THE FREQUENCY DEPENDENCE OF COMPONENT SEPARATION IN PULSAR INTEGRATED PROFILES. II. HIGH FREQUENCIES*

BY J. GIL

Institute of Physics, Pedagogical University of Rzeszów**

(*Received May 21, 1981*)

A model for pulsar radiation at high radio-frequencies is presented. It is argued that the reason for the appearance of the break frequency may be the existence of a hollow-zone near the magnetic axis where the strong coherent radiation is not produced. This model explains the occurrence of the break frequency at approximately the same values for component separation, mean spectrum and the modulation index.

PACS numbers: 97.60.Gb

1. Introduction

For pulsars with two or more-component profiles, the observed component separation (defined as the angular distance between component peaks) is generally frequency dependent. At low frequencies the separation $\Delta\varphi$ varies approximately as ν^{-p}, where the separation index $0.15 \leqslant p \leqslant 0.68$, but above some break frequency ν_b the separation is either constant or slowly increasing with frequency [1].

The mean pulsar spectra often consists of two power-law segments separated by a break frequency, with the higher-frequency segment being steeper. In pulsars where both break frequencies exist, they appear at about the same frequencies [1].

A similar break frequency appears also in the modulation index (which measures the degree of intensity fluctuations from pulse to pulse), with larger modulations above it. This break frequency coincides with the break frequencies in the component separation and/or the mean spectra within the estimated errors [9].

Several possible mechanisms were proposed for the explanation of the existence of the break frequency ν_b in component separation [2]. The first possibility is that all frequencies above ν_b are emitted from the same radius. The second is that the magnetic field is non-dipolar near the surface of a neutron star, where the higher frequencies could

* This paper is supported in part by Interdisciplinary Research Project MR I.7.

** Address: Instytut Fizyki WSP, Rejtana 16A, 35-311 Rzeszów, Poland.

be produced. Thirdly, it is possible that high-frequency ($\nu > \nu_b$) emission is dominated by a second emission component which does not participate in the radius-to-frequency mapping. Yet another reason is possible: the radius-to-frequency map holds in the whole emission region but the condition $\alpha(\nu_b)-\alpha(\nu) \lesssim \gamma^{-1}$ where $\nu > \nu_b$ is satisfied for each dipolar field line which contribute to the peak intensity [3]. Here $\alpha(\nu)$ is the angle between the magnetic axis and the tangent to the field line at the point where the radiation at frequency ν originates and γ is the Lorentz factor of particles emitting by curvature mechanisms. This condition requires that all frequencies higher than ν_b be emitted from a given field line in approximately the same direction (to within the accuracy of the beam-width of the curvature radiation). One can show that if the coherent curvature radiation is emitted with the local plasma frequency and $\gamma > 100$, then the above condition can be satisfied within a region of the open field lines.

Only the second proposition among those presented here could explain why, for some pulsars, the component separation increases with increasing frequency above ν_b. However, this hypothesis may be unjustified, because in the complicated multipolar magnetic field no steady conditions for the developing of plasma instability (which is expected to be a source of the observed emission) should exist [7].

In the previous paper (Gil [4], paper I hereafter), a general approach to the problem of frequency dependence of component separation has been developed in the frame-work of magnetic pole models. In this paper a natural geometrical explanation for the appearance of the break frequencies in the separation, mean spectrum and the modulation index is presented.

2. *The break frequencies*

2.1. Frequency dependence of component separation

The apparent symmetry of the pulsar emission about the profile center suggests that the component separation can be written in the simple form $\Delta\varphi \approx 2\varphi$, where φ is the longitude of one of the component peaks (Fig. 1 in paper I). If the magnetic axis m passes through the observer (radio-telescope) so that $\xi-\theta = 0$ or $\alpha \gg \xi-\theta$ then $\varphi \approx \alpha/\sin\theta$ [4], where α is the angle between the magnetic axis and the line-of-sight, ξ is the angle between the rotation axis Ω and the line-of-sight and θ is the angle between the rotation and magnetic axes. In the magnetic pole models the coherent radiation is emitted tangent to the dipolar field lines. In the majority of these models the coherent emission is assumed to peak near a local plasma frequency. In such a case $\alpha \propto \nu^{-1/3}$ and hence one obtains the frequency dependence of component separation in the form $\Delta\varphi \propto \nu^{-p}$, where the separation index $p = 1/3$ for all pulsars. The above argumentation (which was first presented by Komesaroff [5] and then used in Ruderman and Sutherland's model [6]) includes two special assumptions: (*i*) the component peak at different frequencies is produced along the same dipole field line, (*ii*) the magnetic axis passes through the observer. These assumptions may not be satisfied in the case of an actual pulsar.

In paper I, the general form of the separation index was obtained in the following way: In the magnetic pole models the angle α between the magnetic axis and the line-of-

-sight can be written for peak intensity in the form

$$\alpha(\nu) = 3/2[r(\nu)/A(\nu)]^{1/2}, \tag{1}$$

where $A = A(\nu)$ is the function determining the field line (dipolar field line can be labeled by a parameter $A = r/\sin^2 \vartheta$), r is the radius and ϑ is the polar angle (Fig. 1a) which gives the main contribution to the observed pulse/profile at a frequency ν produced at the radius $r(\nu)$.

If the angle $|\xi - \theta| > 0$, then the longitude φ is not simply proportional to the angle α but can be approximately written in the power-law form

$$\varphi(\alpha, \xi - \theta) \propto \alpha^{p_1(\alpha, \xi - \theta)}, \tag{2}$$

where the coefficient $p_1 = \alpha/\varphi(d\varphi/d\alpha)$.

Writing similarly the radius-to-frequency mapping $r(\nu) \propto \nu^{-2p_2}$ and the function $A(\nu) \propto \nu^{-2p_3}$, one can obtain from Eqs. (1) and (2) and from the empirical relation $\Delta\varphi \propto \nu^{|p}$ the separation index p in the form

$$p = p_1(\nu)\,[p_2 - p_3(\nu)]. \tag{3}$$

The values of the coefficient p_1 depend mainly on α, with the minimum value of this angle, $\alpha_{min} = \xi - \theta$, being a parameter of this dependence. It follows from Eqs. (1) and (2) that p_1 is frequency dependent. In particular, this dependence is significant at higher frequencies that are emitted in a region of smaller angles α, because $p_1 \to \infty$ for $\alpha \to \xi - \theta$. On the other hand $p_1 \equiv 1$ for $\alpha \gg \xi - \theta$ (where the equality holds for $\xi = \theta$). It means that at low frequencies the coefficient p_1 depends weakly on frequency.

The coefficient p_2 depends on the radius-to-frequency mapping $r(\nu) \propto \nu^{-2p_2}$ which operates in the actual pulsar. Since it is expected that smaller frequencies are emitted at greater radii r, then $p_2 > 0$. For example, if the coherent radiation is emitted with the local plasma frequency, then $p_2 = 1/3$.

The coefficient $p_3(\nu)$ depends on the distribution of the emitter energy and/or density in the region of the open field lines. Its sign depends on whether $A \propto \nu^{-p_3(\nu)}$ increases or decreases with frequency. One can note from Eq. (3) that large variations of $A(\nu)$, corresponding to $p_3 \approx p_2$, can be related to the break frequency ν_b, above which $p \approx 0$, so the component separation remains approximately constant.

2.2. Break frequency in the component separation

Let us assume that for frequencies $\nu < \nu_b$, the coefficient $p_3 \equiv 0$ and hence $A(\nu) \equiv$ constant. This corresponds to the case when the subpulse emission occurs most frequently on a tube of field lines with approximately the same value of A (originating at the same distance from the pole). Thus at lower frequencies the separation index $p = p_1 \cdot p_2$. This is illustrated in Fig. 1a. The main contribution to the mean pulsar emission comes from a tube of preferred field lines ($A = A_0$) where the emitter energy and/or density is greatest. For a given frequency ν, the radiation associated with the component peak forms a conical surface with a half-angle $\alpha(\nu) = 3/2\,[r(\nu)/A_0]^{1/2}$. This angle increases with decreasing frequency because of the positive coefficient p_2 in the radius-to-frequency map $r(\nu) \propto \nu^{-2p_2}$. Thus the longitude of the component peak increases with decreasing frequency. For example, the rotating observer passes through the conical surfaces corresponding to the frequencies ν_1 and ν_2 in the longitudes φ_1 and φ_2 respectively. The resulting frequency dependence of component separation is schematically presented in Fig. 1b. It is worth noting that the well known hollow-cone of pulsar emission is not necessary to obtain double-shaped integrated profiles. In the presented model this is a consequence of an

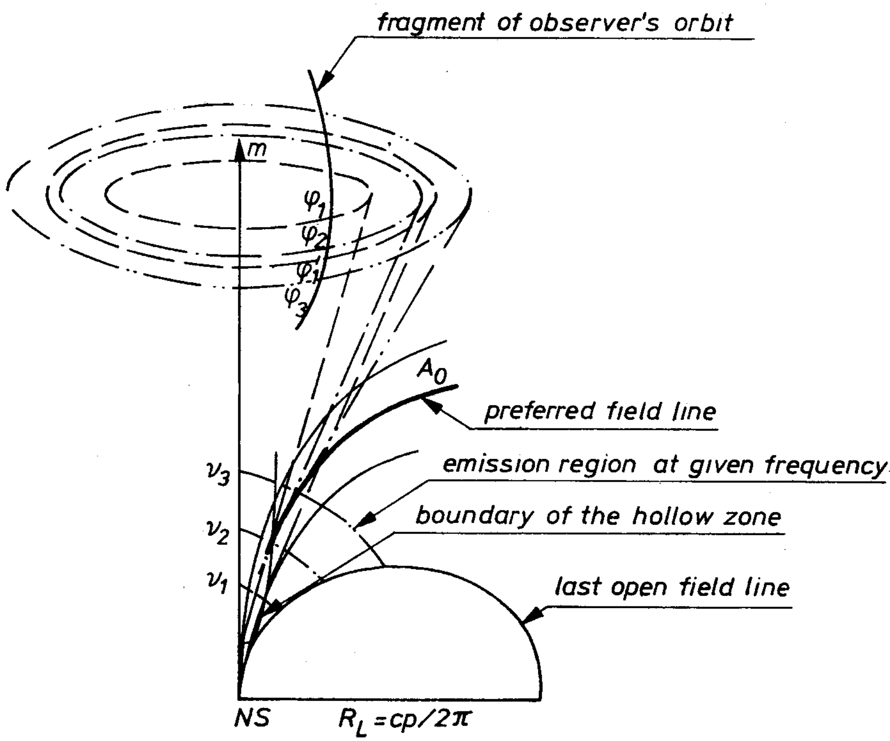

Fig. 1a. A schematic illustration of the model presented in the paper (see text for explanation)

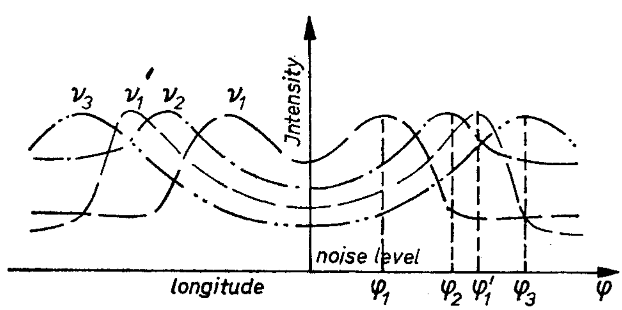

Fig. 1b. A schematic illustration of frequency dependence resulting from Fig. 1a

existence of the radius-to-frequency mapping and the form of the function $A = A(\nu)$. The integrated emission does not fall down to the noise level in the profile centre, as in the case of most of complex integrated profiles. If, with increasing frequency the pulsar spectrum falls down to the noise level sufficiently rapidly, then the profile may be double in the whole spectrum.

In order to illustrate the case when the large variations of $A(\nu)$ can appear, let us consider the following problem. In the magnetic pole models, the hollow-zone around the magnetic axis, where the coherent curvature radiation is not produced, should always exist. This is so when the curvature of field lines is too small and/or conditions for developing the plasma instabilities are not satisfied near the magnetic axis. The boundary of such a zone can intersect a tube of preferred field lines, as illustrated in Fig. 1a. Since the interior of the hollow-zone does not contribute to the observed emission, the function of the preferred field lines in the region corresponding to $\nu > \nu_b$ is performed by the boundary of that zone. One can note from Fig. 1a that starting from $\nu = \nu_2$ the longitude of the component peak may increase with frequency ν (or be constant). For example $\varphi_1'(\nu_1') > \varphi_2(\nu_2)$ although $\nu_1' > \nu_2$ (Figs. 1a and 1b, thin dashed lines). The frequency ν_2 corresponds then to the break frequency ν_b.

Quantitatively, this means that $p_3 = 0$ for $\nu < \nu_b$ and $p_3 \gtrsim p_2$ for $\nu \gtrsim \nu_b$ (see Eqs. (3)). The separation index is $p = p_1 \cdot p_2$ below the break frequency and $p = p_1(p_2 - p_3) \lesssim 0$ above it. Therefore $A(\nu) = A_0$ for $\nu < \nu_b$ and A increases with ν for $\nu \geqslant \nu_b$. Let us estimate the variations of $A(\nu)$ corresponding to $p_3 \approx p_2$. Using the power-law form of the function $A \propto \nu^{-2p_3}$, one can obtain $A_0/A = a^{p_2}$, where $a = \nu/\nu_b > 1$. This condition can be rewritten in the more convenient form $d/d_0 = a^{p_2/2}$. The parameter $d = (R_L/A)^{1/2} \cdot r_p$ is the distance (in units of the polar cap radius $r_p = R^{3/2} \cdot R_L^{-1/2}$) from the pole to the points where the field lines with parameter A originate on the polar cap and $R_L = cP/2\pi$ is the light cylinder radius. For most pulsars demonstrating the break frequency $a < 10$. Thus, for $p_2 = 1/3$ (plasma-resonance) one obtains $d/d_0 < 1.5$. Therefore, if the component peak at the highest frequencies is produced at last open field lines ($d = r_p$), then $d_0 \lesssim 0.6 r_p$. This means that at frequencies lower than ν_b, the emission associated with the component peak is produced on a tube of field lines that originate at a distance less than $0.6 r_p$ from the pole. The above estimation then shows that the value of p_3 can even be significantly greater than the value of p_2. In such a case the separation index could have large negative value, as observed for some pulsars (Table I).

It seems that the form of the function $A = A(\nu)$, both below and above ν_b, may be dependent on various factors. For example, it could be determined by the details of particle injection near the pulsar surface. However, a symmetry about the magnetic axis, appearing in the simple case presented in this paper, is very convenient for the purpose of illustrating the model. Moreover, the symmetrical form of the observed integrated profiles suggests, that this model may be a good approximation of the mean pulsar emission.

The extent of the hollow-zone should be independent of various pulsar parameters (such as period or its derivative), since the curvature of the field lines is a purely geometrical factor. This implies that the smaller values of the break frequency ν_b correspond to smaller angles α (Fig. 1a). On the other hand the separation index p increases with decreasing α

TABLE I

PSR	Separation index		Break frequency MHz		
	Low frequencies	High frequencies	Separation	Mean spectrum	Modulation index
0834+06	0.36 [1]	−0.11 [1]	135 [1]	140 [3]	—
1919+21	0.34 [1]	−0.18 [1]	145 [1]	260 [3]	—
1929+10	0.28 [1]	—	1000 [4]	2800 [3]	1700 [5]
2020+28	0.25 [2]	−0.01 [1]	960 [1]	—	1100 [5]
1133+16	0.26 [1]	0.00 [1]	970 [1]	1400 [3]	600 [5]
0525+21	0.21 [1]	−0.06 [1]	1390 [1]	1080 [3]	—
1237+28	0.18 [1]	0.01 [1]	1410 [1]	—	—
2045−16	0.15 [1]	0.07 [1]	1500 [1]	—	—
2020+28	0.20 [1]	−0.01 [1]	960 [1]	—	1100 [5]
0329+54[a]	0.18 [1]	0.08 [1]	1100 [1]	—	—
0329+54[b]	0.16 [2]	−0.04 [1]	900 [1]	—	—
0329+54[b]	0.08 [1]	−0.04 [1]	900 [1]	—	—
0950+08	0.68 [6]	—	200[c]	—	—

[a] Separation between components 1–3; [b] Separation between components 3–4; [c] Indication of a possible break frequency in Fig. 3 (this paper).

[1] R. N. Manchester, J. H. Taylor, *Pulsars*, Pergamon Press, San Francisco 1977.
[2] V. A. Izvekova, A. D. Kuzmin, V. M. Molofeev, V. M. Shitov, *Aust. J. Phys.* **32**, 25 (1979).
[3] W. Sieber, *Astron. Astrophys.* **28**, 237 (1973).
[4] W. Sieber, R. Reinecke, R. Wielebiński, *Astron. Astrophys.* **38**, 169 (1975).
[5] N. Bartel, W. Sieber, A. Wolszczan, *Astron. Astrophys.* **90**, 58 (1980).
[6] T. H. Hankins, I. M. Cordes, Talk presented at the 95th IAU Symposium on Pulsars, August 1980.

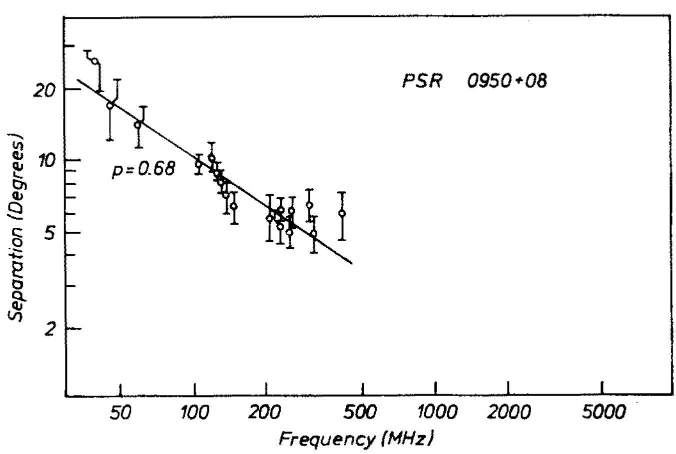

Fig. 2. The observed separation of the main components of PSR 0950+08 (after Hankins et al. 1980, Arecibo preprint NAIC # 144). Author of this paper suggests that the presented data exhibit an existence of the break frequency near 200 MHz

(see comment to Eq. (3) or paper I). This means that larger values of the separation index p should be accompanied by smaller values of the break frequency. Such a correlation can be noted indeed (Table I), particularly if one excludes PSR 0329+54, whose separation data does not correspond to the outermost components. Moreover, the data of PSR 0950+08 (Fig. 2), can be interpreted as exhibiting the break frequency (in the separation of the components of the main pulse) near 200 MHz (this paper). This interpretation is consistent with the existence of the break frequency near 300 MHz for the width of the integrated profile of PSR 0950+08 [8]. It is worth noting that the extremely high value of the separation index $p = 0.68$ for this pulsar is accompanied by very low value of the break frequency (if it indeed exists), $\nu_b \approx 200$ MHz, as predicted in the presented model.

Table I shows that small values of the break frequency are accompanied not only by large values of separation index at low frequencies, but also by large negative values of the separation index at high frequencies. At the same time the largest values of the break frequency are accompanied by smallest values of the separation index at low frequencies and small positive values of separation index at high frequencies. This correlation can be explained in the following way: The fact, that the break frequency has a small value means that frequencies $\nu > \nu_b$ are emitted in the region of smaller angles α, i.e. near the magnetic axis. In this region the function $A(\nu) \propto \nu^{-p_3(\nu)}$ should decrease (along the boundary of the hollow-zone) very fast with ν. In such a case p_3 can exceed p_2 and the separation index $p = p_1(p_2 - p_3)$ attains large negative values. For larger values of the break frequency, the variation of $A(\nu)$ should be slower, so $p_2 \gtrsim p_3$, and the separation index p has its absolute value close to zero.

2.3. Break frequency in the mean spectra

For most pulsars the mean pulse energy decreases towards higher frequencies (usually in the power-law way). This suggests that the emitter energy and/or density decreases (for a given field line) towards the smaller radii where the higher frequencies are produced. Below the break frequency ν_b the component peak is emitted from a tube of preferred field lines, whereas above ν_b, the main contribution to the pulse/profile is produced from field lines lying beyond the preferred line A_0 (Fig. 1a). Thus the mean pulse energy should decrease faster above the break frequency than below it. This implies, whithin the presented model, the existence of the break frequency in the mean spectra with approximately the same value as for component separation. Such a correlation is indeed observed [1].

2.4. Break frequency in the modulation index

The similar break frequency should also appear in the modulation index. In fact, the tube of preferred field lines is composed from lines on which the subpulse emission occurs most frequently. These lines give the main contribution to the mean pulsar intensity at frequencies $\nu < \nu_b$. Since the high-frequency radiation is emitted from field lines which lie beyond the preferred tube (Fig. 1a) then the modulations of intensity from pulse to pulse should be larger above the break frequencies in separation and mean spectra.

3. *Conclusions*

The model of the break frequency presented in this paper predicts the following properties of pulsar emission at high-radio frequencies:

(*i*) The break frequency above which the variation of the considered quantity ceases should appear approximately at the same frequency in the component separation, mean spectrum and the modulation index. Such a correlation is reported in the paper by Bartel et al. [9].

(*ii*) High values of the separation index p at low frequencies (below the break frequency) should be accompanied by small values of the break frequency ν_b. Such a correlation can be noted indeed in the observational data (Table I).

(*iii*) Small values of the break frequency should be accompanied by large negative values of the separation index at high frequencies (above the break frequency), whereas the large values of the break frequency should correspond to the separation index being near zero. The existence of this correlation is also visible in Table I.

REFERENCES

[1] R. N. Manchester, J. H. Taylor, *Pulsars*, Pergamon Press, San Francisco 1977.
[2] J. M. Cordes, Talk presented at the 95th IAU Symposium on Pulsars, August 1980, preprint.
[3] J. Gil, unpublished.
[4] J. Gil, *Acta Phys. Pol.* **B12**, 1081 (1981).
[5] M. M. Komesaroff, *Nature*, **225**, 612 (1970).
[6] M. A. Ruderman, P. G. Sutherland, *Ap. J.* **196**, 51 (1975).
[7] G. Benford, R. Buschauer, *Mon. Not. R. Astron. Soc.* **179**, 189 (1977).
[8] N. Bartel, W. Sieber, D. A. Graham, *Astron. Astrophys.* **87**, 282 (1980).
[9] N. Bartel, W. Sieber, A. Wolszczan, *Astron. Astrophys.* **90**, 58 (1980).