HIGH-ENERGY pp AND $\bar{p}p$ SCATTERING AND GEOMETRICAL SCALING

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The model of geometrical scaling is used to predict the evolution of the diffractive dip-peak structure of pp and $\bar{p}p$ differential cross sections with increasing energy. Previous calculation for pp scattering made by Dias de Deus and Kroll is carried out with new data and their predictions are confirmed. Recent data on $\bar{p}p$ scattering are used to make an analogous analysis for this process as well. It turns out that the $\bar{p}p$ differential cross section behaves analogously, main difference being that, in the $\bar{p}p$ case, the dip-peak structure should not completely disappear with increasing energy.

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1. Introduction

It is well known that, at sufficiently high energies, the differential cross section of proton-proton elastic scattering exhibits a pronounced structure in the interval of the momentum transfer squared between the values $t = -1.3$ GeV^2 and -2.0 GeV^2. Around $t = -1.4$ GeV^2, there is a well-established dip which is followed by a secondary maximum. This structure gradually develops with increasing energy from a slight corrugation below the ISR energies till its maximal appearance at a c.m. energy $\sqrt{s}$ of approximately 20–25 GeV. A quantitative description of this effect was recently given, for instance, by Olsen [1], on the basis of a modified Chou-Yang optical model.

Predictions [2] based on the model of geometrical scaling [3] indicate that, with a further increase of energy, this dip should be gradually smoothed till it completely disappears at $\sqrt{s} = 300$ GeV, while it should later develop again at asymptotic energies.

The basic idea of the model of geometrical scaling is that, at sufficiently high energies, the s- and t-dependences of a hadron-hadron scattering amplitude $F(s, t)$ reduce to a dependence on one single kinematic variable, the scaling parameter τ,

$$\tau = -\frac{t}{t_0} \ln^2 \frac{s}{s_0}. \tag{1.1}$$

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If an amplitude asymptotically saturates the Froissart-Martin bound, i.e. if

$$F(s, 0) \sim s \ln^2 s \tag{1.2}$$

then, as was shown by Auberson, Kinoshita and Martin [4], it follows from the principles of local field theory that the function $\varphi(\tau)$,

$$\varphi(\tau) = \lim_{s \to \infty} \frac{F(s, -t_0 \tau \ln^{-2}(s/s_0))}{F(s, 0)}, \tag{1.3}$$

is analytic in τ and entire of the orde1 $\frac{1}{2}$. In this case, the geometrical scaling model appears as a consequence of general principles, provided that violently oscillating amplitudes are excluded. Let us remark in this connection that the systematic analysis of the p-p scattering data made by Amaldi et al. [5] has led to the result that the total cross section should behave like $\ln^a s$ with $a = 2.1 \pm 0.1$.

Remarkable progress has been made recently in the experimental investigation of the proton-antiproton scattering as well. It has turned out that the differential cross section exhibits a very similar dip-peak structure like in the case of pp scattering. The resemblance goes so far that the t-dependence of $\frac{d\sigma^{\bar{p}p}}{dt}$ at $\sqrt{s} = 9.8$ GeV [6] is experimentally indistinguishable from that of $\frac{d\sigma^{pp}}{dt}$ at $\sqrt{s} = 52.8$ GeV. It is therefore worth revising the geometrical scaling model in relation to the existing pp and also $\bar{p}p$ high-energy data.

We repeated the calculation made by Dias de Deus and Kroll [2] using recent data on pp scattering, and obtained a confirmation of their prediction. The diffractive dip-peak structure of the differential cross section should be gradually smoothed out and it should completely disappear at a c.m. energy $\sqrt{s}$ of approximately several hundreds of GeV. At still higher energies, the structure develops again to its full reappearance at asymptotic energy. The process is controlled by the value of $\varrho(s)$,

$$\varrho(s) = \mathrm{Re}\, F(s, 0)/\mathrm{Im}\, F(s, 0), \tag{1.4}$$

which rises from zero to some positive value, and tends to zero again when $\sqrt{s}$ tends to infinity. Then, using the recent data on $\bar{p}p$ scattering, we made an analogous calculation for this reaction as well. It turns out that the diffractive structure should also be gradually smoothed out with increasing energy but, contrary to the pp case, not completely, again reappearing with a further increase of $\sqrt{s}$.

2. *The method*

Following the standard derivation of geometrical scaling model [3], we introduce the scaling function $\varphi(\tau)$ such that

$$\mathrm{Im}\, F(s, \tau) = \mathrm{Im}\, F(s)\varphi(\tau), \tag{2.1}$$

$$\mathrm{Re}\, F(s, \tau) = \mathrm{Re}\, F(s) \frac{d}{d\tau}(\tau\varphi(\tau)), \tag{2.2}$$

where $\tau = -\sigma(s)t$ and $F(s) = F(s, 0)$, $\sigma(s)$ being the total cross section. The set of quantities F, σ, φ and τ refers either to the pp or to the $\bar{p}p$ scattering.

Combining the relations (2.1) and (2.2) we obtain the following equation [2]:

$$\frac{d\sigma}{dt}(s, t) = \frac{d\sigma}{dt}(s, 0)\left\{\varphi^2(\tau)+\varrho^2(s)\left(\frac{d}{d\tau}(\tau\varphi(\tau))\right)^2\right\}\Big/(1+\varrho^2(s)). \tag{2.3}$$

This is a differential equation for the scaling function $\varphi(\tau)$, all other quantities being known from experiment. Once $\varphi(\tau)$ has been determined at some energy value, then, assuming that the model is valid at all higher energies, one can predict from (2.3) the behaviour of the differential cross section with increasing energy.

In accordance with Ref. [2], we assume that the dip in $\frac{d\sigma}{dt}$ is produced by a zero in $\varphi(\tau)$,

$$\varphi(\tau = \tau_d) = 0, \tag{2.4}$$

where τ_d is the position of the dip. Inserting this into Eq. (2.3) we obtain

$$\frac{d\sigma}{dt}(s, \tau_d)\Big/\frac{d\sigma}{dt}(s, 0) = K^2\varrho^2(s)/(1+\varrho^2(s)) \tag{2.5}$$

where $K = \tau_d \left.\frac{d\varphi}{d\tau}\right|_{\tau=\tau_d}$ is a constant. The left-hand side of (2.5) and the function $\varrho(s)$ are determined by two independent measurements. We found this relation to be in a very good agreement with experimental data (see Sec. 3 for details).

Expanding $\varphi(\tau)$ in powers of $(\tau-\tau_d)/\tau_d$ we have, because of (2.4),

$$\varphi(\tau) = \sum_{n=1}^{\infty} \frac{\tau_d^n}{n!} \left.\frac{d^n\varphi}{d\tau^n}\right|_{\tau=\tau_d} \left(\frac{\tau-\tau_d}{\tau_d}\right)^n. \tag{2.6}$$

We approximate $\varphi(\tau)$ by taking two, three or four terms of (2.6), and insert the approximant into Eq. (2.3). In doing so, we also take into account the fact that $\varrho(s)$ does not exceed the value 0.14, and consider it to be of the same order of magnitude as $(\tau-\tau_d)/\tau_d$.

To determine the form of $\varphi(\tau)$ in the given approximation, we used for pp scattering the data [7] on $\frac{d\sigma}{dt}$ at $\sqrt{s} > 30$ GeV in the intervals

$$1.2\ \text{GeV}^2 \leqslant -t \leqslant 1.7\ \text{GeV}^2 \tag{2.7}$$

and

$$1.2\ \text{GeV}^2 \leqslant -t \leqslant 2.0\ \text{GeV}^2. \tag{2.8}$$

In the case of $\bar{p}p$ scattering, we used the data [6] and [9] on $\frac{d\sigma}{dt}$ at $p = 50$ GeV/c and $p = 100$ GeV/c (i.e., $\sqrt{s} = 9.8$ GeV and 13.8 GeV respectively) in the interval (2.8).

3. *Results and discussion*

It turned out that, in fitting the pp data in the interval (2.7), the third and the fourth derivative of $\varphi(\tau)$ at $\tau = \tau_d$ could be fixed at zero value, while in the interval (2.8) all the four derivatives were important. In the case of $\bar{p}p$ data, the third and the fourth derivative of $\varphi(\tau)$ could be put equal to zero in the whole interval (2.8), which might be connected with relatively larger errors of the input data in this case.

As it was already mentioned, the values of $\varrho(s)$ determined from the fits to $\frac{d\sigma}{dt}$ are in a very good agreement with those obtained by a direct measurement of $\varrho(s)$. This is an interesting fact which speaks in favour of the applicability of this model in this kinematic region. Details are seen from Table I.

TABLE I

Values of ϱ^{pp} and $\varrho^{\bar{p}p}$ obtained by fitting the data of Refs [6, 7 and 9] in comparison with those directly measured (Ref. [5] for pp and Ref. [8] for $\bar{p}p$)

	$\sqrt{s}$ GeV	t-range	$\lvert\varrho(s)\rvert$ obtained from the fit	$\varrho(s)$ measured
pp	30.7	(2.7)	0.023 ± 0.004	0.042 ± 0.011
		(2.8)	0.025 ± 0.003	
pp	44.7	(2.7)	0.062 ± p.005	0.062 ± 0.011
		(2.8)	0.063 ± 0.005	
pp	52.8	(2.7)	0.073 ± 0.007	0.078 ± 0.010
		(2.8)	0.072 ± 0.005	
pp	62.5	(2.7)	0.104 ± 0.032	0.095 ± 0.011
		(2.8)	0.088 ± 0.018	
$\bar{p}p$	9.8	(2.8)	0.024 ± 0.060	
	11.5			0.010 ± 0.018
$\bar{p}p$	13.8	(2.8)	0.050 ± 0.027	
	15.4			0.012 ± 0.020

Results obtained for the pp differential cross section are shown in Fig. 1. The experimental points (corresponding to $\sqrt{s} = 52.8$ GeV) were used to determine $\varphi(\tau)$, the full line representing the corresponding fit. The dashed curve plots the predicted behaviour of $\frac{d\sigma}{dt}$ at $\varrho = 0.14$. We see that the form of $\frac{d\sigma}{dt}$ very strongly depends on the value of ϱ: if ϱ is zero or near to zero, the dip-peak structure of $\frac{d\sigma}{dt}$ is very pronounced, while at ϱ sufficiently large the dependence becomes flat. Thus, to predict the form of $\frac{d\sigma}{dt}$ at higher energies, we have to know, or assume, the energy dependence of the corresponding ratio $\varrho(s)$.

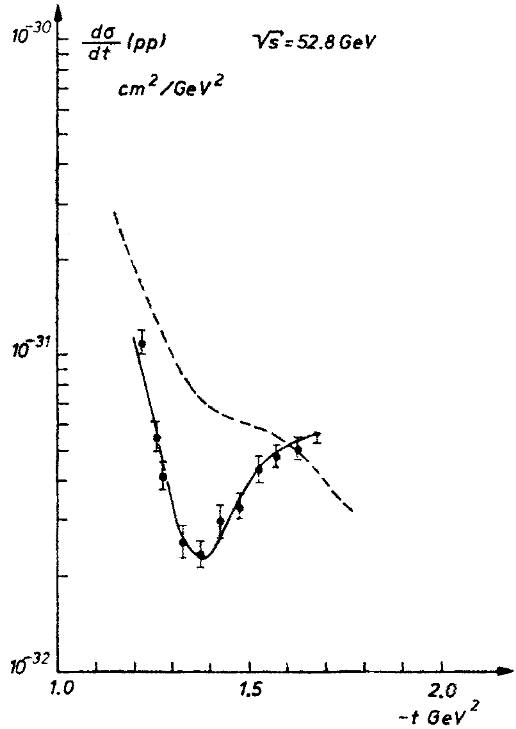

Fig. 1. The pp differential cross section. The full line represents the fit to the 52.8 GeV data, the dashed line the model prediction at $\varrho^{pp} = 0.14$. The dip is smoothed out

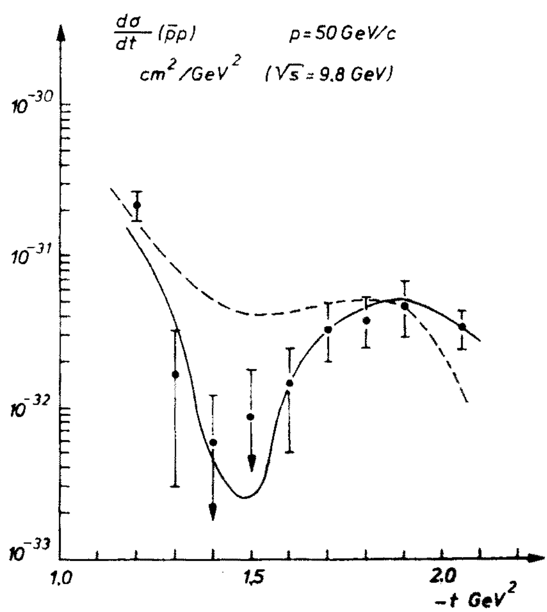

Fig. 2. The $\bar{p}p$ differential cross section. The full line represents the fit to the 50 GeV/c (equivalent of $\sqrt{s} = 9.8$ GeV) data, the dashed line the model prediction at $\varrho^{\bar{p}p} = 0.14$. The dip is still visible

In the ISR energy range, experiments give both ϱ^{pp} and $\varrho^{\bar{p}p}$ positive and increasing. Their expected behaviour at still higher energies can be deduced from general principles of local field theory. Indeed, it can be shown [10] that if at least one of the total cross sections σ^{pp} and $\sigma^{\bar{p}p}$ asymptotically rises and if the amplitude difference $F^{\bar{p}p}(s)-F^{pp}(s)$ is asymptotically negligible with respect to the sum $F^{\bar{p}p}(s)+F^{pp}(s)$, then both $\varrho^{\bar{p}p}(s)$ and $\varrho^{pp}(s)$ must asymptotically tend to zero with increasing energy and, consequently, each of them will have at least one maximum above the known energy range. The values of $\varrho^{\bar{p}p}(s)$ and $\varrho^{pp}(s)$ should gradually become closer to each other with increasing energy. Further, it is expected from phenomenological considerations that the two curves will have a common, extraordinarily flat maximum with a value of approximately 0.14 and a position in the CERN collider energy range. This is why we chose $\varrho = 0.14$ in Fig. 1 and Fig. 2. It is seen that, in the case of pp scattering, the dip in the differential cross section is completely smoothed out at this value.

In Fig. 2, the experimental points of $\frac{d\sigma^{\bar{p}p}}{dt}$ at $\sqrt{s} = 9.8$ GeV and their fit (full line) are shown which were used to determine the scaling function $\varphi(\tau)$ for $\bar{p}p$ scattering. The dashed curve represents the predicted behaviour of $\frac{d\sigma^{\bar{p}p}}{dt}$ at $\varrho^{\bar{p}p} = 0.14$, which is the expected maximum value. The dip is still clearly visible; its full vanishing would require a still higher value of $\varrho^{\bar{p}p}$.

4. *Concluding remarks*

General considerations [4] do not tell the range of s and t values at which the scaling (1.3) should take place. Data on pp scattering suggest that the model of geometrical scaling, which has its rigorous basis at asymptotic energy, is applicable at presently accessible energies, even in the region of relatively large momentum transfers. On the other hand, a similar check of the validity of the model in the case of $\bar{p}p$ scattering is less reliable, because of the lack of data on $\frac{d\sigma^{\bar{p}p}}{dt}$ above $\sqrt{s} = 20$ GeV and, also, because of a low accuracy of the existing data. We nevertheless made an analogous prediction for $\bar{p}p$ scattering as well, although the effects of the model of geometrical scaling may not be very pronounced at the energy $\sqrt{s} = 9.8$ GeV, at which the data have been taken.

An interesting argument in favour of the applicability of the model in this kinematic region follows from Table I, which was not included in the first report [11] on our work. The Table shows a remarkable agreement of the values of $\varrho(s)$ obtained from the model with the measured ones, both for pp and for $\bar{p}p$ scattering. Other arguments are contained in the recent Ref. [12], in which similar results have independently been obtained.

We conclude that the model predicts a full and a partial vanishing of the diffractive dip in the differential cross section of the pp and $\bar{p}p$ scattering, respectively, in the energy range of several hundreds of GeV in the centre-of-mass frame. It does not follow from the results obtained that the model is able to explain the diffractive dip-peak structure in the differential cross section; on the other hand, once such a structure is assumed at a certain energy, its evolution, vanishing and re-appearing is well described within the frame of the model.

In a recent paper [13] P. Kroll establishes the validity of geometrical scaling for πN and $\bar{p}p$ elastic scatteiing. More accurate πN data allow him to check the universality for the imaginary parts of the πN and pp eikonals. Contrary to this, we concentrate on the evolution of the pp and $\bar{p}p$ diffractive structure with increasing energy. Measurements of $\bar{p}p$ scattering on the CERN collider will be a good test of our prediction and of the model of geometrical scaling.

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