DEUTERON STRUCTURE IN THE ELASTIC ELECTRON--DEUTERON SCATTERING

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The relativistic light front dynamics, which unifies descriptions of low and high energy phenomena, is applied to the deuteron. The deuteron wave function is constructed from the Weinberg equation with spin, under the constraints of the quark substructure of the deuteron current. It is shown that the relativistic nucleon impulse approximation of deuteron is insufficient to explain the experimental data for the elastic electron-deuteron scattering at momentum transfers of the order of 8 GeV^2.

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1. *Introduction*

Deuteron is a bound state of two nucleons with a binding energy of about 500 times smaller than the nucleon mass, and a radius about 4 times larger than the nucleon radius. Therefore the nonrelativistic Schrödinger wave function[1] describes very well the deuteron properties. Surprisingly it describes well the deuteron electromagnetic form factors even at momentum transfers t comparable with the nucleon mass.

On the other hand the quark dimensional counting rules [1] predict a flattening of the deuteron elastic form factor according to the rule $F_d(t) \sim t^{-5}$. The experimental data confirm this behaviour already at t of the order of a few GeV^2 [2].

The important question arises whether at these momentum transfers the deuteron current is carried by nucleons, or whether the other intermediate states like meson exchanges, and six quark states have important contributions to this current. Only a consistent relativistic model, incorporating the nonrelativistic knowledge about the deuteron, may answer that question.

In this paper we use the light front dynamics [3]. It provides a unique description of a bound system [4], invariant under three Lorentz boosts [5]. The relativistic deuteron

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[1] The words "wave function" are abbreviated by "wf".

wf is determined by the Weinberg equation [6] which is the light front counterpart of the Bethe-Salpeter equation. In Section 2 we assume that the nucleons in the deuteron are bound by one meson exchanges. The Weinberg equation allows us to incorporate the non-relativistic knowledge in a particularly natural way. The asymptotic tail of the deuteron wf is governed by vector meson exchanges, under constraints of the quark substructure of the deuteron current. The calculation of the deuteron current is presented in Section 3 where we benefit from simplifications of the light front dynamics. In Section 4 we evaluate the contribution of relativistic nucleon impulse approximation to the deuteron Rosenbluth functions A and B, and compare it with experiment. Finally in Section 5 we conclude about the deuteron structure in the elastic electron-deuteron scattering.

2. *Relativistic deuteron wave function*

The Weinberg equation for a two nucleon bound system is invariant under three independent Lorentz boosts [5]. Without the loss of generality we invariantly express the Weinberg equation for the deuteron in terms of the relative three-momentum $\boldsymbol{k}$ of nucleons from *their* center of mass frame of reference. Then for the deuteron at rest the Weinberg equation reduces to the Schrödinger equation with nonlocal energy dependent potential $V(\boldsymbol{k},\boldsymbol{k}')$ [4]. This potential is obtained from the sum of one meson exchanges. The nonlocality and energy dependence of the potential is negligible for relative momenta $\boldsymbol{k}$ small in comparison with the nucleon mass m. Therefore, the Weinberg wf is exactly equal to the Schrödinger one for $|\boldsymbol{k}| \ll m$. By this incorporation of the nonrelativistic knowledge we take into account the light meson exchanges in the potential $V(\boldsymbol{k},\boldsymbol{k}')$. The nucleon-nucleon interaction inside the deuteron at high relative momenta is mediated by heavy vector meson exchanges. Hence we find the relativistic tail of the wf from an approximate solution of the Weinberg equation with the potential $V(\boldsymbol{k},\boldsymbol{k}')$ restricted to heavy vector mesons[2]. Finally, we interpolate between the relativistic tail and the nonrelativistic Schrödinger wf by writing a superposition of two Hulthen wf with appropriate form factors.

The Weinberg equation is

$$\Phi(\boldsymbol{k}) = G_0 \sum_{\substack{\text{spins}\\ \text{isospins}}} \int \frac{d^3k'}{(2\pi)^3\sqrt{m^2+\boldsymbol{k}^2}}\, V(\boldsymbol{k},\boldsymbol{k}')\Phi(\boldsymbol{k}'), \tag{1}$$

where the propagator $G_0 = (m\varepsilon+\boldsymbol{k}^2)^{-1}$ has the same form as in the Schrödinger equation, ε denotes the deuteron binding energy. The potential $V(\boldsymbol{k},\boldsymbol{k}')$ for the ϱ meson exchange is shown in Fig. 1 and given by the following expression

$$V(\boldsymbol{k},\boldsymbol{k}') = gF\bar{u}(p)\gamma^\mu u(p')\chi_p^\dagger\tau_i\chi_{p'}\, gF\bar{u}(n)\gamma_\mu u(n')\chi_n^\dagger\tau_i\chi_{n'} \\ \times[\mu_\varrho^2+(\boldsymbol{k}-\boldsymbol{k}')^2+\alpha(\boldsymbol{k},\boldsymbol{k}')]^{-1}. \tag{2}$$

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[2] The details of calculation depend only on the vector nature of mesons and we shall consider the ϱ meson for illustration.

The nonlocal term α is negligible for small momenta $\boldsymbol{k}, \boldsymbol{k}'$. The nucleon's spinors and isospinors are denoted by u and χ, respectively. The form factor F is chosen to be

$$F = \mu^{\nu}[\mu^2 + (\boldsymbol{k}-\boldsymbol{k}')^2 + \alpha(\boldsymbol{k}, \boldsymbol{k}')]^{-\frac{\nu}{2}}. \tag{3}$$

The exponent ν will be determined later, from the study of the quark substructure of the deuteron current.

The asymptotic tail of the wf $\Phi(\boldsymbol{k})$ is governed by the propagator G_0 and the meson exchange potential, containing the square of the form factor F. On top of that there is the spin and isospin structure of the wf. The isospin structure of the deuteron wf is trivially

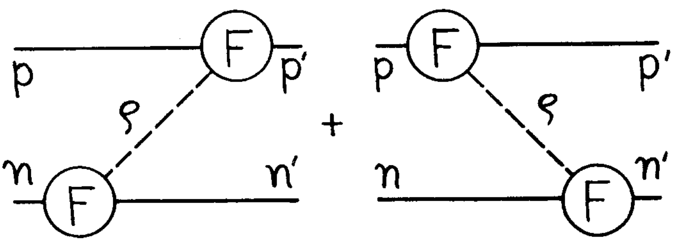

Fig. 1. The Weinberg kernel for the ρ meson exchange

accounted for by the isospin Pauli matrix $i\tau_2$. The approximate spin structure of the deuteron wf is in the form of a constant matrix B, which satisfies the following equation[3]

$$B = \tfrac{1}{4}\gamma^{\mu}(1+\gamma^0)B(1+\gamma^0)\gamma_{\mu}^T.$$

This equation has a solution

$$B = \begin{pmatrix} b & 0 \\ 0 & -b \end{pmatrix},$$

where b is a 2×2 symmetric matrix. From the Schrödinger $\boldsymbol{S}$ state wf at the low momentum $\boldsymbol{k}$ we find $b = \boldsymbol{\sigma}\boldsymbol{S}i\sigma_2$, where the vector $\boldsymbol{S}$ describes the deuteron's spin state. The whole deuteron wf $\Phi(\boldsymbol{k})$ is

$$\Phi(\boldsymbol{k}) = \bar{u}_{\alpha}(p)\bar{u}_{\beta}(n)\,(S\gamma C)_{\alpha\beta}\chi^{\dagger}_{pr}\chi^{\dagger}_{ns}(i\tau_2)_{rs}\varphi_{\mathrm{M}}(\boldsymbol{k}) \tag{4}$$

where the function $\varphi_{\mathrm{M}}(\boldsymbol{k})$ satisfies the scalar Weinberg equation. We take $\varphi_{\mathrm{M}}(\boldsymbol{k})$ in the form of two Hulthen wfs

$$\varphi_{\mathrm{M}}(\boldsymbol{k}) = \frac{Nm^2}{m\varepsilon+\boldsymbol{k}^2}\left(\frac{a}{\mu_1^2+\lambda\boldsymbol{k}^2}+\frac{1-a}{\mu_2^2+\lambda\boldsymbol{k}^2}\right)\left(\frac{\mu^2}{\mu^2+\lambda\boldsymbol{k}^2}\right)^{\nu}, \tag{5}$$

$$\lambda = 1+|k_3|\,(m^2+\boldsymbol{k}^2)^{-\frac{1}{2}},$$

which incorporates the nonrelativistic knowledge of deuteron and has the high energy tail obtained from the Weinberg equation.

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[3] For the wave function concentrated in the region of small relative momenta the sum over spins of nucleon is well approximated by $1/2(1+\gamma^0)$.

The model of the wf $\Phi(\boldsymbol{k})$ given by Eqs. (4) and (5) has the following properties:
1. it is a superposition of two Hulthen wfs,
2. for small momentum $\boldsymbol{k}$ it reduces to the Hulthen-Sugawara soft core S state wf of deuteron [7] with its scalar factor represented by the first term in the bracket in Eq. (5),
3. the second term in the bracket in Eq. (5) represents a contribution of heavy vector meson exchanges; $\mu_2^2 \gg \mu_1^2$,
4. it contains the important form factor describing the composite structure of nucleons,
5. it has the exact asymptotic behaviour of the Weinberg wf. The factor λ originates from the terms denoted by α in Eqs. (2) and (3). The normalization constant N is determined from the condition

$$16m^2 \int \frac{d^3k}{(2\pi)^3} (m^2+\boldsymbol{k}^2)^{-\frac{1}{2}} \varphi_{\mathrm{M}}^2(\boldsymbol{k}) = 1, \tag{6}$$

which follows from the Weinberg equation if we neglect the small energy variation of the Weinberg potential.

3. The deuteron current

We calculate the probability amplitude $\mathfrak{M}_{\mathrm{eD}}$ for elastic electron-deuteron scattering using the old-fashioned light front perturbation theory [3]. The electron transfers a four-momentum $q, (q^2 \equiv t)$ to the deuteron by the emission of the virtual photon in the electron-photon vertex. The sum of all orders of the electron-photon vertex relatively to other vertices gives us the amplitude $\mathfrak{M}_{\mathrm{eD}}$ in the factorized form[4]

$$\mathfrak{M}_{\mathrm{eD}} = j_{\mathrm{e}}^{\mu} \frac{1}{q^2} j_{\mathrm{D}\mu}.$$

In the Breit frame of reference

$$D = (D^-, D^+, -\boldsymbol{q}^{\perp}/2), \quad q = (0, 0, \boldsymbol{q}^{\perp}), \quad D' = D+q$$

the $\mathrm{N\overline{N}}$ pairs do not contribute to the current j_{D} because $q^+ = 0$ [3].

We focus our attention on two main contributions to the deuteron current, depicted in Fig. 2. The contribution of the nucleon impulse approximation (Fig. 2a) is

$$\frac{1}{e} j_{\mathrm{D}}^{\mu} = \int \frac{d^2k^{\perp}dk^+}{16\pi^3} \frac{D^{+2}}{p^{+2}} \frac{D^+}{n^+} \varphi_{\mathrm{D'}}(\boldsymbol{k}')\varphi_{\mathrm{D}}(\boldsymbol{k}) \,\mathrm{Tr}\, [S'^*\gamma(p\gamma'+m)I^{\mu}(p\gamma+m)S\gamma(n\gamma-m)], \tag{7}$$

where

$$I^{\mu} = F_{1S}(q^2)\gamma^{\mu} + \frac{1}{4m}(q\gamma\gamma^{\mu}-\gamma^{\mu}q\gamma)F_{2S}(q^2)$$

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[4] The instantaneous Coulomb-like interaction is included and the deuteron current has to be conserved, $qj_{\mathrm{D}} = 0$.

is the nucleon isoscalar elm. current matrix (Section 4). The deuteron wfs $\varphi_D(\boldsymbol{k})$ and $\varphi_{D'}(\boldsymbol{k}')$ are properly boosted [3, 5] from the deuteron rest frame to the Breit frame, Eq. (A.2). The tricky calculation presented in the Appendix recovers the peculiar structure of the current j_D leading to the standard expression for the cross section.

The asymptotic falloff of the deuteron elastic form factor $F_d(t)$ calculated from the nucleon impulse approximation is $F_d(t) \sim t^{-2-\nu}$, where ν is the exponent of the nucleon--vector meson form factor. The contribution of the Brodsky mechanism, i.e. the reduced

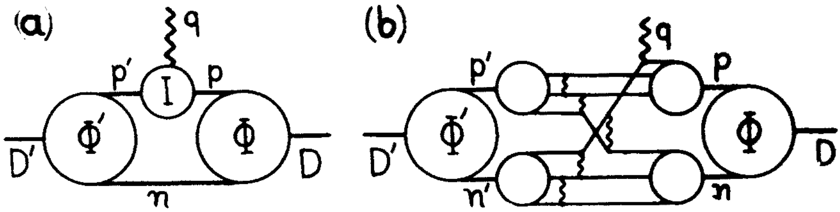

Fig. 2. Contributions to the deuteron current: (*a*) from the nucleon impulse approximation, and (*b*) from the generalized photon-gluon Compton scattering on the struck quark in the reduced form factor picture

form factor mechanism of Fig. 2b, to the deuteron current has asymptotic behaviour [1] $F_d(t) \sim t^{-5}$. According to the quark counting rules this is the slowest possible falloff of the deuteron form factor. The nucleon impulse approximation may not decrease more slowly.

Thus $\nu \geqslant 3$, and the deuteron wf from Eq. (4) is strongly suppressed for the large relative momenta of nucleons, in comparison with the Schrödinger wf. The question whether this damping excludes a saturation of the experimental data by the nucleon impulse approximation itself is answered in the next section.

4. *Numerical analysis of the nucleon impulse approximation*

The Rosenbluth functions A and B are known experimentally for $|t|$ from zero up to 8 GeV^2, and 1 GeV^2, respectively. In Figs. 3 and 4 we present the results for functions A and B for momentum transfers $|t| \lesssim 1$ GeV^2. The qualitative agreement with the data (modulo D state probability) originates from the equality between the Weinberg and the Schrödinger wfs for low relative momenta. The parameter μ_1^2 is equal to 0.05 GeV^2 as in the Hulthen-Sugawara wf [7].

For generality we present results for three standard versions of the nucleon elm isoscalar form factors F_{1S} and F_{2S}, labelled in figures as follows:

Dipole — empirical dipole formula

IJL — vector dominance fit [8]

Dipole + ($F_{1n} = 0$) — dipole fit with the Dirac neutron form factor set equal to zero [9]. This variety of possibilities is a consequence of the lack of independent constraints on the neutron structure.

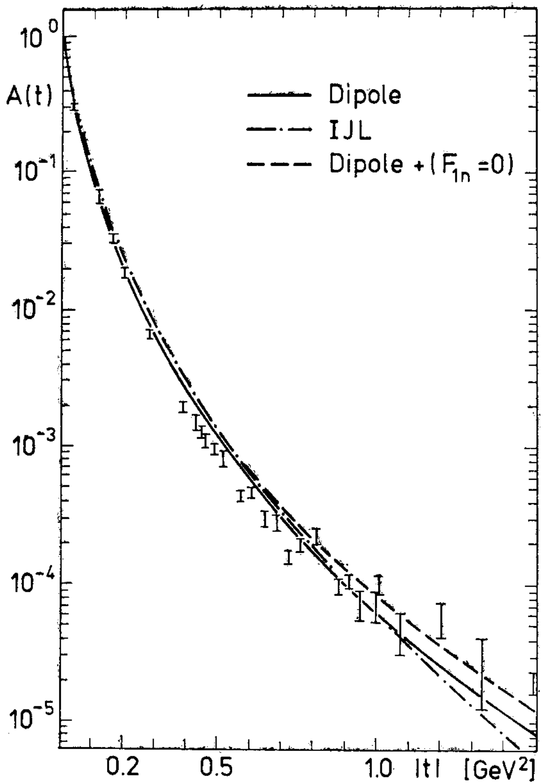

Fig. 3

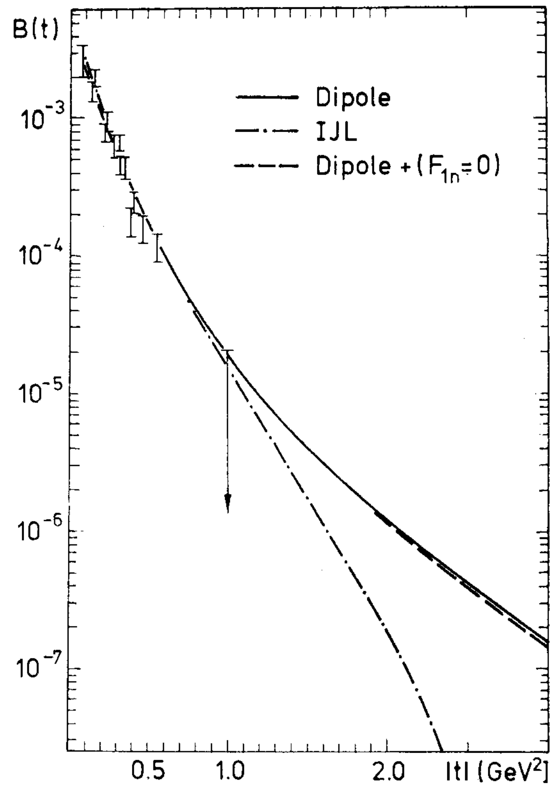

Fig. 4

Fig. 3. The Rosenbluth structure function A compared with experiment. The curves correspond to the different fits to nucleon electromagnetic form factors. See details in the text. The data points are from the Refs. [17–22]

Fig. 4. The Rosenbluth structure function B compared with experiment. The data points are from the Refs. [23, 24]

The important test of the two nucleon impulse approximation is provided by the function A for high momentum transfers (Fig. 5). The parameters ν, μ^2 and μ_2^2 in the wf, Eq. (5), play the decisive role in obtaining an agreement with the data, independently from the neutron ambiguities. To estimate the upper limit of the impulse contribution to the deuteron current we choose for the parameter ν the lowest allowed value $\nu = 3$. For this value of ν the parameter μ^2 is chosen to be 4 GeV^2. The lower values of μ^2 would provide

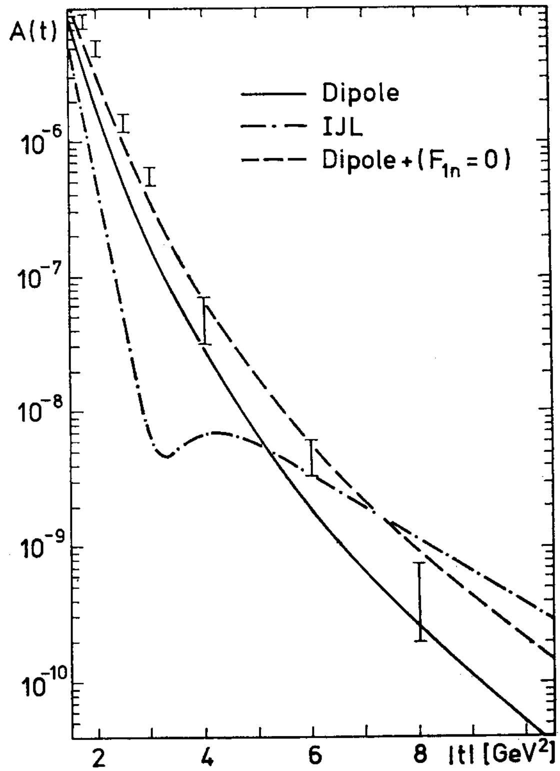

Fig. 5. Structure function A compared with experiment at high momentum transfers. The data points are from the Refs. [16, 22]

a faster damping. This choice of ν and μ^2 is on the limit of agreement with a phenomenology of the meson-nucleon form factors [9, 10]. Then the parameter μ_2^2 has to be much larger than the square of the ϱ meson mass. The curves given in Fig. 5 are for $\mu_2^2 = 2$ GeV^2 and $a = 0.07$.

In principle the heavy vector mesons would be responsible for the flattening of the function A, but their masses should be about twice larger than the ϱ meson mass. Although our estimates are approximate, nevertheless they suggest that the new mechanism of the photon absorption has to be included in the deuteron current. The most natural candidate for it is the reduced form factor mechanism of Brodsky, which is the quark prototype of the meson exchange currents [11]. This is the proper way to take into account six quarks in the electron-deuteron scattering. The external photon picks up the quark and calls for the six quark intermediate state in the deuteron current.

5. *Conclusion*

The light front dynamics provides a consistent description of the deuteron, invariant under three Lorentz boosts. The deuteron wf is determined on the basis of the Weinberg equation with spin. The unique resemblance of the Weinberg equation and the Schrödinger equation is used for the incorporation of the nonrelativistic knowledge about the deuteron. The tail of the wf is determined by vector mesons. The quark substructure of the deuteron current implies that the relativistic generalization of the Schrödinger wf has to contain factors accounting for the nucleon-meson form factor. Then the nucleon impulse approximation needs masses of vector mesons about twice larger than the ϱ meson mass, to saturate the data for large t. This result implies the other contributions suggested by the quark substructure, play considerable role in the deuteron current at momentum transfers of the order of 8 GeV^2.

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APPENDIX

This Appendix contains the main steps of our light front procedure of extracting the deuteron form factors from the Eq. (7). It differs from the procedures used in the Refs. [9, 11–15]. Denote

$$x = \tfrac{1}{2}\, k_3(m^2+\boldsymbol{k}^2)^{-\frac{1}{2}} = k^+/D^+$$

and change variable $\boldsymbol{k}^\perp$ in the integral from Eq. (7) to

$$\boldsymbol{n}^\perp = \tfrac{1}{2}\boldsymbol{D}^\perp - \boldsymbol{k}^\perp,$$

where n denotes the momentum of a passive nucleon. For two polarizations of the deuteron [9] boosted to the Breit frame of reference

$$S_\pm = (S^- = 2\boldsymbol{D}^\perp \boldsymbol{S}^\perp_\pm/D^+, S^+ = 0, \boldsymbol{S}^\perp_\pm), \quad \boldsymbol{S}^\perp_\pm = \frac{-1}{\sqrt{2}}(\pm 1, i),$$

we have

$$Sn = \tfrac{1}{2}\left(\tfrac{1}{2}-x\right)\cdot Sq - \boldsymbol{S}^\perp \boldsymbol{n}^\perp,$$

$$S'^*n = -\tfrac{1}{2}\left(\tfrac{1}{2}-x\right)\cdot S'^*q - \boldsymbol{S}'^{*\perp}\boldsymbol{n}^\perp. \tag{A.1}$$

Introduce

$$\boldsymbol{k}^\perp_\pm = \boldsymbol{n}^\perp \pm \tfrac{1}{2}\left(\tfrac{1}{2}-x\right)\boldsymbol{q}^\perp$$

and

$$k_{\pm}^{2} = (k_{\pm}^{\perp 2}+4x^{2}m^{2})(1-4x^{2})^{-1},$$

then the boosted wfs are

$$\varphi_{D'}(k')\varphi_{D}(k) = \varphi_{M}(k_{+})\varphi_{M}(k_{-}), \quad \lambda = 1+2|x|, \tag{A.2}$$

and we have the following identity for any four-vector a

$$\int d^{2}n^{\perp}\varphi_{D'}(k')\varphi_{D}(k)(a^{\perp}n^{\perp})n^{\perp} \equiv \int d^{2}n^{\perp}\varphi_{D'}(k')\varphi_{D}(k)$$

$$\times\left[a^{\perp}\left(n^{\perp 2}+\frac{(qn)^{2}}{q^{2}}\right)+q^{\perp}\frac{aq}{q^{2}}\left(n^{\perp 2}+2\frac{(qn)^{2}}{q^{2}}\right)\right].$$

Using this identity and Eq. (A.1) for $\mu = +, \perp$ components of the deuteron current from Eq. (7), we calculate the trace and recover a standard Lorentz structure [9]

$$\frac{-1}{e} j_{D}^{\mu} = G_{1}(t)\cdot S'^{*}S\cdot(D'+D)^{\mu}+G_{2}(t)\cdot(S^{\mu}\cdot qS'^{*}-S'^{*\mu}\cdot qS)$$

$$-G_{3}(t)\frac{qS'^{*}\cdot qS}{2M^{2}}(D'+D)^{\mu}. \tag{A.3}$$

The form factors $G_i(t)$ are expressed as integrals of lengthy expressions multiplied by the product of wfs and the nucleon isoscalar form factors. For $\mu = -$ component and the third deuteron polarization S_0, the deuteron current has to have the same structure with the form factors $G_i(t)$ evaluated in Eq. (A.3). Thus the cross section for the elastic electron-deuteron scattering is expressed by the two Rosenbluth structure functions A and B simply related to the form factors $G_i(t)$ [9]

$$\frac{d\sigma}{d\Omega_{e'}} = \left(\frac{d\sigma}{d\Omega_{e'}}\right)_{NS}\left(A(t)+B(t)\,\mathrm{tg}\,\frac{\Theta_{e'}}{2}\right) \equiv \left(\frac{d\sigma}{d\Omega_{e'}}\right)_{NS} F_{d}^{2}(t).$$

REFERENCES

[1] S. J. Brodsky, B. T. Chertok, *Phys. Rev.* **D14**, 3003 (1976).

[2] R. G. Arnold, C. E. Carlson, F. Gross, *Phys. Rev. Lett.* **38**, 1516 (1977).

[3] T.-M. Yan, *Phys. Rev.* **D7**, 1780 (1973); J. F. Gunion, S. J. Brodsky, R. Blankenbecler, *Phys. Rev.* **D8**, 287 (1973); G. P. Lepage, S. J. Brodsky, *Phys. Rev.* **D22**, 2157 (1980).

[4] J. M. Namysłowski, *Phys. Rev.* **D18**, 3676 (1978); P. M. Fishbane, J. M. Namysłowski, *Phys. Rev.* **D21**, 2406 (1980).

[5] L. Susskind, in: Lectures in Theoretical Physics Vol. 11D, *Mathematical Methods in Theoretical Physics*, edited by K. T. Mahanthappa and W. F. Brittin, Gordon and Breach, New York 1969; St. Głazek, The connection between the light front dynamics and the infinite momentum frame, Warsaw University preprint, IFT/15/82, December 1982.

[6] S. Weinberg, *Phys. Rev.* **150**, 1313 (1966).

[7] L. Hulthen, M. Sugawara, *Encyclopedia of Physics* 39, p. 92, Springer 1957.

[8] F. Iachello, A. Jackson, A. Lande, *Phys. Lett.* **43B**, 191 (1973).

[9] R. G. Arnold, C. E. Carlson, F. Gross, *Phys. Rev.* **C21**, 1426 (1980).
[10] M. Gari, H. Hyuga, *Nucl. Phys.* **A264**, 409 (1976).
[11] M. Chemtob, CEN-Saclay preprint, DPh-T/115, December 1981.
[12] A. Fernandez-Pacheco, J. A. Grifols, T. Schmidt, *Nucl. Phys.* **B151**, 518 (1979).
[13] L. L. Frankfurt, M. I. Strikman, *Nucl. Phys.* **B148**, 107 (1979).
[14] M. Chemtob, *Nucl. Phys.* **A336**, 299 (1980).
[15] L. L. Frankfurt, M. I. Strikman, *Phys. Rep.* **76**, No. 4 (1981).
[16] R. G. Arnold et al., Contrib. to IX Intern. Conference on High Energy Physics and Nuclear Structure, Versailles 1981.
[17] D. J. Drickey, L. N. Hand, *Phys. Rev. Lett.* **9**, 521 (1962).
[18] D. Benaksas, D. Drickey, D. Frerejacque, *Phys. Rev.* **148**, 1327 (1966); *Phys. Rev. Lett.* **13**, 353 (1964).
[19] S. Galster et al., *Nucl. Phys.* **B32**, 221 (1971).
[20] J. I. Friedman, M. W. Kendall, P. A. Gram, *Phys. Rev.* **120**, 992 (1960).
[21] J. Elias et al., *Phys. Rev.* **177**, 2075 (1969).
[22] R. G. Arnold et al., *Phys. Rev. Lett.* **35**, 776 (1975).
[23] R. E. Rand et al., *Phys. Rev.* **D8**, 3229 (1973).
[24] F. Martin et al., *Phys. Rev. Lett.* **38**, 1320 (1977).