NUCLEON-QUARK PHASE TRANSITION IN HEAVY ION COLLISIONS*

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Although the details of such a phase transition are not fully understood, the thermodynamical data of the transition can be calculated from known parts of the equation of state in a self-consistent way. The result is that at moderate temperatures the transition begins at $n = 5-6n_0$ and the necessary density is lower for temperatures between 120 and 200 MeV. Some possibilities for observation are also discussed, with the conclusion that clear evidence has not been found until now.

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1. *Introduction*

The topic of this School is "Application of the Gauge Theory". This lecture, maybe, consists of more application and, consequently, less gauge theory than the average one. Namely, Quantum Chromodynamics is a gauge theory. To calculate the pressure of a quark-gluon plasma is then an application, and to calculate the data of the phase transition from the pressure function is an application of an application of a gauge theory. Nevertheless, in contrast to some axioms of the contemporary higher mathematics, an application of an application is again an application, thus this lecture satisfies the conditions of the School.

From the dawn of the quark theory everybody was very interested, how the quarks can build up hadrons, and how they could be released. The nucleon-quark phase transition runs through such steps, and it would be very important to describe them. Nevertheless the author does not think himself to be the proper person to investigate these individual processes. It seems more useful to choose a restricted goal: to obtain the (thermodynamical) data of the corresponding states between which the phase transition happens. Possessing such data one will be able to decide if free quarks occur in hot stages of heavy ion collisions or in neutron stars, and if they occur, one will be able to take them into account.

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The situation seems to be slightly paradoxical: if the structural differences between the nuclear matter and the quark-gluon plasma are great, then one can expect first order phase transition, but in first order transitions the system generally avoids the most "complicated" states (by taking a Maxwellian short cut). That is, in many cases, if the structural change is too great and one cannot calculate the intermediate states, one does not have to do it.

In this lecture it will be shown (or tried to be shown) that this program can be realized for a nucleon-quark transition. In fact, the author belonged to a group calculating this phase transition. He must confess that he is an outsider in QCD, and, although he learnt some elementary QCD from the other members of the group, all possible deficiencies in those parts of the presentation have to be attributed solely to him.

In this scheme of calculation one cannot prove that individual assumptions are true, but it is possible to verify their self-consistency, and this will be done. In Sects. 2 and 3 the general thermodynamical formulation of first order phase transitions is given, Sect. 4 contains the dynamical equations for mixed systems, Sect. 5 discusses the general features of the transition, Sect. 6 gives the equation of state for nuclear matter, while Sect. 7 gives it for quark-gluon plasma, and Sect. 8 is focused on the value of the "bag constant". The results of the calculations can be found in Sect. 9. Sect. 10 contains some considerations for possible observable effects, and, finally, Sect. 11 is a brief conclusion.

2. *First order phase transitions and thermodynamics*

Consider a thermodynamical system whose (thermodynamical) state is determined by the complete set of independent extensives $\{X^i\}$. In the entropic convention this set does not contain the entropy S, while the form of the entropy function

$$S = S(X^i);\ i = 0, \dots, Q;\ X^0 = V \tag{2.1}$$

characterizes the behaviour of the system. Since S is also an extensive parameter (i.e. it is additive in a fictitious process when one divides and reunites a system in equilibrium), it is homogeneous and linear in X^i, that is

$$S = \sum_{i=0}^{Q} Y_r X^r;\ Y_i \equiv \frac{\partial S}{\partial X^i}, \tag{2.2}$$

(where we introduced the intensive data Y_i). On the other hand, from Eq. (2.1),

$$dS = \sum_{i=0}^{Q} Y_r dX^r. \tag{2.3}$$

Now consider a closed system. (Here the term "closed" means that the extensive data are fixed.) According to the Second Law, the system is evolving toward the state of maximal entropy, thus this final state must be the equilibrium. Dividing the system into two fictitious subsystems one can write:

$$S_1(X_1^i)+S_2(X_2^i)+\sum_{r=0}^{Q}\lambda_r(X_1^r+X_2^r-X^r) = \max \tag{2.4}$$

and hence

$$Y_{1i} = Y_{2i} \tag{2.5}$$

is obtained as a necessary condition of the equilibrium. The intensives must possess homogeneous distributions in the system.

Nevertheless, Eq. (2.5) is the condition of the extremum. It is a maximum if and only if the matrix

$$M_{ik} = \frac{\partial^2 S}{\partial X^i \partial X^k} \tag{2.6}$$

is negative definite at the point X^i. If M_{ik} is indefinite, then the equilibrium is (thermodynamically) unstable. Fixing the extensives at such an unstable point, rapid and unpredictable changes can be expected in each volume element of the system.

It is quite possible that these rapid changes lead to a stable equilibrium. Namely, from Cond. (2.4) we have not obtained anything for the homogeneity of the extensive densities. Introducing these densities as

$$x^i \equiv \frac{1}{V} X^i; \, i \neq 0, \qquad s \equiv \frac{1}{V} S = s(x^i), \tag{2.7}$$

Eqs. (2.2), (2.3) yield

$$Y_i = \frac{\partial s(x^k)}{\partial x^i}; \, i \neq 0, \qquad Y_0 = +\frac{p}{T} = s - \sum_{r=1}^{Q} \frac{\partial s}{\partial x^r} x^r, \tag{2.8}$$

$$M_{ik} = \begin{bmatrix} s_{,\varrho\delta} x^\varrho x^\delta & -s_{,\alpha\varrho} x^\varrho \\ -s_{,\beta\varrho} x^\varrho & s_{,\alpha\beta} \end{bmatrix}; \qquad \alpha = 1, \ldots, Q,$$

(i.e. the system is stable if $[s_{,\alpha\beta}]$ is negative definite). If Eq. (2.5) can be satisfied by different extensive densities $\{x_1^i\}$ and $\{x_2^i\}$ and if in these points $[s_{,\alpha\beta}]$ is negative definite, then such an inhomogeneous (mixed) state of the system is a stable equilibrium.

The simplest example is a cold system of one component. In order to describe it, first we arrive at the energy convention:

$$s \leftrightarrow \varepsilon, \qquad Y_0 \leftrightarrow Y_0/Y_1, \qquad Y_1 \leftrightarrow 1/Y_1, \qquad Y_{\alpha \geqslant 2} \leftrightarrow -Y_\alpha/Y_1. \tag{2.9}$$

Thus for a system of one component,

$$\varepsilon = \varepsilon(s, n), \qquad Y_0 = -p = \varepsilon - s\varepsilon_{,s} - n\varepsilon_{,n}, \qquad Y_1 = T = \varepsilon_{,s}, \qquad Y_2 = \mu = \varepsilon_{,n}. \tag{2.10}$$

For $T \to 0$, from the Third Law,

$$s \to 0, \qquad \varepsilon_{,ss} > 0. \tag{2.11}$$

Then, at $T = 0$, the only possible instability comes from $\varepsilon_{,nn} = \mu_n < 0$. Let us assume that

$$\mu_{,n} > 0 \text{ if } n < n_3 \text{ or } n > n_4, \qquad \mu_{,n} < 0 \text{ if } n_3 < n < n_4.$$

Hence it is seen that between n_3 and n_4 μ and p decrease, while ε increases slower than linearly.

Since μ decreases between n_3 and n_4, the condition $\mu(n_1) = \mu(n_2)$ can be fulfilled for density pairs outside of the unstable region (n_3, n_4), and the situation is similar for p. There is equilibrium, if the equality is valid for both intensives, whence

$$\varepsilon'(n_1) = \varepsilon'(n_2), \quad n_1\varepsilon'(n_1)-\varepsilon(n_1) = n_2\varepsilon'(n_2)-\varepsilon(n_2), \tag{2.12}$$

and the geometrical meaning of these conditions is that the tangents of the curve $\varepsilon = \varepsilon(n)$ belonging to the two different densities coincide [1].

Thus, if the average density of the system is between n_3 and n_4, the homogeneous equilibrium is thermodynamically unstable, there is a separation, and the stable configuration consists of domains in which the density is either n_1 or n_2. However, if the average density is outside of the unstable region but between n_1 and n_2, both the homogeneous and the inhomogeneous configurations are stable, only the second one has lower energy.

Performing a very slow compression from an initial state $n < n_1$, first the only possible equilibrium configuration consists of one phase, however, except for nonphysical idealizations, there are fluctuations in the matter too. If the $n = n_2$ configuration is produced even in a small volume as fluctuation, reaching $n = n_1$ it can coexist. Since $\mu_1 = \mu_2$ and $p_1 = p_2$, there is an indifferent equilibrium between the neighbouring domains $n = n_1$ and $n = n_2$. For $n > n_1$ there are two possibilities: either the second phase remains small and practically the whole matter becomes denser (when the energy density changes according to the function $\varepsilon(n)$), or the densities remain constant, but the nucleus of the second phase increases (and ε moves along the double tangent). Since the reorganization of the matter happens only on the surfaces of the domains, the system follows the energetically preferred second path, if the compression is sufficiently slow, If not, it can remain on the curve $\varepsilon(n)$ until $n = n_3$, but after this density the homogeneous state is unstable, there are rapid changes in the whole volume, and finally the system jumps onto the double tangent. For further compression the weight of the $n = n_1$ domains decreases, and the first phase vanishes at $n = n_2$, where the phase transformation has finished.

3. *Gibbs conditions*

For one component system at finite temperature Eq. (2.5) yields:

$$\mu_1 = \mu_2, \quad p_1 = p_2, \quad T_1 = T_2, \tag{3.1}$$

which are the Gibbs conditions for equilibrium of phases. It is the simplest to evaluate them when T and μ are the independent variables. In order to arrive at such a description, let us introduce the quantity:

$$\zeta \equiv \varepsilon - Ts - \mu n = -p. \tag{3.2}$$

Then

$$d\zeta = d\varepsilon - Tds - sdT - \mu dn - nd\mu = -dp, \tag{3.3}$$

whence

$$dp = nd\mu + sdT, \quad p = p(T, \mu). \tag{3.4}$$

Thus the function $p(T, \mu)$ is equivalent with $s(\varepsilon, n)$ or $\varepsilon(s, n)$ for characterizing the behaviour of the system. The system is stable if the matrix $[p_{,\alpha\beta}]$ is positive definite. The only non-trivial Gibbs condition has the unclear form

$$p(\mu_0, T_0) = p(\mu_0, T_0). \tag{3.5}$$

Nevertheless, this condition is to be read as not an identity but an equation for (μ_0, T_0). E.g. if for a fixed value of T the curve $p = p(\mu)$ has a loop, then cond. (3.5) is valid in the point of self-intersection, but the two values of $p_{,\mu} = n$ differ, thus, indeed, there is a first order phase transition.

For higher order phase transitions there are no loops in the equation of state (3.4). Nevertheless, as we shall see, if the nucleon-quark phase transition were not of first order, the transition would cause very poor observable effects.

In some cases one is practically unable to calculate the whole $p(\mu, T)$ function because for the two phases different approximations can and have to be used. In this case the loop cannot be seen. However, if the calculated (and calculable) parts of the function contain the self-intersection, we can describe the slow processes because, as we have seen, then the system does not enter into the loop.

4. *The dynamical equations*

It is well-known that during a first order phase transition the dynamics of the system is affected by the transition. In order to get the correct equations of motion one has to use the original balance equations. For thermodynamically simple systems there are two sets of balance equations, namely the balance equations for the four-momentum and for the particle numbers. The four-momentum always fulfils a local conservation law

$$T^{ir}{}_{;r} = 0 \tag{4.1}$$

for closed systems, because Eq. (4.1) is a consequence of the Einstein equation of the general relativity [2]. (T^{ik} is the energy-momentum tensor of the matter.) The particle numbers may have sources because of chemical processes [3]:

$$(nu^r)_{A;r} = \nu_A. \tag{4.2}$$

If the matter is a continuum, i.e. there is everywhere a unique velocity field u^i, then there are $N+1$ unknown thermodynamical quantities, namely s and n_A ($A = 1, \ldots, N$ for the independent components), and 3 unknown hydrodynamical quantities (the independent components of the velocity). On the other hand, the system (4.1–2) contains just $N+4$ equations. The components orthogonal to u^i of Eq. (4.1) yield the equation of motion for u^i, while the remaining fourth component is the energy (or entropy) equation.

For a one component fluid the energy momentum tensor has the form:

$$T^{ik} = \varepsilon u^i u^k + p(g^{ik} + u^i u^k) + q^i u^k + q^k u^i + \tau^{ik},$$

$$u^r u_r = -1, \quad q_r u^r = \tau_{ir} u^r = 0, \tag{4.3}$$

where g_{ik} is the metric tensor, q_i is the thermal flux and τ_{ik} is the viscous part of the stress tensor. Sufficiently near to the global equilibrium q_i and τ_{ik} generally have the forms:

$$q_i = -\kappa(T_{,r} + Ta_r h_i^r), \quad \tau_{ik} = -h_i^r h_k^s[\eta(u_{r;s} + u_{s;r}) + \eta' h_{rs} u^q{}_{;q}],$$
$$h_{ik} \equiv g_{ik} + u_i u_k, \quad a_i \equiv u_{i;r} u^r, \tag{4.4}$$

where κ, η and η' are the coefficients of the irreversible processes, they vanish for perfect fluids. In this special case we get from Eq. (4.1):

$$(\varepsilon + p) u^i{}_{;r} u^r + p_{;r}(g^{;r} + u^i u^r) = 0,$$
$$\dot{\varepsilon} + (\varepsilon + p) u^r{}_{;r} = 0, \quad \dot{\varepsilon} \equiv \varepsilon_{,r} u^r. \tag{4.5}$$

(In these formulae ε is the density of the total relativistic energy.)

For the more general case when the coefficients of the heat conduction and viscosity do not vanish, here we do not want to discuss the equations of motion. The other two equations have the form [5]

$$\dot{n} + n u^r{}_{;r} = 0, \quad \dot{\varepsilon} + (\varepsilon + p) u^r{}_{;r} = \Sigma, \quad \Sigma \equiv -q^r{}_{;r} + q_{(r;s)} u^r u^s - \tau^{rs} u_{(r;s)}. \tag{4.6}$$

For one phase p fulfils the Eq. (2.10) whence

$$\dot{s} + s u^r{}_{;r} = (nT)^{-1} \Sigma. \tag{4.7}$$

For two phases ε and n are the averaged data, thus for them Eq. (2.10) is not valid. The independent variables are n and the data of the phases n_1, n_2, s_1 and s_2. According to the Gibbs conditions p is the same for both phases, (2.10) is valid within a domain, and

$$\varepsilon = \frac{1}{n_2 - n_1} [(n_2 - n)\varepsilon_1(s_1, n_1) + (n - n_1)\varepsilon_2(s_2, n_2)]. \tag{4.8}$$

The system (2.10), (3.1), (4.6) yields 5 equations for the 5 variables as follows:

$$\dot{n} + n u^r{}_{;r} = 0, \tag{4.9}$$
$$\mathrm{K}\dot{\xi} = \mathbf{w},$$

where:

$$\xi \equiv \begin{bmatrix} n_1 \\ n_2 \\ s_1 \\ s_2 \end{bmatrix}, \tag{4.10}$$

$$\mathrm{K} \equiv \begin{bmatrix} (n_2 - n)\left(\mu - \dfrac{\varepsilon_2 - \varepsilon_1}{n_2 - n_1}\right); & (n - n_1)\left(\mu - \dfrac{\varepsilon_2 - \varepsilon_1}{n_2 - n_1}\right); & (n_2 - n)T \;; & (n - n_1)T \\ n_1\gamma_1 + s_1\alpha_1 \;; & -n_2\gamma_2 - s_2\alpha_2 \;; & n_1\alpha_1 + s_1\beta_1; & -n_2\alpha_2 - s_2\beta_2 \\ \alpha_1 \;; & -\alpha_2 \;; & \beta_1 \;; & -\beta_2 \\ \gamma_1 \;; & -\gamma_2 \;; & \alpha_1 \;; & -\alpha_2 \end{bmatrix},$$

and

$$w = \begin{bmatrix} (n_2-n_1)\Sigma - \frac{\dot{n}}{n}\, T(n_1 s_2 - n_2 s_1) \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

with

$$\alpha_A \equiv (\varepsilon_{,sn})_A; \qquad \beta_A = (\varepsilon_{,ss})_A; \qquad \gamma_A = (\varepsilon_{,nn})_A. \tag{4.11}$$

If $\Sigma = 0$ (i.e. for perfect fluid), a straightforward but circuitous calculation shows that the average specific entropy s/n

$$s = \frac{1}{n_2 - n_1}\left[(n_2 - n)s_1 + (n - n_1)s_2\right] \tag{4.12}$$

remains constant during the phase transition. Another consequence is that for a cold perfect fluid $w = 0$, thus then the effective compressibility is infinite.

5. *Some general remarks on the possible types of the transition*

As we have seen, the equation of state $p = p(T, \mu)$ (or its any equivalent) completely determines the data of the phase transition. However, since our present knowledge on the equation of state is limited in some extent, it is useful first to discuss the possible qualitatively different types of the first order phase transitions. In this Section we restrict ourselves to one component fluids, and use n and T as independent variables. Then the proper thermodynamical potential is the free energy density [6]

$$f = f(T, n). \tag{5.1}$$

The nontrivial Gibbs criteria have the form

$$(f_{,n})_{n_1,T} = (f_{,n})_{n_2,T}, \qquad n_1(f_{,n})_{n_1,T} - f(n_1, T) = n_2(f_{,n})_{n_2,T} - f(n_2, T), \tag{5.2}$$

thus we have two equations for n_1, n_2 and T, whence

$$n_1 = n_1(T), \quad n_2 = n_2(T), \quad n_2 \geqslant n_1. \tag{5.3}$$

We are interested in the qualitative behaviour of these functions.

By taking the total T derivatives of Eqs. (4.2) one gets:

$$\frac{dn_i}{dT} = f_{,nn}^{-1}(n_i, T)\left[\frac{f_{,T}(n_2, T) - f_{,T}(n_1, T)}{n_2 - n_1} - f_{,nT}(n_i, T)\right]. \tag{5.4}$$

The first factor on the right hand side is always positive because the (n_1, T) and (n_2, T) points are in stable regions. Now if we can calculate the free energy density everywhere, including the unstable region, then Eq. (5.4) can be written into a more explicit form in the variables

$$\tfrac{1}{2}(n_1+n_2) = \bar{n},$$
$$\tfrac{1}{2}(n_2-n_1) = \Delta. \tag{5.5}$$

Namely, then

$$f_{,nn}(n_{(\frac{1}{2})}, T)\frac{dn_{(\frac{1}{2})}}{dT} = \pm f_{,Tnn}(\bar{n}, T)\Delta - \tfrac{1}{3} f_{,Tnnn}(\bar{n}, T)\Delta^2 + O(\Delta^3). \tag{5.6}$$

Assuming that this expansion rapidly converges, i.e. that the Δ^3 terms are negligible, we get:

TABLE I

Dominating term	Sign: Positive	Sign: Negative
$f_{,Tnn}$	a) $n_{1,T} > 0$, $n_{2,T} < 0$	b) $n_{1,T} < 0$, $n_{2,T} > 0$
$f_{,Tnnn}$	c) $n_{1,T} < 0$, $n_{2,T} < 0$	d) $n_{1,T} > 0$, $n_{2,T} > 0$

Case (a) means that the density difference is decreasing with the temperature. For atomic matter $f_{,Tnn} > 0$ can be more or less expected because the thermal motion works against the compressibility, but for other types of matter this inequality is not necessarily valid.

Eq. (5.6) contains $f(T, n)$ in the unstable region, and we assumed even that it is a sufficiently flat function there. Nevertheless this is a quite moderate requirement. The van der Waals model system is a good example for this. If the low density limit of such a system is the ideal gas with $\gamma = 5/3$, then the equation of state has the form [7, 8]

$$f = nT\ln\left[\frac{n}{n_0-n}\left(\frac{T_0}{T}\right)^{3/2}\right] - an^2. \tag{5.7}$$

Hence, for $\bar{n}$ and Δ we get the following (partially transcendent) equations:

$$\Delta^2 = (n_0-\bar{n})^2 - \frac{n_0^2 T}{2a\bar{n}},$$

$$0 = -4a\Delta + T\ln\left(\frac{\bar{n}n_0+n_0\Delta+\Delta^2-\bar{n}^2}{\bar{n}n_0-n_0\Delta+\Delta^2-\bar{n}^2}\right) + \frac{2n_0T\Delta}{(n_0-\bar{n})^2-\Delta^2}. \tag{5.8}$$

Obviously f exists and is a smooth function everywhere below n_0, even in the unstable region. Eq. (5.8) does not have real solutions (i.e. there is not first order transition) if

$$T > T_{cr} \equiv \tfrac{8}{27} a n_0. \tag{5.9}$$

At $T = T_{cr}$ there is a second order transition with

$$n_{cr} = \frac{1}{3} n_0. \tag{5.10}$$

Just below T_{cr} Δ is small, and

$$f_{,Tnn}(n_{cr}, T_{cr}) = \frac{27}{4}, \quad f_{,Tnnn}(n_{cr}, T_{cr}) = 0. \tag{5.11}$$

Thus expansion (5.6) converges for the van der Waals system near to T_{cr}. According to Eq. (5.11), the Case (a) in Table I is van der Waals-like, while Type (b) may be referred to as the anti-van-der-Waals transition.

It can be seen that even the qualitative features of the phase diagram cannot be predicted without knowing some fundamental characteristics of the equation of state in the unstable region. Nevertheless, at least the signs of $f_{,Tnn}$ and $f_{,Tnnn}$ can be estimated for a quark-gluon system from perturbative QCD calculations, and thence for high temperatures $f_{,Tnn} < 0$, as we shall see, thus one cannot expect van der Waals type transition for quark systems.

6. *The nuclear matter*

We have seen that the thermodynamical data of the phase transition can be determined if the function $p(T, \mu)$ is known everywhere (at least outside of the region of coexisting phases). If QCD is a good idea, it should yield the equation of state even for nuclear matter, but recently this way seems inefficient. Thus there remains the possibility to calculate the two sides of the transition by means of different methods, and to hope that both approximations remain valid until the corresponding transition density.

First consider the nuclear matter. We are convinced that a normal nucleus is practically not a quark-gluon plasma, the phase transition happens at higher densities. Thus, first, intricate models giving interesting details of the existing nuclei are not necessary here, and some of these models, breaking down at high densities as a causal, even cannot be used. Since, in addition, in energetic heavy ion collisions $T \sim 100$ MeV, the safest candidates for the description are the mean field theories, of which the most obvious is Walecka's model [9, 10] containing one scalar and one vector meson. (If need be, pions could be incorporated too [11].) In this model the equation of state has the form:

$$p(T, \mu) = \frac{1}{2} c_v^2 n^2 - \frac{1}{2c_s^2}(1-x)^2$$
$$+ \frac{\gamma}{3(2\pi)^3} \int (f_+(\nu, T; \boldsymbol{k}) + f_-(\nu, T; \boldsymbol{k})) \frac{k^2 d^3k}{\sqrt{k^2+x^2}}, \tag{6.1}$$

where

$$c_v^2 = 266.9, \quad c_s^2 = 195.7, \tag{6.2}$$

are the dimensionless coupling constants, $\gamma = 2$ for $N_p = 0$, and 4 for $N_p = N_n$. $f_\pm$ stand for the Fermi distributions

$$f_\pm = \left\{1+\exp\left[\frac{1}{T}(\sqrt{k^2+x^2} \mp \nu)\right]\right\}^{-1} \tag{6.3}$$

while the remaining auxiliary quantities are defined as

$$\nu \equiv \mu - c_v^2 n,$$

$$n = \frac{\gamma}{(2\pi)^3}\int [f_+(\nu, T; \boldsymbol{k}) - f_-(\nu, T; \boldsymbol{k})]d^3k, \tag{6.4}$$

$$1 - \frac{1}{x} + \frac{\gamma}{(2\pi)^3} c_s^2 \int \frac{1}{\sqrt{k^2+x^2}} [f_+(\nu, T; \boldsymbol{k}) - f_-(\nu, T; \boldsymbol{k})]d^3k = 0.$$

It can be seen that n is the difference of the baryon and antibaryon densities.

At $T = 0$, $p(n \to \infty) \simeq \frac{1}{2} c_v^2 n^2 = \frac{1}{2c_v^2}\mu^2$, while at low densities $p(T \to \infty) \sim T^4$ (because of the dominating "blackbody radiation" of the baryon-antibaryon pairs).

It is difficult to tell if there is some reason (except for the phase transition itself) because of which this description would break down at a few nuclear densities, and, on the other hand, only very poor observational checks are available for the nuclear equation of state at high densities. Even the coupling constants are fitted to the bulk properties of the usual nuclear matter.

7. *The perturbative QCD region*

In the QCD the quark-gluon plasma can be handled more and more conveniently with increasing density, because of the asymptotic freedom. Thus one may try a perturbative calculation at high densities. This fact gives us a possibility for a self-consistent treatment of the phase transition. Namely, from the asymptotic freedom one expects

$$p \sim n^{4/3} \sim \mu^4 \tag{7.1}$$

thus, if the phase transition exists, the quark plasma has the higher density. If the density gap in the transition is sufficiently wide, then the quark phase can be within the perturbative region. (Of course, there is a possibility that the perturbative treatment is not possible at all, e.g. because some terms are divergent. Nevertheless, here our point of view is rather practical; we need some data about the phase transition, thus we do not want to argue against the only presently open way to calculate these data.)

There are alternative ways to calculate the pressure of the quark-gluon plasma, nevertheless the scheme is more or less general, except some neglections which might be necessary from technical reasons. Since the author belonged to a group investigating the phase transition, he feels a strong temptation to follow the steps of their own particular work [12] here, and, indeed, this way seems to be the simplest (at least from psychological reasons) for him. Probably the improvement of the calculation concerning the neglections would need only extra work, which can be made if needs be.

First, let us remember that we are interested mainly in heavy ion processes. Then the initial state is nuclear matter with $n_p \simeq n_n$, and the time scale is ca. 10^{-22}s, i.e. electromagnetic and weak interactions are forbidden. Hence, in the quark-gluon plasma

$$n_u = n_d; \qquad n_s = 0 \tag{7.2}$$

(and we fully neglect the heavier quarks). Of course, there may be $s\bar{s}$ pairs. Nevertheless, $m_s \simeq 280$ MeV [13], thus at moderate temperatures their contribution is small. Therefore here we neglect them. (This, of course, means that the neutron star problem, where the time scale is very long, would need a different approach.)

The second step is to expand $p = p(\mu_Q, T)$ into a power series in the quark-gluon coupling g^2. Recently there are some divergence problems in the g^6 terms [14], thus one has to stop anyway at g^4. However, as the final results will show, at moderate temperatures it is self-consistent to use the g^2 terms only, and these terms have already been calculated. Namely [15].

$$p = \frac{8\pi^2}{45} T^4 \left(1 - \frac{15}{16}\frac{g^2}{\pi^2}\right) + \frac{7\pi^2}{30} T^4 \left(1 - \frac{25}{42}\frac{g^2}{\pi^2}\right)$$
$$+2\left(\frac{\mu_Q^2}{2} T^2 + \frac{\mu_Q^4}{4\pi^2}\right)\left(1 - \frac{g^2}{2\pi^2}\right). \tag{7.3}$$

Here the first term is the blackbody contribution of the gluons, while the second and third come from the u and d quarks which are massless. Because of the charge symmetry of the nuclear matter, the quark-gluon plasma is symmetric in the (u $\leftrightarrow$ d) exchange, thus

$$\mu_u = \mu_d = \mu_Q = \tfrac{1}{3}\mu, \qquad p = p(T, \mu_Q). \tag{7.4}$$

The third step is to determine g^2, which is not a constant but also a function of μ_Q and T. At $T = 0$ Chapline and Nauenberg suggest [16] that

$$g^2 = \frac{24\pi^2}{33 - 2N_f} [\ln (k_F/\Lambda_F)]^{-1}, \tag{7.5}$$

where k_F is the quark Fermi momentum and Λ_F is a constant. In the lowest order a similar result can be obtained for the "screened charge", with μ_Q instead of k_F [13]. Nevertheless, if the terms above g^2 are neglected in p, then it is sufficient to calculate g^2 itself until g^0 terms, and then μ_Q is proportional with k_F.

It is a quite different problem, what is the proper formula for g^2 if $T > 0$. Nevertheless, as again the final results will show, for the investigated temperature range μ_Q is essentially greater at the transition point than T, thus

$$g^2 = \tfrac{24}{29}\pi^2[\ln (\mu_Q/\Lambda)]^{-1} \tag{7.6}$$

seems to be a fair approximation. The scale parameter Λ should be determined from deep-inelastic lepton-nucleon scattering measurements, but recently its value is very poorly known, it is somewhere between 100 and 500 MeV.

The fourth step is to correct p by some terms taking nonperturbative effects into account. There is at least one such term which is necessary even in the perturbative region, namely it is the energy gap between the "physical" vacuum and the perturbative Fock vacuum in the QCD.

Sometimes it is believed that such a term could be ruled out by means of the gravitational theory. Thus first we show that this energy gap is permitted even in the general relativity. Consider the Einstein equation. Its most general form is

$$R_{ik}-\tfrac{1}{2}\, g_{ik}R = \lambda g_{ik}+\kappa T_{ik}, \tag{7.7}$$

where R_{ik} is the Ricci tensor containing g_{ik} and its first and second derivatives, while λ and κ are constants. In order to get the Newtonian gravity as a limit, $\kappa = -8\pi\, \mathrm{G}/\mathrm{c}^4$. The cosmological constant λ is to be determined from large-scale observations. Since in the usual language $T_{ik} = 0$ in vacuo, λ is a characteristic date of the vacuum, if it is not 0, even the vacuum can make the spacetime curved.

Now consider a (perfect) fluid. Its energy-momentum tensor is of the form

$$T_{ik} = \frac{1}{c^2}(\varepsilon+p)u_i u_k+p g_{ik}. \tag{7.8}$$

Thus one can remove λ from the Einstein equation by correcting ε and p as

$$\varepsilon \to \varepsilon+\lambda/\kappa; \quad p \to p-\lambda/\kappa. \tag{7.9}$$

By other words, a combined shift of the energy scale

$$\varepsilon \to \varepsilon+C; \quad p \to p-C; \quad \lambda \to \lambda-\kappa C \tag{7.10}$$

does not alter even the gravitational effects.

If we choose $\lambda = 0$ by definition, then ε and p can be measured for a physical system via its gravitational effects. For the "usual" vacuum of the intergalactic space cosmological observations give the limit [17]

$$|p(0)| \simeq 10^{-9}\ \mathrm{erg/cm^3} \simeq 10^{-36}\ \mathrm{eV/fm^3}. \tag{7.11}$$

It would be very difficult, however, to obtain an analogous observational limit for the perturbative Fock vacuum of the QCD, because there are probably no gravitational sources in the perturbative region recently, except for, maybe, the cores of the neutron stars. Thus there is neither a priori reason, nor direct evidence against an additive constant term in p:

$$p \to p-B = p-(\hbar c)^{-3}\Lambda_B^4. \tag{7.12}$$

There are even some suggestions that, because of nonperturbative instanton effects B would be to be replaced by a function of μ_Q and T [12, 16, 18], but these corrections are very sensitive to the choice of the maximal instanton size, thus here we do not incorporate them into the pressure.

8. *The value of Λ_B*

It is easy to see either from Eqs. (3.2–4) or from Eq. (7.10) that $B = (\hbar c)^{-3}\Lambda_B^4$ is just the phenomenological bag parameter of the bag model. Since the bag parameter is closely related to the potential between nonrelativistic quarks, it can be determined from heavy particle spectroscopy. First it was calculated by an MIT group from hadron spectroscopy including p, Δ and ω, and the result was $\Lambda_B = 145$ MeV [19, 20]. Later Chin recalculated this parameter by using p, π and the slope of the Regge trajectory, and obtained $\Lambda_B = 190$ MeV [21]. On the other hand, Hasenfratz, Horgan, Kuti and Richard investigated ψ and γ excitations with the result $\Lambda_B = 235$ MeV [22]. Since of all the systems $C\bar{C}$ and $B\bar{B}$ are the simplest (e.g. they are practically nonrelativistic), we believe that the latest result is the most probable. Anyway, some calculations for the smaller values can be found in the literature, thus we can concentrate our attention on the case $\Lambda_B = 235$ MeV.

Nevertheless, since there exist different candidates for Λ_B, it is useful to discuss how the value of the bag constant influences the data of the transition. Of course the correct answer would need the performation of the correct calculations for different values of Λ_B, thus here we choose a simplified model system. Namely, consider the case $T = 0$. Here we know the $n \to \infty$ behaviours of the equations of state (6.1–4) and (7.3), and thence we choose

$$p_{nu} = \frac{m_n^2 c}{2c_v^2 \hbar^3}(\mu-\mu_0)^2 + (\mu-\mu_0)n_0, \qquad \mu_0 = m_n c^2 - E_b, \tag{8.1}$$

where n_0 is the nuclear density and E_b is the binding energy and

$$p_{qu} = (\hbar c)^{-3}\left[\frac{1}{162\pi^2}\mu^4 - \Lambda_B^4\right]. \tag{8.2}$$

Although these pressure formulae are oversimplified, their trends are correct: p_{nu} yields $p = 0$ and $\mu = \mu_0$ at $n = n_0$, and it coincides with Walecka's result when $n \to \infty$, while p_{qu} is the $g^2 = 0$ limit of Chapline and Nauenberg's approximative formula [16, 23]. The system of equations (8.1–2) is sufficiently simple for investigations, and, in spite of the neglections the equations of state are quite legal from thermodynamical viewpoints.

Requiring condition (3.5) we get that for too small Λ_B's the crossing point is below $\mu = \mu_0$. However, in this region $p < 0$, thus here the matter is hydrodynamically (not thermodynamically) unstable [5, 24, 25]. Thus in this case one could not expect a phase transition through equilibrium states. It is interesting that the critical value is very near to $\Lambda_B = 145$ MeV. With increasing Λ_B both transition densities grow, together with the density jump. Thus the quark side of the transition moves deeper and deeper into the perturbative region, and the transition is more and more in the asymptotic region of both systems, i.e. the results become more and more independent of the uncertainties and neglections. While near to $\Lambda_B = 145$ MeV the two curves have almost common tangents, the crossing point is not very well determined, and strongly changes between different calculations because of e.g. the different nuclear models, at $\Lambda_B = 235$ MeV the tangents (i.e. the densities) are very different at the transition (Eqs. (8.1–2) yield $n_{nu} = 3.8\, n_0$, $n_{qu} = 10.2\, n_0$), and the intersection is well marked.

Although these conclusions have been obtained from a special and simplified model, it is well known that different calculations give very different transition densities. For example, Baym and Chin listed some results for different neutron matter models, and the transition density on the neutron side varies between 1.2/fm^3 and 3.7/fm^3 [1]. Similarly, Chapline and Nauenberg used three different models for neutron matter, and they got the following three transition mass densities on the neutron side: $2.7 \cdot 10^{15}$ g/cm^3; $6.5 \cdot 10^{15}$ g/cm^3; and $13 \cdot 10^{15}$ g/cm^3, respectively. These great variations seem to verify the previous conclusion.

Of course, the true value of Λ_B is independent of our hopes. Nevertheless, the higher values would be, as we have seen, more profitable for the calculation, and, although the evidences are poor, there are good arguments for a high bag constant. In this situation it would be some kind of masochism not to try to use this high value. On the other hand, naturally, the high bag constant makes the task of the experimental physicists more difficult.

9. *The results*

Here we enumerate the results of some calculations and at least try to compare them. Although our main goal was to investigate the (*nucleon matter*) ↔ (*quark-gluon plasma*) transition in heavy ion collisions, first we survey the results of the calculations for (*neutron*) ↔ (*quark-gluon plasma*) in neutron stars, which is obviously a closely related problem, and which is more investigated. In Table II we compare 4 calculations which cover 16 different choices of the parameters. The first column identifies the calculation, the second one shows Λ_B (the letter "*e*" means that Λ_B is not an original parameter of the calculation), the third column gives the coupling constant (or, if the calculation uses running g^2, Λ, and then g^2 is determined by Eq. (7.5)). It the next two columns the transition densities can be seen, or, if it has not been given in the original paper, the mass densities, while the last column identifies the neutron matter model. Question marks denote that the data cannot be found in the paper.

Freedman and McLerran's data seem to suggest second order phase transition, but this is an artifact of an incorrect condition for the phase transition:

$$\left(\frac{\varepsilon}{n}\right)_1 = \left(\frac{\varepsilon}{n}\right)_2 \tag{9.1}$$

instead of the Gibbs conditions (cf. Eq. (3.1)), thus their results yield only estimations.

TABLE II

Transition densities in different calculations for the neutron ↔ quark process at $T = 0$. Further explanations can be found in the text

Calculation	Λ_B	$g^2/4\pi$ (Λ, MeV)	n_1/n_0 (ϱ_1/ϱ_0)	n_2/n_0 (ϱ_2/ϱ_0)	Neutron matter model
Chapline-Nauenberg, Ref. [23]	145	2.2	(10.8)	?	Pandharipande-Smith (3)
Same	125	3.0	(16.0)	?	Same
Same	145	2.2	(26.0)	?	Bethe-Johnson (I)
Same	125	3.0	(36.0)	?	Same
Same	145	2.2	(52.0)	?	Bethe-Johnson (VH)
Same	125	3.0	(72.0)	?	Same
Baym-Chin, Ref. [1]	145	2.2	7.1	11.8	Mean field
Same	145	2.2	10.0	15.3	Pandharipande-Smith solid
Same	145	2.2	19.4	34.7	Bethe-Johnson (VN)
Same	145	2.2	21.8	41.8	Reid
Chapline-Nauenberg, Ref. [16]	212e	(300)	12.6	21.2	Bethe-Johnson (VH)
Same	283e	(400)	15.2	29.8	Same
Same	212e	(300)	4.1	7.6	Pandharipande-Smith solid
Same	283e	(400)	6.5	17.6	Same
Freedman-McLerran, Ref. [13]	145	(100)	2.0	2.0	Bethe-Johnson (VH)
Same	0	(275)	1.8	1.8	Same

TABLE III

Transition densities for the *(nuclear matter)* ↔ *(quark-gluon plasma)* process at $T = 0$. The notation is the same as for the previous table

Calculation	Λ_B	$g^2/4\pi$ (Λ, MeV)	n_1/n_0 (ϱ_1/ϱ_0)	n_2/n_0 (ϱ_2/ϱ_0)	Nuclear matter model
Chin, Ref. [21]	145	2.2	5.0	?	Mean field
Same	190	0.68	7.7	?	Same
Kuti, Lukács, Polónyi, Szlachányi, Ref. [12]	235	(100)	5.7	14.8	Same
Same	235	(200)	5.9	17.2	Same
Same	235	(300)	6.7	18.3	Same

Neglecting these results the minimal transition density for neutron matter is $4.1 n_0$, while the maximal one is $21.8 n_0$. It is difficult to a certain extent to draw definite conclusions from so various results.

Now, indeed, we are going to discuss some results for the *(nuclear matter)* ↔ *(quark-gluon plasma)* transition in heavy ion collisions. The first such calculation was made by Chin [21] in 1978, and we compare those results with the data of a Budapest group (J. Kuti,

B. Lukács, J. Polónyi, K. Szlachányi, 1980) [12]. As we mentioned, the Budapest calculation followed the pattern described in Sects. 6–7, and used $\Lambda_B = 235$ MeV. Chin's calculation differed at the points that

a) he used constant g^2; and

b) the value of Λ_B was smaller.

First we compare the results at $T = 0$. The notation is the same as in Table I; Chin's paper does not give the densities on the quark side. It can be concluded that the n_1 values are more or less concordant, and they are rather low compared to the corresponding densities of neutron systems.

Nevertheless, the thermal dependences of the densities are different in the different calculations. In Fig. 1 we compare n_1/n_0 for three sets of parameters, namely, for Chin's

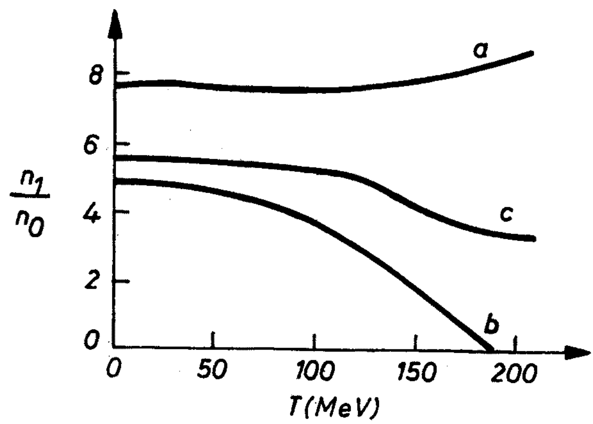

Fig. 1. The transition densities for the nuclear phase versus temperature. The curves a and b have been calculated by Chin (Ref. [21]), for $a - \Lambda_B = 145$ MeV, $g^2/4\pi = 2.2$, while for $b - \Lambda_B = 190$ MeV, and $g^2/4\pi = 0.68$. The curve c is a result of a Budapest group (Kuti, Lukács, Polónyi, Szlachányi, Ref. [12]), $\Lambda_B = 235$ MeV, it used running g^2, in which $\Lambda = 100$ MeV

two calculations and for one Budapest calculation (with $\Lambda = 100$ MeV). The three curves clearly differ, although there are some similarities between the Budapest results and Chin's curve for $\Lambda_B = 190$ MeV below $T = 120$ MeV. In the Budapest calculations (100 MeV $< \Lambda <$ 500 MeV) $n_1(T)$ never reaches 0 (the calculations were extended until $T = 290$ MeV, and there $n_1(T)$ was always increasing), which result seems to be physical. There is no such a "critical" temperature, where the phase transition automatically takes place, although there is a temperature near to 200 MeV at which n_1 is minimal. Of course, it is well known that some calculations predict a "melting point" $T_c \simeq m_\pi$ [26], but for $q\bar{q}$ systems, in which $\mu_Q = 0$ ("zero component fluids"), i.e. in which the density is not an independent quantity.

In Sect. 6 we promised to show that two approximations were compatible with a self-consistent description, namely, there we neglected the temperature dependence of the running g^2, and similarly the terms containing g^4. In the first point our argument was that for moderate temperatures μ_Q is essentially greater than T. In order to verify this, here we give some data of the transition points for $\Lambda = 100$ MeV:

T, Mev	50	150	200
$\mu_Q/m_n c^2$	0.73	0.57	0.37.
$p/m_n c^2$, fm^{-3}	0.76	0.50	0.28

It can be seen that, indeed, T is small compared to μ_Q even at the transition point for moderate temperatures (although the ratio increases with T).

Our second guess was that the contribution of the g^4 terms was negligible. Of course, one cannot compare the g^4 and g^2 terms, if the former has not been calculated. Nevertheless, such a comparison is immediately possible for the terms surviving at $T = 0$. These terms are listed in Ref. [13], and the proper combination in them is $\alpha_c/\pi = g^2/16\pi^2$, whose coefficients are in the order of 1. Thus, at least for low temperatures, the calculation is self-consistent if $g^2/16\pi^2$ is small in the transition point. According to the calculations, for $\Lambda = 100$ MeV, $g^2{}_{tr}/16\pi^2$ starts from 0.02 when $T = 0$, it remains almost constant until $T = 120$ MeV, then the growth accelerates, and the last calculated value at $T = 290$ MeV is 0.06. This result seems to be reassuring.

Having more or less convinced ourselves of the reliability of the calculation, at the end of this Section we are going to look at the phase diagram (on the p, n plane). First, consider the isotherms. They are quite regular for both phases below $T = 290$ MeV, but at this temperature some strange kind of instability appears in the quark phase. Namely, there is a turning point on the isotherm where n is minimal, and for smaller pressures $dp/dn < 0$, moreover, the transition pressure is lower than the pressure belonging to the turning point, thus the compressed system meets this unstable region just after the phase

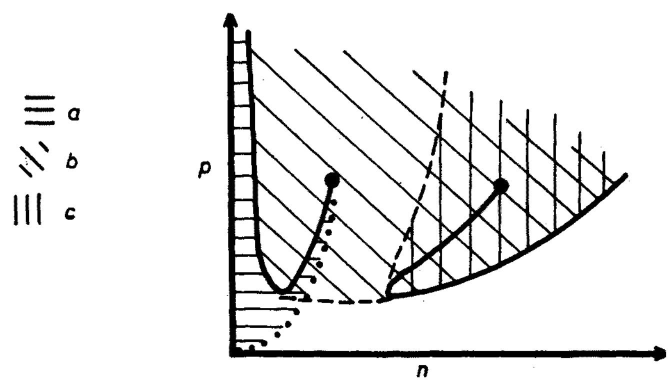

Fig. 2. The structure of the phase diagram of the nucleon-quark system according to the calculation described in Ref. [12]. It can be seen that the type of the transition is definitely different from the usual one obtained for van der Waals systems. Pure nuclear phase can exist in the region a, pure quark-gluon phase can be found in c, while b denotes the region where the two phases coexist. Continuous lines denote the states on the phase transition curve, while the dotted line is the $p(n)$ function for nuclear matter at $T = 0$. The broken lines are some nontrivial consequences of the calculation. The lower part of the diagram is empty, because the system has a minimal nonvanishing pressure at $T = 0$. Some regions are doubly represented, but n and p do not necessarily determine the thermodynamical state, because $n = N/V$, and p and $(-V)$ are conjugate quantities

transition. If this result were physical, there would be "microscopic collapse" [24] at the end of the transition, but in order to accept such a behaviour more evidences would be necessary. Such an effect cannot occur if g^2 is constant, so maybe at this temperature the T-dependence of g^2 must not be neglected. Nevertheless, it is more honest to confess that presently we cannot fully interpret this instability.

Finally, Fig. 2 shows the topology of the phase diagram, which is clearly not similar to the van der Waals case. There are three main differences, namely:

a) There is a region on the (n, p) plane which is empty.

b) The left hand side of the phase equilibrium curve is steeper.

c) There are doubly-occupied regions.

Although these features might seem very peculiar, they have explanations. For (*a*), one should observe from Eqs. (6.1–4) and (7.3) that both systems have nonvanishing pressure at $n \neq 0$, $T = 0$. Point (*b*) is a consequence of the asymptotic freedom, while point (*c*) is not a paradox: for fixed particle number n and V are equivalent quantities. Since p and $(-V)$ are conjugate thermodynamical quantities, they cannot fully characterize the thermodynamical state of the matter. Using n and T, the regions of the phases would be clearly separated.

10. Possibilities for observation?

Let us assume the best, i.e. that we have been successful to convince the reader of the existence of the phase transition, and of the reliability of the obtained data. Even then we ought to mention some ways to check the result. We want to play fairly, but cannot create new possibilities for experiments. As the results have shown, for the phase transition some 5–$7n_0$ density (or $3-4n_0$ but $T \simeq 200$ MeV) is necessary. As far as we know, there are only two such situations when one may hope such circumstances (and there was one more).

a. Cosmology

It is a general opinion that in some past stages of the evolution the Universe possessed a density well above n_0. In the most obvious models as e.g. Friedmannian dust or radiation-dominated solution there exists a time value when the density is infinite (Big Bang) [2]. Of course in rigorous terms "infinite" means "above the validity of the theory" but that is quite high. Although there are models without singularity, e.g. sometimes viscosity can prevent the singularity [27–31], there are few doubts about densities higher than nuclear, and, at least, we have evidences as the He surplus of population II stars, that the Universe had a density 10^{30} times higher than the present density [32]. Thus, certainly, the Universe passed through a phase transition, however, if the confinement is complete at low densities, there cannot be recent spoors. Observing far regions of the Universe we get information from the past. Nevertheless, optical observations cannot reach this time, because it happened before the end of the plasma stage. In principle the primordial neutrino background carries some information, but today it is not a serious proposal to observe this radiation.

b. Neutron stars

There is a forlorn hope that quark matter can occur in the centre of neutron stars. Nevertheless, this hope is decaying as the years go by. In order to make the picture clearer, consider a static, spherically symmetric configuration. Then a consequence of the Einstein equation of the general relativity is the following equation of hydrostatic equilibrium [24]:

$$\frac{dp}{dr} = -\gamma \frac{\left(\varrho + \frac{p}{c^2}\right)\left(m + 4\pi r^3 \frac{p}{c^2}\right)}{r\left(r - 2\frac{\gamma}{c^2} m\right)}, \tag{10.1}$$

$$m(r) = 4\pi \int_0^r \varrho(r') r'^2 dr'.$$

If the matter is cold (which is obvious for final states of the evolutions) and consists of one component, then

$$\varrho = \varrho(n) \to p(n) = p(\varrho(n)). \tag{10.2}$$

Of course, the form of the function $p(\varrho)$ is model-dependent. Having fixed it, Eq. (10.1) can be numerically integrated for any $\varrho(0)$. However, some solutions are unstable against radial oscillations. In the classical limit $\gamma M/c^2 \ll R$ the stability condition is [33]

$$\frac{n}{p}\frac{dp}{dn} = \frac{n^2 \frac{d^2\varrho}{dn^2}}{n\frac{d\varrho}{dn} - \varrho} > \tfrac{4}{3}. \tag{10.3}$$

In the relativistic region special considerations are necessary, however for each investigated model the result has been that after the neutron peak of the $M(\varrho(0))$ curve the decreasing slope is unstable. Thus quark cores cannot exist if the central density of the configuration of the maximal mass is below the transition density ϱ_1.

In the first calculation $\varrho(0, \max)$ was $9.98.10^{15}$ g/cm^3, which might be enough [24]. Nevertheless, that calculation neglected the nuclear forces. Chin and Baym have compared ϱ_1 and $\varrho(0, \max)$. The result is that the maximal central density is too small by a factor $\geqslant 2$ [1, 33]. Of course, they used particular models for nuclear forces.

Even if $\varrho(0)$ could reach ϱ_1, this would happen near to the maximum of the $M(\varrho(0))$ curve, where some characteristic quantities of the neutron stars are almost constant. Thus, probably, the phase transition could not cause great changes in the (theoretically) observable quantities of the stars. Moreover, the quark core could not be too great, so e.g. the heat capacity could not be strongly affected.

In fact, the heat capacity seems to be a good signal, and there was an attempt in comparing the model thermal evolutions to observations, but the results were not decisive [34].

Generally the adiabatic index defined by (10.3) is lower for quark matter than for neutron matter, thus there is a tendency for instability anyway.

c. Heavy ion collisions

Heavy ion processes, at least, are true laboratory experiments, not simply observations, as the previous ones. However, in these experiments one can detect only the final fragments of the fireball. Thus a double question arises:

1) Does the density reach n_1 in the process?

2) Does any surviving signal of the quark phase exist?

We think that the answer for the first question is yes. Namely, the calculations yield the following maximal densities and temperatures for different bombarding energies:

$E_{\mathrm{bomb}}/nucl.$ (GeV)	0.1 [35]	0.5 [35]	2.1
$n_{\max}/n_0$	1.8	2.7	> 5.5 [36]
$T_{\max}$ (MeV)	15	40	$\simeq 120$ [37]

Compared these numbers with the data of Fig. 1 one can see that there are good possibilities at 2.1 GeV/nucleon bombarding energies.

If the compression has produced a mixed state above n_1, the matter becomes softer. Namely, Eqs. (4.8–11) show that at $T = 0$ the mixture follows a $p =$ const. path, if there are no irreversible processes, so the effective compressibility is inifinite. Of course, the compressibility is finite for $T \neq 0$, and then detailed numerical calculations would be necessary, nevertheless it does not seem impossible even to reach the quark phase.

The thermodynamical formulation of the phase transition given in Sects. 2 and 3 is an equilibrium formalism. Since the time scale of a heavy ion process is not very long compared to the time scale of the individual interactions, there are some doubts about the regularity of the transition. Today it would be very difficult to answer this question, however, we must emphasize that some "hysteresis" effects (as for undercooling) are quite possible, but having reached the unstable regions the matter cannot remain on the original trajectory. While in the stable region the growth of the surviving phase happens through surface processes (which can be slow if the surface is small), in the unstable region the new phase is produced in each volume element by great fluctuations. Of course, nobody would be very surprised if some moderate fluctuations occur in the experimental results.

Nevertheless, one would need also a signal of the phase transition in the final, observable stage of the heavy ion collision. Here we are going to discuss three proposed signals: incomplete shock waves, entropy excesses and "abnormal" rates in the particle production.

A Frankfurt group has investigated the propagation of shock waves in mixed systems [38]. They conclude that the phase transition is incompatible with the propagations of sharp shock waves, thus the shock fronts smear out. Unfortunately, viscosity causes a similar effect [5, 35], thus further investigation would be necessary for distinctive marks.

Siemens and Kapusta found too low deuteron to proton ratios in energetic heavy ion reactions [39]. Hence they concluded that there was an entropy excess, and proposed phase transition as one of the possible explanations. Nevertheless, it is necessary to emphasize that the entropy excess cannot be directly measured, and the finite volume of the deuteron automatically leads to a lower deuteron to proton ratio. In fact, 10 fm^3 effective volume explains the observed ratios [40]. Maybe, more compact light nuclei, as He^3, t and α would be more appropriate to determine the entropy of the fireball. On the other hand, as we saw in Sect. 4, the phase transition in itself does not cause any entropy excess.

The third possibility is to measure the strange particle production of the collision. The K and Λ production rate is definitely higher in a hot quark-gluon plasma than in the hadronic phase [41]. In a hadrochemical model calculation the ratio K^+/p turned out to be an almost linear function of the bombarding energy up to 3 GeV/nucleon [42]. If experiments showed a breaking on this curve, that might be a signal for the quark plasma. Of course, one cannot expect too great differences, because the existence of the quark phase is limited both in space and in time.

11. Conclusions

From the data of Refs. [12] and [21] one can draw the conclusion that, if the temperature is not higher than 120 MeV, the nucleon-quark phase transition starts somewhere at $n = 5\text{–}6n_0$, and the central region of the heavy ion collision just reaches this density at $E_{\rm bomb}/A \simeq 2$ GeV according to Ref. [36]. For higher energies the necessary density seems to be even lower because of the higher temperature. The density, at which the transition ends is ca. $15\, n_0$, and, although the mixed state of the matter is relatively soft, without dynamical calculations, it cannot be decided if the matter reaches pure quark phase at reasonable beam energies.

The calculations using running g^2 do not confirm Chin's result for a limiting temperature.

Unfortunately, until now no clear (or even hopeful) evidence for the formation of a quark phase has been found.

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REFERENCES

[1] G. Baym, S. A. Chin, *Phys. Lett.* **62B**, 241 (1976).
[2] H. P. Robertson, T. W. Noonan, *Relativity and Cosmology*, W. B. Saunders, Philadelphia 1969.
[3] J. Ehlers, in the volume *Relativity, Astrophysics and Cosmology*, ed. by W. Israel, D. Reidel Publ. Comp., Dordrecht–Boston, p. 1.
[4] In this Section we use such units that $c = 1$.
[5] L. P. Csernai, B. Lukács, KFKI-1979–58; or: L. P. Csernai, B. Lukács, Proc. EPS Topical Conf. on Large Amplitude Collective Motions, Keszthely 1979.
[6] B. Lukács, KFKI-1978–82.
[7] T. M. Reed, K. E. Gubbins, *Applied Statistical Mechanics, Thermodynamics and Transport Properties of Fluids*, McGraw-Hill, Inc.

[8] T. Biró, H. W. Barz, B. Lukács, J. Zimányi, KFKI–1981–90.
[9] J. D. Walecka, *Ann. Phys.* **83**, 491 (1974).
[10] J. D. Walecka, *Phys. Lett.* **59B**, 109 (1975).
[11] R. F. Sawyer, *Phys. Rev. Lett.* **29**, 382 (1972); D. J. Scalapino, *Phys. Rev. Lett.* **29**, 386 (1972); see also Ref. [21].
[12] J. Kuti, B. Lukács, J. Polónyi, K. Szlachányi, *Phys. Lett.* **95B**, 75 (1980).
[13] B. Freedman, L. McLerran, *Phys. Rev.* **17D**, 1109 (1978).
[14] See e.g. J. Polónyi's lecture on this School.
[15] O. K. Kalashnikov, V. V. Klimov, *Phys. Lett.* **88B**, 328 (1979).
[16] G. Chapline, M. Nauenberg, *Phys. Rev.* **16D**, 450 (1977).
[17] It follows from the fact that $\lambda \neq 0$ cannot be proven in the cosmology when using the trivial gauge $\varrho(0) = 0$, thus λ cannot dominate the matter in the Universe.
[18] C. G. Källman, *Phys. Lett.* **94B**, 272 (1980).
[19] T. DeGrand et al., *Phys. Rev.* **12D**, 2060 (1975).
[20] K. Johnson, *Acta Phys. Pol.* **6B**, 865 (1975).
[21] A. S. Chin, *Phys. Lett.* **78B**, 552 (1978).
[22] P. Hasenfratz, R. R. Horgan, J. Kuti, J. M. Richard, CERN TH 2837 (1980).
[23] G. Chapline, M. Nauenberg, *Nature* **264**, 235 (1976).
[24] B. K. Harrison, K. S. Thorne, M. Wakano, J. A. Wheeler, *Gravitation Theory and Gravitational Collapse*, The University of Chicago Press, Chicago–London 1965.
[25] P. Danielewicz, *Nucl. Phys.* **314A**, 465 (1979).
[26] F. Karsch, H. Satz, *Phys. Rev.* **22D**, 480 (1980).
[27] G. L. Murphy, *Phys. Rev.* **8D**, 4231 (1973).
[28] M. Heller, Z. Klimek, L. Suszycki, *Astropys. Space Sci.* **20**, 205 (1973).
[29] M. Heller, L. Suszycki, *Acta Phys. Pol.* **5B**, 345 (1974).
[30] B. Lukács, *Gen. Relativ. Gravitation* **7**, 653 (1976).
[31] B. Lukács, *Acta Phys. Hung.* **51**, 117 (1981).
[32] R. V. Wagoner, W. A. Fowler, F. Hoyle, *Ap. J.* **148**, 3 (1967).
[33] G. Baym, *Neutron Stars and the Properties of Matter at High Density*, Nordita, 1977.
[34] K. Brecher, A. Burrows, *Ap. J.* **236**, 241 (1980).
[35] L. P. Csernai, B. Lukács, J. Zimányi, *Lett. Nuovo Cimento* **27**, 111 (1980).
[36] A. A. Amsden, A. S. Goldhaber, F. H. Harlow, J. R. Nix, *Phys. Rev.* **17C**, 2080 (1978).
[37] I. Montvay, J. Zimányi, *Nucl. Phys.* **346A**, 490 (1979).
[38] J. Hofmann, B. Müller, W. Greiner, *Phys. Lett.* **82B**, 195 (1979).
[39] P. J. Siemens, J. I. Kapusta, *Phys. Rev. Lett.* **43**, 1486 (1979).
[40] T. Biró, H. W. Barz, B. Lukács, J. Zimányi, KFKI–1981–90.
[41] T. Biró, J. Zimányi, KFKI–1981–69.
[42] T. Biró, B. Lukács, J. Zimányi, H. W. Barz, KFKI–1982–02.