SOFT OR HARD WAVE FUNCTION IN THE END-POINT REGION?

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The tail of the nucleon wave function at $x \to 1$ is investigated. The dominance of this region by soft and hard effects is separately considered. Both contributions produce definite SU(6) symmetry breaking patterns which are compared with deep inelastic and large angle elastic scattering data. The analysis suggests the dominance of soft effects at present energies and momentum transfers.

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The end-point region ($x \to 1$) of the wave function [1] plays an important role in many hadronic processes. It contributes naturally to the deep inelastic scattering at Bjorken x close to 1, it seems also to dominate in exclusive processes [2] at large momentum transfers. It is therefore of great importance what is the structure of this tail of the wave function, in particular whether one is able to use perturbative methods in this region. The last question means in other words, whether the part of the wave function responsible for one of the quarks carrying almost all of the hadron momentum appears due to a hard interaction between the constituents. If this is the case one can predict the behaviour of many processes dominated by the end-point of the wave function. A standard example is the ratio of neutron to proton structure functions at $x \to 1$. If the opposite situation takes place i.e. the end-point region is produced by soft effects one is limited to phenomenological models of the wave function.

In this paper we investigate phenomenological consequences of both possibilities confronting them with observables resulting from inclusive and exclusive processes. We consider the nucleon wave function, for which many constraints can be obtained. The SU(6) proton wave function can be written in terms of the valence quarks

$$\Psi^{\mathrm{p}} = \frac{1}{\sqrt{54}}\left[\sqrt{27}\, B_0^{\mathrm{u}+} + \sqrt{12}\, B_1^{\mathrm{d}-} + \sqrt{3}\, B_1^{\mathrm{u}+} - \sqrt{6}\, B_1^{\mathrm{u}-} - \sqrt{6}\, B_1^{\mathrm{d}+}\right]. \tag{1}$$

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Each $B_I^{q\lambda}$ is separately normalized to 1. The quark q and its spin projection λ onto the proton spin are explicitly written, the lower index $I = 0$ or 1 stands for the isospin (and spin) of the noninteracting diquark. The dependence on the momentum fractions x_i and transverse momenta $k_{\perp i}$ of all constituents is implicitly assumed. All $B_I^{q\lambda}$ are the same functions of x of the struck quark.

The result of perturbative QCD is the SU(6) symmetry breaking term. It follows from the fact that the gluon exchanged between two quarks of parallel helicities is longitudinal and thus suppressed as $x \to 1$ as compared to the transverse gluon exchange, possible between two quarks with opposite helicities [3]. This leads in deep inelastic scattering to the suppression of the configuration where two quark lines in the nucleon, not attached to the photon, are parallel and consequently the spin of the struck quark is predominantly parallel to the nucleon spin as $x \to 1$. In terms of the proton structure fun-
on

$$\nu W_2^p = F_2^p = \tfrac{1}{81}\,[18A_0^{u+} + 2A_1^{d-} + 2A_1^{u+} + 4A_1^{u-} + A_1^{d+}], \tag{2}$$

with the indices defined as in Eq. (1) (x and Q^2 dependence suppressed) the perturbative result reads [4]

$$A_I^{q-}(x) = a(1-x)^2 A_I^{q+}, \tag{3}$$

where a is a free parameter. In the following we shall omit dummy indices

$$A^-(x) = a(1-x)^2 A^+(x).$$

Eq. (3) should be compared with the SU(6) symmetric relation $A^-(x) = A^+(x)$.

To translate Eq. (3) for the behaviour of the wave function we write

$$\begin{aligned} B^+(x_i, k_{\perp i}) &\equiv C^+(x_i, k_{\perp i}) + \gamma(x)\tilde{C}^+(x_i, k_{\perp i}), \\ B^-(x_i, k_{\perp i}) &\equiv C^-(x_i, k_{\perp i}), \\ C^+(x_i, k_{\perp i}) &= C^-(x_i, k_{\perp i}), \end{aligned} \tag{4}$$

where x is the momentum fraction of the struck quark. In the SU(6) symmetry limit $\gamma(x) = 0$. We assume that C^+ and $\tilde{C}^+$ do not interfere i.e.

$$\int dx_1 dx_2 dx_3 d\vec{k}_{\perp 1} d\vec{k}_{\perp 2} d\vec{k}_{\perp 3} C^+(x_i, k_{\perp i})\tilde{C}^+(x_i^!, k_{\perp i})$$
$$\delta(1-x_1-x_2-x_3)\delta(\vec{k}_{\perp 1}+\vec{k}_{\perp 2}+\vec{k}_{\perp 3}) = 0.$$

Remembering that at $x \to 1$ the valence quarks dominate the structure function and consequently

$$F_2(x) \sim \sum_k \int dx_1 dx_2 dx_3 d\vec{k}_{\perp 1} d\vec{k}_{\perp 2} d\vec{k}_{\perp 3} |B(x_i, k_{\perp i})|^2$$
$$\delta(1-x_1-x_2-x_3)\delta(\vec{k}_{\perp 1}+\vec{k}_{\perp 2}+\vec{k}_{\perp 3})\delta(x-x_k),$$

one obtains from (3)

$$\gamma(x) = \sqrt{-1 + \frac{1}{a(1-x)^2}}\,. \tag{5}$$

The resulting proton wave function reads

$$\Psi^{p} = \frac{1}{\sqrt{54}}\sqrt{\frac{3}{3+2\gamma^2}}\,[\sqrt{27}\,C_0^{u+}+\sqrt{12}\,C_1^{d-}+\sqrt{3}\,C_1^{u+}-\sqrt{6}\,C_1^{u-}-\sqrt{6}\,C_1^{d+}]$$

$$+\frac{1}{\sqrt{36}}\sqrt{\frac{2\gamma^2}{3+2\gamma^2}}\,[\sqrt{27}\,\tilde{C}_0^{u+}+\sqrt{3}\,\tilde{C}_1^{u+}-\sqrt{6}\,\tilde{C}_1^{d+}]. \tag{6}$$

Its structure can be symbolically written

$$\Psi = \sqrt{1-\beta^2}\,\Psi^{SU(6)}+\beta\Psi^{\overline{SU(6)}},$$

where

$$\beta(x) = \sqrt{\frac{2\gamma^2(x)}{3+2\gamma^2(x)}}.$$

The first term is SU(6) symmetric and differs from Eq. (1) by the normalization only, the second one represents the symmetry breaking.

Phenomenological consequences following from Eqs (3) and (4) can be compared with the deep inelastic data on neutron to proton structure functions [5], the ratio of down to up quarks in the proton [6] and the asymmetry [7]

$$A^{\gamma p} \equiv \frac{\sigma^{1/2}-\sigma^{3/2}}{\sigma^{1/2}+\sigma^{3/2}},$$

in which σ^{λ} is the γp cross section with total helicity λ. The perturbative predictions following from Eqs (2) and (3) read

$$\frac{F_2^n(x)}{F_2^p(x)} = \frac{3\gamma^2(x)+6}{7\gamma^2(x)+9},$$

$$\frac{d(x)}{u(x)} = \frac{\gamma^2(x)+3}{5\gamma^2(x)+6},$$

$$A^{\gamma p}(x) = \frac{7\gamma^2(x)+5}{7\gamma^2(x)+9},$$

and are shown in Fig. 1. The parameter a is chosen between 1 and 3/2 as suggested by the difference $F_2^p-F_2^n \approx 1/3$. One sees that the curves are compatible with the data except the ratio F_2^n/F_2^p which is predicted to high at $x \to 1$. Because we do not include the sea quark contribution the results are valid only for large x ($x > 0.5$).

Assuming soft effects in the end-point region one faces a technically harder problem than the above consideration. In fact no clear methods are known in dealing with low momentum transfer region and consequently one is using some phenomenologically motivated assumptions. In case their predictions work better than the ones obtained above with perturbative methods, we believe that the end-point region is dominated by soft effects.

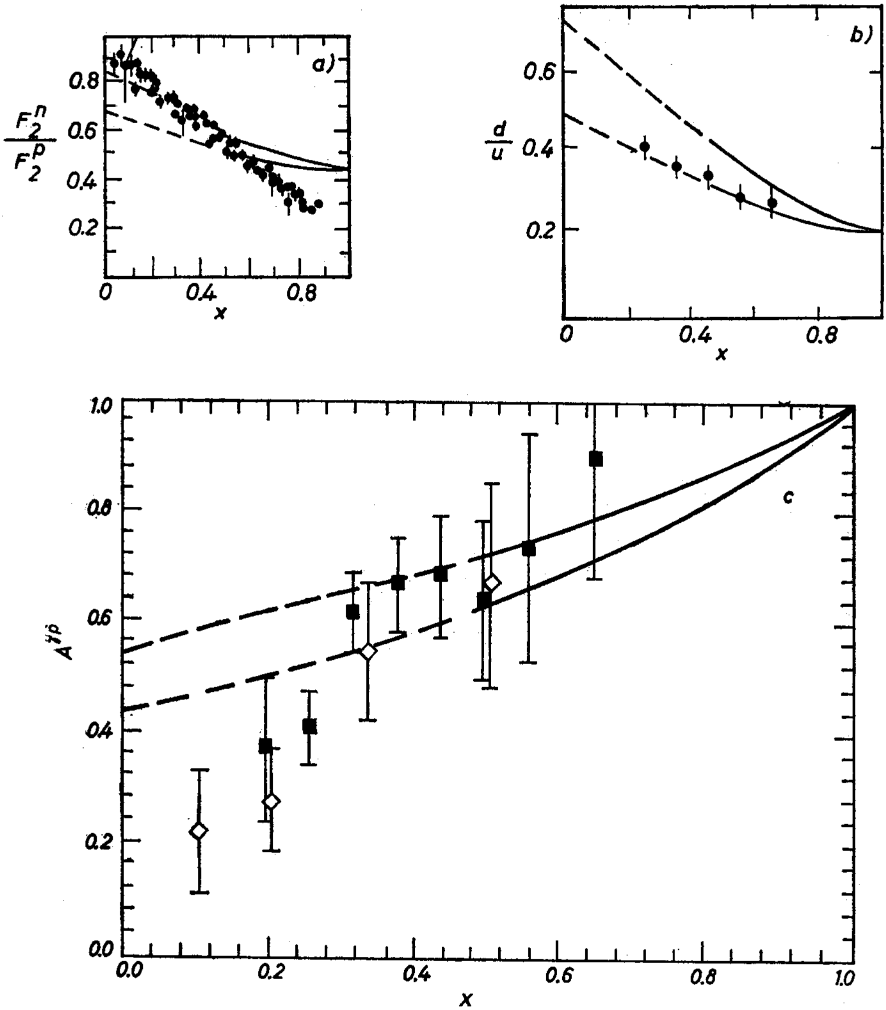

Fig. 1. The comparison of the data on a) the ratio of the neutron to proton structure functions [5], b) the ratio of the down to up quarks in the proton [6], c) the asymmetry $A^{\gamma p}$ [7] with the perturbative predictions (Eq. (6)). The parameter a changes between 1 and 3/2

The assumption we use is taken from the Carlitz and Kaur model [8] of deep inelastic scattering which breaks SU(6) symmetry by the relation

$$A_1(x) = a(1-x)A_0(x). \tag{7}$$

This formula is motivated by the behaviour of N and Δ Regge trajectories [8] which also display SU(6) symmetry breaking.

Translating Eq. (7) for the nucleon wave function means

$$\begin{aligned} B_0(x_i, k_{\perp i}) &\equiv C_0(x_i, k_{\perp i}) + \gamma(x)\tilde{C}_0(x_i, k_{\perp i}), \\ B_1(x_i, k_{\perp i}) &\equiv C_1(x_i, k_{\perp i}), \\ C_0(x_i, k_{\perp i}) &= C_1(x_i, k_{\perp i}). \end{aligned} \tag{8}$$

It is again assumed that C_0 and $\tilde{C}_0$ do not interfere. The resulting wave function constructed in analogy to Eq. (5) reads

$$
\begin{aligned}
\Psi^{\mathrm{p}} &= \sqrt{1-\beta^2}\,\Psi^{\mathrm{SU(6)}} + \beta\Psi^{\overline{\mathrm{SU(6)}}} \\
&= \frac{1}{\sqrt{54}}\sqrt{\frac{2}{2+\gamma^2}}\,[\sqrt{27}\,C_0^{\mathrm{u}+} + \sqrt{12}\,C_1^{d-} + \sqrt{3}\,C_1^{\mathrm{u}+} - \sqrt{6}\,C_1^{\mathrm{u}-} - \sqrt{6}\,C_1^{\mathrm{d}+}] \\
&\quad + \frac{1}{\sqrt{27}}\frac{\gamma}{\sqrt{2+\gamma^2}}\,[\sqrt{27}\,C_1^{\mathrm{u}+}], \qquad (9)
\end{aligned}
$$

where

$$
\beta(x) = \frac{\gamma(x)}{\sqrt{2+\gamma^2(x)}}; \qquad \gamma(x) = \sqrt{-1+\frac{1}{a(1-x)}}.
$$

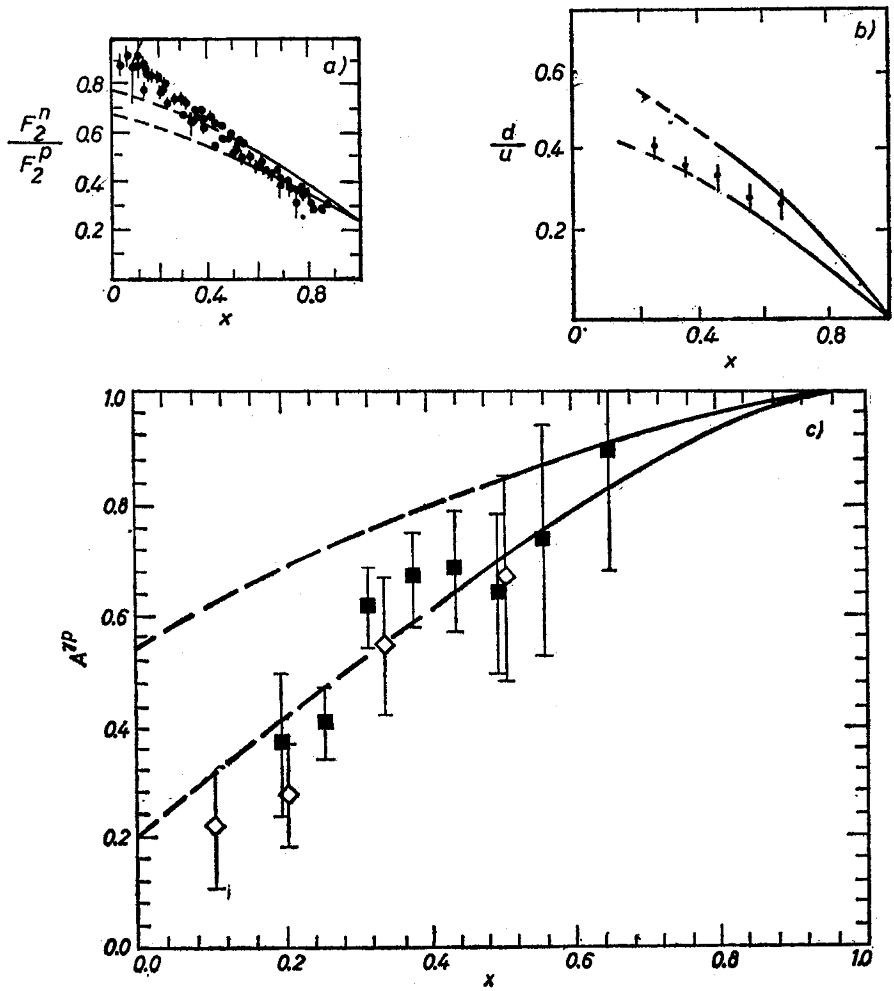

Fig. 2. The same data as in Fig. 1 compared with the nonperturbative model (Eq. (10))

The deep inelastic observables

$$\frac{F_2^n(x)}{F_2^p(x)}=\frac{\gamma^2(x)+4}{4\gamma^2(x)+6},$$

$$\frac{d(x)}{u(x)}=\frac{2}{3\gamma^2(x)+4},$$

$$A^{\gamma p}(x)=\frac{6\gamma^2(x)+5}{6\gamma^2(x)+9}\;. \tag{10}$$

are compared with the data in Fig. 2 showing good agreement. Another place sensitive to the behaviour of the nucleon wave function is exclusive scattering. Its spin and flavour part is best tested in spin observables such as spin-spin asymmetris in elastic nucleon--nucleon scattering. In fact the explanation [9] of the behaviour of A_{NN} at large t in 12 GeV/c pp elastic scattering [10] suggests the dominance of the end-point contribution at these energy and momentum transfers. This result which was obtained with the SU(6) symmetric wave function and assumed soft interaction in part of the elastic scattering diagram gets modified if one includes the wave function of Eq. (9). Assuming that the average momentum fraction $\langle x\rangle$ of the interacting quark is somewhere in the allowed

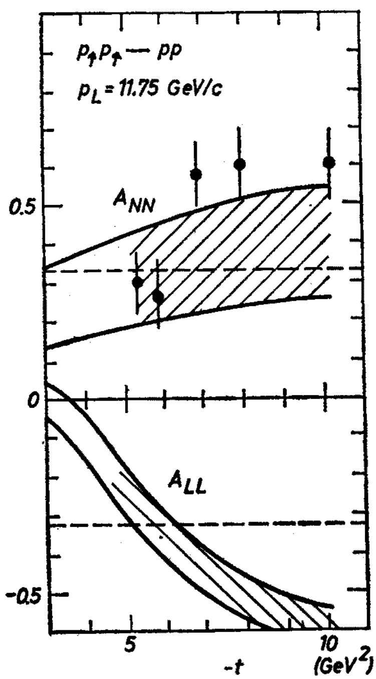

Fig. 3. The data on the elastic nucleon-nucleon asymmetry A_{NN} at large momentum transfers. The shaded area is allowed by the nonperturbative approach [9] with the wave function of Eq. (9). The broken line is the perturbative prediction of Ref. [11]

region [2]

$$x \gtrsim 1 - \frac{\lambda}{\sqrt{-t}} \approx 1 - \frac{1}{\sqrt{10}} \approx 0.65,$$

where λ is the scale characterizing soft momentum transfer ($\langle x \rangle \in (0.7, 0.9)$ is taken) we give in Fig. 3 the predictions for the range in which A_{NN} and A_{LL} change. It is seen that the asymmetry A_{NN} becomes smaller although it is still compatible with the data at 12 GeV/c. It is also clear that with increasing momentum transfer, and consequently increasing $\langle x \rangle$ the asymmetry A_{NN} decreases to the limiting value $A_{NN}(-t \to \infty) = 1/9$. We stress that the last prediction holds in case when the wave function departs from SU(6) symmetry as given by Eq. (9).

The spin-spin asymmetries have been also calculated [11] assuming the elastic scattering to be dominated by hard effects. Although this calculation is not closely related to the end-point region (it rather assumes the dominance of the "end point" in transverse momentum) we quote in Fig. 3 the result $A_{NN} = 1/3$ which is twice too small at $|t| = 10$ GeV^2. It is worth noting that SU(6) symmetry breaking does not influence this result, which follows essentially from the helicity conservation in each quark-gluon vertex.

To summarize the above remarks, we attempted to collect phenomenological arguments to distinguish whether the $x \to 1$ behaviour of the wave function is dominated at present energies by soft or hard effects. It turned out that one can easily translate all considered inclusive and exclusive results into the behaviour of the wave function for which a definite SU(6) symmetry breaking pattern emerged. Present experimental results in deep inelastic and large angle elastic scattering suggest the dominance of soft interaction in the end-point region.

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