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ON THE PROBLEM OF MULTIPLICITY OF PARTICLES IN NUCLEON-NUCLEUS INTERACTIONS IN MULTIPLE SCATTERING THEORY

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In the framework of multiple scattering theory the expressions are obtained for the mean number of inelastic collisions of leading hadron and the mean multiplicity of shower particles $\langle n_S \rangle_{pA}$ in proton-nucleus interactions; change of type of the leading hadron (transfer of leading property from a nucleon to a pion) is taken into account. The comparison of calculations with experimental data shows that the multiplicity excess of shower particles in nucleon-nucleus collisions as compared with the nucleon-nucleon ones cannot be entirely due to multiple inelastic interactions of the incident nucleon but is apparently also due to inelastic interactions of secondary particles, mainly the leading pion collisions.

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The importance of investigating the hadron-nucleus interaction characteristics from the viewpoint of obtaining information on the dynamics of elementary hadron-hadron interaction has been emphasized in many works. Existing theoretical approaches to this problem, however, are far from being complete. In particular there are no more or less strict quantitative predictions on the influence of the finiteness of space-time interval of hadron production on the process of the particle multiple production on nuclear targets. In order to understand the nature of the space-time structure of hadron formation in the process of particle production within the nucleus we think it useful to develop and refine the predictions of a more traditional approach — multiple scattering theory (MST). The determination of the MST application region, the degree of correspondence of its predictions to experimental data will allow one to determine also those characteristics of hadron-nucleus interactions where the consideration of the hadron production space-time structure is necessary.

In the works [1], carried out in recent years, a number of experimental characteristics of hadron-nucleus interactions have been described within the framework of MST. Parti-

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cularly consideration of multiple inelastic collisions of incident hadron in nucleus, making allowance for the energy loss and fluctuations of the latter, leads to predictions for the particle production mean multiplicity close to experimental values. In the mentioned works, however, the secondary inelastic interactions of other particles, among them — the one to which the leading property is transferred in one of the inelastic interaction acts of incident hadron, have not been considered.

In the present work we have attempted to take into account within the framework of MST, for the case of proton-nucleus interaction, the contribution to the multiplicity of produced "shower" particles of multiple inelastic interactions of leading hadrons differing in their type from the incident ones (proton). First, in inclusive processes $pN \to N^{lead}X$ the transitions of proton to leading neutron are considered, the inclusive spectra of which differ considerably from the ones for the leading proton. Second, processes $NN \to \pi^{lead}X$ and subsequent multiplication of the leading pion in nucleus are taken into account.

To carry out calculations of the mean multiplicity of shower particles $\langle n_S \rangle_{pA}$ in pA--interactions, taking into account the mentioned processes, the following information on the hadron-nucleon inelastic interactions is necessary:

a) Inclusive spectra $\frac{d\sigma}{dx}$ of elementary reactions $NN \to NX$, $NN \to \pi X$, $\pi N \to \pi X$ $\left(x = \frac{p_{||}}{p_h}\right.$ where p_h is the primary momentum, $p_{||}$ is the longitudinal momentum of secondary $\left.\text{particle}\right)$.

Making use of experimental data on the reactions $pN \to pX$ and $pN \to nX$ (see e.g. [2]) we have obtained the following parametrization for the inclusive spectrum (integrated over the transverse momentum component) of nucleon in the centre-of-mass system NN in the range of positive $x^* = \frac{2p_{||}^*}{\sqrt{s}}$

$$\frac{d\sigma}{dx^*} = [22+175(x^*-0.45)^2]\theta(0.7-x^*)+30[1+10(x^*-0.8)^2]\theta(x^*-0.7). \tag{1}$$

Expression (1), which describes the nucleon spectrum in the reactions $pN \to NX$ within the limits of experimental errors, satisfies the following sum rules:

$$\int_0^1 x^* \frac{d\sigma}{dx^*} dx^* = \sigma_{NN}^{in}(1-K), \tag{2}$$

where $K = 0.5$ is the inelasticity coefficient, i.e. the mean share of initial energy spent on meson production; and

$$\int_0^1 \frac{d\sigma}{dx^*} dx^* = \sigma_{NN}^{in}, \tag{3}$$

where the production of nucleon-antinucleon pair is neglected. Obviously, expression (1), owing to isotopic invariance, will describe the spectra of the reactions πN → NX as well.

The sum spectrum for the reactions NN → $\pi^{\pm}_{0}$X has been parametrized according to the results of the works [2] in the form

$$\frac{d\sigma^{NN\to\pi X}}{dx^*} = \frac{31.1}{\sqrt{x^{*2}+\langle x_\perp^2\rangle}}(1-x^*)^3(2-2x^*+x^{*2}), \tag{4}$$

where $\langle x_\perp^2\rangle = 2(m_\pi^2+\langle p_\perp^2\rangle)/s$, m_π is the pion mass, $\langle p_\perp^2\rangle$ is its mean transverse momentum.

Finally the sum spectrum πN → $\pi^{\pm}_{0}$X [2] has been parametrized in the form

$$\frac{d\sigma^{\pi N\to\pi X}}{dx^*} = [25(1-x^*)^{1.8}+23.5(1-x^*)^{2.8}]\frac{\theta(0.5-x^*)}{\sqrt{x^{*2}+x_\perp^2}} + \frac{4.8}{x^*}\theta(x^*-0.5). \tag{5}$$

The spectra (4) and (5) also satisfy the corresponding sum rules analogous to (2) and (3).

In our subsequent calculation we shall assume that the spectra (1), (4), (5) are of scaling character for the whole energy range of the leading hadron when passing through the nucleus.

b) Inclusive spectra of leading hadrons $\left(\frac{d\sigma}{dx}\right)^{\text{lead}}$ or, in other words, probabilities $\eta(x)$ of being a leader for the hadron of the given type carrying away the part x of the initial energy in the elementary act.

It is obvious that in the region $0.5 < x < 1$ (in l.s.) $\eta(x) = 1$. Experimental data on the functions $\eta(x)$ in the region $x < 0.5$ are lacking. However, it is obvious that at sufficiently small $x < x_{\min}$ the probability of being a leader is negligibly small: $\eta(x) \approx 0$. In the region $x_{\min} < x < 0.5$ $\eta(x)$ is a monotonically growing function. In the simplest case the function $\eta(x)$ may be presented in the following form:

$$\eta_{hN} = \begin{cases} 1 & \text{at} \quad 0.5 < x < 1 \\ \left(\dfrac{x-x_{\min}}{0.5-x_{\min}}\right)^{\beta_{hN}} & \text{at} \quad x_{\min} < x < 0.5 \\ 0 & \text{at} \quad x < x_{\min} \end{cases} \tag{6}$$

The absolute lower restriction on the value x for the leading hadron may be found out from the condition that its longitudinal momentum in the elementary act in c.m.s. cannot be negative; then in l.s. for the leading nucleon we have $x > x_{\min} = \frac{M}{\sqrt{s}}$, where M is the nucleon mass. At the incident nucleon momentum, e.g. $p = 100$ GeV/c, $x_{\min} \approx 0.07$. At multiple collisions of the leading nucleon in nucleus its energy decreases and, e.g. at $p = 20$ GeV/c, $x_{\min} \approx 0.15$. To make the calculations simpler in the future we shall assume that in all collision acts $x_{\min} \approx 0.1$ for both leading nucleon and pion. A more exact estimation of $x_{\min}$ at present does not seem possible. Note, however, that some little changes in the value of $x_{min} \sim 0.05 \div 0.15$ practically (within several per cent) do not affect the final results of calculations.

The parameter β_{NN} in (6) in the case of NN-interaction is determined from the sum rule meaning that the mean number of leading particles (nucleon or pion) is equal to one:

$$\int_{x_{min}}^{1} \frac{1}{\sigma_{NN}^{in}} \left(\frac{d\sigma}{dx}\right)^{NN \to h^{lead} X} dx = \int_{x_{min}}^{1} \eta_{NN}(x) \left[\frac{1}{\sigma_{NN}^{in}} \left(\frac{d\sigma}{dx}\right)^{NN \to NX} + \frac{1}{\sigma_{NN}^{in}} \left(\frac{d\sigma}{dx}\right)^{NN \to \pi X}\right] dx = 1. \quad (7)$$

From (6), (7) and (1), (4) we find $\beta_{NN} = 0.88$.

In (7) an assumption is made that the probability of being a leader for the given hadron (nucleon or pion) depends only on the variable x and does not depend on its type. The first term in (7) denotes the mean probability for the nucleon to preserve the leading property, the second term — the mean probability for the pion to gain the leading property in NN-interactions; their estimate values (using (1) and (4)) are 0.646 and 0.364 respectively, i.e. the probability for the pion to be a leader in the reaction NN → hX is not small. The direct check of this prediction in correlational experiments is of certain interest.

In the case of πN-interaction the parameter $\beta_{\pi N}$ in (6) is determined from the sum rule

$$\int_{x_{min}}^{1} \frac{1}{\sigma_{\pi N}^{in}} \left(\frac{d\sigma}{dx}\right)^{\pi N \to h^{lead} X} dx = \int_{x_{min}}^{1} \frac{1}{\sigma_{\pi N}^{in}} \left(\frac{d\sigma}{dx}\right)^{\pi N \to \pi X} \eta_{\pi N}(x) dx = 1. \quad (8)$$

The corrections taking into account the transfer of leading property to nucleon in πN--interactions are included in (8); in other words in the present work the process of double change of the leading particle of the type $N^{lead} \to \pi^{lead} \to N^{lead}$, having an insignificant relative probability, mainly because of a very small probability of the leading property transfer to nucleon in πN-collisions, is not considered [2].

From (6), (8) and (5) we find $\beta_{\pi N} = 0.66$.

c) Energy dependence of the charged particle mean multiplicity in elementary hadron-nucleon interactions $\langle n(E) \rangle_{hN}$.

To make the estimations simpler the following parametrization, which agrees with experimental data in the energy region up to several hundred GeV (see e.g. [3]), has been used:

$$\text{for pN-interactions } \langle n(E) \rangle_{pN} = C_p \left(\frac{E}{E_0}\right)^{\alpha_p},$$

where

$$C_p = 1.51, \quad E_0 = 1 \text{ Gev}, \quad \alpha_p = 0.323; \quad (9)$$

$$\text{for } \pi\text{N-interactions } \langle n(E) \rangle_{\pi N} = C_\pi \left(\frac{E}{E_0}\right)^{\alpha_\pi},$$

where

$$C = 2.22, \quad E_0 = 1 \text{ GeV}, \quad \alpha_\pi = 0.25.$$

The shower particle mean multiplicity according to [4] and in correspondence with the expression (1) of the present work has been taken equal to

$$\langle n_S(E)\rangle_{hN} = \langle n(E)\rangle_{hN} - 0.5, \tag{9a}$$

where 0.5 is the mean multiplicity of low energy charged hadrons (having velocities $v < 0.85$ c).

The approximations made (points a), b), c)) considerably simplify the calculation of the shower particle mean multiplicity $\langle n_S\rangle_{pA}$ in proton-nucleus interactions. For the value $\langle n_S\rangle_{pA}$ the following expression has been obtained in the framework of the MST approach:

$$\langle n_S(E)\rangle_{pA} = \langle n(E)\rangle_{pN} \frac{N(0, \sigma_1)}{N(0, \sigma_{NN}^{in})} + \langle n(E)\rangle_{\pi N} \frac{q_0}{1-q_3} \tag{10}$$

$$\frac{[N(0, \sigma_2) - N(\sigma_2, \sigma_3)]}{N(0, \sigma_{NN}^{in})} - (\bar{\nu}' - 1)\langle n\rangle_{hN}^{ch. lead} - \bar{\nu}' 0.5,$$

where $\sigma_1 = \sigma_{NN}^{in}(1-q_1)$, $\sigma_2 = \sigma_{NN}^{in}(1-q_2)$, $\sigma_3 = \sigma_{\pi N}^{in}(1-q_3)$.

$$q_1 = \int_{x_{min}}^{1} \frac{1}{\sigma_{NN}^{in}} \left(\frac{d\sigma}{dx}\right)^{NN\to NX} \eta_{NN}(x) x^{\alpha_p} dx,$$

$$q_2 = \int_{x_{min}}^{1} \frac{1}{\sigma_{NN}^{in}} \left(\frac{d\sigma}{dx}\right)^{NN\to NX} \eta_{NN}(x) x^{\alpha_\pi} dx,$$

$$q_3 = \int_{x_{min}}^{1} \frac{1}{\sigma_{\pi N}^{in}} \left(\frac{d\sigma}{dx}\right)^{\pi N\to \pi X} \eta_{\pi N}(x) x^{\alpha_\pi} dx,$$

$$q_0 = \int_{x_{min}}^{1} \frac{1}{\sigma_{NN}^{in}} \left(\frac{d\sigma}{dx}\right)^{NN\to \pi X} \eta_{NN}(x) x^{\alpha_\pi} dx, \tag{11}$$

$$N(\sigma_a, \sigma_b) = \int \frac{e^{-\sigma_a T(\bar{b})} - e^{-\sigma_b T(\bar{b})}}{\sigma_b - \sigma_a} d^2 b. \tag{12}$$

The first and the second terms in (10) mean respectively the mean number of charged particles produced by a nucleon which has preserved its leading property, and by a pion which has gained the leading property. The third term in (10) is introduced to exclude the intermediate leading charged hadrons from the shower particle mean multiplicity; $\langle n\rangle_{hN}^{ch. lead}$ is the fraction of charged hadrons among the leading ones; it depends weakly on the initial energy and is approximately $\langle n\rangle_{hN}^{ch. lead} \approx 0.7$ (see e.g. [5]). The mean number of the leading hadron inelastic collisions with consideration of its type change is denoted by $\bar{\nu}'$

$$\bar{\nu}' = \frac{A\sigma_{\pi N}^{in}}{\sigma_{pA}^{prod}} + \frac{\sigma_{pN}^{in} - \sigma_{\pi N}^{in}}{\sigma_{pA}^{prod}} N(0, \sigma'), \tag{13}$$

where

$$\sigma' = \sigma_{pN}^{in}(1-\bar{\eta}),$$

$$\bar{\eta} = \int_{x_{min}}^{1} \frac{1}{\sigma_{NN}^{in}} \left(\frac{d\sigma}{dx}\right)^{NN\to NX} \eta_{NN}(x)dx,$$

$$\sigma_{pA}^{prod} = \int (1-\exp\{-\sigma_{pN}^{in}T(\bar{b})\})d^2b.$$

Note that the expression (13) turns into a conventional expression for the mean number of the incident nucleon inelastic interactions

$$\bar{\nu} = \frac{A\sigma_{pN}^{in}}{\sigma_{pA}^{prod}}, \tag{14}$$

if one puts $\sigma_{\pi N}^{in} = \sigma_{pN}^{in}$ or adopts that the nucleon does not transmit the leading property to pion: $\bar{\eta} = 1$ (i.e. if the second term in sum rule (7) is not considered). Note that since

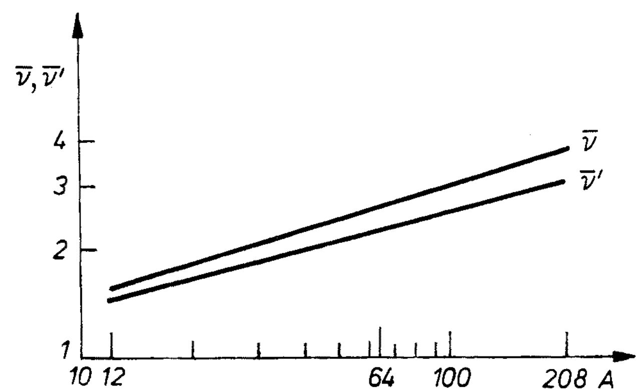

Fig. 1

$\sigma_{\pi N}^{in} < \sigma_{NN}^{in}$, then $\bar{\nu}' < \bar{\nu}$. The comparison of values of $\bar{\nu}'$ and $\bar{\nu}$ for pA-interaction is given in Fig. 1. The last summand in (10) is the mean multiplicity of recoil protons not belonging to the shower type.

The effective numbers $N(\sigma_a, \sigma_b)$ entering the expressions (10) and (13) were calculated using the Fermi distribution for the one-particle nuclear density

$$\varrho(r) = \frac{\varrho_0}{1+\exp\left(\frac{r-R}{c}\right)},$$

where $R = 1.12\,A^{1/2}$ fm, $C = 0.545$ fm, ϱ_0 is determined from the condition of normalization $\int \varrho(r)dr = A$. $T(\bar{b}) = \int_{-\infty}^{\infty} \varrho(\bar{b}, z)dz$ is the nuclear density projection on the impact parameter plane. The inelastic cross sections in NN and πN collisions are taken equal to $\varrho_{NN}^{in} = 31$ mb, $\sigma_{\pi N}^{in} = 20$ mb.

The results of calculations of the ratio of the shower particle mean multiplicities $R_A = \langle n_S(E)\rangle_{pA}/\langle n_S\rangle_{pp}$ in pA and pp-interactions at the initial energy $E = 100$ GeV using the expressions (9) and (10), are presented in the Table (the line next to last). In upper lines the results of calculations in more rough approximation are presented for comparison: without consideration of the incident proton energy loss (first line); with consideration of energy loss but without consideration of their fluctuations (second line); with consideration of the energy loss fluctuations but with a rough approximation for the nucleon inelastic spectrum $\left(\frac{d\sigma}{dx}\right)^{NN\to NX} = \text{const}$ (third line); with a more exact account of the nucleon inclusive spectrum (expression (1)) but without changing the leading hadron type (fourth line). In the last case the values of R_A turn out considerably lower (particularly for heavy nuclei) than the experimental ones [4] at $E = 100$ GeV (last line). From our point of view such a discrepancy is a direct indication of the fact that a notable contribution to the shower particle multiplicity comes from the inelastic interactions of se-

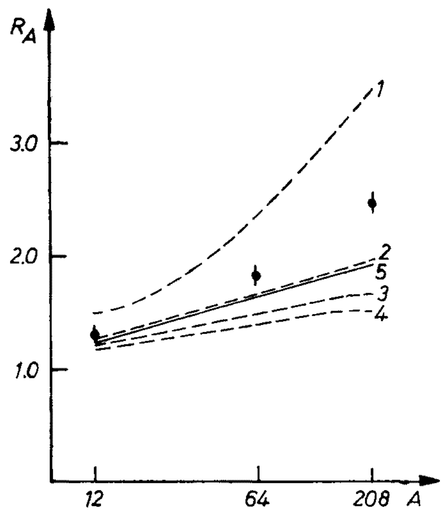

Fig. 2

condary hadrons; one may expect that among them the highest relative contribution would have the inelastic interactions of the leading pion, whose energy in the given interaction act exceeds the energies of other particles including the "parental" nucleon. In the fifth line of the Table together with the contribution to R_A of the leading nucleon, the contribution of the pion to which the leading property is transferred is considered. It should be emphasized that on doing so the possible inelastic collisions of the nucleon which is no more a leader are not considered. Note that the ratio of the leading pion and nucleon contributions to R_A (the ratio of the second and first summands in (10)) is the larger the larger is the target A mass number: for A increase from 12 to 208 it grows from 0.08 to 0.33.

The calculated and experimental values of R_A given in the Table are presented in Fig. 2 for illustration. Note that even the most exact approximation (curve 5) leads to slightly lower values of R_A than the experimental data. This means that inelastic secondary collisions of nonleading hadrons in nucleus also make a certain contribution to the shower particle multiplicity.

TABLE

Results of calculations of $R_A = \langle n_S(E)\rangle_{pA}/\langle n_S\rangle_{pp}$ in various approximations

	C^{12}	Cu^{64}	Pb^{208}
1. Without consideration of energy loss	1.504	2.363	3.479
2. Without consideration of energy loss fluctuations	1.275	1.677	1.995
3. With consideration of energy loss fluctuations $\frac{d\sigma^{PN \to NX}}{dx} = \text{const}$	1.221	1.510	1.679
4. With consideration of energy loss $\frac{d\sigma^{PN \to NX}}{dx} \neq \text{const}$ (Eq. (1))	1.193	1.434	1.538
5. Consideration of the change of the leading hadron type (Eq. (10))	1.256	1.637	1.964
6. R_A^{exp}	1.3±0.04	1.85±0.05	2.48±0.06

In conclusion let us make a short summary. The results of the present work show that the rise of the shower particle multiplicity in the nucleon-nucleus collisions, as compared with the nucleon-nucleon one, cannot be completely due to the multiple inelastic collisions of the incident nucleon only, and there arises a necessity to take into account the inelastic interactions of secondary (newly produced) particles. We have partially taken into account such interactions by considering a more general case of the leading hadron (including the ones differing from the incident hadron by their type) propagation in nucleus. The agreement of the theory with the experiment considerably improves, though experimental values of R_A still exceed the calculational ones; this apparently indicates to the necessity of taking into account inelastic interactions of other nonleading secondary particles. For scarcity of experimental data in the present work we have used certain approximations for inclusive spectra of nucleons and pions, and probabilities of their being a leader in HN interactions; the passage of unstable leading hadrons — baryon and meson resonances through the nucleus is not considered. A more detailed and exact information on inclusive and correlational characteristics of hadron-nucleon interactions will allow one to evaluate more precisely the role played by the secondary inelastic interactions in the multiple production of particles on the nucleus, and in the end, from the comparison with experiment obtain the region of application of the traditional theory of multiple scattering containing no parameters of the hadron-production process space-time structure.

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