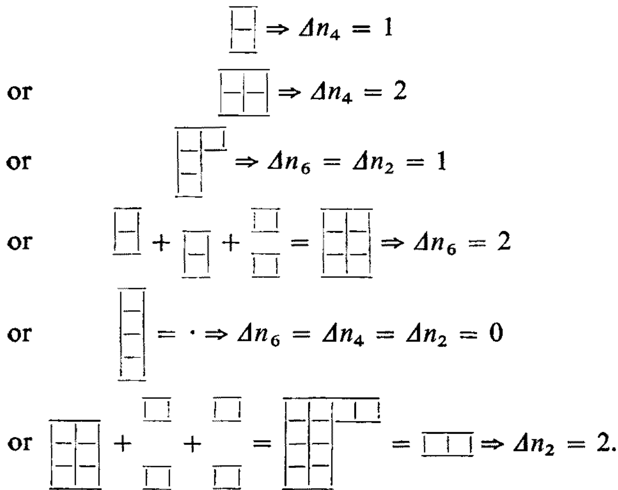
(49)

Then adding a new 70 =  in a symmetric way to an already symmetrized product of 70's requires a complete overlap (or mismatch) of one or both "double boxes" of the new 70 with those of the last 70 previously laid down, i.e.



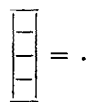
(50)

Suppose we already have an arbitrary representation of the type (47) with n_6+n_2 even. Then we can clearly add to it



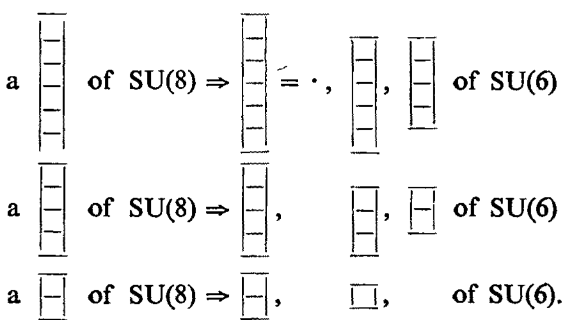
(51)

Games like this enable us to construct any other representation with n_6+n_2 even, and in particular adding an arbitrary number of


(52)

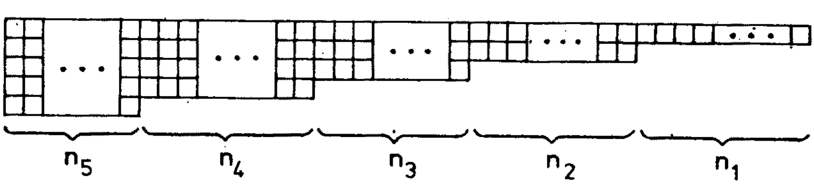
enables to replicate any such representation an arbitrary number of times.

Given an infinite number of every representation of the type (42, 43), if we break SU(8) down to SU(6) it is then easy to see that the symmetric products of $\underline{70}$'s contain an infinite number of every type of representation of SU(6). Reverting to conventional single box Young tableau notation,



(53)

Hence we can construct an arbitrary representation of SU(6)



(54)

with no restriction on the numbers of times $(n_5, n_4, n_3, n_2, n_1)$ that the various heights of columns appear. This means that all unitary representations of noncompact E_7 obtained by multiplying a "base" representation $R(\lambda)$ by arbitrary symmetric products of $\underline{70}$ scalar fields contain every representation of SU(6) an infinite number of times.

It is known [66] that an irreducible representation of non-compact E_7 cannot contain more than k copies of any k-dimensional representation of SU(8) However, it is very probable that supersymmetry cannot be represented using a set of irreducible E_7 representations at each helicity. (Recall that the representations $R(\lambda)$ in Table I are reducible as far as SU(8) is concerned.) Nevertheless, one can ask whether it is true that an irreducible unitary representation of non-compact E_7 contains every representation of SU(6) an infinite number of times. The answer is clearly yes, since there is an infinite set of distinct SU(8) representations obtained in the E_7 series which all contain the same SU(6) representation. To see this, recall from (48) that a $\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \end{array}$ of SU(8) contains a singlet of SU(6), as does a $\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}$ of SU(8), so that we can always get more copies of any given SU(6) representation by going to SU(8) representations with more $\underline{70}$ scalars and taking the appropriate element in the reduction with respect to SU(6).

Infinite dimensional representations are peculiar objects for which the conventional arguments about real vs complex representations become rather ambiguous, since there infinitely many ways of establishing correspondences between different helicity states. Consider for example the trivial SU(3) example of discrete infinities of $\underline{3}$ and $\underline{\bar{3}}$ representations. It would be natural to pair them up as follows:

$$\begin{array}{cccc} \underline{3} & \underline{3} & \underline{3} & \ldots \\ \updownarrow & \updownarrow & \updownarrow & \\ \underline{\bar{3}} & \underline{\bar{3}} & \underline{\bar{3}} & \ldots \end{array} \qquad (55)$$

so that no massless state survives. However, one could equally well set up the correspondence

$$\begin{array}{cccccccc} \underline{3} & & \underline{3} & & \underline{3} & & \ldots \\ & \swarrow\!\!\nearrow & & \swarrow\!\!\nearrow & & \swarrow\!\!\nearrow & \\ \underline{\bar{3}} & & \underline{\bar{3}} & & \underline{\bar{3}} & & \ldots \end{array} \qquad (56)$$

in which one massless triplet survives, or indeed any finite number of $\underline{3}$ or $\underline{\bar{3}}$ representations. The infinite-dimensional representations of SU(6) that we have look *a priori* very chiral, but we can use the liberty of helicity matching exemplified by (55) and (56) to leave any arbitrary finite subset of massless helicity states.

It is easy to set up a systematic procedure for matching unwanted helicity states with states of identical internal group ($\leqslant$ SU(6)) representation content but different helicities, as required if these unwanted states are to be able to acquire large masses $O(m_{PL})$. Start with an arbitrary representation $R(\lambda_0)$ of some helicity λ_0 from our original massless SU(8) supercurrent multiplet, and a set of non-compact E_7 representations with helicities including λ_0 and $-\lambda_0$. Find for each helicity in the set the smallest number of $\underline{70}$'s which give a copy of the desired representation $R(\lambda_0)$ and remove this copy of $R(\lambda_0)$ leaving behind at each helicity an infinite number more copies of $R(\lambda_0)$, as argued above. Repeat the procedure with the first unwanted state of helicity $\lambda_0 - 1$, belonging say to a representation $R(\lambda_0 - 1)$. And so on down through all the helicities down to $-\lambda_0$. Iteration can remove all the unwanted helicity states from our original supercurrent multiplet. We can then go on to do the same matching for unwanted helicity states in the newly introduced non-compact E_7 multiplets. Since there are an infinite number of copies of every representation R at every helicity, we can continue this process indefinitely, always finding a possible set of helicity partners for any unwanted helicity state.

In this way we can find possible ways of giving masses to all unwanted helicity states, leaving behind any arbitrary desired finite, null or even infinite subset of desired helicity states. Of course this simple group-theoretical argument gives no inkling of any criterion for deciding which (if any) desired helicity states avoid acquiring large masses. However, it does suggest that the structure of composite states in the $N = 8$ extended supergravity may be sufficiently rich to dispose of all the unwanted helicity states in the massless supercurrent multiplet we discussed previously.

IV. OPEN PROBLEMS IN SUPERUNIFICATION

We have seen that despite all the interest and activity in supersymmetric versions of various unified gauge theories, it is still by no means clear that supersymmetry is relevant to Nature (or vice versa). *A fortiori*, it is even less clear what the supergap either should be or may be.

In section II we saw that a supergap [8, 29] lower than 10^2 GeV cannot yet be excluded, even if it may seem implausible to some of us. We also saw that a supergap [9–11] of order 10^3 to 10^4 GeV may help solve various problems of mass scale hierarchies and dynamical symmetry breaking, but no completely satisfactory model yet exists. Looking further afield, the search for proton decay may tell us [13, 37, 38] whether supersymmetry is relevant at all at energies $\lesssim 10^{15}$ GeV.

In section III we turned to attempts at unification in supergravity theories [2,4]. At this point we do not even know the complete catalogue of such theories: can the $N \geqslant 6$ supergravities be gauged, and if so, is this useful? We saw that unification in extended supergravity seems to require the use of composite states. However, difficult questions are then raised about the fate of unwanted helicity states. Much of section III was devoted to possible answers to these questions: maybe some were never bound, maybe some are light but interact undetectably weakly, maybe all the unwanted states found "partner" helicity states and acquired large masses $O(m_{PL})$.

The investigation of these possibilities requires much more understanding of the dynamics of supersymmetric theories than we now possess. We do not know [11] whether non-perturbative dynamical breaking of supersymmetry is even possible, let alone whether the apparent symmetries of extended supergravities might be broken in the bizarre ways that we seem so find necessary.

However, it is at least possible to imagine that in extended supergravity we do have the ultimate physical theory, and what we have to do next is learn to solve it.

We would like to thank P. Frampton, M. Gell-Mann and M. Günaydin for useful discussions.

REFERENCES

[1] D. Volkov, V. P. Akulov, *Phys. Lett.* **46B**, 109 (1973); J. Wess, B. Zumino, *Nucl. Phys.* **B70**, 39 (1974).

[2] For a review of supersymmetry, see: P. Fayet, S. Ferrara, *Phys. Rep.* **32C**, 249 (1977); and for a review of supergravity, see: P. Van Nieuwenhuizen, *Phys. Rep.* **68**, 189 (1981).

[3] J. Ellis, M. K. Gaillard, L. Maiani, B. Zumino, *Unification of the Fundamental Particle Interactions*, ed. by S. Ferrara, J. Ellis and P. Van Nieuwenhuizen, Plenum Press, New York 1980, p. 69; J. Ellis, M. K. Gaillard, B. Zumino, *Phys. Lett.* **94B**, 343 (1980).

[4] E. Cremmer, B. Julia, *Phys. Lett.* **80B**, 48 (1978); *Nucl. Phys.* **B159**, 141 (1979).

[5] S. L. Glashow, *Nucl. Phys.* **B22**, 579 (1961); S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967); A. Salam, Proc. 8th Nobel Symposium, Stockholm 1968, ed. by N. Svartholm, Almqvist and Wiksells, Stockholm 1968, p. 367.

[6] For a prototype grand unified theory (GUT), see: H. Georgi, S. L. Glashow, *Phys. Rev. Lett.* **32**, 438 (1974).

[7] For reviews of GUTs, see: J. Ellis, *Gauge Theories and Experiments at High Energies*, ed. by K. C. Bowler and D. G. Sutherland, Scottish Universities Summer School in Physics, Edinburgh 1981, p. 201; P. Langacker, *Phys. Rep.* **72C**, 185 (1981).

[8] P. Fayet, *Unification of the Fundamental Particle Interactions*, ed. by S. Ferrara, J. Ellis and P. Van Nieuwenhuizen, Plenum Press, New York 1980, p. 517; and talk presented at the 1981 Rencontre de Moriond, Ecole Normale Supérieure preprint LPTENS 81/9 (1981).

[9] S. Dimopoulos, S. Raby, I. T. P., Santa Barbara preprint (1981).

[10] M. Dine, W. Fischler, M. Srednicki, I. A. S., Princeton preprint (1981).

[11] E. Witten, Princeton University preprint (1981).

[12] S. Dimopoulos, H. Georgi, Harvard University preprint HUTP-81/A022 (1981).

[13] N. Sakai, Tohoku University preprint TU/81/225 (1981); K. Harada, N. Sakai, Tohoku University preprint TU/81/226 (1981); R. K. Kaul, Tata Institute preprint, TIFR/TH/81-32 (1981); R. K. Kaul, P. Majumdar, Indian Institute of Science, Bangalore preprint (1981).

[14] M. Gell-Mann, Talk at the 1977 Washington Meeting of the American Physical Society (unpublished).

[15] T. Curtright, P. G. O. Freund, *Supergravity*, ed. by P. Van Nieuwenhuizen and D. Z. Freedman, North-Holland, Amsterdam 1979, p. 197.

[16] P. H. Frampton, *Phys. Rev. Lett.* **46**, 881 (1981). The need for infinite sets of supermultiplets and the possible rôle of unitary representations of non-compact groups was mentioned in: B. Zumino, Proc. 1980 Madison Int. Conf. on High Energy Physics, ed. by L. Durand and L. G. Pondrom. A. I. P., New York 1981, p. 964; and *Superspace and Supergravity*, ed. by S. W. Hawking and M. Roček, Cambridge University Press, Cambridge 1981, p. 423.

[17] J. P. Derendinger, S. Ferrara, C. A. Savoy, CERN preprint TH-3025 (1981) (unpublished) and *Nucl. Phys.* **B188**, 77 (1981).

[18] J. E. Kim, H. S. Song, Seoul National University preprint (1981).

[19] For some influential references, see: S. Weinberg, *Phys. Rev.* **D13**, 974 (1976); **D19**, 1277 (1979); L. Susskind, *Phys. Rev.* **D20**, 2619 (1979); and for a recent review, see: J. Ellis, Talk presented at the 1981 SLAC Summer Institute, CERN preprint TH-3139 (1981).

[20] E. Gildener, S. Weinberg, *Phys. Rev.* **D13**, 3333 (1976); E. Gildener, *Phys. Rev.* **D14**, 1667 (1976).

[21] S. Dimopoulos, J. Ellis, *Nucl. Phys.* **B182**, 505 (1981).

[22] J. Ellis, M. K. Gaillard, D. V. Nanopoulos, P. Sikivie, *Nucl. Phys.* **B182**, 529 (1981), and references therein.
[23] J. Wess, B. Zumino, *Phys. Lett.* **49B**, 52 (1974); J. Iliopoulos, B. Zumino, *Nucl. Phys.* **B76**, 310 (1974); S. Ferrara, J. Iliopoulos, B. Zumino, *Nucl. Phys.* **B77**, 413 (1974); See also E. Witten, Ref. [11].
[24] Some progress in this direction may be possible; S. Dimopoulos, F. Wilczek, I. T. P., Santa Barbara preprint (1981).
[25] K. Harada, N. Sakai, Ref. [13].
[26] For a review of composite models, see M. A. B. Bég, Talk presented at the 1981 Lisbon International Conference on High Energy Physics, Rockefeller University preprint RU 81/B/9 (1981).
[27] S. W. Hawking, D. N. Page, C. N. Pope, *Phys. Lett.* **86B**, 175 (1979); *Nucl. Phsy.* **B170**, (FS1) 283 (1980).
[28] B. F. Whiting, talk presented at the 1981 Heisenberg Symposium, Munich, based on work done in collaboration with S. Hawking (unpublished).
[29] G. R. Farrar, Proceedings of the Cornell Z^0 Theory Workshop, ed. by M. E. Peskin and S.-H.H. Tye, CLNS 81-485 (1981), p. 453.
[30] P. Fayet, *Phys. Lett.* **84B**, 416 (1979); *Nucl. Phys.* **B187**, 184 (1981).
[31] P. Fayet, M. Mezard, Ecole Normale Supérieure preprint LPTENS 81/2 (1981).
[32] M. Veltman, *Acta Phys. Pol.* **B12**, 437 (1981).
[33] W. Fischler, H. P. Nilles, J. Polchinski, S. Raby, L. Susskind, *Phys. Rev. Lett.* **47**, 757 (1981); A simple proof of this result is given in B. Zumino, Talk given at the 1981 Lisbon Int. Conf. on High Energy Physics, CERN preprint TH-3167 (1981).
[34] R. Peccei, H. R. Quinn, *Phys. Rev. Lett.* **38**, 1440 (1977); *Phys. Rev.* **D16**, 1791 (1977); S. Weinberg, *Phys. Rev. Lett.* **40**, 223 (1978); F. Wilczek, *Phys. Rev. Lett.* **40**, 279 (1978).
[35] D. A. Dicus, E. W. Kolb, V. L. Teplitz, R. V. Wagoner, *Phys. Rev.* **D18**, 1829 (1978); *Phys. Rev.* **D22**, 839 (1980).
[36] A. J. Buras, J. Ellis, M. K. Gaillard, D. V. Nanopoulos, *Nucl. Phys.* **B135**, 66 (1978).
[37] S. Dimopoulos, S. Raby, F. Wilczek, I. T. P., Santa Barbara preprint (1981).
[38] L. Ibañez, G. G. Ross, Oxford University preprint OU 81–65 (1981); M. B. Einhorn, D. R. T. Jones, University of Michigan preprint UM-HE 81–55 (1981).
[39] D. Z. Freedman, A. Das, *Nucl. Phys.* **B120**, 221 (1977); E. Cremmer, J. Scherk (unpublished).
[40] B. De Wit, H. Nicolai, *Nucl. Phys.* **B188**, 98 (1981).
[41] A. D'Adda, P. Di Vecchia, M. Lüscher, *Nucl. Phys.* **B146**, 63 (1978); E. Witten, *Nucl. Phys.* **B149**, 285 (1979).
[42] E. R. Nissimov, S. J. Pacheva, *Comptes Rendus de l'Académie Bulgare des Sciences* **32**, 1475 (1979); *Lett. Math. Phys.* **5**, 67, 333 (1981).
[43] D. Amati, R. Barbieri, A. C. Davis, G. Veneziano, *Phys. Lett.* **102B**, 408 (1981).
[44] Different approaches to the trace representations have been taken by other authors (Refs [16–18]).
[45] D. V. Nanopoulos, D. A. Ross, *Nucl. Phys.* **B157**, 273 (1979).
[46] For a recent review and references see: D. N. Schramm, Talk presented at the Tenth Texas Symposium on Relativistic Astrophysics, Baltimore 1980, Enrico Fermi Institute preprint EFI-81-03 (1981).
[47] This point has recently been re-emphasized by M. Glück and E. Reya, University Dortmund preprint DO-TH 81-06 (1981) with similar conclusions, though we disagree with the assumptions they used to calculate the effective β-function below the Planck mass, believing that the effective gauge group should be taken as SU(5).
[48] J. Ellis, M. K. Gaillard, A. Peterman, C. T. Sachrajda, *Nucl. Phys.* **B164**, 253 (1980).
[49] S. Coleman, E. Weinberg, *Phys. Rev.* **D7**, 1888 (1973).
[50] For an influential recent paper on implementing the Peccei-Quinn mechanism at mass scales $\gg m_W$, see: M. Dine, W. Fischer, M. Srednicki, *Phys. Lett.* **104B**, 199 (1981); For a minimal SU(5) version of the idea, see: M. Wise, H. M. Georgi, S. L. Glashow, *Phys. Rev. Lett.* **47**, 402 (1981); For a supersymmetric GUT version, see: H. P. Nilles, S. Raby, SLAC preprint SLAC-PUB-2743 (1981).

[51] J. Ellis, M. K. Gaillard, D. V. Nanopoulos, S. Rudaz, CERN preprint TH-3121/LAPP-TH-42 (1981).
[52] See also: M. Veltman, T. Sterling, University of Michigan preprint UM-HE 81-2 (1981) for a four-dimensional analysis.
[53] S. Weinberg, E. Witten, *Phys. Lett.* **96B**, 59 (1980).
[54] M. K. Gaillard, B. Zumino, LAPP preprint TH-37 (1981).
[55] For a recent analysis of experimental constraints on potentials due to the exchanges of light, weakly-interacting particles see: G. Gibbons, B. F. Whiting, *Nature* **291**, 636 (1981).
[56] J. P. Ader, M. Capdeville, H. Navelet, *Nuovo Cimento* **56A**, 315 (1968); see also M. T. Grisaru, H. N. Pendleton, P. Van Nieuwenhuizen, *Phys. Rev.* **D15**, 996 (1977).
[57] D. Z. Freedman, A. Das, Ref. [39]; S. Deser, B. Zumino, *Phys. Rev. Lett.* **38**, 1433 (1977).
[58] G. Steigman, K. A. Olive, D. N. Schramm, *Phys. Rev. Lett.* **43B**, 239 (1979); K. A. Olive, D. N. Schramm, G. Steigman, *Nucl. Phys.* **B180**, (FS2) 497 (1981).
[59] M. T. Grisaru, H. J. Schnitzer, Brandeis University preprint (1981).
[60] H. E. Haber, I. Hinchliffe, E. Rabinovici, *Nucl. Phys.* **B172**, 458 (1980).
[61] E. Rabinovici, private communication (1980).
[62] A. O. Barut, Proceedings 1966 Boulder Summer Institute, p. 125.
[63] M. Günaydin, C. Saçlioğlu, Bonn University preprint Bonn HE-81-11 (1981).
[64] V. Kac, *Math. U.S.S.R. Izv. Ser. Math.* **32**, 1271 (1968); R. Moody, *Bull. Ann. Math. Soc.* **73**, 217 (1967); *J. of Algebra* **10**, 211 (1968).
[65] M. B. Green, J. H. Schwarz, *Nucl. Phys.* **B181**, 502 (1981).
[66] H. Hecht, W. Schmid, *Invent. Math.* **31**, 129 (1975) prove Blattner's conjecture on the multiplicity of representations. We thank M. Günaydin for drawing our attention to this point.