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THE MAXWELLIAN TENSORS AND THE SUPERENERGY TENSORS IN A QUADRATIC GRAVITATIONAL THEORY*

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In this paper we consider the so-called Maxwellian tensors for the curvature and for the torsion of the space-time and the total superenergy and superspin tensors in the theory of gravitation with quadratic Lagrangian $L_g = \varrho\Omega^{i}_{\cdot j} \wedge \eta_{i}^{\cdot j} + \bar{a}\Omega^{i}_{\cdot j} \wedge * \Omega^{j}_{\cdot i} + \alpha\Theta^{i} \wedge * \Theta_{i}$.

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1. Introduction

In paper [1] a general method of constructing Maxwellian tensors for physically important tensor fields which are antisymmetric in some pair of their indices was presented. In this previous paper the antisymmetry of the tensor field for the first pair of indices was assumed. However, this assumption is not necessary.

In this paper we will use Trautman's notation [2] of the curvature and torsion tensors. Therefore, we will consider the tensor fields which are antisymmetric in the last pair of their indices. For such a tensor field with the components

$$H_{Mab} = (-)H_{Mba}, \tag{1}$$

where M denotes any set of indices, the Maxwellian tensor generally defined in [1] has the following components

$$M_{MN\cdot a}^{\cdot\cdot b\cdot} = H_{M\cdot\cdot}^{\cdot bc}H_{Nac}^{\cdot\cdot\cdot} + H_{N\cdot\cdot}^{\cdot bc}H_{Mac}^{\cdot\cdot\cdot} - \frac{1}{2}\,\delta_{a}^{b}H_{M\cdot\cdot}^{\cdot dc}H_{Ndc}^{\cdot\cdot\cdot}. \tag{2}$$

We are interested in the following two tensor fields which are antisymmetric in their two last indices: the curvature tensor field R with components

$$R^{i\cdot\cdot\cdot}_{\cdot klm} = (-)R^{i\cdot\cdot\cdot}_{\cdot kml} : \Omega^{i\cdot}_{\cdot k} = \frac{1}{2}\,R^{i\cdot\cdot\cdot}_{\cdot klm}\vartheta^{l} \wedge \vartheta^{m} \tag{3}$$

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and the torsion tensor field Q with components

$$Q^{i\cdot\cdot}_{\cdot kl} = (-)Q^{i\cdot\cdot}_{\cdot lk} : \Theta^i = \tfrac{1}{2}\, Q^{i\cdot\cdot}_{\cdot kl}\vartheta^k \wedge \vartheta^l. \tag{4}$$

In the above formulae $\Omega^{i\cdot}_{\cdot j}$ is the curvature two-form and Θ^i is the torsion two-form of the metric, linear connection $\omega^{i\cdot}_{\cdot k} = \Gamma^i_{\cdot kl}\vartheta^l$. ϑ^i means the field of the co-frames. The Maxwellian tensor for the curvature tensor field R has the components

$${}_RM^{\cdots b\cdot}_{klpq\cdot a} = R^{\cdot\cdot bc}_{kl\cdot\cdot}R^{\cdots\cdot}_{pqac}+R^{\cdot\cdot bc}_{pq\cdot\cdot}R^{\cdots\cdot}_{klac}-\tfrac{1}{2}\,\delta^b_a R^{\cdot\cdot dc}_{kl\cdot\cdot}R^{\cdots\cdot}_{pqdc} \tag{5}$$

while the Maxwellian tensor for the torsion tensor field has the components

$${}_QM^{\cdot\cdot b\cdot}_{pq\cdot a} = Q^{\cdot bc}_{p\cdot\cdot}Q^{\cdots}_{qac}+Q^{\cdot bc}_{q\cdot\cdot}Q^{\cdots}_{pac}-\tfrac{1}{2}\,\delta^b_a Q^{\cdot dc}_{p\cdot\cdot}Q^{\cdots}_{qdc}\,. \tag{6}$$

In the framework of General Relativity (GRT) and in the framework of the Einstein-Cartan theory of gravitation (ECT) only a special contraction of the Maxwellian tensor for the field R, namely the Bel-Robinson tensor in GRT and in ECT and the generalized Bel-Robinson tensor [3] in ECT may be physically interpreted in a natural manner [3].

The aim of this paper is, among other things, to investigate the physical role of the Maxwellian tensors for curvature and torsion in the framework of the theory of gravitation with the quadratic Lagrangian

$$L_g = \varrho\Omega^{i\cdot}_{\cdot j} \wedge \eta^{\cdot j}_{i\cdot}+\bar{a}\Omega^{i\cdot}_{\cdot j} \wedge *\,\Omega^{j\cdot}_{\cdot i}+\alpha\Theta^i \wedge *\,\Theta_i. \tag{7}$$

In the above expression ϱ, $\bar{a}$, α are suitable constants; η_{ik} denotes the pseudotensorial two-form introduced by Trautman [2] and $*$ is the Hodge-star-operator [4].

2. *The theory of gravitation with the quadratic Lagrangian*

$$L_g = \varrho\Omega^{i\cdot}_{\cdot j} \wedge \eta^{\cdot j}_{i\cdot}+\bar{a}\Omega^{i\cdot}_{\cdot j} \wedge *\,\Omega^{j\cdot}_{\cdot i}+\alpha\Theta^i \wedge *\,\Theta_i$$

This theory contains two gauge fields: the curvature field $\Omega^{i\cdot}_{\cdot j}$ and the torsion field Θ^i with the potentials $\omega^{i\cdot}_{\cdot k}$ and ϑ^i respectively. The fundamental equations of the theory have the following form

$$D*\Theta_b = (-)\frac{1}{2\alpha}\,t_b-\frac{\varrho}{2\alpha}\,\Omega^{jk}_{\cdot\cdot} \wedge \eta^{\cdots}_{bjk}-\frac{\bar{a}}{\alpha}\left(\frac{\delta^p_b}{4}\,R^{ijrt}_{\cdots\cdot}R^{\cdots\cdot}_{jirt}\right.$$

$$\left.-R^{ij\cdot t}_{\cdot\cdot b\cdot}R^{\cdot\cdot p\cdot}_{ji\cdot t}\right)\eta_p-\left(\frac{\delta^p_b}{4}\,Q^{itr}_{\cdots}Q^{\cdots}_{itr}-Q^{i\cdot\cdot}_{\cdot br}Q^{\cdot pr}_{i\cdot\cdot}\right)\eta_p, \tag{8}$$

$$D*\Omega^{l\cdot}_{\cdot i} = (-)\frac{\alpha}{2\bar{a}}\,(\vartheta^l \wedge *\,\Theta^{l}_{i}-\vartheta_i \wedge *\,\Theta^l)-\frac{\varrho}{2\bar{a}}\,\Theta_k \wedge \eta^{\cdot lk}_{i\cdot\cdot}-\frac{1}{4\bar{a}}\,S^{\cdot l}_{i\cdot}. \tag{9}$$

Here t_b is the energy-momentum three-form of matter and $S^{\cdot l}_{i\cdot}$ is the three-form of the intrinsic angular momentum ($\equiv$ spin) of matter. These two three-forms are defined by the following form of the variation of matter Lagrangian

$$L_m = L_m[\phi^A, D\phi^A]$$

$$\delta L_m = \delta \vartheta^i \wedge t_i + \tfrac{1}{2}\, \delta \omega^{i}_{\cdot j} \wedge S_{i\cdot}^{\cdot j} + \delta \phi^A \wedge L_A + \text{an exact form.}$$

(For details, see Trautman [2] or Adamowicz [5].)

From (8) and (9) the tensorial equations follow:

$$\nabla_k Q_{b\cdot\cdot}^{\cdot pk} + Q_{b\cdot\cdot}^{\cdot pk} Q^{l\cdot\cdot}_{\cdot kl} + \tfrac{1}{2}\, Q_{b\cdot\cdot}^{\cdot lk} Q^{p\cdot\cdot}_{\cdot lk} = (-)\frac{1}{2\alpha}\, {}_m t^{p\cdot}_{\cdot b}$$

$$+ \frac{\varrho}{\alpha}\left(R^{p\cdot}_{\cdot b} - \frac{\delta^p_b}{2} R\right) - \left(\frac{\delta^p_b}{4}\, Q^{itr}_{\cdot\cdot\cdot} Q^{\cdot\cdot\cdot}_{itr} - Q^{i\cdot\cdot}_{\cdot br} Q_{i\cdot\cdot}^{\cdot pr}\right)$$

$$- \frac{\bar{a}}{\alpha}\left(\frac{\delta^p_b}{4}\, R^{ijrt}_{\cdot\cdot\cdot\cdot} R^{\cdot\cdot\cdot\cdot}_{jirt} - R^{ij\cdot\cdot}_{\cdot\cdot bt} R_{ji\cdot\cdot}^{\cdot\cdot pt}\right), \tag{10}$$

$$\nabla_m R^{l\cdot pm}_{\cdot i\cdot\cdot} + R^{l\cdot pt}_{\cdot i\cdot\cdot} Q^{k\cdot\cdot}_{\cdot tk} + \tfrac{1}{2}\, R^{l\cdot tn}_{\cdot i\cdot\cdot} Q^{p\cdot\cdot}_{\cdot tn} = (-)\frac{\alpha}{2\bar{a}}\left(Q_{i\cdot\cdot}^{\cdot pl} - Q^{lp\cdot}_{\cdot\cdot i}\right)$$

$$- \frac{1}{4\bar{a}}\, {}_m S^{p\cdot l}_{\cdot i\cdot} - \frac{\varrho}{2\bar{a}}\left(Q^{kl\cdot}_{\cdot\cdot k}\delta^p_i + Q^{k\cdot\cdot}_{\cdot ki} g^{lp} + Q^{p\cdot l}_{\cdot i\cdot}\right). \tag{11}$$

${}_m t^{p}_{\cdot b}$ and ${}_m S^{p\cdot l}_{\cdot i\cdot}$ which occur in (10), (11) are the components of the energy-momentum and spin tensor of matter defined by the following decompositions

$$t_i = \eta_{p\, m} t^{p}_{\cdot i}, \qquad S^{\cdot l}_{i\cdot} = \eta_{p\, m} S^{p\cdot l}_{\cdot i\cdot}.$$

(For details, see Trautman [2] or Adamowicz [5].)

(8) and (9) (or (10) and (11)) are the system of the differential equations of the third order on the 34 unknown functions: the 10 components of the metric tensor g of space-time and the 24 components of the defect tensor κ (see [5]).

The gravitational theory with the quadratic Lagrangian (7) seems to be correct from the formal point of view (see [5]) and it is, in our opinion, the most satisfactory model of the theory of gravitation with quadratic Lagrangian. This theory unifies the ideas of GRT with the ideas of the gauge field theory.

In vacuum and in the case of vanishing torsion the equations of the theory take the following form

$$2R_{bi\cdot\cdot}^{\cdot\cdot jp} R^{i}_{\cdot j} + 2R^{i}_{b} R^{p}_{i} - \delta^p_b R_{ik}^{\cdot\cdot} R^{ik}_{\cdot\cdot} = \frac{\varrho}{\alpha} R^p_b \tag{12}$$

$$\nabla_m R^{l\cdot mp}_{\cdot i\cdot\cdot} = 0 \tag{13}$$

and are equivalent to the vacuum Einstein equations

$$G_{ik} = 0. \tag{14}$$

Owing to that, the theory does not admit the non-physical solutions which exist, in vacuum and when torsion vanishes, in the theory of gravitation with the quadratic Lagrangian

$$L_g = \bar{a}\Omega^{i\cdot}_{\cdot j} \wedge *\Omega^{j\cdot}_{\cdot i} + \alpha\Theta^i \wedge *\Theta_i \tag{15}$$

and in the theory with the quadratic Lagrangian considered by Hehl et al. [6, 7]. In the problem of the static and spherical symmetric gravitational field in vacuum the fundamental equations of the theory admit, among other things, the Schwarzschild solution, with zero torsion.[1]

Moreover, in the theory with Lagrangian (15) and in the quadratic theory considered by Hehl et al. [6, 7], there exist fundamental difficulties with the "weak field approximation" and its correspondence to the Newtonian theory.

3. *The Maxwellian tensors for curvature and torsion in the considered gravitational theory*

From the fundamental equations (8), (9) of the theory it is obvious that the curvature tensor field R and the torsion tensor field Q have their own energy-momentum tensors.

The energy-momentum tensor of the curvature tensor field has the following components

$$\begin{aligned} {}_R T^{k\cdot}_{\cdot b} &= 2\varrho G^{k\cdot}_{\cdot b} - \bar{a}2(R^{ij\cdot\cdot}_{\cdot\cdot bt} R^{\cdot\cdot kt}_{ij\cdot\cdot} \\ &- \tfrac{1}{4}\,\delta^k_b R^{ijrt}_{\cdot\cdot\cdot\cdot} R^{\cdot\cdot\cdot\cdot}_{ijrt}) = :2\varrho G^{k\cdot}_{\cdot b} - \bar{a}_R M^{k\cdot}_{\cdot b}. \end{aligned} \tag{16}$$

In (16), $G^{k\cdot}_{\cdot b}$ are the components of the Einstein tensor $G: G^{k\cdot}_{\cdot b} = R^{k\cdot}_{\cdot b} - \frac{1}{2}\delta^k_b R$ and ${}_R M^{k\cdot}_{\cdot b}$ denotes the contraction of the components (4) of the Maxwellian tensor for the curvature tensor field R in the indices k, p and l, q. Thus, the above mentioned contraction of the Maxwellian tensor for the curvature tensor R occurs in the theory as the quadratic (in R) part of the energy-momentum tensor of the curvature tensor field R.

On the other hand, the energy-momentum tensor of the torsion field Q has the components

$$ {}_Q T^{k\cdot}_{\cdot b} = \alpha 2(Q^{i\cdot\cdot}_{\cdot br} Q^{\cdot kr}_{i\cdot\cdot} - \tfrac{1}{4}\,\delta^k_b Q^{itr}_{\cdot\cdot\cdot} Q^{\cdot\cdot\cdot}_{itr}) =: \alpha_Q M^{k\cdot}_{\cdot b}. \tag{17}$$

and is proportional to the contraction ${}_Q M^{k\cdot}_{\cdot b}$ of the Maxwellian tensor (5) for the field Q in the indices p, q.

The tensors with the components (16) and (17) together with the energy-momentum tensor of matter defined by the decomposition

$$t_i = \eta_{pm} t^{p\cdot}_{\cdot i}$$

are the sources of the gauge field Θ^i.

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[1] This problem will be discussed elsewhere.

4. Conservation laws and superenergy tensors in the theory

The field equations (8), (9) may be transformed to the superpotential form

$$d*\Theta_b = (-)\frac{1}{2\alpha}t_b + \omega^{k\cdot}_{\cdot b}\wedge *\Theta_k - \frac{\varrho}{2\alpha}\Omega^{j\cdot}_{\cdot k}\wedge \eta_{bj\cdot}^{\cdot\cdot k}$$

$$-\frac{\bar{a}}{2\alpha}\left({}_RM^{p\cdot}_{\cdot b} - \frac{\alpha}{\bar{a}}\,{}_QM^{p\cdot}_{\cdot b}\right)\eta_p, \tag{18}$$

$$d*\Omega^{l\cdot}_{\cdot i} = (-)\frac{\alpha}{2\bar{a}}(\vartheta^l\wedge *\Theta_i - \vartheta_i\wedge *\Theta^l) + \omega^{p\cdot}_{\cdot i}\wedge *\Omega^{l\cdot}_{\cdot p}$$

$$-\omega^{l\cdot}_{\cdot p}\wedge *\Omega^{p\cdot}_{\cdot i} - \frac{\varrho}{2\bar{a}}\Theta_k\wedge \eta_{i\cdot\cdot}^{\cdot lk} - \frac{1}{4\bar{a}}S_{i\cdot}^{\cdot l} \tag{19}$$

from which follow the continuity equations

$$d\left[t_b - 2\alpha\omega^{k\cdot}_{\cdot b}\wedge *\Theta_k + \varrho\Omega^{j\cdot}_{\cdot k}\wedge \eta_{bj\cdot}^{\cdot\cdot k} + \bar{a}\left({}_RM^{p\cdot}_{\cdot b} - \frac{\alpha}{\bar{a}}\,{}_QM^{p\cdot}_{\cdot b}\right)\eta_p\right] \equiv 0, \tag{20}$$

$$d[S_{i\cdot}^{\cdot l} + 2\alpha(\vartheta^l\wedge *\Theta_i - \vartheta_i\wedge *\Theta^l) - 4\bar{a}\omega^{p\cdot}_{\cdot i}\wedge *\Omega^{l\cdot}_{\cdot p} + 4\bar{a}\omega^{l\cdot}_{\cdot p}\wedge *\Omega^{p\cdot}_{\cdot i} + 2\varrho\Theta_k\wedge \eta_{i\cdot\cdot}^{\cdot lk}] \equiv 0. \tag{21}$$

From these equations we can get, with help of the Stokes integral theorem [8], the following integral conservation laws

$$\iint_{\partial\Omega}\left[t_b - 2\alpha\omega^{k\cdot}_{\cdot b}\wedge *\Theta_k + \varrho\Omega^{j\cdot}_{\cdot k}\wedge \eta_{bj\cdot}^{\cdot\cdot k} + \bar{a}\left({}_RM^{p\cdot}_{\cdot b} - \frac{\alpha}{\bar{a}}\,{}_QM^{p\cdot}_{\cdot b}\right)\eta_p\right] = 0, \tag{22}$$

$$\int_{\partial\Omega}[S_{i\cdot}^{\cdot l} + 2\alpha(\vartheta^l\wedge *\Theta_i - \vartheta_i\wedge *\Theta^l) - 4\bar{a}\omega^{p\cdot}_{\cdot i}\wedge *\Omega^{l\cdot}_{\cdot p} + 4\bar{a}\omega^{l\cdot}_{\cdot p}\wedge *\Omega^{p\cdot}_{\cdot t} + 2\varrho\Theta_k\wedge \eta_{i\cdot\cdot}^{\cdot lk}] = 0. \tag{23}$$

Here $\partial\Omega$ is the boundary of an arbitrary, four-dimensional domain Ω in space-time.

The expressions contained inside the quadratic brackets in formulae (20) and (21) represent the conserved total energy-momentum and spin three-forms (non-tensorial) of matter and gravitation.

In terms of the components, equations (18) and (19) have the form

$$\partial_d[\alpha\sqrt{|g|}\,\delta^{dq}_{bc}Q_{a\cdot\cdot}^{\cdot bc}] = \sqrt{|g|}\,({}_mt^{q\cdot}_{\cdot a} + {}_gt^{q\cdot}_{\cdot a}), \tag{24}$$

$$\partial_b[2\bar{a}\delta^{bq}_{kp}\sqrt{|g|}\,R^{l\cdot kp}_{\cdot i\cdot\cdot}] = \sqrt{|g|}\,({}_mS^{q\cdot l}_{\cdot i\cdot} + {}_gS^{q\cdot l}_{\cdot i\cdot}), \tag{25}$$

where

$$\frac{1}{2\alpha}\,{}_gt^{q\cdot}_{\cdot b} = (-)\Gamma^k_{\cdot bl}Q_{k\cdot\cdot}^{\cdot ql} - \frac{\varrho}{\alpha}(R^{q\cdot}_{\cdot b} - \delta^q_b R)$$

$$+\frac{\bar{a}}{2\alpha}\left({}_RM^{q\cdot}_{\cdot b} - \frac{\alpha}{\bar{a}}\,{}_QM^{q\cdot}_{\cdot b}\right), \tag{26}$$

$$\frac{1}{4\bar{a}}\,{}_gS^{q\cdot l}_{\cdot i\cdot} = \frac{\alpha}{2\bar{a}}\,(Q^{\cdot ql}_{i\cdot\cdot} - Q^{lq\cdot}_{\cdot\cdot i}(-\Gamma^p_{\cdot ia}R^{l\cdot qa}_{\cdot p\cdot\cdot} + \Gamma^l_{\cdot pa}R^{p\cdot qa}_{\cdot i\cdot\cdot}$$

$$+\frac{\varrho}{2\bar{a}}\,(Q^{kl\cdot}_{\cdot\cdot k}\delta^q_i + Q^{k\cdot\cdot}_{\cdot ki}g^{lq} + Q^{q\cdot l}_{\cdot i\cdot}). \tag{27}$$

The sum $\sqrt{|g|}\,({}_mt^{q\cdot}_{\cdot a} + {}_gt^{q\cdot}_{\cdot a})$ is the conserved energy-momentum complex of matter and gravitation while the sum $\sqrt{|g|}\,({}_mS^{q\cdot l}_{\cdot i\cdot} + {}_gS^{q\cdot l}_{\cdot i\cdot})$ is the conserved spin complex of matter and gravitation in the theory.

These complexes are not intrinsic tensors. Therefore, it is sensible to introduce in the theory the total superenergy and the total superspin tensor of matter and gravitation.

These tensors should be constructed in the normal coordinate system NCS(P; $\tilde{\omega}$) of the Riemannian part of the metric connection of space-time[2] exactly in the same way as in the framework of GRT and ECT (See [3]).
We construct them from the sum ${}_mt^q_{\cdot b} + {}_gt^q_{\cdot b}$ and from the sum ${}_mS^{q\cdot l}_{\cdot i\cdot} + {}_gS^{q\cdot l}_{\cdot i\cdot}$.

As a result, we get the two following tensors: the total superenergy tensor of matter and gravitation S with the components

$$S^p_{\cdot b} = 2\alpha(g^{rs} - 2v^rv^s)\,[(5\tilde{\nabla}_{(s|}\tilde{R}^k_{\cdot(b||r)|c)}$$
$$+\tilde{\nabla}_{(b|}\tilde{R}^k_{\cdot(rs)|c)})Q^{\cdot pc}_{k\cdot\cdot} + \tfrac{4}{3}\,\tilde{R}^k_{\cdot(b|(r||c)}\tilde{\nabla}_{|s)}Q^{\cdot pc}_{k\cdot\cdot}$$
$$+\tilde{\nabla}_{(r}\tilde{\nabla}_{s)}T^p_{\cdot b} - \tfrac{1}{3}\,\tilde{R}^i_{\cdot(r|b|s)}T^p_{\cdot i} + \tfrac{1}{3}\,\tilde{R}^p_{\cdot(r|i|s)}T^i_{\cdot b}], \tag{28}$$

where

$$T^p_{\cdot b} := \frac{\varrho}{\alpha}\,G^p_{\cdot b} - \frac{\bar{a}}{2\alpha}\left({}_RM^p_{\cdot b} - \frac{\alpha}{\bar{a}}\,{}_QM^p_{\cdot b}\right) + \kappa^{k\cdot\cdot}_{\cdot bc}Q^{\cdot pc}_{k\cdot\cdot} - \frac{1}{2\alpha}\,{}_mt^p_{\cdot b} \tag{29}$$

and the total superspin tensor of matter and gravitation $\tilde{S}$ with the components

$$S^{\cdot lk}_{i\cdot\cdot} = 4\bar{a}\,(g^{bd} - 2v^bv^d)\,[\tfrac{1}{6}(5\tilde{\nabla}_{(d|}\tilde{R}^p_{\cdot(i||b)|a)}$$
$$+\tilde{\nabla}_{(i|}\tilde{R}^p_{\cdot(bd)|a)})\tilde{R}^{l\cdot ka}_{\cdot p\cdot\cdot} + \tfrac{4}{3}\,\tilde{R}^p_{\cdot i(a(b)}\tilde{\nabla}_{d)}\tilde{R}^{l\cdot ka}_{\cdot p\cdot\cdot}$$
$$-\tfrac{1}{6}\,(5\tilde{\nabla}_{(d|}\tilde{R}^l_{\cdot(p||b)|a)} + \tilde{\nabla}_{(p|}\tilde{R}^l_{\cdot(bd)|a)})\tilde{R}^{p\cdot ka}_{\cdot i\cdot\cdot}$$
$$-\tfrac{4}{3}\,\tilde{R}^l_{\cdot p(a(b)}\tilde{\nabla}_{d)}\tilde{R}^{p\cdot ka}_{\cdot i\cdot\cdot} + \tilde{\nabla}_{(b}\tilde{\nabla}_{d)}T^{\cdot lk}_{i\cdot\cdot} + \tfrac{4}{3}\,\tilde{R}^l_{\cdot(b|p|d)}T^{\cdot pk}_{i\cdot\cdot}$$
$$+\tfrac{1}{3}\,\tilde{R}^k_{\cdot(b|p|d)}T^{\cdot lp}_{i\cdot\cdot} - \tfrac{1}{3}\,\tilde{R}^p_{\cdot(b|i|d)}T^{\cdot lk}_{p\cdot\cdot}], \tag{30}$$

where

$$T^{\cdot lk}_{i\cdot\cdot} := (-)\,\frac{\alpha}{2\bar{a}}\,(Q^{\cdot kl}_{i\cdot\cdot} - Q^{lk\cdot}_{\cdot\cdot i}) - \frac{\varrho}{2\bar{a}}\,(Q^{ql\cdot}_{\cdot\cdot q}\delta^k_i + Q^{q\cdot\cdot}_{\cdot qi}g^{lk} + Q^{k\cdot l}_{\cdot i\cdot}) - \frac{1}{4\bar{a}}\,{}_mS^{k\cdot l}_{\cdot i\cdot}. \tag{31}$$

In the above espressions $\tilde{\nabla}$ means the covariant derivative with respect to the Riemannian part of the full metric connection of space-time and $\tilde{R}^{i\cdot\cdot\cdot}_{\cdot klm}$ denotes the components

[2] The choice of NCS(P; $\tilde{\omega}$) is dictated first of all by the maximal analytic simplicity of the calculated expressions (For further argumentation, See [3]).

of the curvature tensor calculated from the Riemannian part of the connection; $\kappa^{k\cdot\cdot}_{\cdot ab}$ are the components of the defect tensor (see, eg. [5]).

Multiplying (28) and (30) by $\sqrt{|g|}$ we get the components of the corresponding superenergy and superspin complexes.

It is seen from the formulae given above that if the torsion vanishes in vacuum, then the components of the total superenergy tensor of matter and gravitation also vanish. Therefore, in this case, the total superenergy tensor of matter and gravitation given by (28) has no value.

5. *The case of vacuum and vanishing torsion*

Let us consider the theory more precisely in the degenerate case: vacuum and $\Theta = 0$. Then, as was mentioned in Section 2, the equations of the theory reduce to the vacuum Einstein equations, $G_{ik} = 0$. As a consequence, in this degenerate case the energy-momentum tensors of the tensor fields R and Q and the total superenergy tensor of matter and gravitation vanish. Therefore, in this case, these tensors have no value as tensers describing gravitational field. However, this degenerate form of the theory may be considered as the standard Einstein theory in vacuum with a Riemannian connection $\tilde{\omega}^{i\cdot}_{\cdot k}$ as a gravitational field and we can assign to the gravitational field $\tilde{\omega}^{i\cdot}_{\cdot k}$ an energy-momentum pseudotensor (see, e.g., [3, 4]). The superenergy tensor of the field $\tilde{\omega}^{i\cdot}_{\cdot k}$, calculated from this pseudotensor in the way described in [3], contains the contraction of the Maxwellian tensor for the curvature tensor field R namely, the Bel-Robinson tensor (see [3]).

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