QUARK MIXING AND THE CABIBBO ANGLE IN THE CHIRAL $SU_4 \times SU_4$ BROKEN SYMMETRY

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The problem of a separate and then simultaneous mixing of quarks in doublet (d, s) and (u, c) in the chiral $SU_4 \times SU_4$ symmetry is discussed. A method which allows one to calculate the rotation angle is introduced. It is possible that the measured Cabibbo angle is effectively the sum of angles connected with the simultaneous mixing of quarks in the electrically charge subspaces $+2/3$ and $-1/3$ respectively for the maximal allowed breaking of the chiral $SU_4 \times SU_4$ symmetry.

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It is known that the Cabibbo angle has been introduced into SU_3 symmetry to explain the suppression of processes in which strangeness is not conserved [1]. The Cabibbo angle is connected with the mixing of d and s quarks for weak interactions of hadrons. Its value, calculated by Oakes [2], does not contradict the experimental data. Before the charmed particles were discovered Glashow, Iliopoulos and Maiani have suggested the generalization of a strong interaction symmetry to SU_4 [3]. The charged weak current is then given as follows

$$-\gamma_5)Aq, \tag{1}$$

where

$$A = \begin{pmatrix} 0, & 0, & \cos\theta, & \sin\theta \\ 0, & 0, & -\sin\theta, & \cos\theta \\ 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0 \end{pmatrix}, \quad q = \begin{pmatrix} u \\ c \\ d \\ s \end{pmatrix}. \tag{2}$$

The current (1) can be expressed in another form

$$J_\mu = (\bar{u}, \bar{c})\gamma_\mu(1-\gamma_5)\begin{pmatrix} \cos\theta, & \sin\theta \\ -\sin\theta, & \cos\theta \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix} \tag{3}$$

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so, quark mixing is described by an orthogonal matrix. On the grounds of Eqs. (3) we cannot come to a conclusion about quarks in which the doublets are mixed. If the matrix A is generalized to the following form

$$A = \begin{pmatrix} 0 , & 0 , & \cos\theta , & \sin\theta \\ 0 , & 0 , & -\sin\theta , & \cos\theta \\ \cos\phi , & \sin\phi , & 0 , & 0 \\ -\sin\phi , & \cos\phi , & 0 , & 0 \end{pmatrix}, \tag{4}$$

the quarks in the doublets (u, c) and (d, s) are mixed independently. The zeros in Eqs. (4) are associated with the fact that the neutral currents which change the strangeness and/or charm are not observed. So, the current (1) can be given in the following form

$$J_\mu = (\bar{u}, \bar{c})\gamma_\mu(1-\gamma_5)\begin{pmatrix} \cos(\theta+\phi), & \sin(\theta+\phi) \\ -\sin(\theta+\phi), & \cos(\theta+\phi) \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix}. \tag{5}$$

If the currents only are taken into consideration we cannot solve the problem if the quarks are mixed in one or both doublets. This is not unexpected because the currents are built as a bilinear combination of quark states and the angles θ and ϕ can always be substituted the effective angle $(\theta+\phi)$. To solve the problem the Gell-Mann, Oakes, Renner (GMOR) model [4] will be used.

2

The charged weak current in SU_3 symmetry can be written as follows

$$J_\mu(\theta) = \cos\theta(J^1_\mu + iJ^2_\mu) + \sin\theta(J^4_\mu + iJ^5_\mu) \tag{6}$$

θ — the Cabibbo angle

$$J^k_\mu = \bar{q}\gamma_\mu(1-\gamma_5)\lambda^k q \qquad q = \begin{pmatrix} u \\ d \\ s \\ c \end{pmatrix}. \tag{7}$$

The current (6) can be obtained from the isospin component of the current $(J^1_\mu + iJ^2_\mu)$ by rotation through an angle 2θ about the seventh axis in SU_3 space according to

$$J_\mu(\theta) = e^{-2i\theta F^7}(J^1_\mu + iJ^2_\mu)e^{2i\theta F^7}, \tag{8}$$

where

$$F^k = \int d^3x q^+(x)\frac{\lambda^k}{2}q(x). \tag{9}$$

The charged weak current in SU_4 symmetry (3) can be expressed in the following form

$$\begin{aligned} J_\mu(\theta) = {} & \cos\theta(J^1_\mu + iJ^2_\mu) + \sin\theta(J^4_\mu + iJ^5_\mu) \\ & -\sin\theta(J^{11}_\mu - iJ^{12}_\mu) + \cos\theta(J^{13}_\mu - iJ^{14}_\mu). \end{aligned} \tag{10}$$

The current (10) can be obtained by rotation of the components $\Delta S = \Delta C$ through an angle 2θ about the seventh axis in SU_4 space

$$J_\mu(\theta) = e^{-2i\theta F^7}(J_\mu^1 + iJ_\mu^2 + J_\mu^{13} - iJ_\mu^{14})e^{2i\theta F^7}. \tag{11}$$

The transformation (11) changes the strangeness but not the charm because

$$[F^7, q_1] = [F^7, q_4] = 0\,. \tag{12}$$

The transformation (11) is connected with the mixing of d and s quarks (as in the case of SU_3 symmetry). In SU_4 symmetry the mixing in electric charge subspace $+2/3$ can be taken into consideration. This is not possible in SU_3 symmetry where only one state with the $+2/3$ charge exists. The possibility of expressing the current (10) by the transformation which changes charm but not strangeness should exist. The transformation has been described by Ebrahim in Ref. [6].

$$J_\mu(\phi) = e^{-2i\phi F^{10}}(J_\mu^1 + iJ_\mu^2 + J_\mu^{13} - iJ_\mu^{14})e^{2i\phi F^{10}} \tag{13}$$

$$[F^{10}, q_2] = [F^{10}, q_3] = 0\,. \tag{14}$$

The transformation (13) is connected with the mixing of u and c quarks. The fact that there exist two transformations giving the current (10) but connected with different generators of the SU_4 group changing strangeness or charm respectively suggests that independent mixing in both doublets is possible.

It is known that the Cabibbo angle is connected with strangeness nonconservation in weak interactions. The formula describing the value of the Cabibbo angle has been obtained by Oakes [2] in the procedure of symmetry breaking. Namely the $SU_3 \times SU_3$ symmetry in the limit of the exact $SU_2 \times SU_2$ symmetry is broken. The $SU_2 \times SU_2$ subsymmetry is no longer exact. The symmetry is broken by the rotation of the $SU_2 \times SU_2$ invariant hamiltonian density through angle 2θ about the seventh axis. Then the pion becomes massive. The symmetry breaking is connected with the mixing of d and s quarks. The rotation angle θ, as a measure of symmetry violation, is a function of the mass and the decay constant of the pion and of the mass and the decay constant of the kaon as well (it is connected with the mixing of the strange quark and the strangeness nonconservation). If the breaking of the chiral $SU_2 \times SU_2$ symmetry, the mass of the pion, the Cabibbo angle as well as a strangeness and charm nonconservation have a common origin then it seems that as a result of $SU_4 \times SU_4$ symmetry breaking in the limit of the exact $SU_2 \times SU_2$ subsymmetry by the rotation of the $SU_2 \times SU_2$ invariant hamiltonian density through an angle 2ϕ about the tenth axis the angle ϕ connected with the mixing of u and c quarks as a measure of a symmetry violation should be a function of the mass and decay constant of the pion (breaking of the $SU_2 \times SU_2$ symmetry) and a function of the mass and decay constant of a charmed meson (charm nonconservation).

The cases of the separate and then simultaneously mixing of quarks in the subspaces of electric charge will be considered below.

3

If the electromagnetic mass splitting of $u-d$ quarks is neglected the hamiltonian density breaking the chiral $SU_4 \times SU_4$ symmetry is given in the form

$$H = c_0 u^0 + c_8 u^8 + c_{15} u^{15}, \tag{15}$$

where c_0, c_8, c_{15} are constants, u^a $(a = 0, 1, \ldots, 15)$ are the scalar components of the $(\bar{4},4)+(4,\bar{4})$ representation of the chiral $SU_4 \times SU_4$ group. On the grounds of the GMOR model the following relation for masses of the pseudoscalar mesons can be obtained [8].

$$\langle 0| [\bar{Q}^a, \bar{D}^b] |0\rangle = -i\delta^{ab} f_a^2 m_a^2 - i \int \frac{dq^2}{q^2} \varrho^{ab}$$

$$= -i \Big\{ \langle u^0 \rangle_0 \left(\frac{c_0}{2} \delta^{ab} + \frac{c_8}{\sqrt{2}} d_{a8b} + \frac{c_{15}}{\sqrt{2}} d_{a15b} \right)$$

$$+ \langle u^8 \rangle_0 \left(\frac{c_0}{\sqrt{2}} d_{a8b} + c_8 d_{a8c} d_{b8c} + c_{15} d_{a8c} d_{b15c} \right)$$

$$+ \langle u^{15} \rangle_0 \left(\frac{c_0}{\sqrt{2}} d_{a15b} + c_8 d_{a15c} d_{b8c} + c_{15} d_{a15c} d_{b15c} \right) \Big\}, \tag{16}$$

where

$$\varrho^{ab} = (2\pi)^3 \sum_{n \neq a} \delta^4(p_n - q) \langle 0|\bar{D}^a|n\rangle \langle n|\bar{D}^b|0\rangle \tag{17}$$

f_a — decay constants, $\langle u^i \rangle_0$ — vacuum expectation value of the operator u^i. Because the vacuum expectation values of operators u^8, u^{15} and the spectral density ϱ^{ab} are proportional to the squared parameters of symmetry breaking, they are further neglected [8]. Approximately from Eqs. (16) we obtain

$$m_a^2 f_a^2 \delta^{ab} = \frac{1}{\sqrt{2}} \left(\frac{c_0}{\sqrt{2}} + c_8 d_{a8b} + c_{15} d_{a15b} \right) \langle u^0 \rangle_0. \tag{18}$$

The masses of the mesons are given as follows

$$m_\pi^2 f_\pi^2 = \frac{1}{2\sqrt{3}} (\sqrt{3}\, c_0 + \sqrt{2}\, c_8 + c_{15}) \langle u^0 \rangle_0, \tag{19a}$$

$$m_K^2 f_K^2 = \frac{1}{2\sqrt{3}} \left(\sqrt{3}\, c_0 - \frac{c_8}{\sqrt{2}} + c_{15} \right) \langle u^0 \rangle_0, \tag{19b}$$

$$m_D^2 f_D^2 = \frac{1}{2\sqrt{3}} \left(\sqrt{3}\, c_0 + \frac{c_8}{\sqrt{2}} - c_{15} \right) \langle u^0 \rangle_0. \tag{19c}$$

In the limit of the exact chiral $SU_2 \times SU_2$ subsymmetry there is the following constraint

$$\sqrt{3}\, c_0 + \sqrt{2}\, c_8 + c_{15} = 0 \tag{20}$$

so the pion is massless.

Let us make some remarks. The task of the Cabibbo angle calculation in SU_4 symmetry using the procedure of symmetry breaking has been done in Ref. [6]. In Ebrahim's earlier paper [5] the parameters of the $SU_4 \times SU_4$ symmetry breaking have been found.

$$\frac{c_8}{c_0} = -\frac{2\sqrt{2}}{\sqrt{3}} \frac{m_K^2 f_K^2 - m_\pi^2 f_\pi^2}{m_K^2 f_K^2 + m_D^2 f_D^2}, \tag{21a}$$

$$\frac{c_{15}}{c_0} = -\frac{1}{\sqrt{3}} \frac{3m_D^2 f_D^2 - m_K^2 f_K^2 - 2m_\pi^2 f_\pi^2}{m_K^2 f_K^2 + m_D^2 f_D^2}. \tag{21b}$$

In Ref. [6] the numerical values of parameters (21) have been used to calculate the rotation angle (interpreted as the Cabibbo angle). The $SU_2 \times SU_2$ invariant hamiltonian density breaking $SU_4 \times SU_4$ symmetry has been rotated through an angle 2θ about the seventh axis and the coefficients of the operators u^a $(a = 0, 8, 15)$ have been identified with the parameters of symmetry breaking

$$H_{SB}(\Delta S = 0) = c_0 u^0 + \frac{\sqrt{3}}{2} c_8 \sin^2 \theta u^3$$

$$+ c_8(1 - \tfrac{3}{2} \sin^2 \theta) u^8 - (\sqrt{3}\, c_0 + \sqrt{2}\, c_8) u^{15}. \tag{22}$$

It seems to us that there are some errors in the numerical calculations of the author. The use of the numerical values of the parameters (21) has not been necessary. On the grounds of theoretical formulas only, indeed from the Eqs. (7) in Ref. [5] and the Eqs. (10) in Ref. [6], it follows that

$$\sin^2 \theta = \frac{2m_\pi^2 f_\pi^2}{2m_K^2 f_K^2 + m_\pi^2 f_\pi^2}. \tag{23}$$

Then the value of θ is given by

$$\sin^2 \theta = (0.215)^2 \tag{24}$$

instead of

$$\sin^2 \theta = -0.04 \tag{25}$$

from Eqs. (10) in Ref. [6]. Formula (23) has the same form as in SU_3 symmetry. In agreement with our expectation the angle θ is described by parameters of the pion and the strange meson.

In Ebrahim's method the $SU_4 \times SU_4$ symmetry breaking hamiltonian density is parametrized by the factors c_0, c_8, c_{15}. The parameters of symmetry breaking are expressed by the masses and decay constants of the mesons and they are fixed (Eqs. (7) in Ref. [5]). In the limit of the exact $SU_2 \times SU_2$ subsymmetry the factors c_0, c_8, c_{15} should satisfy the constraint (20) but it is possible only if $m_\pi = 0$ namely the parameters of symmetry breaking are not expressed by real (measured in experiment) masses of mesons. In Ref. [6] Ebrahim breaks the $SU_4 \times SU_4$ symmetry in the limit of the exact $SU_2 \times SU_2$ subsymmetry by the rotation of the $SU_2 \times SU_2$ invariant hamiltonian density through an angle 2θ about the seventh axis. The factors of the rotated hamiltonian density are identified with the parameters of symmetry breaking (Eqs. (7) in Ref. [5]). Solving a set of equations the author gets the factors c_0, c_8, c_{15} dependent on the rotation angle and on the real mesons masses already. The masses of mesons standing in the formula which describes the parameters of symmetry breaking are determined by the method of symmetry breaking and they have a real value for the real realisation of the symmetry breaking only. In this case the rotation angle does not matter a parameter of the symmetry violation. It seems to us that such an interpretation is not satisfactory. The expression of meson masses as a function of the rotation angle (as a measure of symmetry violation) seems to be more natural. In the present paper the other interpretation of the symmetry breaking and the method of calculating the rotation angle is proposed. We describe our method as follows.

Before the $SU_4 \times SU_4$ symmetry in the limit of the exact $SU_2 \times SU_2$ subsymmetry is broken the masses of mesons have been expressed by the factors c_0, c_8, c_{15} which satisfy the constraint (20). After symmetry breaking a new set of factors c'_0, c'_8, c'_{15} dependent on the old factors c_0, c_8, c_{15} and on the rotation angle is introduced. The new factors are identified with the coefficients by the operators u^i of the rotated hamiltonian density (22).

$$c'_0 = c_0, \tag{26a}$$

$$c'_8 = c_8(1-\tfrac{3}{2}\sin^2\theta), \tag{26b}$$

$$c'_{15} = -\sqrt{3}\,c_0-\sqrt{2}\,c_8. \tag{26c}$$

Meson masses are expressed by new factors and they are the function of the rotation angle as a measure of symmetry violation.

$$m_\pi^2 f_\pi^2 = \frac{1}{2\sqrt{3}}(\sqrt{3}\,c'_0+\sqrt{2}\,c'_8+c'_{15})\,\langle u^0\rangle_0 = -\frac{\sqrt{3}}{2\sqrt{2}}c_8\sin^2\theta\langle u^0\rangle_0 \tag{27a}$$

$$m_K^2 f_K^2 = \frac{1}{2\sqrt{3}}\left(\sqrt{3}\,c'_0-\frac{c'_8}{\sqrt{2}}+c'_{15}\right)\langle u^0\rangle_0 = -\frac{\sqrt{3}}{2\sqrt{2}}c_8(1-\tfrac{1}{2}\sin^2\theta)\,\langle u^0\rangle_0 \tag{27b}$$

$$m_D^2 f_D^2 = \frac{1}{2\sqrt{3}}\left(\sqrt{3}\,c'_0+\frac{c'_8}{\sqrt{2}}-c'_{15}\right)\langle u^0\rangle_0 = \left(c_0+\frac{\sqrt{3}}{2\sqrt{2}}c_8(1-\tfrac{1}{2}\sin^2\theta)\right)\langle u^0\rangle_0. \tag{27c}$$

It seems to be more natural that the meson masses are functions of the parameters of symmetry breaking (27) than inversely the parameters of symmetry breaking are functions of meson masses which are not consistent with the experimental data and are dependent on the method of symmetry breaking. This interpretation is consistent with the fact that the mass generation of the mesons is a consequence of symmetry breaking. From Eqs. (27) we obtain the formula for the angle θ as in Eqs. (23). Let us consider the other variant of symmetry breaking described in Ref. [7]. Ebrahim, using his method, breaks the $SU_4 \times SU_4$ symmetry in the limit of the exact $SU_3 \times SU_3$ symmetry by the rotation of $SU_3 \times SU_3$ invariant hamiltonian density about the fourteenth axis in SU_4 space. The rotation angle θ' is identified with the Cabibbo angle. The formula describing the angle θ' should be given as follows

$$\sin^2\theta' = \tfrac{3}{2}\frac{m_K^2 f_K^2}{m_D^2 f_D^2 + m_K^2 f_K^2}, \tag{28}$$

(in Eqs. (4a) in Ref. [7] there is the factor 3/2). The rotation of a hamiltonian density about the fourteenth axis is considered in Ref. [9] too. The D meson is interpreted as a Goldstone boson. Putting aside the agreement of the numerical value of the angle θ' with the experimental data it seems to us that the angle connected with the rotation about the fourteenth axis cannot be interpreted as the Cabibbo angle, because the rotation is performed inside the doublet (s, c). Then the states with the different electric charges are mixed. The interpretation that the D meson is a Goldstone boson is also unsatisfactory. If the $SU_4 \times SU_4$ symmetry is broken in such a way that the $SU_2 \times SU_2$ subsymmetry is still exact, so the K meson becomes massive but the pion is still massless. Such a symmetry breaking cannot be accepted, results contradict the experimental data. The next breaking of the exact $SU_3 \times SU_3$ symmetry is connected with the mixing of s and c quarks. The rotation angle cannot be interpreted as the Cabibbo angle for the reasons given above. It seems that the hierarchy of symmetry breaking is extended and the breaking of the $SU_4 \times SU_4$ symmetry taken as a whole cannot be connected with the Cabibbo angle. This is possible, however, for $SU_4 \times SU_4$ symmetry breaking in the limit of exact $SU_2 \times SU_2$ subsymmetry. Then results are in agreement with our expectation.

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Our method described above is used to calculate an angle ϕ which is connected with the rotation about the tenth axis in SU_4 space. Then the $SU_4 \times SU_4$ symmetry is broken by the rotation of the $SU_2 \times SU_2$ invariant hamiltonian density through an angle 2ϕ about the tenth axis.

$$\begin{aligned} H_{SB}(\Delta C = 0) &= c_0 u^0 + \left(\sqrt{2}\, c_0 + \frac{\sqrt{3}}{2} c_8\right)\sin^2\phi u^3 \\ &+ \left(c_8 + \frac{1}{2\sqrt{2}}\left(\frac{4}{\sqrt{3}} c_0 + \sqrt{2}\, c_8\right)\sin^2\phi\right) u^8 \\ &+ \left(-\sqrt{3}\, c_0 - \sqrt{2}\, c_8 + \left(\frac{4}{\sqrt{3}} c_0 + \sqrt{2}\, c_8\right)\sin^2\phi\right) u^{15}. \end{aligned} \tag{29}$$

Using the factors from hamiltonian density (29) the masses of mesons are given as follows

$$m_\pi^2 f_\pi^2 = \left(c_0 + \frac{\sqrt{6}}{4} c_8\right)\sin^2\phi \langle u^0\rangle_0, \tag{30a}$$

$$m_K^2 f_K^2 = -\left(\frac{\sqrt{6}}{4} - \tfrac{1}{2}\left(c_0 + \frac{\sqrt{6}}{4} c_8\right)\sin^2\phi\right)\langle u^0\rangle_0, \tag{30b}$$

$$m_D^2 f_D^2 = (1 - \tfrac{1}{2}\sin^2\phi)\left(c_0 + \frac{\sqrt{6}}{4} c_8\right)\langle u^0\rangle_0, \tag{30c}$$

so

$$\sin^2\phi = \frac{2m_\pi^2 f_\pi^2}{2m_D^2 f_D^2 + m_\pi^2 f_\pi^2}\,. \tag{31}$$

In agreement with our expectation the angle ϕ is a function of the mass of the pion (as a measure of the $SU_2 \times SU_2$ violation) and is connected with the parameters of the charmed meson (mixing in (u, c) doublet). For the mass $m_D = 1862$ MeV and $f_D/f_\pi = 0.974$ [10] one gets

$$\sin\phi = 0.076. \tag{32}$$

The small value of the angle ϕ is the effect of the large mass of the charmed quark. From (32) results that only mixing in the (u, c) system is excluded, the value of the angle ϕ contradicts the experimental data. The simultaneous mixings in both doublets are, however, still possible. Fritzsch [11] considers also the mixing in (u, c) system. The mixing angle is calculated on the grounds of quark masses and does not contradict the results obtained above. Although the value of the angle ϕ is relatively small, it is significant: the sum of the angles $\theta+\phi$ is larger than the value of the angle measured experimentally, called Cabibbo angle. This fact cannot be explained by the limits of experimental errors. Let us note that the angles (23) and (31) are calculated for the case where quarks are mixed separately. The angles from formula (5) cannot be identified with those from Eqs. (23) and (31). In the case of simultaneous mixing in both doublets the relation between the angles is more complicated. To find the relation, the $SU_2 \times SU_2$ invariant hamiltonian density is rotated through an angle 2ϕ about the tenth axis and afterwards by an angle 2θ about the seventh axis. The sequence of the rotations is insignificant, because

$$[F^7, F^{10}] = 0. \tag{33}$$

The rotated hamiltonian density is given by

$$\begin{aligned} H_{SB}(\Delta S = \Delta C = 0) &= c_0 u^0 \\ &+ \left(\frac{\sqrt{3}}{2} c_8 \sin^2\theta + \left(\sqrt{2}\, c_0 + \frac{\sqrt{3}}{2} c_8\right)\sin^2\phi\right) u^3 \\ &+ \left(\frac{1}{\sqrt{3}}\left(\sqrt{2}\, c_0 + \frac{\sqrt{3}}{2} c_8\right)\sin^2\phi + c_8(1 - \tfrac{3}{2}\sin^2\theta)\right) u^8 \\ &+ \left(-\sqrt{3}\, c_0 - \sqrt{2}\, c_8 + \frac{2\sqrt{2}}{\sqrt{3}}\left(\sqrt{2}\, c_0 + \frac{\sqrt{3}}{2} c_8\right)\sin^2\phi\right) u^{15}. \end{aligned} \tag{34}$$

The meson masses are given as follows

$$m_\pi^2 f_\pi^2 = \frac{1}{2\sqrt{3}}\left\{\sqrt{6}\left(\sqrt{2}\, c_0 + \frac{\sqrt{3}}{2} c_8\right)\sin^2\phi - \frac{3}{\sqrt{2}} c_8 \sin^2\theta\right\}\langle u^0\rangle_0, \tag{35a}$$

$$m_{K}^{2}f_{K}^{2} = \frac{1}{2\sqrt{3}}\left\{\frac{3}{\sqrt{6}}\left(\sqrt{2}\,c_{0}+\frac{\sqrt{3}}{2}\,c_{8}\right)\sin^{2}\phi - \frac{3}{\sqrt{2}}\,c_{8}(1-\tfrac{1}{2}\sin^{2}\theta)\right\}\langle u^{0}\rangle_{0}, \tag{35b}$$

$$m_{D}^{2}f_{D}^{2} = \frac{1}{2\sqrt{3}}\left\{2\sqrt{3}\,c_{0}-\frac{3}{\sqrt{6}}\left(\sqrt{2}\,c_{0}+\frac{\sqrt{3}}{2}\,c_{8}\right)\sin^{2}\phi + \frac{3}{\sqrt{2}}\,c_{8}(1-\tfrac{1}{2}\sin^{2}\theta)\right\}\langle u^{0}\rangle_{0}. \tag{35c}$$

Now the angles θ and ϕ cannot be described independently. The following relation is obeyed.

$$2m_{\pi}^{2}f_{\pi}^{2}+2(m_{K}^{2}f_{K}^{2}+m_{D}^{2}f_{D}^{2})\sin^{2}\theta\sin^{2}\phi = (2m_{K}^{2}f_{K}^{2}+m_{\pi}^{2}f_{\pi}^{2})\sin^{2}\theta+(2m_{D}^{2}f_{D}^{2}+m_{\pi}^{2}f_{\pi}^{2})\sin^{2}\phi \tag{36}$$

or equivalently

$$1+\left(\frac{1}{\sin^{2}\theta_{0}}+\frac{1}{\sin^{2}\phi_{0}}-1\right)\sin^{2}\theta\sin^{2}\phi = \frac{\sin^{2}\theta}{\sin^{2}\theta_{0}}+\frac{\sin^{2}\phi}{\sin^{2}\phi_{0}}, \tag{37}$$

where

$$\sin^{2}\theta_{0} = \frac{2m_{\pi}^{2}f_{\pi}^{2}}{2m_{K}^{2}f_{K}^{2}+m_{\pi}^{2}f_{\pi}^{2}}, \tag{38a}$$

$$\sin^{2}\phi_{0} = \frac{2m_{\pi J}^{2}\,{}_{\pi}^{2}}{2m_{D}^{2}f_{D}+m_{\pi}^{2}f_{\pi}^{2}}. \tag{38b}$$

The angles θ and ϕ from Eqs. (37) concern a simultaneous mixing in doublets (d, s) and (u, c) respectively and they can be identified with those from Eqs. (5). The condition (37) limits the values of the angles θ and ϕ. The maximal values of the angles θ_0 and ϕ_0 are given by Eqs. (38). The value of the function

$$f(\theta, \phi) = \sin(\theta+\phi) \tag{39}$$

is also limited. A numerical calculation shows that there is an extremum (a maximum) of function (39) on the condition (37) for

$$\theta_{m} = 0.20452$$
$$\phi_{m} = 0.02575 \tag{40}$$

so

$$\sin(\theta_{m}+\phi_{m}) = 0.2282. \tag{41}$$

It is worth noting that the extremum of function (39) on condition (37) can be identified with the measured Cabibbo angle. It is not excluded that symmetry breaking is realized in the maximal allowed case, so the effective angle of mixings would correspond to the maximum of the function (39).

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