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ON THE GRAVITATIONAL THEORY WITH QUADRATIC LAGRANGIAN $L_g = \varrho\Omega^{i\cdot}_{\cdot j} \wedge \eta_{i\cdot}^{\cdot j} + \bar{a}\Omega^{i\cdot}_{\cdot j} \wedge * \Omega^{j\cdot}_{\cdot i} + \alpha\Theta^i \wedge * \Theta_i$ †

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In the paper we consider the most simple spherically symmetric and cosmological solutions to the equations of the gravitational theory with the quadratic Lagrangian $L_g = \varrho\Omega^{i\cdot}_{\cdot j} \wedge \eta_{i\cdot}^{\cdot j} + \bar{a}\Omega^{i\cdot}_{\cdot j} \wedge * \Omega^{j\cdot}_{\cdot i} + \alpha\Theta^i \wedge * \Theta_i$.

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1. Introduction

The theory of gravitation with the quadratic Lagrangian $L_g = \varrho\Omega^{i\cdot}_{\cdot j} \wedge \eta^{i\cdot}_{\cdot j} + \bar{a}\Omega^{i\cdot}_{\cdot j} \wedge * \Omega^{j\cdot}_{\cdot i} + \alpha\Theta^i \wedge * \Theta_i$ was considered initially in the paper [1] as the most general model of the gravitational theory which closely relates the ideas of the General Relativity Theory (GRT) and the ideas of the gauge field theory.

If we apply the standard Palatini variational procedure to the full action, gravitation and matter, then we get the following fundamental equations of the theory[1]:

$$\bar{a}D * \Omega^{j\cdot}_{\cdot i} = (-)\frac{\alpha}{2}(\vartheta^j \wedge * \Theta_i - \vartheta_i \wedge * \Theta^j) - \frac{\varrho}{2}\Theta_k \wedge \eta_{i\cdot\cdot}^{\cdot jk} - \frac{S_{i\cdot}^{\cdot j}}{4},$$

$$\alpha D * \Theta_l = (-)\frac{t_l}{2} - \frac{\varrho}{2}\Omega^{jk}_{\cdot\cdot} \wedge \eta_{ljk} + \bar{a}\left(\frac{\delta^p_l}{4} R^{ijrm}_{\cdot\cdot\cdot\cdot} R^{\cdot\cdot\cdot\cdot}_{ijrm}\right.$$

$$\left. - R^{ij\cdot\cdot}_{\cdot\cdot lt} R^{\cdot\cdot pt}_{ij\cdot\cdot}\right)\eta_p + \alpha\left(Q^{a\cdot\cdot}_{\cdot lr} Q^{\cdot pr}_{a\cdot\cdot} - \frac{\delta^p_l}{4} Q^{atr}_{\cdot\cdot\cdot} Q^{\cdot\cdot\cdot}_{atr}\right)\eta_p. \tag{1}$$

From (1) there follow the tensorial equations

$$\bar{a}(\nabla_m R_{li\cdot\cdot}^{\cdot\cdot pm} + R_{li\cdot\cdot}^{\cdot\cdot pt} Q^{k\cdot\cdot}_{\cdot tk} + \tfrac{1}{2} R_{li\cdot\cdot}^{\cdot\cdot tn} Q^{p\cdot\cdot}_{\cdot tn})$$

$$= \frac{\alpha}{2}(Q_{i\cdot l}^{\cdot p\cdot} - Q_{l\cdot i}^{\cdot p\cdot}) - \frac{\varrho}{2}(Q^{k\cdot\cdot}_{\cdot lk}\delta^p_i + Q^{k\cdot\cdot}_{\cdot ki}\delta^p_l + Q^{p\cdot\cdot}_{\cdot il}) - \tfrac{1}{4} S^{p\cdot\cdot}_{\cdot il},$$

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[1] The notation here is the same as in [1].

$$\alpha(\nabla_k Q_{b\cdot\cdot}^{\cdot pk} + Q_{b\cdot\cdot}^{\cdot pk} Q^{l\cdot\cdot}_{\cdot kl} + \tfrac{1}{2} Q_{b\cdot\cdot}^{\cdot lk} Q^{p\cdot\cdot}_{\cdot lk})$$

$$= (-)\tfrac{1}{2} t^{p\cdot}_{\cdot b} + \varrho\left(R^{p\cdot}_{\cdot b} - \frac{\delta^p_b}{2} R\right) - \alpha\left(\frac{\delta^p_b}{4} Q^{itr}_{\cdot\cdot\cdot} Q^{\cdot\cdot\cdot}_{itr} - Q^{i\cdot\cdot}_{\cdot br} Q_{i\cdot\cdot}^{\cdot pr}\right)$$

$$-\bar{a}\left(R^{ij\cdot\cdot}_{\cdot\cdot bt} R_{ij\cdot\cdot}^{\cdot\cdot pt} - \frac{\delta^p_b}{4} R^{ijrt}_{\cdot\cdot\cdot\cdot} R^{\cdot\cdot\cdot\cdot}_{ijrt}\right). \tag{2}$$

If

$$t_l = S_{i\cdot}^{\cdot j} = \Theta_l = 0, \tag{3}$$

then the equations of the theory reduce to the vacuum Einstein equations

$$R_{ik} = 0. \tag{4}$$

Inside cf matter, even if we put $\Theta^i = S_{i\cdot}^{\cdot j} = 0$, the full system of the equations of the theory does not correspond with the Newtonian gravitational theory (NGT): the equations with $D * \Omega$ which are of the third order destroy this correspondence. In the consequence of these equations, with $\Theta^i = 0$, we get in the Newtonian limit the equation $\Delta\varphi = \text{const}$ which is satisfied by the potential $\varphi = kr^2/2 + C$, $k = \text{const}$, $C = \text{const}$ of the elastic force $\vec{F} = (-)\,\text{grad}\,\varphi = (-)k\vec{r}$ instead of the Poisson equation $\Delta\varphi = \varrho$. Thus, the full system (1) of the field equations of the theory cannot describe the macroscopic gravitation.

However, if we choose the constants ϱ, $\bar{a}$, α in the following way

$$(i) \qquad \varrho = \frac{c^4}{16\pi G}, \quad \bar{a} = \varrho A = \varrho\frac{\hbar G}{c^3} = \frac{\hbar c}{16\pi}, \quad \alpha = \varrho, \tag{5}$$

or in the way

$$(ii) \qquad \varrho = \frac{c^4}{16\pi G}, \quad \bar{a} = \varrho A = \varrho\frac{\hbar G}{c^3} = \frac{\hbar c}{16\pi},$$

$$\alpha = \varrho C = \varrho\frac{H}{c}\sqrt{\frac{\hbar G}{c^3}} = \frac{Hc}{16\pi}\sqrt{\frac{\hbar c}{G}}, \tag{6}$$

then, in the limit $\hbar = 0$ we get the theories which for incoherent perfect fluid correspond with the Newtonian theory as well as GRT does.

In (5) and (6) $\hbar$ is the Planck constant; c is the value of the velocity of light in vacuum, G is the Newtonian gravitational constant and H is the Hubble constant.

Thus, with the above choice of the constants $\varrho, \bar{a}, \alpha$, and putting $\hbar = 0$ into the field equations (1) or (2) one obtains the macroscopic gravitational theoiy. This theory, if we choose the constants $\varrho, \bar{a}, \alpha$ as in (*ii*), coincides with the Einstein-Cartan theory

(ECT) of gravitation [2, 3]. On the other hand, the choice of the constants $\varrho, \bar{a}, \alpha$ given by (*i*) leads us in the limit $\hbar = 0$ to a new macroscopic gravitational theory with the equations

$$Q_{i\cdot l}^{\cdot p\cdot} - Q_{l\cdot i}^{\cdot p\cdot} + Q_{\cdot lk}^{k}\delta_i^p + Q_{\cdot ki}^{k\cdot\cdot}\delta_l^p + Q_{\cdot il}^{p\cdot\cdot} = (-)\frac{S_{\cdot il}^{p\cdot\cdot}}{2\varrho},$$

$$\nabla_k Q_{b\cdot\cdot}^{\cdot pk} + Q_{b\cdot\cdot}^{\cdot pk} Q_{\cdot kl}^{l\cdot\cdot} + \tfrac{1}{2} Q_{b\cdot\cdot}^{\cdot lk} Q_{\cdot lk}^{p\cdot\cdot} - G_{\cdot b}^{p\cdot}$$

$$+ \frac{\delta_b^p}{4} Q_{\cdot\cdot\cdot}^{itr} Q_{itr}^{\cdot\cdot\cdot} - Q_{\cdot br}^{i\cdot\cdot} Q_{i\cdot\cdot}^{\cdot pr} = (-)\frac{1}{2\varrho} t_{\cdot b}^{p\cdot}. \tag{7}$$

The macroscopic gravitational theory with the equations (7) is based on the Lagrangian

$$L_g = \varrho(\Omega_{\cdot k}^{i\cdot} \wedge \eta_{i\cdot}^{\cdot k} + \Theta^i \wedge * \Theta_i) \tag{8}$$

with

$$\varrho = \frac{c^4}{16\pi G},$$

and for classical spin and for incoherent perfect fluid it corresponds with NGT as well as GRT and ECT do.

The dynamical equations of the new macroscopic gravitational theory

$$D * \Theta_l = (-)\frac{t_l}{2\varrho} - \tfrac{1}{2}\Omega_{\cdot\cdot}^{jk} \wedge \eta_{ljk} + \left(Q_{\cdot lr}^{a\cdot\cdot} Q_{a\cdot\cdot}^{\cdot pr} - \frac{\delta_l^p}{4} Q_{\cdot\cdot\cdot}^{atr} Q_{atr}^{\cdot\cdot\cdot}\right)\eta_p$$

are of the second order and have the form of the equations of a gauge field theory. Moreover, in vacuum, equations (7) reduce to the vacuum Einstein equations (4). In general, for spinless matter equations (7), similarly as the ECT equations, reduce to the Einstein equations

$$G_{\cdot b}^{p\cdot} = \frac{1}{2\varrho} t_{\cdot b}^{p\cdot}. \tag{9}$$

The choice (*i*) of the constants $\varrho, \bar{a}, \alpha$ has some advantages because the constants $\varrho = \alpha, \bar{a}$, are constructed from the fundamental constants $\hbar, c, G$ only; on the other hand, the choice (*ii*) of these constants gives the theory with A approximately equal to C ($A \approx 10^{-66}\ \mathrm{cm}^2$; $C \approx 10^{-60}$) and leads to a simpler macroscopic gravitational theory (ECT).

The full system of the field equations (1) (or (2)) is of the third order and, in our opinion, gives a classical, microscopic gravitational theory.

Thus, the full system with $\varrho, \bar{a}, \alpha$ given by (*i*) or (*ii*) and its solutions should have a physical meaning in microphysics and in the singularity problem.

2. *Remarks on the spherically symmetric solutions*

The O(3) symmetry admits the following, nonvanishing components of the metric, g, and the torsion, Θ, in the "spherical" coordinates $x^0 = ct$, $x^1 = r$, $x^2 = \vartheta$, $x^3 = \varphi$:

$$g_{00} = e^{\nu(r,t)}, \quad g_{11} = (-)e^{\lambda(r,t)}, \quad g_{22} = (-)r^2, \quad g_{33} = (-)r^2 \sin^2 \vartheta,$$

$$Q^{0\cdot\cdot}_{\cdot 01} = f(r, t), \quad Q^{1\cdot\cdot}_{\cdot 10} = h(r, t), \quad Q^{2\cdot\cdot}_{\cdot 20} = Q^{3\cdot\cdot}_{\cdot 30} = k(r, t),$$

$$Q^{2\cdot\cdot}_{\cdot 12} = Q^{3\cdot\cdot}_{\cdot 13} = g(r, t). \tag{8}$$

With the help of (8) we can write down the field equations (2) of the theory. If we do that, we get a very complicated system of nine nonlinear differential equations of the third order. The system is too complicated to be presented here.

These equations admit only two nonvanishing components $S^{0\cdot\cdot}_{\cdot 10}$ and $S^{1\cdot\cdot}_{\cdot 10}$, of the classical spin

$$S^{i\cdot\cdot}_{\cdot jk} = u^i S_{jk}; \quad S_{jk} = (-)S_{kj}.$$

However, these components must vanish in the consequence of the conditions

$$u^l S_{lk} = 0.$$

Therefore, the O(3) symmetry and the field equations of the theory both exclude the classical spin (in the following we restrict ourselves to the classical model of spin). In consequence we can take the energy-momentum tensor of the perfect fluid in the same form as in GRT, i.e., in the form

$$t^{j\cdot}_{\cdot i} = (\varepsilon + p)u^j u_i - p\delta^j_i. \tag{9}$$

2.1. Remarks on the exterior solutions

Outside of a spherically symmetric distribution of matter, i.e., in the domain where $t^{j\cdot}_{\cdot k} = 0$, we obtain a very complicated system of nine nonlinear differential equations of the third order on the six unknown functions $\nu(r, t)$, $\lambda(r, t)$, $f(r, t)$, $g(r, t)$, $h(r, t)$, $k(r, t)$.

With the help of Trautman's differential identities [2], [4] we can prove that this system may possess no more than six independent equations. In consequence the system may possess solutions with dynamical torsion.

Up to now we have not obtained any solution to the system with torsion neither using traditional methods nor using methods typical for the gauge field theory [5], i.e., using a suitable Ansatz for $\Omega^{i\cdot}_{\cdot k}$. (All Ansatzes lead inconsistent systems of equations.) The investigation of this problem is continued.

If we put $\Theta^i = 0$, then the equations of the theory reduce to the vacuum Einstein equations and, as the solution, we get the exterior Schwarzschild solution only.

2.2. Remarks on the interior solutions

Inside of matter we have, in general, the system of nine nonlinear differential equations of the third order for nine unknown functions: $\lambda(r, t)$, $\nu(r, t)$, $f(r, t)$, $g(r, t)$, $h(r, t)$, $k(r, t)$,

$\varepsilon(r, t)$, $p(r, t)$, $u^1(r, t)$. The system is too complicated to be presented here. Up to now we have not been able to obtain a solution to the system with dynamical torsion.

If we confine ourselves to the static sphere (we do this in the following), then we get the system of nine nonlinear differential equations of the third order on eight unknown functions: $\lambda(r)$, $\nu(r)$, $f(r)$, $g(r)$, $h(r)$, $k(r)$, $\varepsilon(r)$, $p(r)$. The system is as complicated as in the general case and it is very hard to obtain its exact solution with dynamical torsion if it exists.

However, assuming the static case and putting the torsion equal to zero we simplify the system of the field equations to a tolerable form:
The equations with $D * \Theta$

$$e^{-\lambda}\left(\frac{\lambda'}{r}-\frac{1}{r^2}\right)+\frac{1}{r^2}+\frac{\bar{a}}{\varrho}\left[(-)e^{-2\lambda}\frac{\nu'^2}{2r^2}-e^{-2\lambda}\left(\frac{\lambda'\nu'}{4}-\frac{\nu''}{2}-\frac{\nu'^2}{4}\right)^2\right.$$
$$\left.+\frac{1}{r^4}(e^{-\lambda}-1)^2+e^{-2\lambda}\frac{\lambda'^2}{2r^2}\right]-\frac{\varepsilon}{2\varrho}=0,$$

$$(-)e^{-\lambda}\left(\frac{\nu'}{r}+\frac{1}{r^2}\right)+\frac{1}{r^2}+\frac{\bar{a}}{\varrho}\left[(-)e^{-2\lambda}\frac{\lambda'^2}{2r^2}-e^{-2\lambda}\left(\frac{\lambda'\nu'}{4}-\frac{\nu''}{2}-\frac{\nu'^2}{4}\right)^2\right.$$
$$\left.+e^{-2\lambda}\frac{\nu'^2}{2r^2}+\frac{1}{r^4}(e^{-\lambda}-1)^2\right]+\frac{p}{2\varrho}=0,$$

$$\frac{e^{-\lambda}}{2}\left(\frac{\lambda'}{r}-\frac{\nu'}{r}-\nu''-\frac{\nu'^2}{2}+\frac{\lambda'\nu'}{2}\right)+\frac{\bar{a}}{\varrho}\left[e^{-2\lambda}\left(\frac{\lambda'\nu'}{4}-\frac{\nu''}{2}-\frac{\nu'^2}{4}\right)^2\right.$$
$$\left.-\frac{1}{r^4}(1-e^{-\lambda})^2\right]+\frac{p}{2\varrho}=0. \tag{10}$$

The equations with $D * \Omega$

$$\frac{\lambda'^2\nu'}{4}-\frac{\lambda'\nu''}{2}-\frac{\lambda'\nu'^2}{4}+\frac{\nu'''}{2}-\frac{\lambda''\nu'}{4}-\frac{\lambda'\nu''}{4}+\frac{\nu'\nu''}{2}-\frac{\nu'}{r^2}$$
$$+\frac{\nu''}{r}-\frac{\lambda'\nu'}{2r}+\frac{\nu'^2}{2r}=0,$$

$$\frac{e^{-\lambda}}{r}\left(\frac{\lambda'\nu'}{4}+\frac{\nu'^2}{4}-\frac{\lambda'^2}{2}+\frac{\lambda''}{2}+\frac{1}{r^2}\right)-\frac{1}{r^3}=0. \tag{11}$$

In equations (10)–(11)

$$\lambda' := \frac{d\lambda}{dr}, \quad \lambda'' := \frac{d^2\lambda}{dr^2} \text{ etc.}$$

Equations (10)–(11) form a system of five nonlinear differential equations of the third order on the four unknown functions: $\lambda(r)$, $\nu(r)$, $\varepsilon(r)$, $p(r)$. The system surely possesses solutions. This is seen because if $S_{i.}^{\cdot j} = \Theta^i = 0$, then the field equations (2) of the theory reduce to the form

$$\nabla_l R_{ip} - \nabla_i R_{lp} = 0,$$

$$\varrho\left(R^{p\cdot}_{\cdot b} - \frac{\delta^p_b}{2} R\right) = \tfrac{1}{2} t^{p\cdot}_{\cdot b} - \bar{a}\left(\frac{\delta^p_b}{4} R^{ijrt}_{\cdot\cdot\cdot\cdot} R^{\cdot\cdot\cdot\cdot}_{ijrt} - R^{ij\cdot\cdot}_{\cdot\cdot bt} R^{\cdot\cdot pt}_{ij\cdot\cdot}\right). \tag{12}$$

The system (12) is in turn satisfied, e.g., by every solution of the system

$$R_{ik} = \Lambda g_{ik}, \quad \Lambda = \text{const} \tag{13}$$

with the equation of state $p = (-)\varepsilon$.

We can most easily obtain these solutions in the following way: we look for all spherically symmetric solutions to equations (11) and select those which simultaneously satisfy equations (10). If we do that, then we get the following solutions to the system (10)–(11):
(*i*)

$$ds^2 = c^2 dt^2 - \left(1 - \frac{r^2}{R^2}\right)^{-1} dr^2 - r^2 d\Omega^2,$$

$$R = \text{const}, \quad d\Omega^2 = d\vartheta^2 + \sin^2\vartheta d\varphi^2,$$

$$\varepsilon = \frac{6}{R^2}\left(\varrho + \frac{\bar{a}}{R^2}\right), \quad p = \frac{2}{R^2}\left(\frac{\bar{a}}{R^2} - \varrho\right), \quad p = \frac{\varepsilon}{3} - \frac{4\varrho}{R^2}. \tag{14}$$

This solution may be physically valid only in the following two cases (we assume that the equation of state cannot depend on $R \equiv$ radius of the sphere):

1. $p = \dfrac{\varepsilon}{3}, \quad R = \infty,$

2. $p = 0, \quad R = \sqrt{\dfrac{\bar{a}}{\varrho}} = \sqrt{\Lambda} = \sqrt{\dfrac{\hbar G}{c^3}}.$

In the first case we get an empty Minkowskian world with $p = \varepsilon = 0$ and in the second case we get a very interesting solution describing a particle with the following parameters

$$R = \sqrt{\frac{\bar{a}}{\varrho}} = \sqrt{\Lambda} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \times 10^{-33}\ \text{cm},$$

$$\varepsilon = 12\varrho\Lambda^{-1} = \frac{3c^7}{4\pi\hbar G^2},$$

$$m_0 = \tfrac{4}{3}\pi R^3 \frac{\varepsilon}{c^2} = \sqrt{\frac{\hbar c}{G}} \approx 10^{-5}\text{g}, \quad r_g = \frac{2Gm_0}{c^2} = 2R. \tag{15}$$

Particles of that kind, called maximons, were previously considered by Markov [6].

The maximons have some interesting properties [6]; especially, their gravitational interaction is able to create bounded states even for electrically charged maximons.

The bounded states of $N > 2$ maximons may have arbitrarily small rest mass depending on the mean distance between constituting maximons.

It is interesting that maximons appear in the considered gravitational theory as special solutions to the field equations.

(*ii*)

$$ds^2 = \left(1-\frac{r^2}{R^2}\right)c^2dt^2 - \left(1-\frac{r^2}{R^2}\right)^{-1} dr^2 - r^2 d\Omega^2,$$

$$R = \text{const}, \qquad \varepsilon = \frac{2\varrho}{R^2}, \qquad p = (-)\varepsilon,$$

$$m_0 = \tfrac{4}{3}\pi R^3 \frac{\varepsilon}{c^2} = \frac{c^2 R}{6G}, \qquad r_g = \frac{2Gm_0}{c^2} = \frac{R}{3}. \tag{16}$$

We see that it is possible to put $\varepsilon = p = 0$ in this case if and only if $R = \infty$, i.e., if the space-time is Minkowskian.

The solution (16) may be joined smoothly to the exterior Schwarzschild solution on the surface of the sphere $r = R$. It seems to us that the solution (16) may represent a neutron star because the typical parameters of a neutron star [7] are compatible with this solution.

3. *Remarks on the homogeneous and isotropic cosmology with O(3) isotropy group*

Let us study the fundamental equations of the theory for the homogeneous and isotropic cosmology with the O(3) isotropy group at every point. Taking the Robertson--Walker linear element in the form [8, 9]

$$ds^2 = dx^{02} - a^2(t)\left[d\chi^2 + R^2(\chi)(d\vartheta^2 + \sin^2\vartheta d\varphi^2)\right], \tag{17}$$

where

$$R(\chi) = \begin{cases} \sin\chi & \text{if} \quad k = 1, \\ \chi & \text{if} \quad k = 0, \\ \operatorname{sh}\chi & \text{if} \quad k = -1, \end{cases}$$

and the following nonzero components of torsion Θ^i

$$Q^0_{\cdot 0 \dot 1} = f(\chi, t), \qquad Q^1_{\cdot 1 \dot 0} = h(\chi, t), \qquad Q^2_{\cdot 0 \dot 2} = Q^3_{\cdot 0 \dot 3} = (-)k(\chi, t),$$
$$Q^2_{\cdot 1 \dot 2} = Q^3_{\cdot 1 \dot 3} = g(\chi, t), \tag{18}$$

we can explicitly write down the field equations (2).

We get a very complicated system of nine nonlinear differential equations of the third order. The system is too complicated to be presented here.

The system admits only one component, $S^{0\cdot\cdot}_{\cdot 10}$, of the classical spin. However, this component must vanish in the consequence of the conditions $u^l S_{lk} = 0$. Therefore, the classical spin is ruled out here from the consideration. In consequence we have a system of nine nonlinear differential equations of the third order on seven unknown functions $a(t)$, k, $p(t)$, $\varepsilon(t)$, $f(\chi, t)$, $g(\chi, t)$, $h(\chi, t)$. Probably this system is consistent only in the case of vanishing torsion. This problem will be examined elsewhere.

In this paper we restrict ourselves to the cosmological equations of the theory in the case $\Theta^i = 0$. Then we have the following equations:

The case $k = 0$

$$c^4\varepsilon = 6\varrho c^2 \frac{\dot{a}^2}{a^2} + 6\frac{\bar{a}}{a^2}\left(\frac{\dot{a}^4}{a^2} - \ddot{a}^2\right),$$

$$(-)c^4 p = 2\varrho \frac{c^2}{a^2}\left(2\ddot{a} + \frac{\dot{a}^2}{a}\right) + 2\frac{\bar{a}}{a^2}\left(\ddot{a}^2 - \frac{\dot{a}^4}{a^2}\right),$$

$$\frac{\dddot{a}}{a} + \frac{\dot{a}\ddot{a}}{a^2} - 2\frac{\dot{a}^3}{a^3} = 0. \tag{19}$$

The case $k = 1$

$$c^4\varepsilon = 6\varrho \frac{c^2}{a^2}(\dot{a}^2 + c^2) + 6\frac{\bar{a}}{a^2}\left[\frac{(c^2+\dot{a}^2)^2}{a^2} - \ddot{a}^2\right],$$

$$(-)c^4 p = 2\varrho c^2\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{c^2}{a^2}\right) + 2\frac{\bar{a}}{a^2}\left[\ddot{a}^2 - \frac{(c^2+\dot{a}^2)^2}{a^2}\right],$$

$$\frac{\dddot{a}}{a} + \frac{\dot{a}\ddot{a}}{a^2} - 2\frac{\dot{a}^3}{a^3} - 2c^2\frac{\dot{a}}{a^3} = 0. \tag{20}$$

The case $k = (-)1$

$$c^4\varepsilon = 6\varrho \frac{c^2}{a^2}(\dot{a}^2 - c^2) + 6\frac{\bar{a}}{a^2}\left[\frac{(c^2-\dot{a}^2)^2}{a^2} - \ddot{a}^2\right],$$

$$(-)c^4 p = 2\varrho c^2\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} - \frac{c^2}{a^2}\right) + 2\frac{\bar{a}}{a^2}\left[\ddot{a}^2 - \frac{(\dot{a}^2-c^2)^2}{a^2}\right],$$

$$\frac{\dddot{a}}{a} + \frac{\dot{a}\ddot{a}}{a^2} - 2\frac{\dot{a}^3}{a^3} + 2c^2\frac{\dot{a}}{a^3} = 0. \tag{21}$$

The above systems of equations surely possess solutions because, e.g., every suitable solution to the equations

$$R_{ik} = \Lambda g_{ik} \quad \text{with } \Lambda = \text{const}$$

satisfies the equations of these systems with $p = (-)\varepsilon$.

Up to now we have obtained only some special solutions of the systems (19)–(21). These solutions are:

(*i*) In the case $k = 1$ we have a static maximon solution with the parameters

$$a = \sqrt{A} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \times 10^{-33}\ \text{cm},$$

$$\varepsilon = 12\varrho A^{-1} = \frac{3c^7}{4\pi\hbar G^2}, \quad p = 0,$$

$$m_0 = \tfrac{4}{3}\pi a^3 \frac{\varepsilon}{c^2} = \sqrt{\frac{\hbar c}{G}}, \quad r_g = \frac{2Gm_0}{c^2} = 2a. \tag{22}$$

The static solution with

$$a^2 = (-)A, \quad \varepsilon = 12\varrho A^{-1} = \frac{3c^7}{4\pi\hbar G^2}, \quad p = 0 \tag{23}$$

exists also in the case $k = (-)1$. However, in this case, the volume and, therefore, the mass of the solution are infinite.

The investigation of the non-static solutions of the equations (20) and (21) is continued.

(*ii*) In the case $k = 0$ there exists the following solution (in the case there are not admissible the solutions with $a = \text{const}$; $\varepsilon, p \neq 0$)

$$a = \text{const} \times t^{1/2}, \quad \varepsilon = \frac{3\varrho}{2c^2} t^{-2}, \quad p = \frac{\varepsilon}{3}. \tag{24}$$

This solution is the same as the corresponding cosmological solution obtained in the framework of GRT (see, e.g., [8]). Moreover, there exists the solution with

$$a = Be^{bt}, \quad B = \text{const}, \quad b = \text{const};$$

$$\varepsilon = \frac{6\varrho}{c^2} b^2 = \text{const}, \quad p = (-)\varepsilon. \tag{25}$$

The solution (25) is the same as the de Sitter solution of the GRT equations with cosmological constant Λ [10]. However in the considered gravitational theory we may put $p = \varepsilon = 0$ if and only if $b = 0$, i.e., if $a = \text{const}$.

The case $k = 0$, $p = 0$ is investigated. Probably we have in this case a cosmological model without singularity.

4. *Conclusions*

In our opinion, the considered model of the gravitational theory does have some interesting features. It seems to be the best model of the gravitational theory which combines the ideas of GRT and the ideas of gauge field theory. If we construct the constants ϱ, $\bar{a}$, α

solely from the fundamental constants $\hbar$, c, G then we get, in the limit $\hbar = 0$, the macroscopic theory of gravitation which differs from the ECT.

It seems to us that the full system of the equations of the theory gives some kind of microscopic gravitational theory.

Note Added in Proof

The equations of the theory have, in the case of O(3) symmetry, only torsionless solutions. The proof will be given in the paper submitted to *Gen. Relativ. Gravitation.*

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