KANTOWSKI-SACHS GRAVITATIONAL INSTANTON SOLUTION

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We present a new self-dual solution to Euclidean gravity which is of the Kantowski-Sachs group type and may be considered as a gravitational instanton.

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1. *Introduction*

The discovery of pseudoparticle (instanton) solutions to the Euclidean SU(2) Yang-Mills theory (Belavin et al. 1975; Rajamaran 1982) has suggested the possibility that analogous solutions might occur in Einstein's theory of gravitation. Since the Yang-Mills pseudoparticles posses self-dual field strength, one likely possibility is that gravitational pseudoparticles are characterized by self-dual curvature.

Starting from the idea that the Yang-Mills potential is asymptotically a pure gauge (see Actor 1979 for a detailed description of Yang-Mills solutions), a gravitational instanton should have an asymptotically flat metric. Four types of non-compact instantons (a gravitational instanton with a cosmological constant $\Lambda > 0$ is necessarily compact) have been discovered (for recent reviews see Eguchi et al. 1980; Pope 1981; Perry 1982): asymptotically Euclidean (AE), asymptotically locally Euclidean (ALE), asymptotically flat (AF) and asymptotically locally flat (ALF). In general a gravitational instanton can be defined as a complete Riemannian manifold (with Euclidean signature) (M, g) of dimension four with finite action satisfying the Einstein Equations $R_{\mu\nu} = \Lambda g_{\mu\nu}$, where $R_{\mu\nu}$ is the Ricci tensor.

A variety of solutions of Einstein's field equations with instanton-like properties have been discovered. The corresponding metrics of many of these solutions may be cast into the Euclidean equivalent of the Bianchi type-IX space-time. Thus one is forced to consider the complete set of Bianchi types I–IX.

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In papers published previously (Lorenz 1983a, b, c) we presented new self-dual solutions for the Bianchi types I–IX. In this paper we consider the Euclidean Kantowski-Sachs space-time (Kantowski 1966; Kantowski and Sachs 1966), which is closely related to the Bianchi type-III model.

2. *Metric, field equations and solutions*

The Kantowski-Sachs metric is a special case of the metrics

$$ds^2 = dt^2 + S^2(t)dr^2 + R^2(t)(d\theta^2 + f^2(\theta)d\phi^2), \tag{1}$$

where $f(\theta)$ is $\sinh\theta$, θ or $\sin\theta$, respectively, for Bianchi type-III, type-I or Kantowski--Sachs. By requiring that the Riemann tensor $R^{\mu}{}_{\nu\alpha\beta}$ is self-dual we obtain the following field equations

$$\ddot{S} = \ddot{R} = \dot{S}\dot{R} = 0, \qquad \dot{R}^2 = -f''/f, \tag{2}$$

where $(\ )^{\cdot} = d/dt$ and $(\ )' = d/d\theta$. We have $k = f''/f = 1, 0, -1$ in case of Bianchi type-III, type-I or K-S. The case $k = 0$ gives flat space solutions. According to the Positive Action Theorem (Hawking 1979; Schoen and Yau 1979) the only asymptotically Euclidean instanton is flat space. The case $k = 1$ has been already discussed by us (Lorenz 1983c) and gives only complex solutions. However, in the Euclidean Kantowski-Sachs space-time considered here we obtain the following real solution of Equations (2):

$$S = a, \qquad R = b(t - t_1), \tag{3}$$

where a, b, t_1 are constants of integration obeying $b^2 = 1$.

Our solution possesses the interesting feature that the components of the curvature two-forms

$$\theta^{\mu}{}_{\nu} = \tfrac{1}{2} R^{\mu}{}_{\nu\alpha\beta}\sigma^{\alpha} \wedge \sigma^{\beta} \tag{4}$$

all vanish whereas the curvature

$$R^* = -f''/(fR^2) \tag{5}$$

of the two-dimensional surfaces $(t, r = \text{const})$ is always positive in a finite region but falls off as $1/t^2$ at infinity.

Defining the new coordinates (X, Y, Z, W) by

$$X = (t-t_1)\sin\theta\cos\phi, \quad Y = (t-t_1)\sin\theta\sin\phi, \quad Z = (t-t_1)\cos\theta, \quad W = ar, \tag{6}$$

the metric (1) can be transformed into flat space-time. In these new coordinates our solution turns out to be identical with the flat space solution obeying the Positive Action Theorem.

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