THE ACTION PRINCIPLE FOR THE LONGITUDINAL ELECTROMAGNETIC FIELD

By A. Staruszkiewicz

Institute of Physics, Jagellonian University, Cracow*

(*Received August 17, 1982*)

An argument is presented which allows to determine the numerical coefficient with which the longitudinal part of the electromagnetic field should enter into the total action.

PACS numbers: 12.20.-m

1. *Introduction*

The usually accepted action of the electromagnetic field,

$$-\frac{1}{16\pi}\int d^4xF^{\mu\nu}F_{\mu\nu},$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ contains only the so called transverse field. Several authors [1–3] found it desirable to add the longitudinal field as well. The action containing the longitudinal field has the form

$$-\frac{1}{16\pi}\int d^4x\{F^{\mu\nu}F_{\mu\nu}+2\gamma(\partial^\mu A_\mu)^2\},$$

where γ is a dimensionless constant.

For different values of γ one obtains different theories. For example, from the equations of motion,

$$\Box A_\mu+(\gamma-1)\partial_\mu(\partial^\lambda A_\lambda) = 0$$

one sees that the potential A_μ will fulfill the wave equation only for $\gamma = 1$. For this reason, I suppose, in most textbooks one assumes without comment $\gamma = 1$ [4].

The aim of this paper is to present an argument, aesthetic rather than physical, that

$$\gamma = \frac{e^2}{4\pi},$$

where e is the elementary charge.

---

* Address: Instytut Fizyki UJ, Reymonta 4, 30-059 Kraków, Poland.

2. *The argument*

Our argument is an attempt to reconcile the results of Ref. [5] with those of Ref. [6]. In [5] we obtained, *assuming* $\gamma = 1$, that apart from the elementary charge e there is another distinguished charge $e' = 4\pi/e$. In [6] we argued that e' must be commensurate with e. The simplest way to reconcile both results is to put $e' = e$, on grounds that all charges are alike. One has then $e = \sqrt{4\pi}$, a nice prediction but not supported by the experimental evidence which shows that e is much smaller than that. Another way is to put $e' = ne = 4\pi/e$, n being a natural number. The experimental value of e is best recovered for $n = 1722$. The formula $e^2 = 4\pi/1722$ was actually suggested by Rosen [7] who gave also a certain explanation of the number 1722. Although this is an attractive possibility, one feels that commensurability of two charges with the very large factor 1722 is not aesthetically appealing.

For this reason we propose to solve the problem as follows. Instead of assuming a priori $\gamma = 1$ we take an arbitrary γ, repeat the argument of Ref. [5] and impose the condition $e' = e$, on grounds that all charges are alike. In this way we obtain

$$\gamma = \frac{e^2}{4\pi}.$$

3. *The gauge invariant form of the total action*

The principle of gauge invariance says that the potential A_μ should enter into equations of mathematical physics only in the gauge invariant combination $A_\mu + \partial_\mu(S/e)$, where S is a scalar field called phase. Introducing the phase and putting $\gamma = e^2/4\pi$ we obtain for the total action the expression

$$-\frac{1}{16\pi}\int d^4x\left\{F^{\mu\nu}F_{\mu\nu} + \frac{1}{2\pi}(e\partial^\mu A_\mu + \Box S)^2\right\}.$$

The electric current is easily found to be

$$j_\mu = -e\frac{1}{16\pi^2}\,\partial_\mu(\Box S + e\partial^\nu A_\nu).$$

For $A_\mu = 0$ one has

$$j_\mu = -e\frac{1}{16\pi^2}\,\partial_\mu \Box S,$$

where

$$\Box\Box S = 0$$

because of the charge conservation law.

Let us calculate the total charge

$$Q = \int d^3x j_0.$$

It is sufficient to calculate it for spherically symmetric phase. The most general spherically symmetric solution of the equation $\Box\Box S = 0$ has the form

$$S = f(t+r)+f(t-r)+\frac{1}{r}[g(t+r)-g(t-r)],$$

where $t = x^0$, $r = \sqrt{(x^1)^2+(x^2)^2+(x^3)^2}$ and f and g are arbitrary functions. Hence

$$\partial_0\Box S = -\frac{2}{r}[f''(t+r)-f''(t-r)]$$

and

$$Q = \frac{e}{2\pi}\int_0^\infty drr\frac{d}{dr}[f'(t+r)+f'(t-r)].$$

Integrating by parts and assuming that f' vanishes at + and − infinity more rapidly than its argument, we have

$$Q = -\frac{e}{2\pi}\int_0^\infty dr\frac{d}{dr}[f(t+r)-f(t-r)]$$

$$= -\frac{e}{2\pi}[f(+\infty)-f(-\infty)]$$

$$= -\frac{e}{4\pi}[S(+\infty)-S(-\infty)],$$

where $\pm\infty$ in the last line represents respectively positive and negative time-like infinity.

The last equality says that the total charge Q will be a multiple of the elementary charge e if the total change of phase between positive and negative time-like infinity is a multiple of 4π.

REFERENCES

[1] W. Heisenberg, W. Pauli, *Z. Phys.* **56**, 1 (1929).
[2] P. A. M. Dirac, V. Fock, B. Podolsky, *Physikalische Zeitschrift der Sowjetunion* **2** (1932). Reprinted in *Selected Papers on Quantum Electrodynamics*, ed. by Julian Schwinger, Dover. Publ., New York.
[3] G. Rideau, *J. Math. Phys.* **19**, 1627 (1978).
[4] G. Källén, *Quantum Electrodynamics*, Springer Verlag 1972, page 17.
[5] A. Staruszkiewicz, *Acta Phys. Pol.* **B10**, 601 (1979).
[6] A. Staruszkiewicz, *Acta Phys. Pol.* **B13**, 617 (1982).
[7] G. Rosen, *Int. J. Theor. Phys.* **18**, 305 (1979).