BOUND STATES AND LORENTZ-POINCARÉ SYMMETRY

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A hypothesis of the "relation-continuum" C_4 is put forward, closely connected with isolation of physical systems, which extends to finite universal constant c the absolute nature of the Galilean relative coordinates and the absolute Newtonian time. Points of C_4 continuum are directly unobservable and the relativistic symmetry L_4 of directly observable space-time events becomes the limiting case of the C_4-symmetry. Consequently, though the possibility of the hypothesis of C_4-continuum is due to quantum physics, the modifications it implies come with finite universal constant $\hbar/c$ and concern the description of the internal structure of bound states only. The C_4-symmetry of relations, as weaker than the Lorentz-Poincaré L_4-symmetry of events, makes "more room" for quantum dynamical models. The Feynman graphs phenomenology with form factors (vertex functions) of non-point particles left for experimental determination can be connected with the C_4-framework which determines their analytic structure. The C_4-effects then would reveal themselves only in these processes in which composite particles participate. Therefore, the "good" quantum electrodynamics of point-particles is left unmodified. Two off-mass-shell effects are analyzed in the relatively low-energy processes which are connected with the mass-dependent localization of the centre-of-mass of composite particle "M". They seem to be crucial for the hypothesis itself.

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1. *Relations v. events*

Present quantum theory, though going deeper into the nature of the structure of matter than the classical ($\hbar = 0$) theory does, maintains the classical continuum of directly observable events $X_\mu = (\boldsymbol{X}, ict)$ as an elementary background of all physics. Within this picture there should be no room for the well-known quantum asymmetry between the momentum "p" and the "x" languages [1] favourising the "p" one. Doubtless the Lorentz-Poincaré symmetry L_4 of events must rule the asymptotic zone where any micro-structure is measured indirectly, i.e. in terms of free four-momenta of scattered particles, because this symmetry is imposed by the "classical" measuring devices. However, this is not a com-

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pelling reason why the L_4-symmetry is to be extrapolated onto the internal structure of isolated systems. The uncertainty relations

$$\Delta P_\mu \sim \hbar/\Delta X_\mu \quad (P_\mu = (\boldsymbol{P}, iE/c)) \quad (\mu, \nu, \ldots = 1, 2, 3, 4) \tag{1.1}$$

themselves show the (quantum) origin of the "$x-p$" asymmetry and, simultaneously, provide us with an argument against the elementary character of the L_4-symmetry. Indeed, the better determined the coincidence of events is ($\Delta X_\mu \to 0$), the larger becomes the uncontrollable distortion of the base ($\Delta P_\mu \to \infty$) thus destroying, also in an uncontrollable way, the "quantum-potential" [2] structure "ψ" of the isolated object under measurement. Let us remember that the space-time localization is always accompanied by the "factual" [2], irreversible process of the "reduction of the wave packet" [3] which cannot be obtained from the unitary, time-reversible development of any "quantum-potential" state "ψ". Therefore, any measuring apparatus capable of causing "factual" space-time coincidences of events cannot be described completely, i.e. by a "pure" quantum state "ψ" [4]. On the other hand, it is evident that direct "p" measurements of the scattered particles in the asymptotic region do not conflict with "quantum-potentiality" of e.g. the eigenstates of energy of isolated systems which determine their structure. Thus the "p" and not "x" language is compatible with the state of isolation of the investigated system and its "quantum" structure.

Within the classical framework ($\hbar = 0$) the duality of "factual-potential" vanishes together with the wave-corpuscular duality, because everything is here "actualized" by means of the energy-momentum-free "classical photons" [5]. Consequently, the event-continuum of direct measurement must remain elementary.

Moreover, in a framework which accounts for finiteness of both, $\hbar$ and c an exact determination of the event coincidence, i.e. $\Delta X_\mu \to 0$, is inescapably connected with the transfer of an infinite amount of inertia $\mathcal{M}$. In fact,

$$\Delta\mathcal{M} = \Delta\mathcal{W}/c^2 \sim (\hbar/c^2)(1/\Delta t) \xrightarrow[\Delta t \to 0]{} \infty \text{ hence } \mathcal{M} \xrightarrow[\Delta X_\mu \to 0]{} \infty\,. \tag{1.2}$$

One can doubt therefore, if the classical L_4-symmetry of events accounts correctly for the symmetry of internal structure of isolated ("quantum") system with finite inertia. As seen from (1.2) this objection would concern neither the classical framework ($\hbar = 0$) nor the non-relativistic (NR) one ($c \to \infty$); it would concern present framework where finite $\hbar/c$ must be taken into account.

Our aim is to show that the uncertainty principles simultaneously admit the hypothesis of a new continuum C_4, called the relation-continuum, which precedes the event-continuum L_4. According to this hypothesis the *directly unobservable* relations between two "possible" quantum particles, and not the directly observable "possible" events of L_4, found the first metrical relations. C_4 means the Euclidean four-dimensional continuum composed of the three-dimensional relation-space C_3 and the internal-time continuum θ. Let C_3 be parametrized by the Cartesian coordinate $\boldsymbol{y}$ denoting the space-relation between two "possible" quantum particles and let τ denote the absolute variable which parametrizes θ and enumerates the quantum-canonical evolution of the state $|\psi)$. The assumed canonical formalism in C_4 automatically imposes the simultaneity in τ, much as is the case in the NR framework, where the absolute Newtonian time t^G plays the role of τ.

The C_4-symmetry means the 4-parameter symmetry group of the rotations in C_3, without translations, and the translations in the internal-time τ hence $C_4 = C_3 \otimes \theta$ and

$$y'_j = O_{jk} y_k, \qquad \tau' = \tau + a_0 \quad (j, k, \ldots = 1, 2, 3). \tag{1.3}$$

O_{jk} is the 3×3 orthogonal matrix and a_0 the translation parameter.

The unobservable nature of the points in C_4 allows the fundamental assumption, namely that all *metrical relations established in C_4 are a priori absolute*. Since directly observable events of L_4 will get the status of the limit of C_4-relations, and not vice-versa, the C_4-absoluteness precedes the L_4-absoluteness. In other words, the C_4-absoluteness does not imply the L_4-invariance. For example, the *two* C_4-absolute intervals

$$\begin{aligned} r = |\boldsymbol{y}| &= C_4\text{-inv.} = C_4\text{-absolute} \\ \Delta\tau &= C_4\text{-inv.} = C_4\text{-absolute} \end{aligned} \tag{1.4}$$

cannot be a priori expressed by any L_4-invariants since L_4 deals with the only *one* a priori L_4-absolute four-interval

$$x_\mu^2 = (x_{2\mu} - x_{1\mu})^2 = L_4\text{-inv.} = L_4\text{-absolute}. \tag{1.5}$$

Simultaneously, this proves that C_4 and L_4 geometries are non-isomorphic. For the same reasons, the L_4 and G_4 continua of events are non-isomorphic either, because in the NR limit $c \to \infty$ the discontinuity emerges in the number of absolute intervals, from *one* in L_4 to *two* in G_4. Moreover, the NR relation-continuum C_4^G can be spanned on the Galilean relative space coordinate $\boldsymbol{y}^G$ and τ^G which, up to the translation constant, can be identified with the absolute Newtonian time t^G. This proves the isomorphy between C_4^G and G_4. Indeed, since

$$\boldsymbol{y}^G = (\boldsymbol{x}_2^G - \boldsymbol{x}_1^G)|_{\Delta t^G = 0}, \qquad \Delta\tau^G = \Delta t^G, \tag{1.6}$$

then, in contrast to (1.4), both C_4^G-absolute intervals become simultaneously the G_4-absolute quantities as

$$\begin{aligned} r^G = |\boldsymbol{y}^G| &= C_4^G\text{-inv.} = G_4\text{-inv.} = \begin{Bmatrix} C_4^G \\ G_4 \end{Bmatrix}\text{-absolute} \\ \Delta\tau^G = \Delta t^G &= C_4^G\text{-inv.} = G_4\text{-inv.} = \begin{Bmatrix} C_4^G \\ G_4 \end{Bmatrix}\text{-absolute.} \end{aligned} \tag{1.7}$$

This reflects the singular character of the Galilean group [6] with its "neutral element" alien to the L_4-group and shows that all C_4-effects must vanish in the NR limit $c \to \infty$.

Let us consider the simplest two-body system, where the relation-coordinate $\hat{\boldsymbol{y}}$ (operator) and the canonically conjugate to it relation-momentum $\hat{\boldsymbol{q}}$, thus fulfilling the commutation relations

$$[\hat{y}_j, \hat{y}_k] = [\hat{q}_j, \hat{q}_k] = 0, \quad [\hat{y}_j, \hat{q}_k] = i\hbar\delta_{jk}, \tag{1.8}$$

parametrize the internal, C_4-absolute laws of motion of the system. In the Schroedinger representation, which will be postulated throughout the paper, the commutation relations (1.8) are realized by putting $\hat{\boldsymbol{y}} = \boldsymbol{y}$ and $\hat{\boldsymbol{q}} = -i\hbar \, \mathrm{grad}_y$. Assume that dynamics is introduced through the C_4-absolute internal potential V independent of spins and depending on the C_4-absolute distance r only, between the constituents. Thus $V = V(r)$ $(r = |\boldsymbol{y}|)$ and the four generators of the Lie algebra in C_4 are assumed in the following form:

$$\hat{h} = c[(m_1^2c^2+\hat{\boldsymbol{q}}^2)^{1/2}+(m_2^2c^2+\hat{\boldsymbol{q}}^2)^{1/2}]+V(r)$$

$$\hat{j}_k = e_{kls}\hat{y}_l\hat{q}_s+s_{1k}+s_{2k}, \tag{1.9}$$

where the internal Hamiltonian $\hat{h}$ means the translation generator in the internal-time τ, while $\hat{j}_k$ are three rotation generators in C_3, where $\boldsymbol{s}_{1,2}$ are spin matrices of the constituents "1" and "2". Thus $\hbar$ enters the C_4-framework through the canonical commutation relations (1.8), while c, through the analytic form of $\hat{h}$ adjusted to the relativistic kinematics. We postulate that the C_4-framework leaves the L_4-kinematics unmodified.

The rotation invariance of $\hat{h}$ and the analytic form of $\hat{j}_k$ guarantee that the C_4 Lie algebra equalities are fulfilled, as

$$[\hat{h}, \hat{j}_k] = 0, \quad [\hat{j}_k, \hat{j}_l] = i\hbar e_{kls}\hat{j}_s. \tag{1.10}$$

The action-at-a-distance in C_4 implied by $V(r)$ does not conflict with the Einsteinian principle of relativity or causality [7] because r is directly unobservable whence, $V(r)$ cannot propagate any signal. The latter always means the propagation of some discontinuity in the continuum of directly observable events. A direct proof of that $V(r)$ remains consistent with the relativity principle and causality will be given in Section 5.

The thus resulting C_4-absolute Schroedinger equation

$$i\hbar\partial/\partial\tau|\psi) = \hat{h}|\psi) \tag{1.11}$$

leads, for the stationary states, to the internal-energy $W = Mc^2$ eigenvalue problem of $\hat{h}$. According to the absolute nature of C_4, $|\psi)$ and W are a priori absolute, much as $\hat{h}$ itself, i.e.

$$\hat{h}|\psi_M) = W|\psi_M), \quad |\psi_M) = |M)\exp(i\phi^C) = C_4\text{-abs.}$$

$$W = Mc^2 = C_4\text{-abs.}, \quad \phi^C = -W\tau/\hbar = C_4\text{-abs.}. \tag{1.12}$$

The meaning of the C_4-absoluteness is easy to understand in the NR framework, although in this limit $(1/c = 0)$ the C_4-relationism becomes of no physical relevance.

Indeed, denoting $C_4^G = \lim_{c\to\infty} C_4$ and taking into account that $G_4 = \lim_{c\to\infty} L_4$, the following well-known point- and canonical-transformations

$$(i) \qquad \boldsymbol{X}^G = a^G \boldsymbol{x}_1^G + b^G \boldsymbol{x}_2, \quad \boldsymbol{y}^G = \boldsymbol{x}_2^G - \boldsymbol{x}_1^G \quad (a^G = 1 - b^G = m_1/(m_1+m_2))$$

$$(ii) \qquad \boldsymbol{P}^G = \boldsymbol{p}_1^G + \boldsymbol{p}_2^G \quad \boldsymbol{q}^G = a^G \boldsymbol{p}_2^G - b^G \boldsymbol{p}_1^G \tag{1.13}$$

establish the isomorphy between C_4^G and G_4. After subtracting from $\hat{h}$ the rest-energy $(m_1+m_2)c^2$ and letting c go to infinity we get from (1.11) the C_4^G-absolute two-body Schroedinger equation

$$(i\hbar\partial/\partial\tau^G)\psi^G(\boldsymbol{y}^G, \tau^G) = [(\hat{\boldsymbol{q}}^G)^2/2\mu + V^G(r^G)]\psi^G, \tag{1.11'}$$

where $\mu = m_1 m_2/(m_1+m_2)$ is the reduced mass of "1+2" and V^G the usual NR potential; $V^G = V$, $r^G = |\boldsymbol{y}^G|$.

Let us assume that the C_4^G-absolute equation (1.11') is the starting point of the theory and provides us with the C_4^G-absolute differential cross-section $d\sigma/d^3q_{as}^G$ for elastic scattering of "1" and "2", where $\boldsymbol{q}_{as}^G$ means the asymptotic relation-momentum. The form-invariance of the $(\boldsymbol{y}^G, \boldsymbol{q}^G)$-coordination of (1.11') under the Galilean group of transformations implies that no contact exists a priori between the theoretical predictions obtained in the internal relation-continuum C_4^G and the external event-continuum (here G_4), where any direct measurement takes place. In the NR limit this "contact" between the C_4^G-inside and G_4-outside is solved by (1.13), however, in the general case of finite c a *new dichotomy* of all physical characteristics is called for. It divides them into the classical-like (C–L) and the quantum-like (Q–L) which will be defined in Section 3. However, in order to understand why such a dichotomy will appear, let us analyze some essential difference between the scattering and bound states which is seen already in the NR quantum framework.

In the scattering process the asymptotic momenta $\boldsymbol{p}_{1as}^G$ and $\boldsymbol{p}_{2as}^G$ of each of the constituents are directly measurable in the asymptotic zone. By resorting to the c-number boundary condition $\boldsymbol{P}^G = 0$ which determines the CM-system S^* of "1+2" the c-number momentum equalities

$$\boldsymbol{q}_{as}^G = \boldsymbol{p}_{2as}^G = -\boldsymbol{p}_{1as}^G \tag{1.14}$$

following from (1.13) solve the "contact" problem. In fact:

$$C_4^G\text{-abs.} = d\sigma/d^3q_{as}^G = (d\sigma/d^3p_{2as}^G)_{S^*}, \tag{1.15}$$

and now the cross-section can be evaluated in any other laboratory "S".

Suppose that the same system "$M = 1+2$" is now in a C_4^G-absolute bound state $\psi_n^G(\boldsymbol{y}^G)$. Thus, in the asymptotic zone we deal with a single particle "M" whose internal structure $\psi_n^G(\boldsymbol{y}^G)$ *is hidden* from observation. The singular character of the NR limit [6] makes that the Fermi relation-momenta $\boldsymbol{q}^G$ of $\psi_n^G(\boldsymbol{y}^G)$ can be also expressed by the external, Galilean momenta $\boldsymbol{p}_{1,2}^G$ as

$$\boldsymbol{q}^G = \boldsymbol{p}_2^{G*} = -\boldsymbol{p}_1^{G*}. \tag{1.16}$$

However, and this will be of first importance, in the general case of finite c, from the experimental point of view the equalities (1.16) are dead, because $\boldsymbol{q}^G$ is directly unobservable. For example, the disintegration of "M" extracts the constituents on their energy-shells, thus making $\boldsymbol{q}^G$ directly measurable, but this is a dynamical process and, like any other, calls for a theory. In consequence, in the general C_4-framework ($c < \infty$) where no "contact" exists a priori between the external L_4 and the internal C_4 parametrizations like (1.13), we get room for new hypotheses concerning the "contact" problem. Here, the aforementioned new dichotomy will come into play.

No matter if c is finite or infinite, the difference between the scattering and bound states is of "pure" quantum nature. In the classical framework ($\hbar = 0$) the momenta "p" become x-local quantities, cf. App. I, and the classical world-line is automatically "seen" by the "classical photons" [5]. On the other hand, the classical world-line threatens to kill any interaction [8]. Nevertheless, apart from the "no-interaction" theorems, there is no room for a continuum more elementary than the event-continuum, because everything is automatically "actualized" in the classical continuum L_4 of the "classical" measuring devices. However, each real object is "quantum" as independently of its structure it respects the uncertainty relations [9], while $\hbar$ much as $1/c$ is always different from zero. Thus the "classical" nature of measuring devices cannot be due to the classical framework ($\hbar = 0$), as well as the "NR physics" does not result from $c \to \infty$. The "classical" nature of any measuring device results from its heaviness and, which is more important, from the instability of its state of isolation. In fact, with increasing number N of the internal (non "cooled") degrees of freedom of heavy systems the energy gaps ΔW between its different internal states decrease like $\exp(-N)$ [10], thus tending to zero with $N \to \infty$. With that respect it reminds the classical framework limit $\hbar \to 0$ [5], where the energy gaps do not exist at all making an exact isolation of the system a fiction inherent of the Hamiltonian mechanics.

2. *Relation and event continua*

Two limiting procedures explain the status of event-continuum L_4 within the relation-continuum C_4 and hence, they will be called the Lorentz limits I and II. Let us consider them subsequently.

I. Assume that the constituent "1" of an isolated system composed of two interacting particles "1+2" becomes infinitely heavy. Then, in order to deal with finite internal Hamiltonian, we define the renormalized Hamiltonian

$$\hat{H} = \lim_{m_1 \to \infty} (\hat{h} - m_1 c^2) = c(m_2^2 c^2 + \hat{\boldsymbol{q}}^2)^{1/2} + V(\boldsymbol{y}). \tag{2.1}$$

Thus the infinitely heavy constituent "1" disappears from the equation of motion (1.11) creating the notion of e x t e r n a l i t y for the remaining part of the system — here the particle "2".

Let us change the notation by putting

$$(\boldsymbol{y}, ic\tau) = (\boldsymbol{x}_2^*, ict_2^*) = x_{2\mu}^*, \tag{2.2}$$

and let us define the four numbers U_μ^* such that

$$U_\mu^* = (0, 0, 0; (i/c)V(y = x_2^*)). \tag{2.3}$$

Apart from spins, Eq. (1.11) can be rewritten in the form

$$[(-i\hbar\partial/\partial x_{2\mu}^* - U_\mu^*)^2 + m_2^2c^2]\psi(x_{2\mu}^*) = 0$$

or, omitting the superscript "*",

$$[(-i\hbar\partial/\partial x_{2\mu} - U_\mu)^2 + m_2^2c^2]\psi(x_{2\mu}) = 0. \tag{2.4}$$

If $x_{2\mu}$ is identified with the four-coordinate of "2" and U_μ with the external four-vector field, Eq. (2.4) coincides with the L_4-covariant Klein-Gordon equation. From now on the equalities (2.2) and (2.3) indicate the rest-frame S^* of infinitely heavy "1" in L_4.

However, though (2.4) is L_4-covariant hence consistent with the theory of relativity, the presence of external field U_μ makes that the L_4-symmetry is not the internal symmetry group of (2.4). In other words, Eq. (2.4) is not L_4-form-invariant and different analytic forms of the representations of $U_\mu(x_\nu)$ enumerate the 10-parameter variety of the reference frames S in L_4. Thus infinitely heavy source "1" of U_μ gets the classical world-line $x^*(t^*) = 0$ identified here with the origin of S^* and hence, the Lorentz limit I automatically implies that one end of the relation-coordinate y, namely that which indicates infinitely heavy "1", gets the classical localization. In this way the Lorentz limit I implies the fundamental *asymmetry* between the "classical" heavy apparatus "1" at the origin of x_2 and the "quantum" object "2" at the end of x_2. Simultaneously, events of the L_4--continuum become the limits of relations in C_4 — not vice-versa!

Moreover, Eq. (2.4) shows that in the presence of an external field the L_4-covariant equations of motion coincide with the C_4-equations (1.11) with the corresponding internal dynamics.

II. The second Lorentz limit II is complementary to the first one and it consists in the transition to the asymptotic zone $r \to \infty$ in C_3 assuming that

$$V(r) \xrightarrow[r\to\infty]{} 0 \quad (r = |y|). \tag{2.5}$$

Much as in the limit I, assume also that $m_1 \to \infty$, but now, when the constituents "1" and "2" do not interact ($r \to \infty$), this assumption is very weak. It only requires that outside "2" there exists a world of inertia infinitely larger than m_2, which can be called the *condition of measurability in microphysics.* After the renormalization analogous to that from (2.1), the Schroedinger equation (1.11) becomes equivalent to the equation

$$[\nabla_y^2 - (1/c^2)\partial^2/\partial\tau^2 - (m_2c/\hbar)^2]\psi(y, \tau) = 0 \tag{2.6}$$

which is of the analytic form of the Klein-Gordon free-equation. Now, the L_4-symmetry becomes the internal symmetry group of Eq. (2.6), i.e. Eq. (2.6) is L_4-form-invariant, and hence, the heretofore C_4-absolute relation-coordinate $(y, ic\tau)$ can be identified with the L_4-four-coordinate $x_{2\mu} = (x_2, ict_2)$ of "2" in *any* reference frame S in L_4. Instead of a single external field U_μ from the limit I, the limit II creates the 10-parameter variety of the infinitely heavy, "classical" measuring devices "1" whose external fields are hidden in the asymptotic-kinematic zone of the L_4-symmetry. Thus, in the Lorentz limit II the 4-parameter symmetry C_4 becomes split into the 10-parameter symmetry L_4 of the event--continuum. Let us remember that in the limit I the 10-parameter symmetry group L_4 is not the internal symmetry of Eq. (2.4) because of the presence of an external field.

This embranchment of the C_4 into the L_4-symmetry forces one to distinguish between two meanings, the meaning "*A*" and "*B*", of the L_4-absoluteness when one confronts it with the C_4-absoluteness. If the external event-continuum were the Galilean one G_4 then, the G_4-absoluteness of $r^G = |\mathbf{y}^G|$ in the meaning "*A*" denotes the absoluteness of the "proper-length" $L_0 = r^G$ of a *single* measuring rod. In the meaning "*B*" the G_4-absoluteness of r^G denotes the common "proper-lengths" L_0 of *all* identical, but different measuring rods of all reference frames S^G parametrizing G_4. The same concerns the second absolute interval $\Delta t^G = T_0 = \Delta\tau^G$. In order to distinguish between these two meanings of the absoluteness we add the letter, *A* or *B* to the G_4 or L_4 absoluteness of the corresponding absolute quantity.

The discontinuity in the number of absolute intervals from *one* in L_4 to *two* in G_4 implied by the NR limit $c \to \infty$ causes that the two "*A*, *B*-absoluteness" of L_0 and T_0 are manifested in a very different mathematical forms. If $L_0\,(T_0)$ means the proper-length of a single rod then the L_4-covariant expression of $L_0\,(T_0)$ calls for a single four-vector n_μ, such that in the rest-frame S^* of this rod (clock) we have $n^*_\mu = (0, 0, 0;\ i)$. The necessity of introducing some external field n_μ is evident since a priori L_4 deals with only one L_4-absolute four-interval x^2_μ (1.5). Thus, if $x_\mu = x_{2\mu} - x_{1\mu}$ then

$$L_0 = |\mathbf{x}^*| = [x^2_\mu + (n_\mu x_\mu)^2]^{1/2} = L_4\text{-}A\text{-absolute}$$

$$T_0 = \Delta t^* = -n_\mu x_\mu / C = L_4\text{-}A\text{-absolute}. \tag{2.7}$$

However, the "*B*-absoluteness" of L_0 (T_0) must call for the whole *infinite set* of external four-vectors $n^{(S)}_\mu$ where "*S*" indicates the rest-frame *S* of the corresponding rod (clock), such that

$$n^{(S)}_\mu|_S = (0, 0, 0;\ i), \quad (n_\mu \equiv n^{(S^*)}_\mu). \tag{2.8}$$

Then

$$L_0 = |\mathbf{x}||_S = |\mathbf{x}'||_{S'} = \ldots = L_4\text{-}B\text{-absolute}$$

$$T_0 = \Delta t|_S = \Delta t'|_{S'} = \ldots = L_4\text{-}B\text{-absolute}. \tag{2.9}$$

Of course, much as in (2.7), L_0 and T_0 of each of the rods (clocks) can be expressed L_4--covariantly with the help of the corresponding four-vector $n^{(S)}_\mu$.

The embranchment of the 4-parameter symmetry C_4 to the 10-parameter symmetry L_4 of the asymptotic-kinematic zone implies that the C_4-absoluteness of $r = |\mathbf{y}|$ and/or $\Delta\tau$ corresponds to the L_4-*B*-absoluteness. Here the essential asymmetry reveals itself between the $L_4 - x$ and $L_4 - p$ languages [1], because the $L_4 - x$ relations must deal with *two* four-points $x_{1\mu}, x_{2\mu}$, e.g. $x^2_\mu = (x_{2\mu} - x_{1\mu})^2$, whereas the $L_4 - p$ relations exist for a single "four-point" P_μ in the $L_4 - p$ space, e.g. $P^2_\mu = -M^2c^2$. Therefore, we can expect the difference between the "x" and "p" relations which will establish the contact between the C_4-absolute and the L_4-absolute quantities.

Note that the Lorentz limit I creates a *single* physical entity U_μ of the external field which spoils the L_4-symmetry of equation (2.4), but not its L_4-covariant structure! Therefore, in contrast to the limit II, it results in the standard "A-absoluteness", like that from (2.7). In the limit II the situation is quite different. Here the 10-parameter set of the "classical" measuring devices becomes hidden in the L_4-asymptotic zone, however, their existence is implicitly manifested in the L_4-symmetry of the equation (2.6). Thus the weaker 4-parameter symmetry group C_4 with *two* absolute intervals $r = |\mathbf{y}|$ and $\Delta\tau$ becomes split into the stronger, 10-parameter symmetry group L_4 with only *one* absolute interval x_μ^2. This discontinuity in the number of absolute intervals, from two to one, automatically implies that all measuring rods and clocks in L_4 are of the same units as the corresponding units of r and $\Delta\tau$, respectively. This calls for the new "B" meaning of the absoluteness whose mathematical expression requires the infinite set of the L_4-four-vectors $n_\mu^{(S)}$ implicitly characterizing all measuring devices. Thus ($r = |\mathbf{y}|$)

$$C_4\text{-abs.} = r = |\mathbf{x}||_S = |\mathbf{x}'||_{S'} = \dots = L_4\text{-}B\text{-abs.}$$

$$C_4\text{-abs.} = \Delta\tau = \Delta t|_S = \Delta t'|_{S'} = \dots = L_4\text{-}B\text{-abs.}\,. \tag{2.10}$$

In the NR limit, C_4^G and G_4 deal with the same number of the fundamental absolute intervals hence, the "B-absoluteness" of L_0 and T_0 does not call for external fields $n_\mu^{(S)}$; they are "contained" in the very structure of the G_4-symmetry. Since the external fields $n_\mu^{(S)}$ cannot enter the L_4-symmetric equations of motion, the "B-absoluteness" (2.10) shows that the C_4-framework goes beyond the scope of the L_4-one.

3. *Partial relativization of two-body states*

We start with laws of motion in C_4, but now, unlike the NR framework, C_4 is not isomorphic with L_4 and hence the general question arises of the determination of contact between the theoretical predictions in C_4 and the experiment in L_4. This problem will be solved by the a posteriori relativization procedure which consists in the appropriate projection of the C_4-characteristics obtained from the C_4-framework onto the L_4--continuum of events.

For this purpose we must determine the already mentioned new dichotomy of all physical characteristics. A characteristic of the measured object "O" will be called the classical-like (C–L), if its exact measurement can be done without the recoil of "O". Conversely, a characteristic will be called the quantum-like (Q–L), if its determination must be connected with finite recoil of "O". Of course, in the classical framework ($\hbar = 0$) where the "classical photons" exist [5] all characteristics are C–L. Although quantum theory has ruled out the "classical photons" from physical reality, the classical event--continuum L_4 regarded as the first physical continuum implies that even the "quantum--potential" shapes "ψ" must be a priori sunk in L_4. This classical element of the present quantum theory would be eliminated by the C_4-framework, where the "quantum-potential" shapes "ψ" remain a priori hidden in C_4. This seems to be consistent with the experiment because, in general, "ψ", much like the forces, is not "visible" from the L_4--outside. In consequence, the C–L characteristics, as measurable directly without affecting "O" must be relativized directly, much like all characteristics of the present L_4--framework, whereas the Q–L characteristics which are measurable indirectly, i.e. in the "p" language, will be relativized indirectly. Below and in Sections 4 and 5 these two kinds of the a posteriori relativization procedures will be defined and illustrated in several examples.

Suppose that from the equation (1.11) we have determined the C_4-absolute stationary state of the two-body system

$$(\boldsymbol{y}|M) \exp[-iW\tau/\hbar)] = \psi_M(\boldsymbol{y}, \tau) \tag{3.1}$$

which is the eigenstate of $\hat{h}$ to the eigenvalue $W = Mc^2$,

$$\hat{h}(\boldsymbol{y}|M) = W(\boldsymbol{y}|M). \tag{3.2}$$

In Appendix I we show that the four-momentum P_μ of a free particle "M" of mass M is a C–L characteristic and therefore, the C_4-absolute eigenvalue M must be relativized directly. This consists in attaching to M the four-momentum P_μ where

$$P_\mu = (\boldsymbol{P}, iE/c), \quad P_\mu^2 = -M^2c^2 = L_4\text{-inv.}\,. \tag{3.3}$$

As it was to be expected the direct relativization of the "p" scalar M does not require the "B-absoluteness". Besides, if $|M)$ is the eigenstate of $\hat{\boldsymbol{j}}^2$ from (1.9) to the eigenvalue $\hbar^2[s(s+1)]$ then, the spin s of "M" represents the second C–L characteristic and its direct relativization consists in attaching to the state $|M, s)$ the corresponding L_4-amplitude $A(s)$ as in the present theory. Let us emphasize that direct relativization of M concerns a single eigenvalue $W = Mc^2$ of $\hat{h}$. Of course, the internal Hamiltonian $\hat{h}$ itself cannot be relativized, because this would mean the isomorphy of the C_4 and L_4 continua.

Along with P_μ we attach to "M" in the eigenstate $|M)$, i.e. also a posteriori, the *overall four-coordinate* X_μ

$$X_\mu = (\boldsymbol{X}, ict), \tag{3.4}$$

then recognizing $\hat{\boldsymbol{X}}$ and $\hat{\boldsymbol{P}}$ as a pair of the quantum-canonical variables (operators) which fulfil the standard 3-dimensional canonical commutation relations. After determining the internal state of "M" in C_4 we can realize the standard canonical representation of the Poincaré algebra of "M" as a single free particle [11] if the 10 generators are taken in the form

$$\hat{H} = c(M^2c^2 + \hat{\boldsymbol{P}}^2)^{1/2}, \quad \hat{P}_k = \hat{P}_k, \quad \hat{J}_k = e_{kls}\hat{X}_l\hat{P}_s + S_k$$
$$\hat{K}_k = (1/2c^2)(\hat{H}\hat{X}_k + \hat{X}_k\hat{H}(+(\hat{H}+Mc^2)^{-1}e_{kls}\hat{P}_lS_s - \hat{P}_kt. \tag{3.5}$$

Here S_k are finite or infinite representations of spin of the state $|M)$, depending on whether $|M)$ is or is not the eigenstate of $\hat{\boldsymbol{j}}^2$, respectively, and

$$[S_k, S_l] = i\hbar e_{kls}S_s. \tag{3.6}$$

All Q–L characteristics of "M" which are directly unobservable, remain hidden in C_4 and, consequently, they do not enter the L_4 generators.

Direct measurability of the C–L characteristic P_μ implies the direct relativization of the next C–L quantity, namely the phase $\phi^C = -W\tau/h$ of the stationary state (3.1). Neglecting the arbitrary constant phase shifts due to the translation group in the internal-time τ as well as the space-time translation subgroup of L_4 as unmeasurable, the directly relativized phase ϕ^C takes the L_4-inv. form:

$$\phi^C = C_4\text{-abs.} = -W\tau/h = P_\mu X_\mu/h = \phi^L = L_4\text{-abs.}\,. \tag{3.7}$$

Thus we obtain the direct relativization of the external degrees of freedom of "M" as a single particle and the partially relativized state of "M" takes the form

$$(X_\mu, y|\boldsymbol{P}, M; M) \doteq A(\boldsymbol{P})(y|M)\exp[i/h(P_\mu X_\mu)] \tag{3.8}$$

or any superposition of the states (3.8) with different $\boldsymbol{P}$, but $P_\mu^2 = -M^2c^2$. The states (3.8) are sunk in the configuration space $K_2^{C,L}(X_\mu, y)$ spanned partially on the L_4 and partially on the C_4 continua, which is pointed out by the superscripts "C", "L".

Let us emphasize that the question if a particle "M" found in the experiment is a point-particle or has some internal structure concerns the first basis [12] which is always referred to experiment. Indeed, if $|M)$ is a bound state then all experiments where the momentum transfer "t" (the Mandelstam variable) is small enough, "M" can be treated as a point-particle. However, in high-energy collisions with large "t" we always can discover the internal structure of "M" regarded heretofore as a point particle. The uncertainty relations together with the energy-mass relation have discovered the "cosmology" of the "x" point in the complementary energy-momentum "p" space.

4. *Equivalence of the C_4 and L_4 kinematics and bound states*

The Lorentz limit II shows that in spite of the non-isomorphy of the C_4 and L_4 geometries the C_4 and L_4 kinematics are equivalent. Then let us confront these two, L_4 and C_4 frameworks in describing two free particles, assuming that their "trajectories" are the eigenstates of the corresponding momenta.

In the standard L_4-framework we deal from the very beginning with the fully relativized two-particle states ("trajectories")

$$(x_{1\mu}, x_{2\mu}|\boldsymbol{p}_1, m_1, s_1; \boldsymbol{p}_2, m_2, s_2) = A_1A_2\exp[i/h(p_{1\mu}x_{1\mu}+p_{2\mu}x_{2\mu})]$$
$$\text{with}\quad p_{1,2\mu}^2 = -m_{1,2}^2c^2 \tag{4.1}$$

sunk in the 8-dimensional configuration-space $K_1^L(x_{1\mu}, x_{2\mu})$.

On the other hand, the partially relativized states of "M" = "1+2" take the form (3.8). Now the internal state $(\boldsymbol{y}\mid M)$ of "M" is the eigenstate of $\hat{\boldsymbol{q}}$, hence

$$(\boldsymbol{y}|M) = (\boldsymbol{y}|\boldsymbol{q}) = \exp[i/h(\boldsymbol{q}\,\boldsymbol{y})]. \tag{4.2}$$

Inserting in (3.8) A_1A_2 instead of A and (4.2), the partially relativized state (3.8) takes the form

$$(X_\mu, \boldsymbol{y}|\boldsymbol{P}, M; \boldsymbol{q}; s_1, s_2) = A_1A_2 \exp[i/\hbar(P_\mu X_\mu + \boldsymbol{q}\boldsymbol{y})]$$
$$\text{with} \quad -P_\mu^2 = M^2c^2 = [(m_1^2c^2+\boldsymbol{q}^2)^{1/2}+(m_2^2c^2+\boldsymbol{q}^2)^{1/2}]^2. \tag{4.3}$$

So far, the state (4.3) is sunk in the 7-dimensional configuration space $K_2^{\mathrm{C,L}}(X_\mu, \boldsymbol{y})$. However, as the constituents "1" and "2" of "M" are free, each of them reaches the asymptotic zone of L_4-symmetry and therefore, the internal structure $(\boldsymbol{y}|\boldsymbol{q})$ from (4.2) cannot remain hidden in C_4. The direct (a posteriori) relativization concerns here the C_4-absolute phase $\phi^{\mathrm{C}} = (\boldsymbol{q}\boldsymbol{y})/\hbar$. Since

$$P_\mu = p_{1\mu}+p_{2\mu} = (\boldsymbol{P}, iE/c), \tag{4.4}$$

let us introduce the four-coordinate

$$X_\mu = ax_{1\mu}+bx_{2\mu} = (\boldsymbol{X}, ict), \tag{4.5}$$

where the, so far undetermined, weights a, b must be L_4-invariants, because X_μ, $x_{1\mu}$, $x_{2\mu}$ all parametrize four-points in L_4. Besides, we introduce the variables

$$x_\mu = x_{2\mu}-x_{1\mu} = (\boldsymbol{x}, ict), \tag{4.6}$$

$$p_\mu = dp_{2\mu}-ep_{1\mu} = (\boldsymbol{p}, ip_0), \tag{4.7}$$

where the weights d, e, much as a, b, must be L_4-invariants.

It is assumed that similarly as P_μ relativizes a posteriori the C_4-absolute mass M, cf. (3.3), the four-momentum p_μ relativizes, also a posteriori and directly, the C_4-absolute length of $\boldsymbol{q}$. Thus

$$C_4\text{-abs.} = \boldsymbol{q}^2 = p_\mu^2 = L_4\text{-abs.} \tag{4.8}$$

making p_μ of the space-like character. In consequence, the "Breit-like" systems $^*S$ exist where $^*p_0 = 0$ hence $^*\boldsymbol{p}^2 = \boldsymbol{q}^2$ and $^*\boldsymbol{p}^*\boldsymbol{x} = p_\mu x_\mu = L_4$-inv. Now the "$B$-absoluteness" must be taken into account as we deal with the "x" intervals, according to which, cf. (2.10), $|^*\boldsymbol{x}|\,|_{*S} = |\boldsymbol{y}|$ which, together with (4.8), result in the internal phase relativization. Indeed, since $^*\boldsymbol{p}^*\boldsymbol{x} = \boldsymbol{q}\boldsymbol{y}$ we end up with

$$C_4\text{-abs.} = \phi^{\mathrm{C}} = (\boldsymbol{q}\boldsymbol{y})/\hbar = (p_\mu x_\mu)/\hbar = \phi^{\mathrm{L}} = L_4\text{-abs.}. \tag{4.9}$$

Thus the total two-body phase becomes directly relativized as

$$-W\tau+\boldsymbol{q}\boldsymbol{y} = C_4\text{-abs.} = P_\mu X_\mu + p_\mu x_\mu = L_4\text{-abs.}, \tag{4.10}$$

and the fully relativized C_4-"trajectories" (4.3) take the form

$$(X_\mu, x_\mu|\boldsymbol{P}, M; p_\mu; s_1, s_2) = A_1A_2 \exp[i/\hbar(P_\mu X_\mu + p_\mu x_\mu)] \tag{4.11}$$

sunk in the 8-dimensional configuration space $K_2^L(X_\mu, x_\mu)$. The equivalence of the C_4 and L_4 kinematics then requires the "trajectories" (4.1) and (4.11) to coincide which will take place if

$$p_{1\mu}x_{1\mu}+p_{2\mu}x_{2\mu} = P_\mu X_\mu + p_\mu x_\mu. \tag{4.12}$$

Inserting (4.4), (4.5), (4.6) and (4.7) into the right-side of (4.12) and making use of (4.8) (cf. Appendix II) we get $a = 1-b = d = 1-e$ where

$$a = a(M) = \tfrac{1}{2}\left[1+(m_1^2-m_2^2)/M^2\right] = L_4\text{-inv.} \xrightarrow[c\to\infty]{} a^G. \tag{4.13}$$

The NR weight $a^G = m_1/(m_1+m_2)$ becomes independent of the mass M and equal to $a(M)$ for $M = m_1+m_2$, independent of the internal state of "M". Thus, the equivalence of the C_4 and L_4 kinematics manifested in the coincidence of the "trajectories" (4.1) and (4.11),

$$A_1A_2 \exp\left[i/\hbar(P_\mu X_\mu+p_\mu x_\mu)\right] = A_1A_2 \exp\left[i/\hbar(p_{1\mu}x_{1\mu}+p_{2\mu}x_{2\mu})\right], \tag{4.14}$$

implies the following c-number relations between the four-coordinates and four-momenta which parametrize both "trajectories":

(*i*) $$X_{M\mu} \equiv a(M)x_{1\mu}+b(M)x_{2\mu}, \quad x_\mu = x_{2\mu}-x_{1\mu}$$

(*ii*) $$P_\mu = p_{1\mu}+p_{2\mu}, \quad p_\mu = a(M)p_{2\mu}-b(M)p_{1\mu}. \tag{4.15}$$

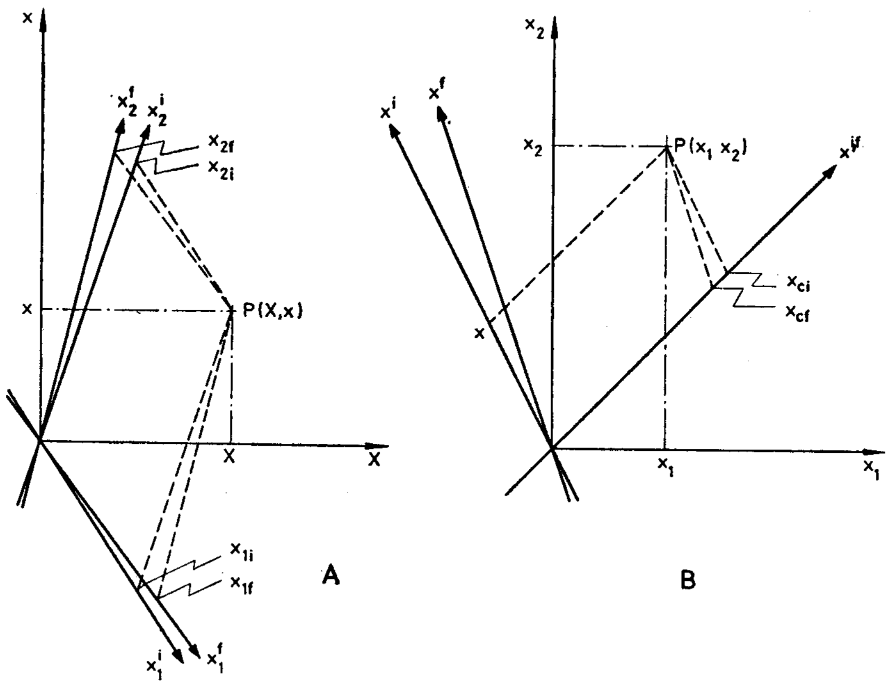

Fig. 1. Non-isomorphy of the configuration spaces (A) $K_2^L(X, x)$ and (B) $K_1^L(x_1, x_2)$ illustrated by two pairs of the "weights" a, b: $a_i = 2/3$, $b_i = 1/3$ and $a_f = 3/4$, $b_f = 1/4$

The M-dependence of the centre-of-mass coordinate $X_{M\mu}$ reflects the hierarchic way of the description of "M" implied by the C_4-framework. First one must determine the internal state of "M" in C_4 which next determines the coordination of "M" as a whole in L_4. Therefore, (4.15*i*) does not represent a point transformation, except from these processes where the absolute mass M of "M" remains unchanged. Thus, in general, the two configuration spaces $K_1^L(x_{1\mu}, x_{2\mu})$ and $K_2^L(X_\mu, x_\mu)$ are not isomorphic which is due to the non-isomorphy of the C_4 and L_4 continua, cf. Fig. 1. Again the singular character of the NR framework can be observed, as the lack of energy-mass relation implies that $a(M) \to a^G$, where a^G is M-independent, and the space parts of (4.15) coincide with (1.13) which establish the isomorphy between C_4^G and G_4.

Let us introduce an auxiliary overall coordinate of "M",

$$\overset{0}{X}_\mu = \overset{0}{a} x_{1\mu} + \overset{0}{b} x_{2\mu} = (\overset{0}{X}, ic\overset{0}{t}) \quad (\overset{0}{b} = 1 - \overset{0}{a}) \tag{4.16}$$

with an arbitrary M-independent $\overset{0}{a}$, which together with $x_\mu = x_{2\mu} - x_{1\mu}$ establishes the isomorphy between the configuration spaces $K_1^L(x_{1\mu}, x_{2\mu})$ and $K_2^L(\overset{0}{X}_\mu, x_\mu)$. The absolute phase $P_\mu X_{M\mu}/\hbar$ can be then rewritten in the form

$$P_\mu X_{M\mu} = P_\mu \overset{0}{X}_\mu + (b(M) - \overset{0}{b})\,(P_\mu x_\mu) = P_\mu \overset{0}{X}_\mu + \hbar \phi_M$$

with the absolute phase-shift ϕ_M equal to

$$\phi_M = (b(M) - \overset{0}{b})\,(P_\mu x_\mu)/\hbar = (\overset{0}{b} - b(M))\,(Mc^2 \Delta t^*)/\hbar = L_4\text{-abs.} \tag{4.17}$$

where $\Delta t^* = t_2^* - t_1^*$ is the relative-time coordinate in the rest-frame S^* of "M". Thus ϕ_M does not affect the C_4-absolute shape $(\boldsymbol{y} \mid M)$ and it vanishes in the single-time formalism like the NR one.

We make the following ANSATZ I as to the description of bound states suggested by the aforementioned kinematic considerations. Assume that the internal-potential $V(r)$ responsible for a bound state $|M_n)$ vanishes for $r = |\boldsymbol{y}| \to \infty$. The eigenmass M_n of $|M_n)$ can be then expressed in the form

$$M_n = (m_1^2 + \boldsymbol{q}_{n:\text{as}}^2/c^2)^{1/2} + (m_2^2 + \boldsymbol{q}_{n:\text{as}}^2/c^2)^{1/2} < m_1 + m_2. \tag{4.18}$$

Here $\boldsymbol{q}_{n:\text{as}}$ is pure imaginary asymptotic momentum, as $\boldsymbol{q}_{n:\text{as}}^2 < 0$. In order to assure the stability of "M_n" it must be assumed that $-\boldsymbol{q}_{n:\text{as}}^2 < c^2 \min(m_1^2, m_2^2)$ as otherwise, M_n would be complex and "M_n" unstable [13]. Therefore M_n of stable bound states must lie in the region

$$|m_1^2 - m_2^2|^{1/2} \leqslant M_n < m_1 + m_2. \tag{4.19}$$

Ansatz I says that $a(M)$ from (4.13) is to be analytically continued onto $M_n < m_1 + m_2$, i.e.

$$a_n = (m_1^2 + \boldsymbol{q}_{n:\text{as}}^2/c^2)^{1/2}/M_n = \tfrac{1}{2}\,[1 + (m_1^2 - m_2^2)/M_n^2] = a(M_n). \tag{4.13'}$$

Thus we maintain the same analytic expression of the centre-of-mass as that from (4.15 *i*),

$$X_{n\mu} = a_n x_{1\mu} + b_n x_{2\mu}, \quad x_\mu = x_{2\mu} - x_{1\mu}, \tag{4.15'ii}$$

and, consequently, the same analytic expression (4.17) of the phase shifts $\phi_{M_n} = \phi_n$ with $M < M_n$

$$\phi_n = (b_n - \overset{0}{b})(P_\mu x_\mu)/h = (\overset{0}{b} - b_n)(W_n \Delta t^*)/h = L_4\text{-abs}.$$

$$P_\mu^2 = -M_n^2 c^2 = -W_n^2/c^2. \tag{4.17'}$$

The partially relativized states of "M" in the 7-dimensional configuration space $K_2^{\mathrm{C,L}}(\overset{0}{X}_\mu, y)$ take the form

$$(\overset{0}{X}_\mu, y|P, M_n; M_n) = A_n \exp[i/h(P_\mu \overset{0}{X}_\mu)] \exp(i\phi_n)(y|M_n). \tag{4.20}$$

Again we see that the phase-shifts ϕ_n do not affect the C_4-absolute states $(y|M_n)$, nor they spoil the orthogonality and closure properties of the states (4.20) in the 6-dimensional configuration space $K_2^{\mathrm{C,L}}(\overset{0}{X}, y)$, because $(M_{n'}|M_n) = \delta_{nn'}$, while $\phi_n = \phi_{n'}$ if $M_n = M_{n'}$. In the NR framework $\Delta t^{\mathrm{G}} = 0$ hence all $\phi_n^{\mathrm{G}} = 0$ and therefore, the C_4-phase-shift effects can be expected only if: 1° finite c is taken into account which calls for multi-time states and, 2° the process in question is inelastic, as then $M_n \neq M_{n'}$ and $\Delta\phi = \phi_{n'} - \phi_n \neq 0$.

Each scattering state $(y|M)$ $(M > m_1 + m_2)$ is a C–L characteristic of "$M = 1+2$" and hence, it must be relativized directly which leads to (4.15 *i*, *ii*). However, any bound state $(y|M_n)$ is a Q–L characteristic of "M" and, as pointed out in Section 1, it remains "hidden" from the external space-time. Consequently, according to ANSATZ II we maintain the x-relations (4.15′*i*), as the centre-of-mass $X_{n\mu}$ parametrizes the directly relativizable (C–L) external motion of "M" as a whole, but we do not maintain the direct relativization of the internal Fermi relation-momentum q of each of the Q–L bound states separately as it must be for the C–L scattering states (5.15 *ii*). For example, if the C_4-absolute form factor of "M" is of the form

$$F_{nn'}(y) = (M_{n'}|y)(y|M_n) = C_4\text{-abs}. \tag{4.21}$$

then, according to the Ansatz II, the (indirect) relativization will concern the whole bilinear form $F_{nn'}(y)$ and not each bound state $(y|M_n)$ and $(y|M_{n'})$ separately. In consequence, besides the phase shift effects due $\Delta\phi \neq 0$ we can also expect the C_4-effects due to the indirect relativization of the elastic form factor $F_{nn}(y)$ when $M_n = M_{n'}$ and $\Delta\phi = 0$ — cf. Sec.6.

5. *Indirect relativization of Q–L characteristics*

Let us consider the elastic scattering of two point-particles without spins ($s_1 = s_2 = 0$) which interact at-a-C_4-distance through the internal-potential $V(y^2)$. In this simple dynamical process the total mass M is a constant of motion, $M_\mathrm{i} = M_\mathrm{f} = M$, and hence $\Delta\phi = \phi_\mathrm{f} - \phi_\mathrm{i} = 0$. Thus the whole problem concerns the relativization of the Q-L internal-potential $V(y^2)$. Let us emphasize that all matrix elements M_{fi} represent the C–L characteristics as they determine the directly observable cross-sections, which let us denote by writting $M_{\mathrm{fi}} = M_{\mathrm{fi}}^{\mathrm{L}}$.

Although the initial and final states of "$M = 1+2$" as the C–L scattering states must be directly relativized ($p_{1,2}^2 = -m_{1,2}^2$)[1], let us tentatively evaluate the matrix element $M_{\rm fi}$ with the help of "wrong" as only partially relativized asymptotic states sunk in the configuration space $K_2^{\rm C,L}(X, \boldsymbol{y})$. In the assumed Born approximation they take the form (4.20) with $(\boldsymbol{y}|M)$ equal to $(\boldsymbol{y}|\boldsymbol{q}_{\rm i,f})$, respectively and $A_n \to A_{1\rm i,f}A_{2\rm i,f}$. Since $M_{\rm i} = M_{\rm f} = M$, by putting $\overset{0}{a} = a(M)$ we get $\overset{0}{X} = X$ and $\varphi_{\rm i,f} = 0$ and thus

$$(X, \boldsymbol{y}|\boldsymbol{P}_{\rm i}, M; \boldsymbol{q}_{\rm i}) = A_{1\rm i}A_{2\rm i} \exp\left[i(P_{\rm i}X+\boldsymbol{q}_{\rm i}\boldsymbol{y})\right]$$

$$(X, \boldsymbol{y}|\boldsymbol{P}_{\rm f}, M; \boldsymbol{q}_{\rm f}) = A_{1\rm f}A_{2\rm f} \exp\left[i(P_{\rm f}X+\boldsymbol{q}_{\rm f}\boldsymbol{y})\right]$$

$$\text{with } P_{\rm i}^2 = P_{\rm f}^2 = -M^2. \tag{5.1}$$

Of course, $M_{\rm i} = M_{\rm f}$ also implies that $\boldsymbol{q}_{\rm i}^2 = \boldsymbol{q}_{\rm f}^2$ and hence, the Mandelstam momentum--transfer (square) variable "t" is equal to

$$t = (\boldsymbol{q}_{\rm i}-\boldsymbol{q}_{\rm f})^2 = C_4\text{-abs.} = L_4\text{-abs.}\,. \tag{5.2}$$

According to the ANSATZ III the amplitude $M_{\rm fi}^{\rm C,L} = ({\rm f}\,|V|\,{\rm i})$ provides us with correct numerical values of the absolute matrix element accounting for the transition between the initial and final states, where

$$M_{\rm fi}^{\rm C,L} = \int d^4X \int d^3y(\boldsymbol{P}_{\rm f}, M; \boldsymbol{q}_{\rm f}|X, \boldsymbol{y})V(\boldsymbol{y}^2)\,(X, \boldsymbol{y}|\boldsymbol{P}_{\rm i}, M; \boldsymbol{q}_{\rm i})$$

$$= (2\pi)^4\,(\overset{+}{A}_{1\rm f}A_{1\rm i})\,(\overset{+}{A}_{2\rm f}A_{2\rm i})\delta^{(4)}(P_{\rm i}-P_{\rm f})\tilde{V}[(\boldsymbol{q}_{\rm i}-\boldsymbol{q}_{\rm f})^2 = t] = C_4\text{-abs.}\,, \tag{5.3}$$

and

$$\tilde{F}(\boldsymbol{q}) = \int d^3yF(\boldsymbol{y}) \exp(-i\boldsymbol{q}\boldsymbol{y}) \tag{5.4}$$

which accounts for the canonical commutation relations (1.8).

In spite of the correct numerical value of the amplitude (5.3), (Ansatz III) its analytic form does not account for the C–L character of any matrix element which requires the L_4-invariant expression of $M_{\rm fi}$. In order to satisfy this geometrical requirement let us take instead of (5.1) the correct, fully relativized "trajectories" sunk in $K_2^{\rm L}(X, x)$, where $\overset{0}{X} = X$ and $x = x_2 - x_1$. By virtue of (4.11) we get

$$(X, x|\boldsymbol{P}_{\rm i}, M; p_{\rm i}) = A_{1\rm i}A_{2\rm i} \exp\left[i(P_{\rm i}X+p_{\rm i}x)\right]$$

$$(X, x|\boldsymbol{P}_{\rm f}, M; p_{\rm f}) = A_{1\rm f}A_{2\rm f} \exp\left[i(P_{\rm f}X+p_{\rm f}x)\right], \tag{5.5}$$

where, according to (4.15), $P_{\rm i,f}, p_{\rm i,f}$ and X, x can be expressed by $p_{1\rm i,f}, p_{2\rm i,f}$ and x_1, x_2, respectively. The manifestly L_4-absolute expression of $M_{\rm fi}^{\rm C,L}$ such that $M_{\rm fi}^{\rm C,L} = M_{\rm fi}^{\rm L}$ requires the 4-dimensional integrals in $K_2^{\rm L}(X, x)$ (or $K_1^{\rm L}(x_1, x_2)$) and consequently, the L_4-invariant expression of the C_4-absolute interaction $V(\boldsymbol{y}^2)$. Then let U denotes the appropriately relativized interaction V which, much as $V(\boldsymbol{y}^2)$, cannot depend on the boundary conditions

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[1] In Sections 5–7 we put $\hbar = c = 1$ and omit the relativistic indices $\mu, \nu, \ldots$. Thus $a_\mu = a$ and $a_\mu b_\mu = ab = \boldsymbol{ab} - a_0 b_0$.

of the collision process. Since x^2 is the only given a priori L_4-invariant interval between the interacting constituents "1" and "2" then $U = U(x^2)$ and hence

$$M_{fi}^{C,L} = M_{fi}^{L} = \int d^4X \int d^4x(\boldsymbol{P}_f, M; p_f|X, x)U(x^2)(X, x|\boldsymbol{P}_i, M; p_i)$$
$$= (2\pi)^4(A_{1f}^{+}A_{1i})(A_{2f}^{+}A_{2i})\delta^{(4)}(P_i - P_f)U(t) = L_4\text{-inv.}, \tag{5.6}$$

where

$$t = (p_i - p_f)^2, \quad P_{i,f} = p_{1i,f} + p_{2i,f},$$

hence

$$C_4\text{-abs.} = \tilde{V}(t) = \tilde{U}(t) = L_4\text{-abs.} \tag{5.7}$$

which determines the indirect relativization of $V(\boldsymbol{y}^2)$ to $U(x^2)$. More precisely the situation is as follows.

Assume that the C_4-absolute "x-shape" $F(\boldsymbol{y})^2$ represents a Q–L characteristic of "O". If the integral (5.4) is convergent we get its momentum representation $\tilde{F}(q^2)$ and, following (4.8) and (5.7) we perform the direct relativization of $\tilde{F}(\boldsymbol{q}^2)$ by determining the L_4-invariant "p-shape" $\tilde{G}(p^2)$ as equal to

$$\tilde{G}(p^2) = \tilde{F}(p^2 = \boldsymbol{q}^2 \geqslant 0). \tag{5.8}$$

$\tilde{G}(p^2)$ for time-like p is determined through the analytic continuation hence, on getting $\tilde{G}(p^2)$ for all p, the L_4-x-form-invariant "x-shape" $G(x^2)$ which relativizes indirectly $F(\boldsymbol{y}^2)$ is given by

$$G^{(D)}(x^2) = (2\pi)^{-4} \int_D d^4p\tilde{G}(p^2)\exp(ipx). \tag{5.9}$$

Here D means the suitable contour of integration in the complex p_0-plane, if that is required by the singularities of $\tilde{G}(p^2)$. We shall say henceforth that F and G represent "the same shape" ("$F = G$"), where F is its representation *from the C_4-inside*, whereas G, *from the L_4-outside*. In particular, $U(x^2)$ and $V(\boldsymbol{y}^2)$ represent "the same" interaction between "1" and "2". Thus the action-at-a-C_4-distance given by $V(\boldsymbol{y}^2)$ as corresponding to the L_4-x-form-invariant propagator $U^{(D)}(x^2)$ remains consistent with the requirements of the relativity and, by suitable choice of D, causality [7].

From (5.8) and (5.9) we find that independently of D

$$\int_{-\infty}^{+\infty} dx_0 G^{(D)}(x^2 = \boldsymbol{x}^2 - x_0^2) = F(\boldsymbol{x}^2) \xrightarrow[x^2 \to y^2]{} F(\boldsymbol{y}^2) \tag{5.10}$$

which determines the C_4-absolute shape $F(\boldsymbol{y}^2)$ from its L_4-counterpart $G^{(D)}(x^2)$. Thus if the integrals (5.4), (5.9) and (5.10) are convergent then, apart from the ambiguity due to D, any absolute x-shape has its two "faces": the internal F and the external G. Note that (5.10) clearly shows the L_4-B-absoluteness of $|\boldsymbol{y}|$, because the L_4-x-form-invariance of $G(x^2)$ implies that, independently of the reference frame S in L_4 where the integral (5.10) is evaluated, we end up with the same analytic form of $F(\boldsymbol{x}^2)$. This exactly is what the "B-absoluteness" means as stated in (2.10).

The three following examples illustrate the indirect relativization procedure and the correspondence between "F" and "G" representations.
1. Assume that $F = V(r)$ is the C_4-absolute Yukawa static potential

$$V(r) = (1/4\pi) \exp(-\kappa r)/r \quad (r = |\mathbf{y}|). \tag{5.11}$$

The integral (5.4) is convergent and $\tilde{V}(q^2) = (\kappa^2 + \mathbf{q}^2)^{-1}$ hence, after analytic continuation, we obtain $\tilde{U}(p^2) = (\kappa^2 + p^2)^{-1}$ and finally

$$U^{(D)}(x) = (2\pi)^{-4} \int_D d^4 p \, \frac{\exp(ipx)}{\kappa^2 + p^2} = \Delta^{(D)}(x; \kappa). \tag{5.12}$$

In particular, if D is the Feynman contour "F", the static Yukawa potential (5.11) has its external "face" given by the Feynman propagator $U^{(\mathrm{F})} = \Delta^{(\mathrm{F})}(x; \kappa)$.
2. Suppose now that $F(\mathbf{y}^2) = \mathbf{y}^2$. Then the integral (5.4) is divergent and therefore, the C_4-absolute internal-space distance represents a shape which has not its external ("face") L_4-representation.
3. Conversely, let the L_4-absolute be the shape of the four-interval $G(x^2) = x^2$. Now the integral (5.10) is divergent and hence, the L_4-invariant four-interval has not its internal ("face") C_4-representation.

6. *Inelastic collision*

Let us consider now the inelastic collision of a point-particle "3" without spin with the particle "M" composed of two point-particles "1" and "2" also without spins, which results in the excitation of "M" from its ground state $(\mathbf{y}|M_{\mathrm{i}}) = \psi_{\mathrm{i}}$ of the absolute mass M_{i} to the final bound state $(\mathbf{y}|M_{\mathrm{f}}) = \psi_{\mathrm{f}}$ of mass M_{f}. For the sake of simplicity we assume that the composite particles "$M_{\mathrm{i,f}}$" are also spinless which implies that the eigenstates $\psi_{\mathrm{i,f}}$ of $\hat{h}$ are spherically symmetric in C_3, i.e. $\psi_{\mathrm{i,f}} = \psi_{\mathrm{i,f}}(\mathbf{y}^2)$.

It is assumed that "3" interacts with "M" through the constituent "1" and, since "1" does not coincide with the centre-of-mass of "M", the corresponding cross-section will indirectly measure the internal Q–L structure of "M" as well as the very interaction V_{13}. In consequence, both these shapes must be indirectly relativized.

In the assumed Born approximation the partially relativized asymptotic states of the three-body system in question are the eigenstates of $\hat{\boldsymbol{P}}$ and $\hat{\boldsymbol{p}}_3$. The indirectly relativized interaction V_{13} to $U((x_3 - x_1)^2)$ implies that the matrix element $M^{\mathrm{L}}_{\mathrm{fi}}$ must be evaluated in the configuration space $K^{\mathrm{L}}_1(x_1, x_2, x_3)$. Thus, instead of the phase-shifts $\phi_{\mathrm{i,f}}$ let us parametrize the initial and final asymptotic states by explicitly introducing the centre-of--mass $X_{\mathrm{i,f}} = a_{\mathrm{i,f}} x_1 + b_{\mathrm{i,f}} x_2$ of "$M_{\mathrm{i,f}}$", respectively hence:

$$\Psi_{\mathrm{i}} = A_{\mathrm{i}} A_{3\mathrm{i}} \exp\{i[P_{\mathrm{i}}(a_{\mathrm{i}} x_1 + b_{\mathrm{i}} x_2) + p_{3\mathrm{i}} x_3]\} \psi_{\mathrm{i}}(\mathbf{y}^2),$$
$$\Psi_{\mathrm{f}} = A_{\mathrm{f}} A_{3\mathrm{f}} \exp\{i[P_{\mathrm{f}}(a_{\mathrm{f}} x_1 + b_{\mathrm{f}} x_2) + p_{3\mathrm{f}} x_3]\} \psi_{\mathrm{f}}(\mathbf{y}^2), \tag{6.1}$$

with

$$a_{\mathrm{i,f}} = \tfrac{1}{2}[1 + (m_1^2 - m_2^2)/M^2_{\mathrm{i,f}}] = 1 - b_{\mathrm{i,f}}.$$

The internal, C_4-absolute structure of "M" still remains hidden in C_4 and, as seen from (6.1), in the process under consideration, it is given by the Q–L form factor

$$F_{fi}(y^2) = (M_f|y)(y|M_i) = {}_{,}\psi_f^+(y^2)\psi_i(y^2), \tag{6.2}$$

as stated in (4.21). Therefore, from ANSATZ II it follows that the indirect relativization concerns the whole form factor $F_{fi}(y^2)$ and not each of the bound states $\psi_{i,f}(y^2)$ separately. Denoting by $G_{fi}((x_2-x_1)^2)$ the external, L_4-representation of the internal form factor (6.2), the manifestly L_4-invariant form of the C–L matrix element in question takes according to the ANSATZ III the following form

$$\begin{aligned} M_{fi}^{L} &= (A_f^+A_i)(A_{3f}^+A_{3i})\int d^4x_1\int d^4x_2\int d^4x_3 \exp\{i[(a_iP_i-a_fP_f)x_1+(b_iP_i-b_fP_f)x_2]\} \\ &\quad\times \exp[i(p_{3i}-p_{3f})x_3]G_{fi}((x_2-x_1)^2)U((x_3-x_1)^2) \\ &= (2\pi)^4(A_f^+A_i)(A_{3f}^+A_{3i})\delta^{(4)}(P_i+p_{3i}-P_f-p_{3f})\tilde{U}(t)\tilde{G}_{fi}(u^2), \end{aligned} \tag{6.3}$$

where

$$\begin{aligned} \Delta &= p_{3i}-p_{3f} = P_f-P_i; \quad \Delta^2 = t \\ u &= b_iP_i-b_fP_f. \end{aligned} \tag{6.4}$$

This amplitude is illustrated in Fig. 2 (A) and the discontinuity of the "thick" line corresponds to the "jump" of the centre-of-mass of "M" from X_i to X_f. This "jump" or, in other words, the phase-shift effect due to $\phi_i \neq \phi_f$, implies that $u \nparallel \Delta$ and hence, u^2 is not proportional to t as in the standard theory. The corresponding "jump-effect" will be evaluated in Section 8.

Let us evaluate the same amplitude (6.3) by parametrizing all quantities in the variables $\overset{0}{X} = \overset{0}{a}x_1+\overset{0}{b}x_2$, $x = x_2-x_1$ and x_3 instead of x_1, x_2, x_3. This would correspond to the standard Feynman graphs phenomenology where the same overall coordinate $\overset{0}{X}$ parametrizes "M_i" and "M_f". Then (6.3) can be rewritten in the form

$$M_{fi}^{L} = (2\pi)^4(A_f^+A_i)(A_{3f}^+A_{3i})\delta^{(4)}(P_i+p_{3i}-P_f-p_{3f})\tilde{U}(t)\int d^4xG_{fi}^{F}(x)\exp(-i\overset{0}{b}\Delta x)$$

which determines the corresponding Feynman form factor $G_{fi}^F(x)$ equal to

$$G_{fi}^F(x) = G_{fi}(x^2)\exp\{i[(\overset{0}{b}-b_f)P_f-(\overset{0}{b}-b_i)P_i]x\}. \tag{6.5}$$

The phase in the exponent of (6.5) is equal to (4.17)

$$\phi_i-\phi_f = (\overset{0}{b}-b_i)M_i\Delta t^{(i)}-(\overset{0}{b}-b_f)M_f\Delta t^{(f)} = \text{abs.},$$

where $\Delta t^{(i,f)}$ denote the relative-time variable in the rest-frames S_i and S_f of "M_i" and "M_f", respectively. When rewritten L_4-covariantly, $\phi_i-\phi_f$ takes the form

$$\phi_i-\phi_f = [(\overset{0}{b}-b_f)P_f-(\overset{0}{b}-b_i)P_i]x = L_4\text{-inv.} \tag{6.6}$$

which coincides with the phase in the exponent of $G_{fi}^F(x)$ from (6.5).

Because of the arbitrariness of $\overset{0}{b}$ we can put $\overset{0}{b} = b_f$ and then

$$G^F_{fi}(x) = G_{fi}(x^2) \exp\left[i(b_i - b_f)P_i x\right]. \qquad (6.5')$$

For elastic collision when $b_i = b_f$ we obtain

$$G^F_{ii}(x) = G_{ii}(x^2) = L_4\text{-x-form-inv.} \qquad (6.7)$$

which means that the C_4 and the Feynman phenomenological form factors coincide and, what is important, both are L_4-x-form-invariant functions.

Of course, Ansatz II, according to which we have constructed the L_4-x-form-invariant form factor $G_{fi}(x^2)$, conflicts with the L_4-framework independently if we deal with the inelastic or elastic collisions. The point is that in the L_4-framework all characteristics are C–L and hence they must be relativized directly. Any shape is a priori sunk in L_4 and so there is no room for the dichotomy of the Q–L and C–L characteristics and con-

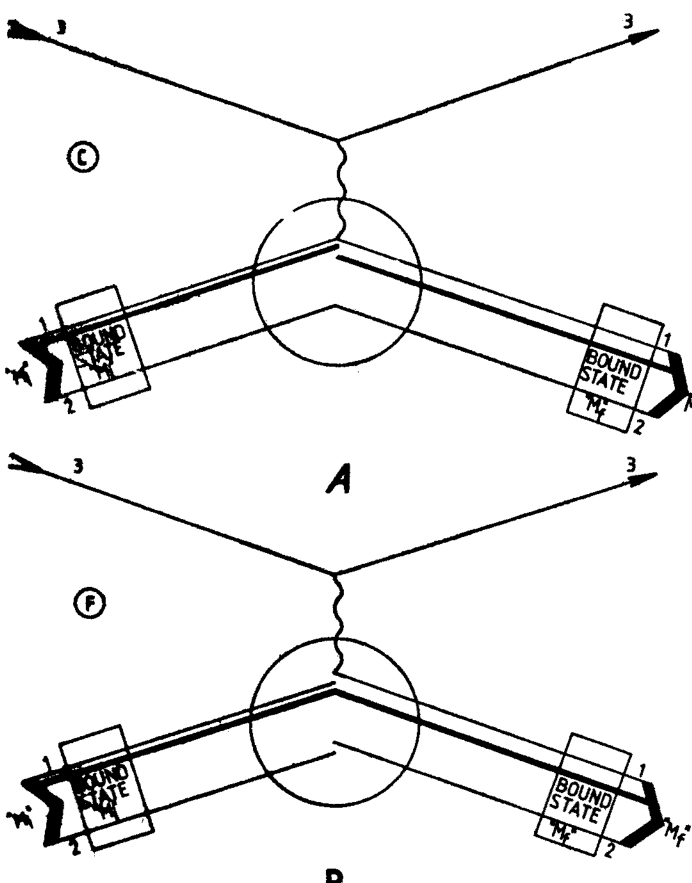

Fig. 2. Inelastic scattering graphs: (A) in the configuration space $K^L_1(x_1, x_2)$ with the "jump" of X_M and (B) in $K^L_2(\overset{0}{X}, x)$ which corresponds to the phenomenology of the Feynman graphs

sequently, for the Ansatz II. Therefore the construction of the "relativistic" form factor $G^{L}_{fi}(x)$ must resort to the (directly relativized) "relativistic wave functions" [14]. In our simple case of spinless constituents and spinless "$M_{i,f}$" the relativistic wave functions should be of the form

$$\psi^{L}_{i,f}(x) = \psi^{L}_{i,f}(x^2, P_{i,f}x), \tag{6.8}$$

where the arguments $P_{i,f}x$ indicate the rest-frames $S_{i,f}$ of "$M_{i,f}$", respectively. Consequently, the "relativistic" form factor G^{L}_{fi} must be spanned on three L_4-scalars,

$$G^{L}_{fi}(x) = G^{L}_{fi}(x^2, P_i x, P_f x) \neq L_4\text{-}x\text{-form-inv.}, \tag{6.9}$$

for inelastic as well as for elastic collisions. For elastic collision we have besides $P_i^2 = P_f^2 = -M^2(M = M_i = M_f)$. Thus the "relativistic" form factors must be entangled in the boundary conditions of the collision process which conflicts with the Feynman (elastic) form factor (6.7). This inconsistency of the L_4-framework is, to some extent, concealed in the momentum representation where we need to know the Fourier component of the form factors for $\Delta = P_f - P_i$ only. Thus

$$\int d^4x G_{fi}(x^2) \exp\left[i(P_f - P_i)x\right] = \tilde{G}_{fi}(t)$$

$$\int d^4x G^{L}_{fi}(x^2, P_i x, P_f x) \exp\left[i(P_f - P_i)x\right] = \tilde{G}^{L}_{fi}(t), \tag{6.10}$$

as $t = \Delta^2$ is the only Mandelstam variable which parametrizes both L_4-absolute integrals (6.10).

If "M" is of finite mass then it cannot be treated as the source of external L_4-field (cf. Lorentz limit I) and the recoil of "M" due to finite t implies that $S_i \neq S_f$. From the x-representation of G^{L}_{fi} (6.9) it is easily seen that the "jump" from S_i to S_f affects the analytic structure of $G^{L}_{fi}(x)$ which means the so-called "relativistic distortions" of the structure of "M" [15].

Let us introduce the dimensionless parameter f,

$$f = t/M^2c^2, \tag{6.11}$$

call it the "Q–L-parameter" whose magnitude is responsible for the deviation of the indirect relativization of a Q–L characteristic implied by the C_4-framework, from its direct relativization implied by the standard theory. In the NR framework ($c \to \infty$) we have $f = 0$ independently of the magnitude of the recoil-velocity of the measured object "O" and therefore, we cannot expect any C_4-effect. This follows from the isomorphy of the C_4^G and G_4 continua. Also in the L_4-framework $f = 0$ as $M \to \infty$ characterizes the Lorentz limit I from Section 2. Indeed, all external fields do not suffer from recoil and hence they must be directly relativized as the C–L characteristics. This shows the singular character of the L_4-framework when dealing with external fields carried by infinitely heavy sources. According to the C_4-framework $f \neq 0$ calls for the indirect relativization and therefore f is the measure of the Q–L "degree" of the described shape.

The L_4-x-form-invariance of the C_4-framework form factors G_{fi} means that the internal structure of "M" does not suffer from any "relativistic distortion". They would be "produced" by the L_4-framework which imposes the C–L character onto the Q–L characteristics. Of course, in the "NR" collisions when $f \ll 1$ the "relativistic distortions" are negligible, however, the penetration of "M" by means of collisions with $f = t/M^2c^2 \ll 1$ is restricted to the periphery of "M", where $r \gg \hbar/Mc$.

From the elastic electron-nucleon collisions we know the phenomenological Feynman form factor $\tilde{G}^{\rm F}_{\rm ii}(t) = \tilde{G}_{\rm ii}(t)$, cf. (6.7), up to $t = 25({\rm GeV}/c)^2 \gg M^2c^2 \cong 1({\rm GeV}/c)^2$ [16]. Since $f = 28 \gg 1$, the recoil velocities of the nucleon are extremely "relativistic" and hence, according to the L_4-framework, the "relativistic distortions" should dominate.

In the standard procedure the static charge distribution $\varrho(\boldsymbol{x}^2)$ of the nucleon is introduced as equal to [17]

$$\varrho(\boldsymbol{x}^2) = eF_{\rm ii}(\boldsymbol{x}^2) = (2\pi)^{-3} e \int d^3 \Delta p \tilde{G}^{\rm F}_{\rm ii}[t = (\Delta \boldsymbol{p})^2] \exp(i \Delta \boldsymbol{p} \boldsymbol{x}) \qquad (\tilde{G}^{\rm F}_{\rm ii}(0) = 1). \tag{6.12}$$

In the L_4-framework $\varrho(\boldsymbol{x}^2)$ represents a static shape of the nucleon in the CM-system S^* of the colliding electron and nucleon, because S^* is the only "Breit-like" system where $\Delta p_0 = 0$. Therefore, $t = (\Delta \boldsymbol{p})^2$ as stated in (6.12), independently of the magnitude of t. However, it is quite obscure why the internal structure of the nucleon should be static in the "accidental" reference frame S^*. Of course, this results in further difficulties if one tries to explain theoretically the experimentally obtained "dipole" form of the form factor $\tilde{G}^{\rm F}_{\rm ii} = \tilde{G}_{\rm ii}$ [16, 17].

In the C_4-framework $\varrho(\boldsymbol{x}^2)$ coincides with the C_4-absolute form factor $\varrho(\boldsymbol{y}^2) = eF_{\rm ii}(\boldsymbol{y}^2)$ sunk in C_3 and hence, all the obscurities disappear. The internal structure of any isolated ("quantum") system "O", in particular the nucleon structure, exists (according to the C_4-framework) in the relation continuum C_4, whereas the event-continuum L_4 is unavoidably the continuum where these structures are measured from the outside of "O".

For example, if $\psi(\boldsymbol{y}^2) = 8\pi^{-1/2}R^{-3/2}\exp(-r/2R)$ were the nucleon wave function in C_4 then, the indirectly relativized form factor $\tilde{G}_{\rm ii}(t) = \tilde{G}^{\rm F}_{\rm ii}(t)$ would take the "dipole fit" form

$$\tilde{G}_{\rm ii}(t) = (1+R^2t)^{-2}. \tag{6.13}$$

From the experiment [16] we get $R = 1.19\,({\rm GeV}/c)^{-1}$ or $R = 2.34\times10^{-14}$ cm and then, the mean-square radius $\bar{R}$ of the nucleon, $\bar{R} = \langle \boldsymbol{y}^2\rangle^{1/2} = \sqrt{12}R = 8.10\times10^{-14}$ cm.

We see then that besides the phase shifts φ_n in the inelastic collisions, the Q–L nature of the form factors, even the elastic ones, provides us with another source of C_4-effects connected with high-momentum transfer collisions. Both effects are crucial for the hypothesis of the C_4-continuum and the Ansatz's I, II, III implied by the C_4-framework.

7. Spectator distributions in disintegration processes

Let us consider the same physical system as in Section 6 assuming however, that the impinging particle "3" distintegrates "M" into its two constituents "1" and "2". We shall neglect the final state interaction between the products of disintegration (Born approximation) which is well justified if t is much larger than the C_4-absolute Fermi relation-momentum square $\boldsymbol{q}^2 = p_{\rm F}^2$ characteristic for the initial bound state $\psi_{\rm i}(\boldsymbol{y}^2)$ of "M" [18].

Thus the final state $\psi_f \sim \exp(i\mathbf{q}_f\mathbf{y})$ in C_4, as the scattering state of "1+2", must be directly relativized and, from (4.9) and (4.15 *ii*) we obtain

$$\psi_f \sim \exp(i\mathbf{q}_f\mathbf{y}) = \exp(ip_f x) \quad (x = x_2 - x_1),$$
$$p_f = a_f p_{2f} - b_f p_{1f} \quad \text{with} \quad p_{1f}^2 = -m_1^2, \quad p_{2f}^2 = -m_2^2. \tag{7.1}$$

Much as in Section 6, let $U((x_3-x_1)^2)$ be the indirectly relativized interaction between "3" and "1", but now, the initial state, as the unique bound state represents the Q–L characteristic of "M" accounting for its internal structure. Thus

$$\psi_i(\mathbf{y}^2) = F_i(\mathbf{y}^2) \tag{7.2}$$

represents the form factor of "M". Its indirect determination becomes possible through the measurement of the momentum distribution of the spectators "2" which, in consequence of dynamical process of disintegration, become directly observable. Therefore $F_i(\mathbf{y}^2)$ must be represented from the L_4-outside by the L_4-x-form-invariant form factor

$$F_i(\mathbf{y}^2) \to G_i((x_2-x_1)^2); \quad \text{"}F_i = G_i\text{"}. \tag{7.3}$$

Omitting the form factor G_i, the directly relativized initial and final asymptotic states of "M" take in the assumed Born approximation the following form in the configuration space $K_1^L(x_1, x_2, x_3)$:

$$\Psi_i = A_i A_{3i} \exp\{i[P_i(a_i x_1 + b_i x_2) + p_{3i} x_3]\},$$
$$\Psi_f = A_{1f} A_{2f} \exp\{i[P_f(a_f x_1 + b_f x_2) + p_f(x_2 - x_1)]\} A_{3f} \exp(ip_{3f} x_3). \tag{7.4}$$

The interaction between "3" and the constituent "1" of "M" implies that, much as in Section 6, the corresponding transition amplitude M_{fi}^L must be evaluated in the configuration space $K_1^L(x_1, x_2, x_3)$. According to (4.15 *ii*) the final scattering states Ψ_f from (7.4) can be rewritten in the form

$$\Psi_f = A_{1f} A_{2f} A_{3f} \exp[i(p_{1f} x_1 + p_{2f} x_2 + p_{3f} x_3)] \tag{7.5}$$

which shows that the weights a_f, b_f disappear from the parametrization of the scattering state. Thus the centre-of-mass X_M is of no physical meaning in the scattering states of "M". In spite of no "jump" $X_i \to X_f$, the C_4-effects will emerge because of the off-mass-shell states of the constituents "1" and "2" in the initial bound state ψ_i.

Following Ansatz III, the L_4-invariant expression of the C–L absolute matrix element M_{fi} of the disintegration process takes the form

$$M_{fi}^L = \int d^4x_1 \int d^4x_2 \int d^4x_3 \Psi_f^+(x_1, x_2, x_3) U((x_3-x_1)^2) G_i((x_2-x_1)^2 \Psi_i(x_1, x_2, x_3)$$
$$= (2\pi)^4 (A_{1f}^+ A_{2f}^+ A_i)(A_{3f}^+ A_{3i}) \delta^{(4)}(P_i + p_{3i} - p_{1f} - p_{2f} - p_{3f}) \tilde{U}(t) \tilde{G}_i(k^2), \tag{7.6}$$

where

$$k = b_i P_i - p_{2f}, \quad P_i^2 = -M_i^2, \quad t = (p_{3i} - p_{3f})^2. \tag{7.7}$$

The C_4-effect evaluated in Section 9 will be due to the structure of the four-vector k.

8. "*Jump-effect*" *in inelastic collision*

The inelastic collision analyzed in Section 6 will be now illustrated by the electron collision with the hydrogen-like atom "M" which becomes excited from its initial ground state $|1S)$ ("M_i") to all lowest excited states $|2S)$ and $|2P; j)$ ($j = 1, 2, 3$) ("M_f").

The internal C_4-absolute states $\psi_{i,f}(y)$ will be evaluated from the Schroedinger equation (1.11′) which we call the C_4^G-approximation to C_4. One could suspect that the evaluation of the c-depending C_4-effects conflicts with the NR equation (1.11′), however, it is not so, because of the singular character of the G_4-group [6]. The point is that the G_4-group deals simultaneously with *two* subsequent terms of the expansion into powers of $1/c^2$ parameter. For example, if M_n is the exact value of the absolute mass of two-body system in the state $|M_n)$ then,

$$M_n = (m_1+m_2)+w_n/c^2 = \text{abs.}, \tag{8.1}$$

where w_n is the (exact) internal-energy value of "M" in the eigenstate $|M_n)$ of $\hat{h}$ after subtracting $M_0 = m_1+m_2$ which means the "neutral element" of the G_4-group. On the other hand, from the NR Schroedinger equation (1.11′), i.e. in the C_4^G-approximation, we obtain $|w_n^G)$ and the eigenvalues w_n^G and, taking into account the Einsteinian energy-mass relation we can write

$$M_n = (m_1+m_2)+w_n^G/c^2+O_n(1/c^4), \tag{8.2}$$

where $O_n(1/c^4)$ accounts for higher-order correction. Thus $|w_n^G)$ and w_n^G account correctly for the $1/c^2$ effects and, moreover, if

$$O_n(1/c^4)c^2/w_n \ll 1, \tag{8.3}$$

which occurs for loosely bound structures like atoms, the C_4^G-approximation accounts correctly for the c-depending C_4-effects up to the accuracy $1/c^2$. Consequently, if $q^2/c^2 \ll \min(m_1^2, m_2^2)$ the relation-momentum spectrum obtained from $|w_n^G)$ approximate well the real spectrum. The above singularity of the G_4-group forces one to distinguish between two "NR limits". The first, on the level of masses, when $c \to \infty$ implies that $M_n \to M_0$ thus the inertia of "M" becomes independent of its internal state M_n and the second, on the level of internal-energies, when $w_n \xrightarrow[c\to\infty]{} w_n^G$, but the rest-energy $W_0 = M_0c^2 \xrightarrow[c\to\infty]{} \infty$. By the C_4^G-approximation to C_4 we mean the second NR limit maintaining, however, W_0 finite. The C_4-effects evaluated in this and the next Sections will be based on the C_4^G-approximation thus accounting for the first-order effects in the expansion parameter $1/c^2$.

The "jump effect" discussed further on is relatively a low-energy process hence the neglect of spin interaction is justified, while "weak" electromagnetic interaction justifies the Born approximation. Moreover, it is assumed that the impinging electron ("3") interacts with the atomic nucleus "A" ("1") only which, as will be shown, is justified by large t's in the region where the "jump effect" should appear. In consequence, we can neglect all exchange effect [19]. Thus

$$m_1 = AM, \quad m_2 = m_3 = m, \tag{8.4}$$

where M and m now denote the nucleon and electron masses, respectively. Under these assumptions the matrix element (6.3) coincides with the transition amplitude of the inelastic "e-atom" collision. Since the lab-energy E_e of impinging electron will be of the order of 1 MeV or even less, while the energy gap δ_Z between the ground and excited states of the atom is of the order of $10\,Z^2$ eV, we shall deal with three energies δ_Z, E_e, AMc^2 of different order of magnitude,

$$\delta_Z \ll E_e \ll AMc^2 = A\,\text{GeV}, \tag{8.5}$$

which simplifies further calculations. Thus the "Q-L-parameter" $f = t/A^2M^2c^2 \lesssim E_e^2/A^2M^2c^4 \approx (m/AM)^2 \ll 1$ and therefore, the Q–L nature of the atomic form factor cannot result in any C_4-effect. The latter will be due to the "jump" $X_i \to X_f$. Strong inequality $E_e/AMc^2 \ll 1$ also implies that the overal CM-system of the impinging electron and atom can be identified with the lab-system where the atom is (almost) at rest before and after the collision. On the other hand, as $E_e \gg \delta_Z$, while the "jump-effect" deals with $t \gg \delta_Z^2/c^2$, the scattering lab-angle θ of the impinging electron will be very well approximated by the corresponding elastic scattering angle when

$$\sin(\theta/2) = t^{1/2}/2p_e. \tag{8.6}$$

Here p_e is the initial lab-momentum of the electron ($E_e = c(m^2c^2+p_e^2)^{1/2} - mc^2$) and since $E_e \gg \delta_Z$ we also can neglect the lab-velocity difference of the scattered electron from before and after the collision. Thus, according to (6.3), the inelastic cross-section for "e-atom" collision omitting the "e–e" interaction takes the following form

$$d\sigma/dt = (d\sigma/dt)_{e-A}|\tilde{G}_{fi}(u_\mu^2)|^2. \tag{8.7}$$

$(d\sigma/dt)_{e-A}$ means the elastic cross-section for the electron-nucleus collision, both regarded as point particles, while $\tilde{G}_{fi}$ is the global form factor for the transition from the initial ground state $|1S)$ to all four excited bound states $|2S)$ and $|2P, j)$. In the C_4^G-approximation all excited states are of the same internal energy w_f^G hence, denoting by w_i^G the ground state internal energy we have

$$(M_f - M_i)c^2 = \delta_Z = w_f^G - w_i^G = \tfrac{3}{8}\mu c^2\alpha^2Z^2 = Z^2\delta = 10.2\,Z^2\,\text{eV}, \tag{8.8}$$

where $\mu = m/(1+m/AM) \cong m$ and $\alpha =$ "1/137". From (6.4) we have

$$u_\mu = b_i P_{i\mu} - b_f P_{f\mu}, \quad P_{i,f\mu}^2 = -M_{i,f}^2c^2$$

and from (4.13′)

$$b_{i,f} = \tfrac{1}{2}[1 - (A^2M^2 - m^2)/M_{i,f}^2] \xrightarrow[c\to\infty]{} b^G = m/(AM+m) \cong m/AM \tag{8.9}$$

hence, taking into account that $m/AM \ll 1$ we end up with

$$u_\mu^2 = (b_i b_f)t - [(A^2M^2 - m^2)/M_i^2]\delta_Z^2/c^2 \cong (b^G)^2 t - \delta_Z^2/c^2. \tag{8.10}$$

If we assumed an opposite situation, namely that the impinging electron collides with the atomic electron while heavy nucleus "A" is the internal spectator then, instead of

u_μ^2, the argument of the same form factor $\tilde{G}_{fi}$ would be equal to ($m \rightleftarrows AM$, $a \cong a^G \cong 1$)

$$u_{e\mu}^2 = t + \delta_Z^2/c^2. \tag{8.11}$$

In the standard theory, instead of (8.10) and (8.11) we have

$$\overset{s}{u}{}_\mu^2 = (b^G)^2 t, \quad \overset{s}{u}{}_{e\mu}^2 = t. \tag{8.12}$$

Since $t \geqslant 0$, $u_{e\mu}^2$ is only shifted towards positive values as compared to $\overset{s}{u}{}_{e\mu}^2$ by a small amount δ_Z^2/c^2, which is practically impossible to detect. However, an experimentally much more optimistic situation occurs when the lighter constituent (electron) is the internal spectator and exactly this case corresponds to the cross-section (8.7) where the "e–e" interaction is neglected. It will be shown below, cf. (8.17), why the "e–e" interaction can be neglected when investigating the "jump-effect". By introducing the critical momentum transfer t_0,

$$t_0 = (b^G)^{-2}\delta_Z^2/c^2 = A^2(M/m)^2\delta_Z^2/c^2 \gg \delta_Z^2/c^2, \tag{8.13}$$

instead of (8.10) we can write

$$u_\mu^2 = (b^G)^2(t - t_0) \tag{8.14}$$

which shows that u_μ changes its character from time-like for $t < t_0$ to space-like for $t > t_0$. The largest deviation of u_μ^2 from $\overset{s}{u}{}_\mu^2$ takes place in the vicinity of $t = t_0$ when $u_\mu^2(t_0) = 0$ and hence, $\tilde{G}_{fi}(0) = 0$ either, because of the orthogonality of $|1S)$ to all four excited states $|2S)$ and $|2P; j)$. The "jump-effect" ($t_0 \neq 0$) then would result in the vanishing of the cross-section (8.7) at $t = t_0$ and its essential modification in the vicinity of $t = t_0$.

After translating into the scattering angle, the critical value of t_0 results in the corresponding critical scattering angle θ_0,

$$\sin(\theta_0/2) = t_0^{1/2}/2p_e = \tfrac{1}{2}AZ^2(M/m)(\delta/cp_e), \quad (\delta_Z = Z^2\delta), \tag{8.15}$$

which shows that θ_0 lies in the physical region ($\theta_0 \leqslant 180°$) if

$$p_e \geqslant \tfrac{1}{2}AZ^2(M/m)(\delta/c) = 9.4\,AZ^2(\text{keV}/c) \quad \text{or} \quad E_e \geqslant 84A^2Z^4\ \text{eV}. \tag{8.16}$$

We see then that the experimentally comfortable scattering angles θ_0 for low values of Z, e.g. for helium ion $A = 2Z = 4$, correspond to the energies of the order of 1 MeV, or even less. Thus the "jump-effect" would be rather a low-energy effect when each of the three constituents of our system can be safely treated mechanically. In Fig. 3 the critical angle θ_0 is plotted vs $x = \dfrac{p_e}{mc}\Big/AZ^2$.

Let us consider now when the neglect of "e–e" interaction in (8.7) is justified. The ratio K of the corresponding amplitudes accounting for the "e–A" and "e–e" interactions, respectively, is equal to

$$K = Z\tilde{G}_{fi}[(b^G)^2 t]/\tilde{G}_{fi}(t),$$

where the factor Z is due to Z-times stronger "e–A" than "e–e" interactions. In analyzing the "jump-effect" when $t \approx t_0$, $t_0 \gg q_F^2 \gg \delta_Z^2/c^2$, and taking $\tilde{G}_{fi}$ from (III. 1) we obtain

$$K = 0.22 Z^6 A^4 (t/t_0)^{5/2}. \tag{8.17}$$

Except the hydrogen atom, even for helium ion, K takes a very large value $K = 3.6 \times 10^3$ $(t/t_0)^{5/2}$ which justifies the neglect of the "e–e" amplitude.

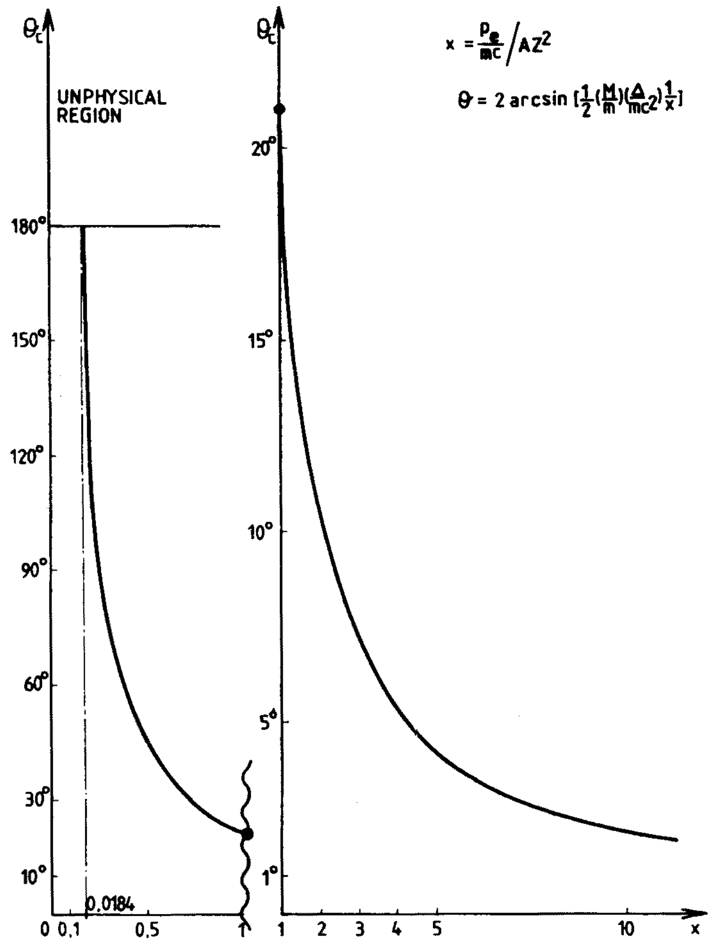

Fig. 3. Critical angle θ_0 vs $x = \dfrac{p_e}{mc} \Big/ AZ^2$. For $x < 0.0184$, θ_0 lies in the unphysical region ($\theta_0 > 180°$)

The difficulty in proving-disproving the "jump-effect" is due first of all to small cross-section (8.7) as

$$|\tilde{G}_{fi}[(b^G)^2 t]|^2 = 4.16 \times 10^{-6} Z^2 (t/t_0). \tag{8.18}$$

Thus, from a large background of e–e interactions and γ-cascades we must pick out these events when the leading electron is of almost the same lab-energy E_e it had before the collision, in coincidence with γ-quantum of energy δ_Z coming from disexcitation of the atom. The experimental facility can be the fact that in presence of the "jump-effect" we should not observe such events for $t = t_0$ ($\theta = \theta_0$). Taking into account that $t = 4p_e^2 \sin^2(\theta/2)$ and $(d\sigma/dt)_{e-A}$ is the Rutherford cross-section, we end up with

$$d^2\sigma/dt d\varphi = 4.16\times 10^{-6} Z^2 \begin{pmatrix} t/t_0 \\ t/t_0 - 1 \end{pmatrix} 2(Ze^2/v_e)^2 (1/t^2). \tag{8.19}$$

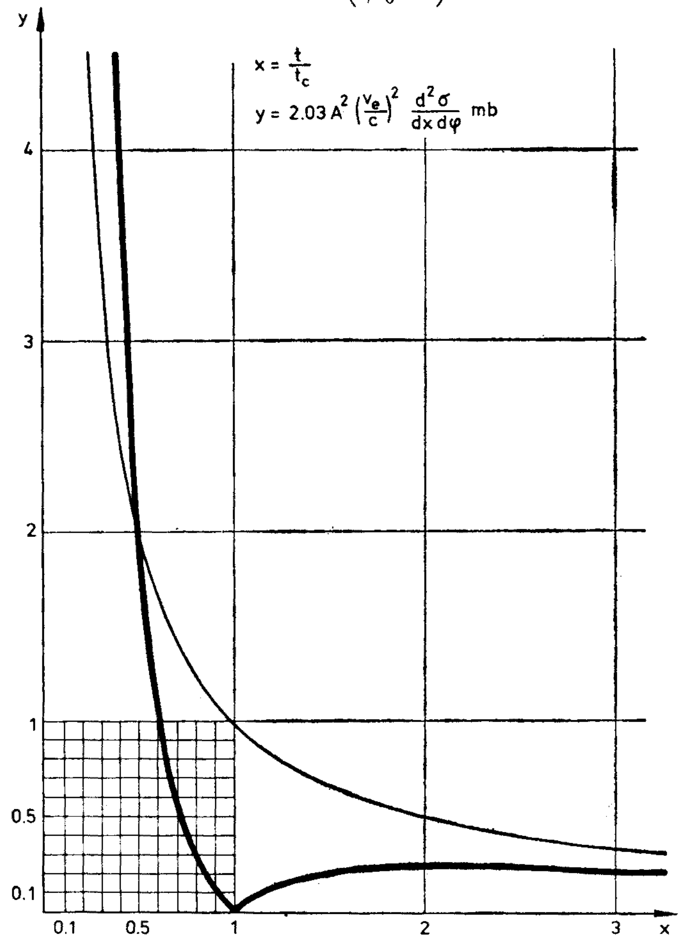

Fig. 4. Inelastic cross-section (8.7) plotted vs $x = t/t_0$ in the vicinity of $x = 1$. Thick curve corresponds to the "jump-effect"; Thin curve, to the standard theory

Here $dt d\varphi = 2p_e^2 d\Omega$ and $d\Omega$ is the element of the spherical angle of the scattered electron, $v_e = p_e/(m+E_e/c^2)$ is the lab-velocity of the impinging electron, while the two values in brackets correspond to the standard theory and the "jump-effect", respectively. These two cross-sections (8.19) are plotted in Fig. 4 vs. $x = t/t_0$ in the vicinity of $x = 1$.

9. C_4-off-mass-shell effect in disintegration process

Another C_4-effect would concern the momentum distribution of spectators in the disintegration process analyzed in Sec. 7. Much as in Sec. 8 let "M" be the hydrogen-like atom initially in the ground state $(y|1S) = \psi_i(y^2)$ which now becomes disintegrated by the impinging electron. Let us consider the sample of collisions with $t \gg p_F^2$, where $p_F = Z\alpha mc$ is the characteristic Fermi relation-momentum of the $|1S)$ state, because then the final-state interaction between the products does not disturb the spectator distribution [18]. In consequence, the matrix element (7.6) describes with a very good approximation the transition amplitude for the disintegration process in question, provided that the "e–e" interaction is neglected. However, if $t \gg p_F^2$ the investigated sample splits into two well-separated groups, first when the spectator is the atomic electron and second, when the nucleus is the spectator while the "e–e" collision takes place. The investigated cross--section will then concern the first group, when the atomic electron is the low-energy spectator particle, while almost the whole value of t is transferred onto heavy "A". In consequence, the L_4-invariant momentum distribution $E_{2f}(d\sigma/d^3p_{2f})$ of the spectatorelectron becomes proportional to the form factor squared $|\tilde{G}_i(k_\mu^2)|^2$ from (7.6) with k_μ as given by (7.7). Thus

$$E_{2f}(d\sigma/d^3p_{2f}) \sim |\tilde{G}_i[(b_iP_{i\mu}-p_{2f\mu})^2]|^2, \tag{9.1}$$

where

$$P_{i\mu}^2 = -M_i^2c^2, \quad p_{2f\mu}^2 = -m^2c^2, \quad b_i = \tfrac{1}{2}[1-(A^2M^2-m^2)/M_i^2]$$

$$M_i = (AM+m)-w_i/c^2, \quad w_i = w_i^G = \tfrac{1}{2}\mu c^2\alpha^2Z^2 = 13.6Z^2 \text{ eV}. \tag{9.2}$$

Since $w_i/mc^2 \ll 1$ and $m \ll AM$ we obtain

$$k_\mu^2 = 2mT\left[1-\left(\frac{AM}{AM+m}\right)\left(\frac{w_i}{mc^2}\right)\right] - \frac{A^2M^2}{(AM+m)^2}\frac{w_i^2}{c^2}$$

$$\cong 2mT(1-w_i/mc^2)-w_i^2/c^2, \tag{9.3}$$

where $T = c(m^2c^2+p^2)^{1/2}-mc^2$ is the lab-kinetic energy of the spectator electron and hence, $p = p_{2f}|_{lab}$.

As seen from (9.3) the constant term $-w_i^2/c^2$ implies that for very small T, namely

$$T < w_i^2/2mc^2 = 1.81\times10^{-4}Z^4 \text{ eV}$$

k_μ becomes time-like. However, this effect is practically impossible to observe and we shall neglect it by putting

$$k_\mu^2 = 2mT(1-w_i/mc^2). \tag{9.3'}$$

Moreover, since G_i is evaluated in the C_4^G-approximation of C_4 — cf. Appendix III — the spectator momenta p must be restricted to the "NR" values when $p^2/m^2c^2 \ll 1$ and hence, $T = p^2/2m$. Then

$$k_\mu^2 = (1-w_i^G/mc^2)p^2. \tag{9.4}$$

In the NR limit $c \to \infty$ $k_\mu^2 \to (\boldsymbol{p}^G)^2$ as it should be according to the standard theory. The c-depending C_4-off-mass-shell effect reveals itself in that the factor $(1 - w_i^G/mc^2)$ is less than 1 and, within the C_4^G-approximation it accounts for the effect in question. The restriction to the "NR" lab-energies T implies that instead of (9.1) we have

$$d\sigma/d^3p \sim |\tilde{\psi}_i[(\lambda_Z \boldsymbol{p})^2]|^2 \quad \text{with}$$

$$\lambda_Z = 1 - w_i^G/2mc^2 \quad (w_i^G/mc^2 \ll 1). \tag{9.5}$$

Taking into account that (App. (III.3))

$$\tilde{\psi}_i(k_\mu^2) = \tilde{G}_i(k_\mu^2) = 8\pi^{1/2} p_F^{5/2}/(p_F^2 + k_\mu^2)^2 \tag{9.6}$$

we end up with

$$d\sigma/d^3p \sim (p_F^2 + \lambda_Z^2 \boldsymbol{p}^2)^{-4}. \tag{9.7}$$

The spherically symmetric distribution (9.7) of the spectator in the lab-system, where the atom is at rest before disintegration, reflects the spherical symmetry of the ground state $(\boldsymbol{y} \mid 1S)$ in C_3. By introducing the Z-scaled, dimension-free momentum

$$\boldsymbol{s} = \boldsymbol{p}/p_F \quad (s = |\boldsymbol{s}|), \tag{9.8}$$

the normalized to unity distributions $P_Z(s)$ of s take the form

$$P_Z(s) = \frac{32}{\pi} \lambda_Z^3 \frac{s^2}{(1 + \lambda_Z^2 s^2)^4}. \tag{9.9}$$

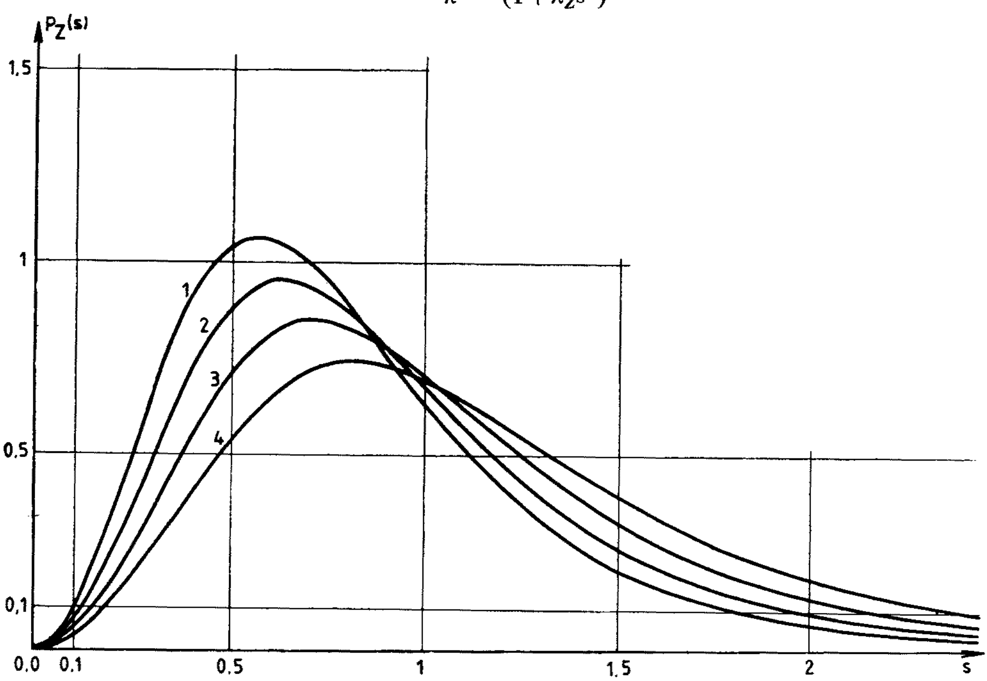

Fig. 5. Momentum distributions $P_Z(s)$ of spectator electrons ($s = p/p_F$) for four values of Z: 1. $Z = 0$ ($\lambda_0 = 1$), 2. $Z = 87$ ($\lambda_{87} = 0.9$), 3. $Z = 123$ ($\lambda_{123} = 0.8$) and 4. $Z = 150$ ($\lambda_{160} = 0.7$)

These functions exhibit the broadening of the distribution of s increasing with the increase of Z, when the atomic electron becomes more and more off its mass-shell. The loosely bound nature of atoms makes the "broadening effect" very weak, as

$$\lambda_Z = 1-1.33\times 10^{-5}Z^2 \tag{9.10}$$

is very close to unity. Of course, for the experimental as well as theoretical reasons Z must be small. In Fig. 5 the distributions $P_Z(s)$ are plotted for different values of Z. The narrowest curve for $Z = 0$ coincides with the universal one of the standard theory when, independently of Z, $\lambda_Z^G = 1$. The other three curves correspond to unrealistically large Z's in order to show the effect clearly.

10. Measuring process and final remarks

Any particular measurement provides us with a C–L absolute quantity $W_{AO}^{(S)}$ which characterizes the two-body relation between the measured object "O" and the "classical" measuring apparatus "$A^{(S)}$". The C–L nature of infinitely heavy "$A^{(S)}$" (the Lorentz limit) implies that the 10-parameter family of *different* but *identical* "$A^{(S)}$" emerges, enumerated by the superscript "S" which indicates the rest-frame S of "$A^{(S)}$" in L_4. The measurement, as such, must account for the fact that *all* measuring devices "$A^{(S)}$'s" lead to the determination of the same absolute properties of a *single* entity "O" factored-out from measuring tools "$A^{(S)}$". The "classical" nature of "$A^{(S)}$'s" implies that $W_{AO}^{(S)}$ are L_4-scalars of the form

$$W_{AO}^{(S)} = A_{\mu\nu\ldots}^{(S)}O_{\mu\nu\ldots} = L_4\text{-abs.} = L_4\text{-inv.}, \tag{10.1}$$

thus "localizing" the properties $O_{\mu\nu\ldots}$ of "O" in the space-time continuum L_4 of "classical" "$A^{(S)}$'s". In other words, the directly measurable properties of "O" must be directly relativized which manifests itself in the L_4-tensor structure of $O_{\mu\nu\ldots}$. Since all "$A^{(S)}$'s" are identical, the analytic form of the representations of "$A^{(S)}$" in S and "$A^{(S')}$" in S' are the same. The external fields $n_\mu^{(S)}$ from (2.8) represent the particular case of a set of L_4-tensor fields $A_{\mu\nu\ldots}^{(S)}$. All L_4-scalars which can be constructed from the S-independent, "relative" properties $O_{\mu\nu\ldots}$ thus represent the L_4-absolute C–L characteristics of "O" itself.

Let us consider the particularly interesting example when $W_{AO}^{(S)}$ means the time-interval indicated in S by the infinitely heavy clock at rest in S: $\Delta t|_S = W_{AO}^{(S)} = L_4$-inv. The infinite heaviness of the clock or measuring rod is required if the corresponding intervals are to be determined accurately [5]. From (2.9) we get $O_\mu = x_\mu$ and hence, $O_\mu O_\mu = x_\mu^2$ is the only L_4-invariant interval generated by (infinitely heavy) clocks and rods, but simultaneously factored-out from their reality.

In the classical framework ($\hbar = 0$), where everything is sunk in L_4, Eq. (10.1) takes place a priori thus making all properties $O_{\mu\nu\ldots}$ of "O" the C–L characteristics. Then, as rightly pointed out by Einstein, the physical realization of the "measuring fields" $A_{\mu\nu\ldots}^{(S)}$ is of no physical matter, and we can start with the L_4-tensors $O_{\mu\nu\ldots}$ neglecting the "relationistic" nature of any experimental data $W_{AO}^{(S)}$. However, in the C_4-framework besides the C–L there exist also the Q–L characteristics which are directly unmeasurable.

Thus the measurability condition (10.1) represents a constraint imposed onto the properties $O_{\mu\nu\ldots}$ of "O", namely that they must be C–L. Therefore, if "O" is a "quantum" object of finite inertia then all L_4-scalars constructed from $O_{\mu\nu\ldots}$ must be parametrized by suitable Mandelstam variables in their physical regions (scattering states). This privileged position of the L_4-p over the L_4-x language which reflects the fundamental asymmetry between the "classical" measuring devices and the "quantum" measured entities simultaneously shows why the Q–L characteristics can exist (in C_4) without falling in conflict with the relativity principle which concerns the C–L characteristics only.

The following operationalistic argument strictly connected with what was said above also inclines to accept the priority of the C_4-relationism over the L_4-relativism. The point is that direct measurement of the length L_0 of a rod or the time-interval T_0 of a clock consists in the determination of the appropriate *two boundary events* which determine two ends of each of these intervals. In fact, any direct measurement must be reduced to the determination of the "actual" coincidences of better or worse localized events. Of course, the thus determined intervals, as submitted to the L_4-symmetry of events, suffer from the contraction and dilatation effects, respectively. However, such operations do not accompany the determination of e.g. the nucleon radius $\bar{R}$ or the proper-life-time $\overset{\circ}{\tau}$ of short--living particles [13]. These C_4-absolute intervals are directly unmeasurable hence their determination is indirect, via the L_4-p language. Again the "quantum" origin of the C_4--continuum hypothesis can be observed, as there is no room for indirect measurement in the classical framework ($\hbar = 0$) where the momenta "p" become the "x-local" quantities, cf. App. I. The homogeneous Lorentz transformation of the four-momenta "p" as opposed to the inhomogeneous Lorentz transformation of events "x" clearly shows that the indirect measurement of $\bar{R}$ does not (and cannot) consist in the determination of two "boundary events" between which the nucleon exists. As a matter of fact, the "p" measurement of $\bar{R}$ implies that the nucleon "O" remains entirely unlocalized with respect to the "classical" measuring rods of the L_4 reference frames S. If $\tilde{G}_{ii}(t) = \tilde{F}_{ii}(t = q^2)$ is the experimentally determined elastic form factor of the nucleon (6.12) then,

$$\bar{R} = \langle y^2\rangle^{1/2} = [\int d^3y y^2 F_{ii}(y^2)]^{1/2} = C_4\text{-abs.}$$
$$= \hbar\sqrt{-6\tilde{F}'_{ii}(q^2 = 0)} = \hbar\sqrt{-6\tilde{G}'_{ii}(t = 0)} = L_4\text{-abs.} \qquad (10.2)$$

We see then that the determination of $\bar{R}$ requires to know only *one* parameter of the slope of $\tilde{G}_{ii}(t)$ at $t = 0$.

A similar situation occurs in the determination of the proper-life-time $\overset{0}{\tau}$ of unstable particles measured indirectly through the uncertainty relation

$$\overset{0}{\tau} = \hbar/\Delta Mc^2 = \hbar/\Delta W = C_4\text{-abs.} = L_4\text{-abs.}. \qquad (10.3)$$

Here ΔM means the width of the C_4-absolute mass-level of unstable particle "M" where, a posteriori, $M = (-P_\mu^2)^{1/2}/c = L_4$-inv. Again it is enough to determine a single parameter ΔM which shows that the life-time $\overset{0}{\tau}$ is a priori unlocalized with respect to the external L_4-time determined by the "classical" clocks. The presence of the Planck constant $\hbar$

in (10.2) and (10.3) proves the "quantum" nature of the C_4-absolute intervals $\bar{R}$ and $\overset{0}{\tau}$ and, in consequence, the "quantum" nature of the very hypothesis of the relation continuum C_4.

It must be remembered that the C_4-hypothesis assumes the priority of the isolation state of the "quantum" system over its observability from the L_4-outside which unavoidably calls for some dissipative process in the observing "medium". Thus the main idea of the C_4-hypothesis can be summed-up as follows: in the L_4-framework the space-time externality "E" of events precedes the internality "I" of the relations, whereas in the C_4--framework this hierarchy is reversed. The internality "I" of the C_4-relations precedes the L_4-eventysm "E" of the observation,

$$L_4: \text{"E"} \to \text{"I"}, \qquad C_4: \text{"I"} \to \text{"E"}. \tag{10.4}$$

The inversion (10.4) is possible and leads to C_4-effects because of the "quantum" energy--gaps which protect the inside "I" of isolated bound states $|M_n)$ from direct observation from the outside "E", thus making room for the Q–L characteristics of isolated entities "O".

In particular, according to (10.4), the mass M_n of "M" as the C_4-absolute eigenvalue of $\hat{h}/c^2$ is first determined from the inside "I" of the system. Its direct relativization and hence, the determination from the L_4-outside "E" given by the equality $M = 1/c(E^2/c^2 - \boldsymbol{P}^2)^{1/2}$ takes place a posteriori as stated in (10.4). Accordingly $M = (E/c^2)|_{\boldsymbol{P}=0}$ must resort to the c-number boundary condition $\boldsymbol{P} = 0$, but the momentum $\boldsymbol{P}$ as well as energy E are secondary to the C_4-absolute M. Finally, since the C_4-symmetry is weaker than that L_4 of the external space-time continuum, the C_4-relationism makes "more room" for dynamical models than the L_4-relativism.

APPENDIX A

C–L character of free four-momenta

The Mandelstam parametrization of cross-sections implies that the asymptotic states of the colliding particles must be (almost) the eigenstates of the momenta. Thus $\Delta P_k \to 0$ and since $\Delta X_k \sim \hbar/\Delta P_k$ the determination of $\boldsymbol{P}$ calls for a very large (infinite) space-time region Ω_4. If $\Delta X_k = a$ and $\Delta t = a/V$, where $V = \partial E/\partial P$ is the velocity of "M", the volume a^4/V of the event Ω_4 which enables us to localize "M" in order to determine its momentum $\boldsymbol{P}$, in principle, must tend to infinity ($a \to \infty$). Moreover, inside Ω_4 the measured $\boldsymbol{P}$ must be a constant of motion, as otherwise the very measured characteristic would become indetermined. The correlation nature of any quantum measurement [20] then implies that the "p" language of the Mandelstam variables calls for infinite L_4-x regions of the asymptotic zone of quantum process. This is consistent with the C_4-continuum hypothesis where the L_4-symmetry rules the asymptotic zone of the "classical" measuring devices. In the classical framework ($\hbar = 0$) $\Omega_4 = a^4/V$ can be reduced to arbitrarily small value without affecting the momentum $\boldsymbol{P}$ of "M" and hence, the momentum $\boldsymbol{P}$ becomes an x-local characteristic of "M" attached to each point of the classical world-line of "M" in L_4.

In order to prove the C–L character of $P_\mu = (\boldsymbol{P}, iE/c)$ of free "M" let us assume that its mass M and the electric charge e are known. In Fig. 6 the two events $\Omega_4^{(1)}$ and $\Omega_4^{(2)}$ are drawn of the same volume a^4/V where "M" becomes "actualized" by means of some

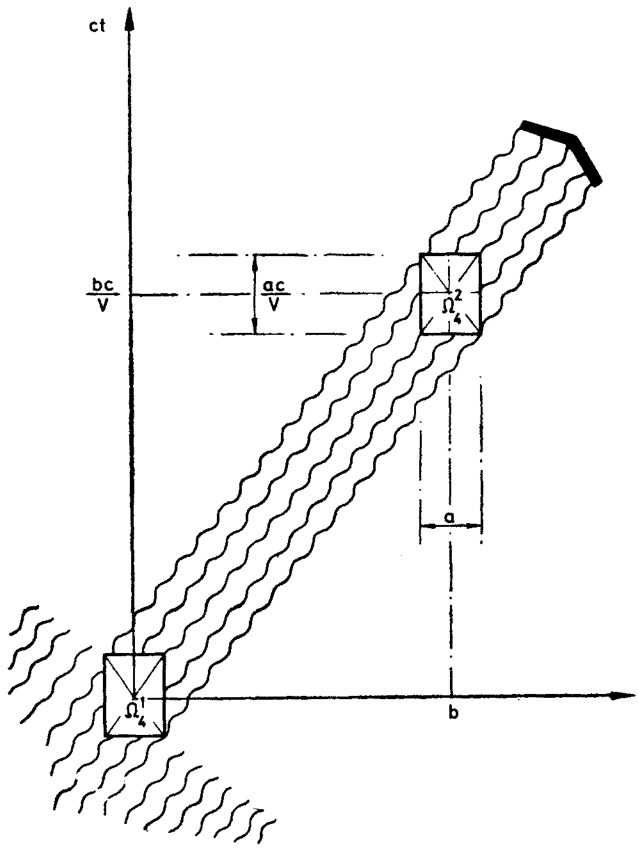

Fig. 6. Direction of the trajectory of "M" indicated by two "actual" events $\Omega_4^{(1,2)}$ separated by the space distance b (in S) and the time interval b/V, which precedes the measurement of the longitudinal momentum P_z of "M"

inelastic process of interaction of "M" with the "medium S" which can be observed. Of course, the uncontrollable momentum transfer must obey the uncertainty relations, hence

$$\Delta P_k^{(1)} \approx \Delta P_k^{(2)} \approx \hbar/a, \tag{A.1}$$

and, after crossing both events $\Omega_4^{(1,2)}$ on the "classical" trajectory, "M" gets the momentum uncertainties ΔP_k of the same order of $\hbar/a$. The two events $\Omega_4^{(1,2)}$ enable us to draw the direction of $\boldsymbol{P}$, but we still assume that the two coincidences of "M" in $\Omega_4^{(1,2)}$ do not provide us with sufficient information about the magnitude of $\boldsymbol{P}$. If $\delta\theta$ denotes the uncertainty of the direction of $\boldsymbol{P}$ then, from Fig. 6 we see that

$$\delta\theta \approx (a/b) + (\hbar/(aP)) \quad (P = |\boldsymbol{P}|), \tag{A.2}$$

where b is the space distance between $\Omega_4^{(1)}$ and $\Omega_4^{(2)}$. Since a and b are arbitrary we can take $(a/b) \ll (\hbar/aP)$ and hence,

$$\delta\theta \approx \hbar/(aP). \tag{A.2'}$$

Now we take the rotated reference frame with the z-axis parallel to $\langle \boldsymbol{P} \rangle$ where the constant magnetic field $\boldsymbol{B}$ takes the form $\boldsymbol{B} = (0, B, 0)$. Thus before "$M$" entering the spectrometer we have

$$\langle P_x \rangle = \langle P_y \rangle = 0, \quad \Delta P_x \approx \Delta P_y \approx \hbar/a. \tag{A.3}$$

By taking a large enough it is seen from (A.3) that the determination of $\boldsymbol{P}$ is reduced to the determination of $P_z = P$, where

$$P_z = P = eBR/c, \tag{A.4}$$

and R is the radius of the trajectory of "M" deflected by the external magnetic field $\boldsymbol{B}$. Consequently, the uncertainty of P_z is proportional to the uncertainty in determining R,

$$\Delta P_z \approx (eB/c)\delta R. \tag{A.5}$$

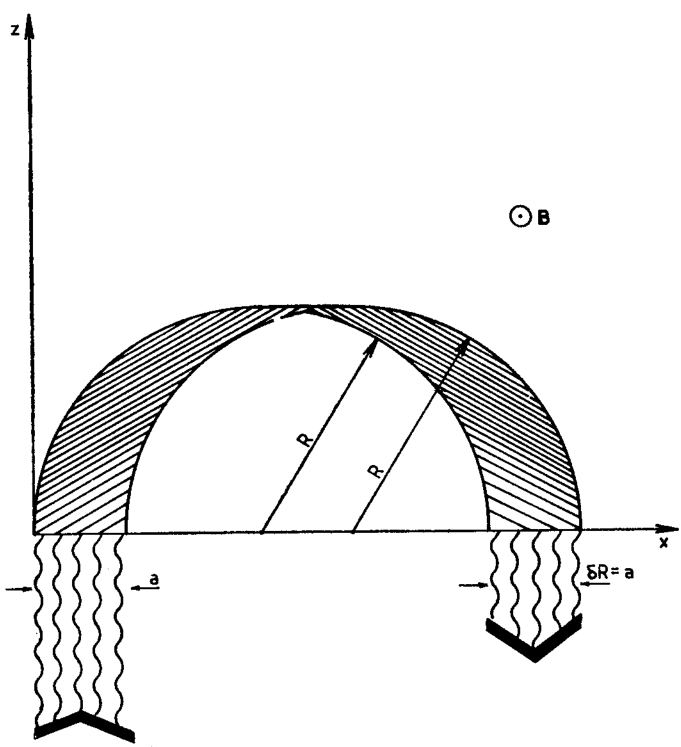

Fig. 7. Measurement of $P_z = P$ by deflecting "M" in the magnetic field $\boldsymbol{B}$. The width a of the diaphragm is large enough, so that $\delta R = a$

If a is large enough, so that the expansion of the wave packet on its halt-circle path can be neglected, then, as seen in Fig. 7, $\delta R = a$ and we get

$$\Delta P_x \approx \Delta P_y \approx \hbar/a, \quad \Delta P_z \approx (eB/c)a, \tag{A.6}$$

and

$$\boldsymbol{P} = (0, 0, eBR/c), \quad E = c(M^2c^2+\boldsymbol{P}^2)^{1/2}. \tag{A.7}$$

From (A.6) we see that the dispersions of $\boldsymbol{P}$ and E become negligible if

$$a \to \infty, \quad B \sim a^{-1-\varepsilon} \xrightarrow[a\to\infty]{} 0, \quad R \sim a^{1+\varepsilon} \xrightarrow[a\to\infty]{} \infty \quad (\varepsilon > 0). \tag{A.8}$$

If M were unknown, some other particle "m" could be recognized as one of the "unit" mass ($m = 1$) and, assuming that "M" and "m" have the same charge e, the above measurement results in

$$M = R_M/R_m. \tag{A.9}$$

Thus the exact and direct measurement of P_μ can be performed without penetrating "M" which proves the C–L character of P_μ of free "M". Notice also that the "Q–L-parameter" $f = t/\mathcal{M}^2c^2 = 0$, because here $\mathcal{M}$ denotes the mass of infinitely heavy magnets which create the external field $\boldsymbol{B}$, cf. the Lorentz limit I.

APPENDIX B

Determination of the "weights" a, b

The four-momenta $p_{1,2\mu}$ of free particles "1" and "2" take the following form in their CM-system S^*:

$$p^*_{1,2\mu} = (\mp\boldsymbol{q}, iM_{1,2}c), \quad M_{1,2} = (m^2_{1,2}+\boldsymbol{q}^2/c^2)^{1/2}, \tag{B.1}$$

where the c-number values $\boldsymbol{q}$ coincide (numerically) with the C_4-absolute relation-momentum eigenvalue of $\hat{\boldsymbol{q}}$. Let us introduce an auxiliary four-momentum $\pi_\mu(\alpha)$ depending on the parameter α, whose 4-length determines the L_4-invariant function of α. Then

$$\pi_\mu(\alpha) = \alpha p_{2\mu}-(1-\alpha)p_{1\mu} \quad \text{and} \quad f(x) = [\alpha p_{2\mu}-(1-\alpha)p_{1\mu}]^2 = L_4\text{-inv.} \tag{B.2}$$

By virtue of (B.1) and denoting $M = M_1+M_2$, instead of (B.2) we get

$$f(\alpha) = \boldsymbol{q}^2-(\alpha M-M_1)^2c^2. \tag{B.2$'$}$$

As known from Section 4, the direct a posteriori relativization of the relation-momentum $\boldsymbol{q}$ calls for a space-like four-vector p_μ such that $\boldsymbol{q}^2 = p^2_\mu$. From (B.2′) we see that π_μ will coincide with p_μ provided that the negative term in (B.2′) vanishes, i.e.

$$\alpha = a(M) = M_1/M = \tfrac{1}{2}[1+(m_1^2-m_2^2)/M^2] = 1-b(M) = L_4\text{-inv.} \tag{B.3}$$

as stated in (4.13), (4.15). Thus

$$\pi_\mu(a(M)) = p_\mu = a(M)p_{2\mu}-b(M)p_{1\mu}. \tag{B.4}$$

The following discontinuity is worth emphasizing which reflects the fundamental discontinuity in the number of absolute intervals in L_4 (one) and C_4 (two). As seen from (B.3) in the Lorentz limit $m_1 \to \infty$ $a(M) \to 1$ and hence $b(M) \to 0$. In spite of that p_μ from (B.4) cannot be identified with $p_{2\mu}$ because $b(M)p_{10}$ remains finite in the limit $m_1 \to \infty$. Thus, even in the Lorentz limit, p_μ maintains the space-like character as $p_\mu^2 = \boldsymbol{q}^2 \geqslant 0$, whereas $p_{2\mu}^2 = -m_2^2c^2$ and $p_{2\mu}$ is time-like.

On the other hand, in the same Lorentz limit $m_1 \to \infty$ performed in the x-representation we end up with the one-body Klein-Gordon equation (2.6) of the particle "2" and in consequence, its "p" representation provides us with time-like $p_{2\mu}$. This descrepancy between the "p" and "x" representations of the Lorentz limit $m_1 \to \infty$ is due to the fact that finite momentum transfer t does not result in finite velocity recoil of "1" and hence, the "Q–L-parameter" $f = t/m_1^2c^2 = 0$. In the "p" representation, even that $m_1 \to \infty$, the reality of "1" enters through the term $\lim_{m_1 \to \infty} b(M)p_{10} \neq 0$, whereas in the "$x$" picture the reality of "1" disappears and we are left with the L_4-event continuum with only one absolute four-interval x_μ^2. Thus the mathematical reference frames in L_4 conceal the second absolute (but not L_4-invariant) property which compels us to distinguish between the L_4-A and L_4-B absolutenesses.

APPENDIX C

I. Inelastic global form factor G_{fi}

The $|1\mathrm{S})$ initial and the $|2\mathrm{S})$, $|2\mathrm{P}; j)$ final states of the hydrogen-like atoms take in the C_4^{G}-approximation the form ($\hbar = 1$)

$$(\boldsymbol{y}|1\mathrm{S}) = \left(\frac{8}{27\pi}\right)^{1/2} q_{\mathrm{F}}^{3/2} \exp\left(-\tfrac{2}{3}\, q_{\mathrm{F}} r\right)$$

$$(\boldsymbol{y}|2\mathrm{S}) = \left(\frac{1}{27\pi}\right)^{1/2} q_{\mathrm{F}}^{3/2} \exp\left(-\tfrac{1}{3}\, q_{\mathrm{F}} r\right)\left(1-\tfrac{1}{3}\, q_{\mathrm{F}} r\right)$$

$$(\boldsymbol{y}|2\mathrm{P};j) = \left(\frac{1}{243\pi}\right)^{1/2} q_{\mathrm{F}}^{5/2} y_j \exp\left(-\tfrac{1}{3}\, q_{\mathrm{F}} r\right),$$

where $r = |\boldsymbol{y}|$, $q_{\mathrm{F}} = \frac{3}{2} Ze^2\mu$, $\mu = m/(1+m/AM)$. One easily finds the q-representations of the corresponding form factors as equal to:

$$\tilde{F}_{2\mathrm{S}:1\mathrm{S}}(\boldsymbol{q}) = \frac{64\sqrt{2}}{81}\, q_{\mathrm{F}}^4 \frac{\boldsymbol{q}^2}{(q_{\mathrm{F}}^2+\boldsymbol{q}^2)^3}$$

$$\tilde{F}_{2\mathrm{P};j:1\mathrm{S}}(\boldsymbol{q}) = \frac{64\sqrt{2}}{81}\, q_{\mathrm{F}}^5 i \frac{q_j}{(q_{\mathrm{F}}^2+\boldsymbol{q}^2)^3}\,.$$

The global, spherically symmetric form factor $\tilde{F}_{fi}(\boldsymbol{q}^2)$ which accounts for the transition to all four excited states is thus determined by

$$|\tilde{F}_{fi}(q^2)|^2 = |\tilde{F}_{2S:1S}(\boldsymbol{q})|^2 + \sum_{j/1}^{3} |\tilde{F}_{2P;j:1S}(\boldsymbol{q})|^2$$

hence,

$$\tilde{F}_{fi}(\boldsymbol{q}^2) = \frac{64\sqrt{2}}{81} q_F^4 \frac{(\boldsymbol{q}^2)^{1/2}}{(q_F^2+\boldsymbol{q}^2)^{5/2}},$$

and

$$\tilde{G}_{fi}(p_\mu^2) = \frac{64\sqrt{2}}{81} q_F^4 \frac{(p_\mu^2)^{1/2}}{(q_F^2+p_\mu^2)^{5/2}} \tag{C.1}$$

which, by virtue of the analytic continuation, is valid for all p_μ. Finally,

$$G_{fi}^{(D)}(x_\mu) = (2\pi)^{-4} \frac{64\sqrt{2}}{81} q_F^4 \int_D d^4p \frac{(p_\mu^2)^{1/2}}{(q_F^2+p_\mu^2)^{5/2}} \exp(ip_\mu x_\mu), \tag{C.2}$$

where the contour D must avoid the singularity at $p_0 = \pm(q_F^2+\boldsymbol{p}^2)^{1/2}$. Moreover, the half-integer exponent "5/2" generates the branching points of the integrand alien to the standard field-theoretical characteristic functions.

II. *Disintegration form factor G_i*

By introducing the Fermi relation-momentum $p_F = Ze^2\mu$ characteristic of the bound state $|1S)$, the form factor $F_i(\boldsymbol{y}^2)$ takes the form

$$F_i(\boldsymbol{y}^2) = (\boldsymbol{y}|1S) = \pi^{-1/2} p_F^{3/2} \exp(-p_F r).$$

Its q-representation is then equal to

$$\tilde{F}_i(\boldsymbol{q}^2) = (\boldsymbol{q}|1S) = 8\pi^{1/2} p_F^{5/2} (p_F^2+\boldsymbol{q}^2)^{-2}$$

and hence, after relativization and analytic extension, one obtains

$$\tilde{G}_i(p_\mu^2) = 8\pi^{1/2} p_F^{5/2} \frac{1}{(p_F^2+p_\mu^2)^2}. \tag{C.3}$$

For the purpose of Section 9 it is enough to know the above L_4-p-representation of G_i, however, now the x-representation of G_i can be expressed by means of the standard "functions" (distributions)

$$\Delta^{(D)}(x_\mu;\kappa) = (2\pi)^{-4} \int_D d^4p \frac{\exp(ip_\mu x_\mu)}{\kappa^2+p_\mu^2}.$$

Indeed,

$$G_i^{(D)}(x_\mu) = -4\pi^{1/2} p_F^{3/2} \partial/\partial p_F[\Delta^{(D)}(x_\mu; p_F)], \tag{C.4}$$

which represents the L_4-x-form-invariant disintegration form factor in the C_4^G-approximation in which $(\boldsymbol{y} \mid 1S)$ is evaluated.

III. Point-particle form factor

The internal "structure" of a point-particle "composed" of two constituents has its form factor in C_4 equal to the 3-dimensional δ-function:

$$F(\boldsymbol{y}) = \delta^{(3)}(\boldsymbol{y}), \quad \tilde{F}(\boldsymbol{q}) = 1. \tag{C.5}$$

The analytic continuation of $\tilde{F}$ to time-like p_μ results in

$$\tilde{G}(p_\mu) = 1 \quad \text{hence} \quad G(x_\mu) = \delta^{(4)}(x_\mu), \tag{C.6}$$

which coincides with the standard form factor of a point-particle.

Let us emphasize the singular position of the point-particle form factor $\delta^{(4)}(x_\mu)$ in L_4 as it exhibits another aspect of the same discontinuity between C_4 and L_4 symmetries. The point is that $\delta^{(4)}(x_\mu)$ is the only L_4-x-form-invariant form factor (in L_4),

$$\delta^{(4)}(l_{\mu\nu} x_\nu) = \delta^{(4)}(x_\mu), \tag{C.7}$$

($l_{\mu\nu}$ are the elements of the homogeneous Lorentz matrix) which implies the vanishing of both, the space and the time intervals between two events $x_{1,2\mu}$. In fact, $x_\mu = x_{2\mu} - x_{1\mu} = 0$ means the coincidence of the two events. However, if $x_{1\mu} \neq x_{2\mu}$ then, $x_\mu^2 = (x_{2\mu} - x_{1\mu})^2$ is the only a priori given L_4-x-form-invariant attached to two events and therefore, any form factor G independent of the boundary conditions must depend on x_μ^2 only; $G = G(x_\mu^2)$. The form factor $G(x_\mu^2)$ remains constant on the Minkowski "circles" $x_\mu^2 = $ const. and, because of the indefinite L_4-metric $x_\mu^2 = 0$ does not imply the coincidence of the events $x_{1\mu}$, $x_{2\mu}$. The equality $x_\mu^2 = 0$ only means that $x_{1\mu}$ and $x_{2\mu}$ are separated by the light-like intervals. Thus there is no continuous transition from $G(x_\mu^2)$ to $\delta^{(4)}(x_\mu)$ which shows the singular position of the point-particles in the L_4-framework.

This discontinuity between $\delta^{(4)}(x_\mu)$ and $G(x_\mu^2)$ is concealed in the momentum representation if the momentum transfers t are too small to detect the internal structure G of "M". From the considerations in Sections 5,6 and 7 we know that the L_4-x-form-invariant functions like $G(x_\mu^2)$ describe properly the Q–L characteristics of the "quantum" systems "O". However, according to the C_4-hypothesis which revers the priority of "E" and "I" as stated in (10.4), the dynamical explanation of the form factor of "M" must take place in C_4. The two C_4-absolute intervals r and $\Delta\tau$ together with the definite metric of C_3-internal-space make that, much as in the NR framework, the C_4-continuum provides us with the proper "first medium" of quantum entities. The translation of the C_4-internal into the L_4-external Q–L shapes ("$F = G$") is necessary if the corresponding absolute shape becomes (indirectly) detectable experimentally.

APPENDIX D

New singularity

The non-local structure of the C_4-operators like $\hat{h}$ implies a new singularity alien to the Lorentz limit I ($m_1 \to \infty$) similarly as to the NR limit ($c \to \infty$). The simplest example which leads to this singularity is the S-state radial Schroedinger equation (1.12). By putting $u(r) = rR(r)$, where $R(r)$ is the radial wave function, we obtain

$$\frac{d^2u}{dr^2} = -\frac{1}{4\hbar^2}[(W-V(r))^2/c^2-2(m_1^2+m_2^2)c^2+(m_1^2-m_2^2)^2c^6/(W-V(r))^2]u(r). \quad \text{(D.1)}$$

If $m_1 \neq m_2$ and if the structure of Eq. (D.1) admits $r = r_c$ such that

$$V(r_c) = W \quad \text{(D.2)}$$

then, besides the "old" singularities at $r = 0$ and $r = \infty$, we get the third one at $r = r_c$ determined implicitly by (D.2).

In the case of the C_4-absolute Coulomb interaction $V = -e^2/r$ ($r = |\mathbf{y}|$) responsible for the hydrogen-atom structure, the value of r_c can be well estimated because of small binding energy. Indeed,

$$r_c = -e^2/W \cong -e^2/(M+m)c^2 \doteq -1.5\times 10^{-16}\ \text{cm}, \quad \text{(D.3)}$$

where the negative value of r_c is due to the attractive force. We see that r_c tends to zero in the Lorentz limit I ($M \to \infty$) as well as in the NR limit ($c \to \infty$) and so, r_c would characterize the C_4-framework. However, the W-dependence of the localization of r_c deserves a separate mathematical treatment.

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