LINEAR RESPONSE OF NUCLEAR MATTER WITH TENSOR FORCES*

By P. Haensel**

Département de Physique Théorique, CEN Saclay, 91191 Gif-sur-Yvette Cedex, France

and A. J. Jerzak

N. Copernicus Astronomical Center, Polish Academy of Sciences, Warsaw***

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The dynamic form factor of nuclear matter in spin and spin-isospin channels is calculated with proper inclusion of tensor forces. The calculation is performed in the long wavelength limit, using the methods of the theory of normal Fermi liquid. Numerical results, obtained for the quasiparticle interaction derived from the realistic nucleon-nucleon potentials show that tensor forces may appreciably modify the linear response of nuclear matter in the spin channel. In particular, for some models of the quasiparticle interaction, the presence of tensor forces leads to instability of the standard ground state of nuclear matter with respect to some small amplitude spin-dependent perturbations.

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1. Introduction

A reliable theoretical description of the properties of infinite nuclear matter is thought to be a first step towards a many-body theory of nuclei. Up to now, most of theoretical effort has been concentrated on the simplest problem, that of the calculation of the ground state properties of nuclear matter starting from an assumed nucleon-nucleon interaction [1].

Incomparably less attention has been paid to the excitation spectrum of infinite nuclear matter. The small amplitude excitations of nuclear matter may be studied (neglecting possible effects of superfluidity) using the general methods of the theory of linear response

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** On leave of absence from N. Copernicus Astronomical Center, Polish Academy of Sciences, Warsaw, Poland.

*** Address: Centrum Astronomiczne M. Kopernika, Polska Akademia Nauk, Bartycka 18, 00-716 Warszawa, Poland.

for normal Fermi liquids [2]. The central quantity in this theory is dynamic form factor which describes linear response of the system to external perturbations. The knowledge of the nuclear matter response function is relevant for the studies of such problems as, e.g., large momentum transfer electron scattering on nuclei, Gamow Teller mode of collective excitation, or hypothetical pion condensation in dense nuclear matter. Finally, the dynamic form factor of nuclear matter is an essential ingredient for the calculation of the diffusion of neutrinos in dense hot matter in the interior of neutron stars and in the central region of supernovae.

In the present paper we shall restrict ourselves to the spin-dependent, long wavelength excitations in nuclear matter. Our calculation will be performed within the framework of the theory of normal Fermi liquids. This problem has been considered previously by other authors. Gogny and Padjen [3] studied the dynamic form factor of nuclear matter, using the Fermi liquid parameters calculated in the Hartree-Fock approximation from the semi-empirical nuclear forces. Similar problem has been studied by Alberico et al. [4] using a broad selection of the Skyrme interactions. Particular emphasis in Ref. [4] has been put on obtaining analytical expressions for the response function, the sum rules and the relative strength of the modes of excitations.

In the calculations of the dynamic form factor of nuclear matter, reported in Refs. [3, 4] (and in most of other existing calculations of this quantity) the tensor component of the nucleon-nucleon interaction has been usually neglected. One of the few exceptions from this rule is the work of Kohno [5], who calculated the effect of tensor force on the lowest-order perturbation contribution to the Fermi gas response function in nuclear matter. However, his calculation has been performed for the large momentum transfer, which is relevant to inelastic electron scattering. Hence, he did not consider the spin and spin-isospin sound modes of collective excitation, appearing in the long wavelength region (i.e., for small momentum transfer). For the tensor force he used the effective nucleon--nucleon interaction parametrized by Sprung [6]. This interaction is based on the reaction matrix in nuclear matter, derived from the Reid soft-core potential.

In the last years several sets of the Fermi liquid parameters of nuclear matter have been calculated starting from the realistic nucleon-nucleon interaction [7, 8]. One of important conclusions, stemming from the results reported in Ref. [7], is that tensor forces may play an important role for the spin dependent excitations in nuclear matter. Tensor forces were included in the paper of Friman and Haensel [9], who studied however only the collective spin-dependent excitations (discrete spectrum). In the present work we study the influence of tensor quasiparticle interaction on the whole spectrum of long wavelength spin-dependent excitations in nuclear matter.

In the next section we recall basic equations of the linear response theory and introduce the notation. The Fermi liquid theory calculation of the dynamic form factors is described in Sects. 3 and 4. In Sect. 5 we briefly present the models of the quasiparticle interaction in nuclear matter, derived from realistic nucleon-nucleon potentials. Numerical results are described in Sect. 6, where, in particular, we study the stability of the ground state of nuclear matter with respect to the spin-dependent perturbations. Finally, Sect. 7 contains a discussion of results and conclusion.

2. *Linear response of nuclear matter to the spin dependent perturbations*

In what follows, we shall consider, for the sake of simplicity, the isospin independent perturbations. Let us considei the linear response of symmetric, unpolarized nuclear matter at $T = 0$ K to external perturbation, which is assumed to be described by the hamiltonian

$$H^{\text{ext}} = \int d^3r\, V(\boldsymbol{r}, t) \cdot \boldsymbol{\sigma}(\boldsymbol{r}) + \text{hermitian conjugate}, \tag{1}$$

where the spin density operatoı

$$\boldsymbol{\sigma}(\boldsymbol{r}) = \sum_i \boldsymbol{\sigma}_i \delta(\boldsymbol{r} - \boldsymbol{r}_i). \tag{2}$$

Assuming

$$V(\boldsymbol{r}, t) = V e^{i(\boldsymbol{k} \cdot \boldsymbol{r} - \omega t)}$$

we get, passing to the momentum representation,

$$H^{\text{ext}}(t) = \boldsymbol{V} \cdot \boldsymbol{\sigma}(-\boldsymbol{k}) e^{-i\omega t} + \text{h.c.}, \tag{3}$$

where

$$\boldsymbol{\sigma}(\boldsymbol{k}) = \int d^3 r e^{-i\boldsymbol{k} \cdot \boldsymbol{r}} \boldsymbol{\sigma}(\boldsymbol{r}). \tag{4}$$

The probability (per unit time and volume) that the perturbing external field transfers the momentum $\hbar \boldsymbol{k}$ and the energy $\hbar\omega$ to the system can be calculated using the Fermi Golden Rule, as

$$P(\boldsymbol{k}, \omega) = \frac{2\pi}{\hbar^2} \sum_{l,j} S_{lj}(\boldsymbol{k}, \omega) V_l V^{j*}, \tag{5}$$

whre l, j are cartesian indices of vector or tensor quantities, and the dynamic form factor for spin-density fluctuations is defined by

$$S_{lj}(\boldsymbol{k}, \omega) = \frac{1}{\Omega} \sum_n \langle n | \sigma^l(-\boldsymbol{k}) | 0 \rangle \cdot \langle 0 | \sigma^j(\boldsymbol{k}) | n \rangle \delta(\omega - \omega_{n0}). \tag{6}$$

Here, Ω is the volume of the system, $|n\rangle$ are the eigenstates of the unperturbed hamiltonian ($H^0 |n\rangle = E_n |n\rangle$) and

$$\omega_{n0} = (E_n - E_0)/\hbar. \tag{7}$$

In general, the tensor S_{ij} can be split into a purely real symmetric component

$$S_{lj}^{(s)} = \tfrac{1}{2}[S_{lj} + S_{jl}] \tag{8}$$

and a purely imaginary antisymmetric part

$$S_{lj}^{(a)} = \tfrac{1}{2}[S_{lj} - S_{jl}]. \tag{9}$$

In consequence of the invariance of the nuclear hamiltonian with respect to space inversion and time reversal, one has

$$S_{lj}(\boldsymbol{k}, \omega) = S_{lj}^{(s)}(\boldsymbol{k}, \omega) \tag{10}$$

and, consequently,

$$P(\boldsymbol{k}, \omega) = \frac{2\pi}{\hbar^2} \sum_{lj} S_{lj}^{(s)}[V^l V^{j*}]^{(s)}. \tag{11}$$

From now on, the index (s) will be omitted.

We define the spin density — spin density response tensor by

$$\sum_j \chi_{lj}(\boldsymbol{k}, \omega) V^j = \langle \sigma^l \rangle_{\boldsymbol{k}\omega}, \tag{12}$$

where

$$e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} \langle \sigma^l \rangle_{\boldsymbol{k}\omega} + \text{c.c.} \equiv \langle \psi(t) | \, \sigma^l(\boldsymbol{r}) \, | \psi(t) \rangle \tag{13}$$

and the (perturbed) wavefunction $\psi(t)$ is calculated using first order time dependent perturbation theory for $H = H^0 + H^{\text{ext}}(t)$. Assuming that nuclear matter was in its ground state in the remote past ($\psi(t) = e^{-\frac{i}{\hbar}E_0 t} |0\rangle$ for $t \to -\infty$) and using the symmetry properties of the unperturbed hamiltonian we can relate the $\chi_{lj}(\boldsymbol{k}, \omega)$ and $S_{lj}(\boldsymbol{k}, \omega)$ tensors through the fluctuation-dissipation theorem

$$\operatorname{Im} \chi_{lj}(\boldsymbol{k}, \omega) = -\frac{\pi}{\hbar} [S_{lj}(\boldsymbol{k}, \omega) - S_{lj}(\boldsymbol{k}, -\omega)]. \tag{14}$$

The theory can be easily generalized to the case of the spin and isospin dependent perturbations of the form

$$H^{\text{ext}} = \sum_{\alpha=1}^{3} V^\alpha e^{-i\omega t} \sum_i \sigma_i \tau_i^\alpha e^{i\boldsymbol{k}\cdot\boldsymbol{r}_i} + \text{h.c.} \,. \tag{15}$$

In the present paper we shall restrict ourselves to the case of symmetric nuclear matter and hence the quantity $S_{lj}^{\alpha\beta}(\boldsymbol{k}, \omega)$, defined by an obvious generalization of Eq. (6), will be proportional to the unit matrix in the isospin space, $S_{lj}^{\alpha\beta} = \delta_{\alpha\beta} S'_{lj}$.

3. *Elements of the Fermi liquid theory of nuclear matter*

The Fermi liquid theory, formulated by Landau [10–12], is semiphenomenological in nature and describes infinite fermion systems near $T = 0$ K. A basic assumption is that the system is normal. Namely, it must have the property that, when the interaction is slowly turned on, the ground state of the system of noninteracting particles develops adiabatically into a ground state of weakly interacting quasiparticles. This system of quasiparticles retains the essential properties of an ideal Fermi gas, e.g., the ground state distribution function for the quasiparticles is the same as for an ideal gas. In the case of nuclear matter at $T = 0$ K, the quasiparticle distribution matrix in the ground state, $n_{\boldsymbol{p}}^0$, reads

$$n_{\boldsymbol{p}}^0 = \theta(p_\mathrm{F}-p) \times (\text{unit } 2\times 2 \text{ matrix in spin and isospin space}). \tag{16}$$

The Fermi momentum p_F is related to the equilibrium nucleon density by $\varrho^0 = 2p_\mathrm{F}^3/3\pi^2\hbar^3$.

The Fermi liquid theory establishes a one-to-one correspondence between low lying excitations in the system of quasiparticles and those in the real system. The theory is valid only when the number of excited quasiparticles is small compared with the total number of particles. In what follows we shall restrict ourselves, for the sake of simplicity, to the case of static and space homogeneous excitations. The total energy E of the system is a functional of the quasiparticle distribution matrix $n_{\boldsymbol{p}}$. However, for a small number of excited quasiparticles, the excitation energy (per unit volume) implied by a small deviation $\delta n_{\boldsymbol{p}} = n_{\boldsymbol{p}} - n_{\boldsymbol{p}}^0$ from the ground state distribution matrix is given by the simple formula, including only terms quadratic in $\delta n_{\boldsymbol{p}}$,

$$\delta E = \mathrm{Tr}_\sigma\, \mathrm{Tr}_\tau \int \frac{d^3p}{(2\pi\hbar)^3}\,(e_{\boldsymbol{p}}^0 + U_{\boldsymbol{p}}(\boldsymbol{\sigma}\cdot\boldsymbol{\tau}))\delta n_{\boldsymbol{p}}(\boldsymbol{\sigma},\boldsymbol{\tau})$$

$$+\tfrac{1}{2}\,\mathrm{Tr}_{\tau_1,\tau_2}\mathrm{Tr}_{\sigma_1,\sigma_2}\int \frac{d^3p_1}{(2\pi\hbar)^3}\int \frac{d^3p_2}{(2\pi\hbar)^3}\cdot f(\boldsymbol{p}_1,\boldsymbol{p}_2)\delta n_{\boldsymbol{p}_1}(\boldsymbol{\sigma}_1,\boldsymbol{\tau}_1)\delta n_{\boldsymbol{p}_2}(\boldsymbol{\sigma}_2,\boldsymbol{\tau}_2) \tag{17}$$

where $e_{\boldsymbol{p}}^0$ is the quasiparticle energy in the absence of other elementary excitations and $U_{\boldsymbol{p}}$ describes the effect of a time and space independent but possibly spin and/or isospin dependent external field applied to the system. The matrix structure of $\delta n_{\boldsymbol{p}}$ has been indicated using Pauli matrices in the spin and isospin spaces, $\boldsymbol{\sigma}_i$, $\boldsymbol{\tau}_i$. The quantity $f(\boldsymbol{p}_1, \boldsymbol{p}_2)$, appearing in Eq. (17), plays a central role in the Fermi liquid theory. This is the quasiparticle interaction, which determines the properties of the low lying excited states of nuclear matter as well as many of its static properties [13].

The quasiparticle interaction in symmetric unpolarized nuclear matter is assumed to be of the form [14]

$$N_0 f(\boldsymbol{p}_1,\boldsymbol{p}_2) = F + F'\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + G\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$

$$+ G'(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) + q^2/p_\mathrm{F}^2[H + H'(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)]S_{12}(\hat{\boldsymbol{q}}), \tag{18}$$

where N_0 is the density of quasiparticle states at the Fermi surface,

$$N_0 = \frac{2m^* p_\mathrm{F}}{\pi^2\hbar^3}, \tag{19}$$

and m^* is the quasiparticle effective mass at the Fermi surface, $p_\mathrm{F}/m^* = (\partial e_p^0/\partial p)_{p=p_\mathrm{F}}$. The tensor operator, coupling quasiparticle momenta to their spins, is defined as

$$S_{12}(\hat{\boldsymbol{q}}) = 3(\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{q}})(\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{q}}) - \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2, \tag{20}$$

where $\boldsymbol{q} = \boldsymbol{p}_1 - \boldsymbol{p}_2$.

Within the approximations of the theory, the quasiparticle momenta are restricted to the Fermi surface, $\boldsymbol{p}_i = p_F \boldsymbol{n}_i$. The quantities F, F', G, G', H and H' depend thus only on the cosine of the angle between $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, $\cos\theta_{12} = \boldsymbol{n}_1 \cdot \boldsymbol{n}_2$. This dependence is expanded in Legendre polynomials,

$$F(\boldsymbol{p}_1, \boldsymbol{p}_2) = \sum_l F_l P_l(\cos\theta_{12}), \tag{21}$$

with similar expansions for the remaining functions.

In what follows we shall restrict ourselves to the case of excitations in the spin channel. The expansion of the matrix $\delta n_p = n_p - n_p^0$ around the unperturbed Fermi surface reads then, including only first order terms,

$$\delta n_p = \frac{\partial n_p^0}{\partial e_p} w(\boldsymbol{n}), \tag{22}$$

where hermitian, traceless matrix $w(\boldsymbol{n})$ may be rewritten in a suitable form [7]

$$w(\boldsymbol{n}) = \boldsymbol{u}(\boldsymbol{n}) \cdot \boldsymbol{\sigma} = \sum_{\mu=-1}^{1} (-)^\mu u^\mu(\boldsymbol{n}) \sigma^{-\mu}.$$

Here, σ^{+1} and σ^{-1} are spin rising and spin lowering operators and $\sigma^0 = \sigma^z$. The part of the quasiparticle interaction relevant for the excitations in the spin channel is

$$\Delta\mathscr{F} = G\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + q^2/p_F^2 H S_{12}(\hat{\boldsymbol{q}}). \tag{23}$$

Let us define a rank two spherical tensor

$$\begin{aligned} \Delta\mathscr{F}_{\mu\mu'} &= \tfrac{1}{4} \mathrm{Tr}_{\sigma_1} \mathrm{Tr}_{\sigma_2}[\Delta\mathscr{F}(-)^{\mu'} \sigma_2^{-\mu'} \sigma_1^{\mu}] \\ &= (G - q^2/p_F^2 H)\delta_{\mu\mu'} + 3Hq^\mu q^{-\mu'}(-)^{\mu'}/p_F^2. \end{aligned} \tag{24}$$

In view of the symmetry properties of the quasiparticle interaction it will be particularly suitable to rewrite the equation of the Fermi liquid theory in the basis of the total angular momentum of the quasiparticle — quasihole pair, J [7]. The matrix elements of $\Delta\mathscr{F}$ in this basis are given by the equation

$$\begin{aligned} \Delta\mathscr{F}^J_{ll'} = \sum_{\substack{m\mu \\ m'\mu'}} (-)^{\mu+\mu'} (lm1-\mu|JM)\,(l'm'1-\mu'|JM) \\ \times \int d\boldsymbol{n}_1 \int d\boldsymbol{n}_2 Y^*_{lm}(\boldsymbol{n}_1) \Delta\mathscr{F}_{\mu\mu'} Y_{l'm'}(\boldsymbol{n}_2). \end{aligned} \tag{25}$$

The explicit formulae for the matrix elements $\Delta\mathscr{F}^J_{ll}$ may be found in Ref. [7].

The generalization to the case of the spin-isospin channel is straightforward, since the quasiparticle interaction is invariant under rotations in isospin space. One should only replace G_l, H_l by G'_l H'_l and use matrix w of the form

$$w(\boldsymbol{n}) = \sum_{\alpha=1}^{3} \boldsymbol{u}^\alpha(n) \cdot \boldsymbol{\sigma}\tau^\alpha. \tag{26}$$

4. Kinetic equation, its solution and the dynamical form factor

In what follows we shall restrict ourselves to the case of the perturbations in the spin channel. In order to calculate the quantity $\langle\boldsymbol{\sigma}\rangle_{k\omega}$, Eq. (13), we should first calculate the change in the quasiparticle distribution matrix, $\delta n_{\boldsymbol{p}}(\boldsymbol{r}, t)$, implied by the external perturbation described by the hamiltonian $H^{\text{ext}}(t)$, Eq. (3). In the $T \approx 0$ K region, where the frequency of collisions between the quasiparticles becomes negligible ($\omega_{\text{coll}} \ll \omega$: the collisionless regime), the kinetic equation for $n_{\boldsymbol{p}}(\boldsymbol{r}, t) = n_{\boldsymbol{p}}^0 + \delta n_{\boldsymbol{p}}(\boldsymbol{r}, t)$, after linearization in $\delta n_{\boldsymbol{p}}$, takes the form

$$\frac{\partial}{\partial t}\,\delta n_{\boldsymbol{p}} + (\nabla_{\boldsymbol{p}} e_{\boldsymbol{p}}^0)\cdot(\nabla_{\boldsymbol{r}}\delta n_{\boldsymbol{p}}) - (\nabla_{\boldsymbol{r}}\delta e_{\boldsymbol{p}})\cdot(\nabla_{\boldsymbol{p}} n_{\boldsymbol{p}}^0) = 0. \tag{27}$$

In the linear approximation, the change in the local quasiparticle energy implied by $H^{\text{ext}}(t)$ is calculated as

$$\delta e_{\boldsymbol{p}_1}(\boldsymbol{\sigma}_1, \boldsymbol{r}, t) = U_{\boldsymbol{p}_1}(\boldsymbol{\sigma}_1, \boldsymbol{r}, t) + \mathrm{Tr}_{\sigma_2}\int \frac{d^3 p_2}{(2\pi\hbar)^3}\, N_0^{-1}\Delta\mathcal{F}(\boldsymbol{p}_1, \boldsymbol{p}_2)\delta n_{\boldsymbol{p}_2}(\boldsymbol{\sigma}_2, \boldsymbol{r}, t), \tag{28}$$

where $\Delta\mathcal{F}$ is the quasiparticle interaction in the spin channel, Eq. (23), and

$$U_{\boldsymbol{p}}(\boldsymbol{\sigma}, \boldsymbol{r}, t) = \boldsymbol{V}\cdot\boldsymbol{\sigma} e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)}. \tag{29}$$

Let us notice, that in our case the potential energy U does not depend on the quasiparticle momentum. The kinetic equation, Eq. (27), will be solved in the long-wavelength limit ($k \ll p_{\text{F}}/\hbar$). The form of Eq. (27) implies the following Ansatz for the $\delta n_{\boldsymbol{p}}$ matrix,

$$\delta n_{\boldsymbol{p}} = \frac{\partial n_{\boldsymbol{p}}^0}{\partial e_{\boldsymbol{p}}}\, w(\boldsymbol{n}) e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)}, \tag{30}$$

where the functional dependence of the 2×2 matrix w on $\boldsymbol{n} = \boldsymbol{p}/p$ is to be found. The ground state of nuclear matter is assumed to be spin isotropic and hence it will be natural to choose $\boldsymbol{k}$ as our z axis (spin quantization axis). Inserting expressions (28–30) into Eq. (27) we obtain an inhomogeneous integral equation for the spherical components $u^{\mu}(n)$, Eq. (22),

$$(s - z)u^{\mu}(\boldsymbol{n}_1) - z\sum_{\mu'}\int \frac{d\boldsymbol{n}_2}{4\pi}\Delta\mathcal{F}_{\mu\mu'} u^{\mu'}(\boldsymbol{n}_2) = -zV^{\mu}, \tag{31}$$

where $z = \boldsymbol{n}_1\cdot\hat{\boldsymbol{k}}$, $\Delta\mathcal{F}_{\mu\mu'}$ is given by Eq. (24) and, to satisfy our assumption that nuclear matter had been in its ground state in the remote past ($t \to -\infty$), we have put $s = \lambda + i\eta$, with real $\lambda = m^*\omega/kp_{\text{F}}$ and η being an infinitesimal positive quantity ($\eta \to +0$).

We expand the angular dependence of the $u^{\mu}(n)$ functions in spherical harmonics [9],

$$u^{\mu}(n) = \frac{z}{s - z}\sum_{lm}\sqrt{\frac{4\pi}{2l+1}}\, u_{lm}^{\mu} Y_{lm}(\boldsymbol{n}). \tag{32}$$

This expansion is inserted into Eq. (31). Introducing the parameters C_{lJ}^{M} related to u_{lm}^{μ} through the linear transformation

$$C_{lJ}^{M} = \sum_{m\nu} (-)^{\nu}(lm1-\nu|JM)\, u_{lm}^{\nu}, \tag{33}$$

we reduce Eq. (31) to a system of linear equations for C_{lJ}^{M},

$$C_{lJ}^{M} - \sum_{J'l'l''} \Delta\mathscr{F}_{ll'}^{J}\alpha_{l'l''}^{JJ'M}(\lambda)C_{l''J'}^{M} = -(-)^{M}V^{-M}\delta_{J,1}\delta_{l,0}. \tag{34}$$

The matrices C_{lJ}^{M} corresponding to different M's are not coupled by the kinetic equation. The matrix elements $\Delta\mathscr{F}_{l'}^{J}$ are given by Eq. (25) and functions $\alpha_{ll'}^{JJ'M}(\lambda)$ are defined by

$$\alpha_{ll'}^{JJ'M}(\lambda) = \sum_{m\nu} (lm\,1-\nu|JM)\,(l'm1-\nu|J'M)\cdot\alpha_{ll'}^{m}(\lambda) \tag{35}$$

with

$$\alpha_{ll'}^{m}(\lambda) \equiv [(2l+1)\,(2l'+1)]^{-1/2}\int d\boldsymbol{n}\,\frac{z}{\lambda-z+i\eta}\cdot Y_{lm}(\boldsymbol{n})Y_{l'm}^{*}(\boldsymbol{n}). \tag{36}$$

Useful formulae for $\alpha_{ll'}^{m}(\lambda)$ are collected in the Appendix.

In the Fermi liquid theory the definition of the spin density — spin density response function can be rewritten in terms of the quantity $w(\boldsymbol{n})$ as

$$\sum_{\mu'} \chi_{\mu\mu'}(\boldsymbol{k},\omega)V^{\mu'} = \mathrm{Tr}_{\sigma}\int \frac{d^3p}{(2\pi\hbar)^3}\,\sigma^{\mu}w(\boldsymbol{n})\,\frac{\partial n_p^0}{\partial e_p} = -N_0\int\frac{d\,\boldsymbol{n}}{4\pi}\,u^{\mu}(\boldsymbol{n}). \tag{37}$$

In the linear approximation the quantities

$$\bar{C}_{lJ}^{M} = C_{lJ}^{M}/V^{-M} \tag{38}$$

are independent of $\boldsymbol{V}$. Using expansion (32) for $u^{\mu}(\boldsymbol{n})$ we obtain following formula for the diagonal elements of the $\chi_{\mu\mu'}$ tensor,

$$\chi_{\mu\mu}(\lambda) = -(-)^{\mu}N_0\sum_{lJ}\alpha_{l0}^{J1,-\mu}(\lambda)\,\bar{C}_{lJ}^{-\mu}, \tag{39}$$

where the parameters $\bar{C}_{lJ}^{-\mu}$ are to be calculated from a system of linear equations, resulting from Eq. (34),

$$\bar{C}_{lJ}^{-\mu} - \sum_{l'l''J'} \Delta\mathscr{F}_{ll'}^{J}\alpha_{l'l''}^{JJ',-\mu}(\lambda)\bar{C}_{l''J'}^{-\mu} = -(-)^{\mu}\delta_{J,1}\delta_{l,0}. \tag{40}$$

The solutions to this equation correspond to a fixed value of $M = -\mu = m-\nu$. With our assumption about the symmetry properties of the unperturbed hamiltonian of nuclear matter, the nondiagonal elements of the $\chi_{\mu\mu'}$ tensor vanish and $\chi_{+1,+1} = \chi_{-1,-1}$. The cartesian components of the spin density — spin density response function can be calculated from

$$\chi_{xx} = \chi_{yy} = \tfrac{1}{2}(\chi_{+1,+1}+\chi_{-1,-1}) = \chi_{+1,+1}, \qquad \chi_{zz} = \chi_{00}, \tag{41}$$

the nondiagonal elements being identically zero. Let us notice, that in the standard case of central quasiparticle interaction one would have $\chi_{xx} = \chi_{yy} = \chi_{zz}$.

In what follows we shall consider the case $\lambda \geqslant 0$, relevant for nuclear matter at $T = 0$. Then, for every $\lambda < 1$ there exists a nontrivial *complex* solution to Eq. (40), yielding a nonvanishing imaginary part of $\chi_{\mu\mu}$. The corresponding contribution to $S_{\mu\mu}(\lambda)$, coming from the single pair excitations (excited states $|n\rangle$ corresponding to one quasiparticle-one quasihole pair), will be denoted by

$$S^{\mathrm{s.p.}}_{\mu\mu}(\lambda) = \theta(1-\lambda)S_{\mu\mu}(\lambda). \tag{42}$$

For $\lambda > 1$ the only contribution to the imaginary part of $\chi_{\mu\mu}$ can result from the existence of nontrivial solutions to the homogeneous counterpart of Eq. (34). Such a nontrivial solution corresponds to an undamped zero-sound mode of collective excitation of nuclear matter in the spin channel, with $M = -\mu$.

Let us rewrite Eq. (40) in the form

$$\sum_{l'J'} A^{JJ'}_{ll'}(\lambda)\bar{C}^{-\mu}_{l'J'} = (-)^{\mu}\delta_{J,1}\delta_{l,0}, \tag{43}$$

where the $A^{JJ'}_{ll'}$ matrix is defined as

$$A^{JJ'}_{ll'}(\lambda) = \sum_{l''} \Delta\mathscr{F}^{J}_{ll''}\alpha^{JJ'-\mu}_{l''l'}(\lambda)-\delta_{ll'}\delta_{JJ'}. \tag{44}$$

The solution to Eq. (43) is given by the formula

$$\bar{C}^{-\mu}_{lJ} = M^{-\mu}_{lJ}(\lambda)/\det A(\lambda), \tag{45}$$

where the numerator is a determinant resulting from the application of the Cramer's rule.

The existence of an undamped zero-sound type mode of collective excitation, propagating at phase velocity $v_0 = p_F\lambda_0/m^*$ ($\lambda_0 > 1$), corresponds to

$$\det A(\lambda_0) = 0. \tag{46}$$

In the vicinity of λ_0

$$\det A(\lambda) \cong \frac{d}{d\lambda}\det A(\lambda)\Big|_{\lambda_0}(\lambda-\lambda_0). \tag{47}$$

In order to determine the behavior of C's in the vicinity of $\lambda = \lambda_0$, we put expression (47) into Eq. (45) and use the prescription $\lambda \to \lambda + i\eta$, in accordance with the asymptotic condition at $t \to -\infty$. Using the Dirac identity

$$\frac{1}{\lambda-\lambda_0+i\eta} = \mathrm{P}\frac{1}{\lambda-\lambda_0} - i\pi\delta(\lambda-\lambda_0) \tag{48}$$

we obtain the final formula for the collective mode contribution to dynamic form factor,

$$S^{\mathrm{coll.}}_{\mu\mu}(\lambda) = \mathscr{U}^{\mu}\hbar\delta(\lambda-\lambda_0), \tag{49}$$

where

$$\mathscr{U}^{\mu} = -N_0(-)^{\mu} \sum_{lJ} \alpha_{l0}^{J1,-\mu}(\lambda_0) \frac{M_{lJ}^{-\mu}(\lambda_0)}{\left.\dfrac{d}{d\lambda} \det A(\lambda)\right|_{\lambda_0}} . \tag{50}$$

Finally, let us remind once again, that in the presence of tensor forces projections of orbital angular momentum and spin on the z-axis (m and $-\nu$ in Eq. (33)) are no longer good quantum numbers. Instead, the excitations are labeled by $M = m-\nu$. Solution contributing to $S_{\mu\mu}$ and $\chi_{\mu\mu}$ are those from the $M = -\mu = 0, \pm 1$ channels, because only in these channels can the value $m = 0$ appear in expansion (32), giving rise to a non-vanishing value of $\langle \sigma^{\mu} \rangle$.

The generalization of the results obtained above to the case of the spin-isospin channel is straightforward and consists in replacing G_l, H_l by G_l', H_l', respectively.

5. *Quasiparticle interaction in nuclear matter*

So far no empirical information on the tensor components in the quasiparticle interaction in nuclear matter is available. We shall therefore use the theoretical results for spin dependent Fermi liquid parameters obtained by Bäckman et al. [7] and Jackson et al. [8], starting from the realistic nucleon-nucleon potentials.

The calculation of Bäckman et al. [7] has been based on perturbational techniques, the basic input being the Brueckner reaction matrix calculated from the Reid soft-core nucleon-nucleon potential [15]. The values of the spin dependent Fermi liquid parameters has been taken from Table 2 of Ref. [7]. In view of large uncertainties in perturbational results for m^* and for the renormalization of the quasiparticle pole, Z, we made the simplest choice $m^* = m$ and $Z = 1$. Also, we did not attempt to include the effects of the so-called induced quasiparticle interaction, for the following reasons. Firstly, in the calculation reported in Ref. [7] the tensor component of the induced interaction has not been constructed, neither the effects of the tensor part of the quasiparticle interaction on the central induced interaction have been included. Secondly, according to Ref. [7], the changes in values of Fermi liquid parameters implied by the inclusion of the induced interaction are, except for F_0, rather small.

The expansion in Legendre polynomials converges rapidly for the functions G and G', where the Fermi liquid parameters for $l > 2$ are small. Hence, we put G_l, $G_l' = 0$ for $l > 4$. However, for H and H' the convergence of Legendre expansion is rather slow. Hence, in order to get reliable results for the solution of Eq. (40) we must retain a large number of partial waves in the H, H' expansions. Actually, we put H_l, $H_l' = 0$ for $l > 8$. The set of spin dependent Fermi liquid parameters, described above, will be hereafter referred to as the RSCI one.

Very recently, the Fermi liquid parameters in nuclear matter have been calculated by Jackson et al. [8] using variational techniques, for the Reid soft-core and the Bethe--Johnson [16] nucleon-nucleon interactions. Actually, for technical reasons, they used the nucleon-nucleon potentials represented in the so-called v_6 form [17]. Their calculations have been performed within the correlated basis functions (CBF) scheme in which the initial approximation was the Jastrow type trial wave function. Then, the effects of the state dependent correlations have been included using second order CBF corrections. For technical reasons, only lowest ($l < 3$ for the central part and $l < 2$ for the tensor part) Fermi liquid parameters could be calculated with sufficient precision. We have put $G_l, G'_l = 0$ for $l > 2$. However, putting $H_l, H'_l = 0$ for $l > 1$ would be unreasonable in view of the slow convergence of the Legendre expansions. Hence, the values of H_l, H'_l for $1 < l < 9$ have been calculated using the one-pion exchange approximation but with a proper value of m^*/m in the density of states. As in the case of the RSCI model, we put $H_l, H'_l = 0$ for $l > 8$. The corresponding sets of the spin dependent Fermi liquid parameters for the Bethe-Johnson and the Reid soft-core interactions will be hereafter referred to as the BJ and RSCII ones, respectively.

6. *Results*

6a. Stability of the ground state

The stability of the ground state of nuclear matter implies that the free energy (per unit volume) has a *minimum* for $n_p = n_p^0$. The criteria of stability in the presence of tensor forces have been derived by Bäckman et al. [7]. In what follows, we shall restrict ourselves to the most interesting case of the perturbations of the quasiparticle distribution matrix in the spin channel. The stability criterion is there equivalent to the requirement for the matrices

$$\langle lJ|A|l'J\rangle = \delta_{ll'} + \Delta\mathscr{F}^J_{ll'} \tag{51}$$

to be positive-definite for each value of J. The matrix elements $\Delta\mathscr{F}^J_{ll'}$ are given by Eq. (25) and the explicit formulae for them may be found in Ref. [7]. In the standard case of purely central quasiparticle interaction one would have $\langle lJ|A|l'J\rangle = \delta_{ll'}[1+G_l/(2l+1)]$ and hence the stability criterion would read

$$1+G_l/(21+1) > 0 \tag{52}$$

for every l.

In the presence of tensor forces we have, for a given J, one 2×2 matrix $\langle lJ|A|l'J\rangle$ ($l, l' = J\pm 1$) that has to be positive-definite and one value $\langle JJ|A|JJ\rangle$ that should be positive. For $J = 0$ we get the condition $\langle 10|A|10\rangle > 0$. The explicit form of the stability criteria for $J < 3$ may be found in Ref. [7]. The stability criteria in the spin-isospin channel are obtained by replacing G_l and H_l by G'_l and H'_l.

Summarizing: the stability of the ground state of nuclear matter requires that the *lowest eigenvalue* of the $\langle lJ|A|l'J\rangle$ matrix be *positive*. It has been pointed out in Ref. [7] that the presence of tensor component in the quasiparticle interaction may significantly lower the value of the $\langle 10|A|10\rangle$ element of the stability matrix, as compared to that for central quasiparticle interaction. This has been confirmed by our calculations with the RSC and BJ Fermi liquid parameters, for which the lowest eigenvalue of the stability matrix is always

$$\langle 10|A|10\rangle = 1-\tfrac{10}{3}H_0+\tfrac{1}{3}G_1+\tfrac{4}{3}H_1-\tfrac{2}{15}H_2. \tag{53}$$

For both the RSCII and BSJ Fermi liquid parameters we found *negative* values of $\langle 10|A|10\rangle$ for $1.2\,\mathrm{fm}^{-1} < k_F < 2\,\mathrm{fm}^{-1}$, i.e., in the whole density interval studied in Ref. [8]. The lowest eigenvalues of A are given in Tables I and II. At the lowest density considered, $k_F = 1.2\,\mathrm{fm}^{-1}$, the (negative) value of $\langle 10|A|10\rangle$ is close to zero. However, the magnitude of $\langle 10|A|10\rangle$ grows rapidly with increasing density. At the highest density considered, $k_F = 2\,\mathrm{fm}^{-1}$ ($\varrho = 3.25\,\varrho_0$, where ϱ_0 is normal nuclear matter density corresponding to

TABLE I

The lowest eigenvalue of the stability matrix A for the RSC Fermi liquid parameters of Ref. [8]

k_F (fm^{-1})	Lowest eigenvalue	
	RSC with OPE for $l > 1$	RSC with $H_l = 0$ for $l > 1$
1.2	−0.035	0.043
1.2550	—	0.0
1.4	−0.263	−0.147
1.6	−0.557	−0.397
1.8	−0.911	−0.697
2.0	−1.378	−1.096

TABLE II

The same as in Table I but for the BJ Fermi liquid parameters of Ref. [8]

k_F (fm^{-1})	Lowest eigenvalue	
	BJ with OPE for $l > 1$	BJ with $H_l = 0$ for $l > 1$
1.2	−0.050	0.020
1.2098	—	0.0
1.4	−0.329	−0.227
1.6	−0.483	−0.350
1.8	−0.657	−0.497
2.0	−0.746	−0.563

$k_F = 1.35\,\mathrm{fm}^{-1}$) the instability in the $J = 0$, $S = 1$, $T = 0$ channel becomes very strong. Let us mention, that at the same time the system is stable with respect to the standard static, space homogeneous perturbations

$$H^{\mathrm{ext}} = V \cdot \sigma(r) \tag{54}$$

relevant for the definition of the static spin susceptibility and the spin symmetry energy of nuclear matter. The deformation of the Fermi surface induced by (54) are there restricted to the $J = 1$, $l = 0,2$ channel [18].

The spin instability in the $J = 0$, $l = 1$ channel is driven by the *tensor force*. More precisely, it results from large, positive values of the parameter H_0. Quite large OPE values of H_l for $1 < l < 4$ amplify further this effect. If one puts $H_l = 0$ for $l > 1$ instead of replacing them by their OPE values, the point of instability shifts towards higher value of k_F. Namely, nuclear matter becomes then unstable for $k_F > 1.210\,\mathrm{fm}^{-1}$ in the case of the BJ model and for $k_F > 1.255\,\mathrm{fm}^{-1}$ in the case of the RSCII model.

6b. Dynamic form factor

In view of the instability of the standard ground state of nuclear matter with the RSCII and the BJ quasiparticle interactions, discussed in the preceding subsection, we have restricted ourselves to the calculations with the RSCI Fermi liquid parameters. The calculations have been done at the saturation density of nuclear matter corresponding to $k_F = 1.35\,\mathrm{fm}^{-1}$.

Our results for the dynamic form factors in the spin and spin-isospin channels are presented in Figs. 1a, b. The plotted quantity is $S_{\mu\mu}(\lambda)$ in the units of $\frac{mp_F}{\pi^2\hbar^2}$, the maximum value of the dynamic form factor for the Fermi gas model, $S_F(\lambda) = \frac{p_F m}{\pi\hbar}\lambda\theta(1-\lambda)$. The contributions from undamped collective modes, corresponding to poles of the linear response functions, are represented by vertical lines. In order to illustrate the effect of tensor forces, the results obtained with $H_l, H_l' = 0$ are also shown.

The relative probability of excitation of a collective mode in a given channel M is given by

$$r_{\mathrm{coll.}}(M = -\mu) = \int\limits_1^\infty S_{\mu\mu}(\lambda)d\lambda / \int\limits_0^\infty S_{\mu\mu}(\lambda)d\lambda = \mathcal{U}^\mu h / \int\limits_0^\infty S_{\mu\mu}(\lambda)d\lambda. \tag{55}$$

The results of our search for undamped collective modes, as well as the relative probability of their excitation are presented in Table III.

The main qualitative effect of tensor force is to remove degeneracy with respect to M: the tensor component of the quasiparticle interaction discriminates between the $M = 0$ and $M = \pm 1$ excitations and the only degeneracy is that with respect to the sign of M. Generally, the effect of tensor force in the spin channel is stronger than in the spin-isospin one; this is due to the fact that H_l's are usually $2\div 3$ times larger than H_l''s. The pole corresponding to undamped collective excitation splits into two poles: one corresponding to $M = 0$ and the other corresponding to $M = \pm 1$ mode. Let us consider first the case

TABLE III

The position of the poles of the linear response functions, λ_0, and their relative contribution to the excitations spectrum, $r_{\mathrm{coll.}}$ (see the text) for the RSC I set of the Fermi liquid parameters, calculated at $k_F = 1.35\,\mathrm{fm}^{-1}$

	Spin channel		Spin-isospin channel	
	λ_0	$r_{\mathrm{coll.}}$ (%)	λ_0	$r_{\mathrm{coll.}}$ (%)
$\mu = \pm 1$	1.1162	59	1.0724	61
$\mu = 0$	1.0308	29	1.1349	76
no tensor forces	1.0734	54	1.0914	68

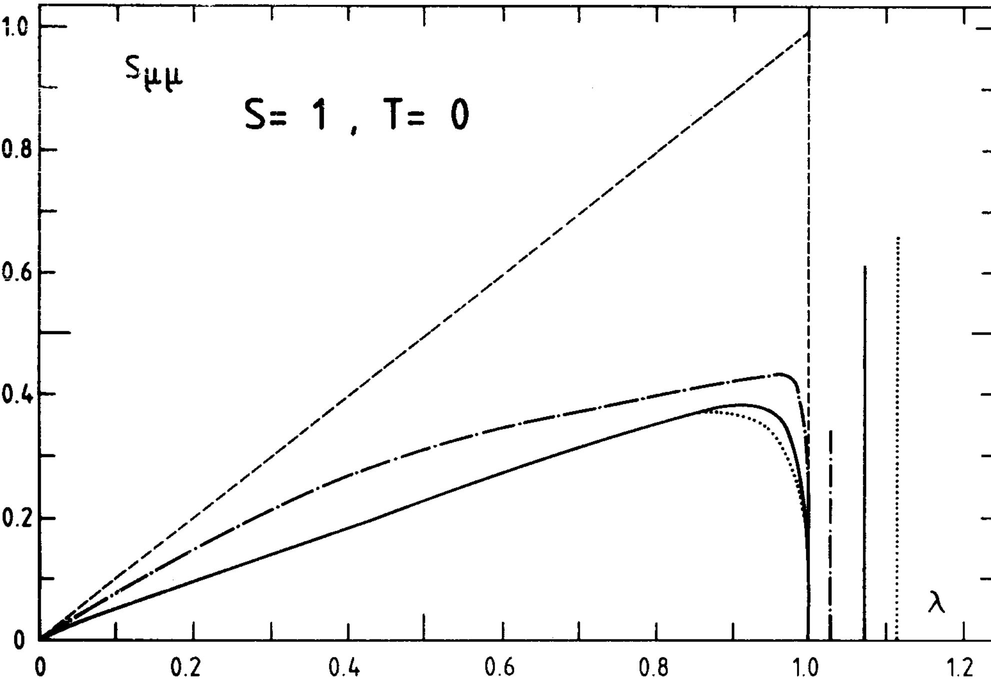
1.0
0.8
0.6
0.4
0.2
0
$s_{\mu\mu}$
S= 1 , T= 0
λ
0
0.2
0.4
0.6
0.8
1.0
1.2

Fig. 1a

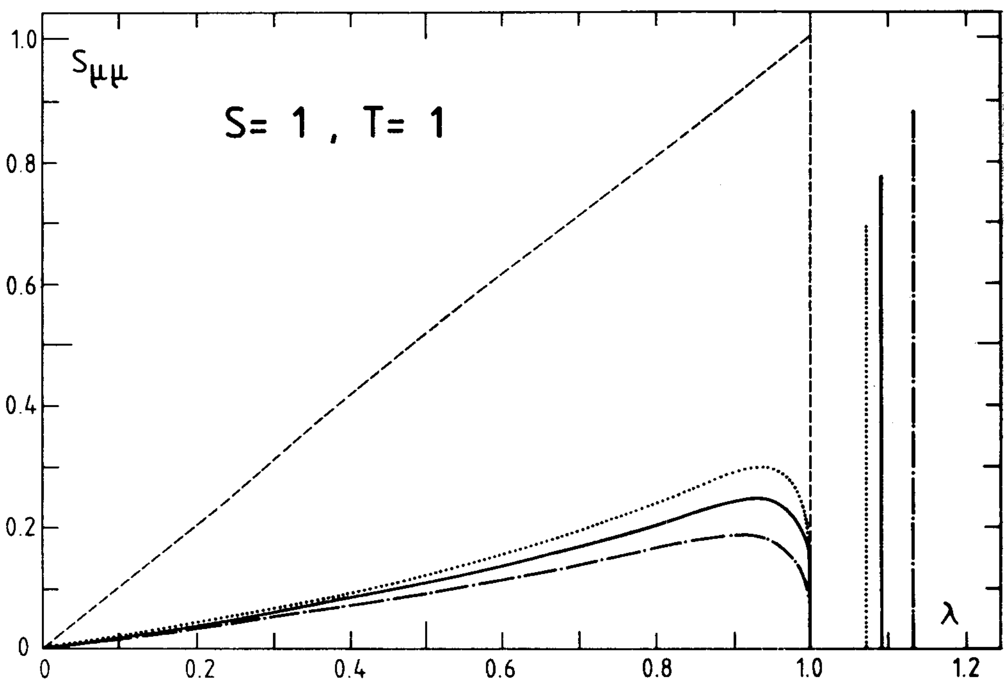
1.0
0.8
0.6
0.4
0.2
0
$s_{\mu\mu}$
S= 1 , T= 1
λ
0
0.2
0.4
0.6
0.8
1.0
1.2

Fig. 1b

of the spin channel. At given value of the wavenumber of collective excitation k, tensor force shifts the energy of the $M = 0$ mode down and that of the $M = \pm 1$ mode up with respect to the energy of excitations in the absence of tensor force. This is accompanied by a significant decrease of the relative strength of the collective $M = 0$ mode (see Fig. 1a and Table III).

In the spin-isospin channel the effect of tensor force is weaker. At fixed k the tensor force shifts up the energy of $M = 0$ mode and shifts down that of the $M = \pm 1$ ones, as compared to the case of purely central quasiparticle interaction. Hence, the effect of tensor force is opposite to that seen in the spin channel. This is due to the fact that H_l' are negative while H_l were positive.

7. Conclusion

In the present paper we have studied, within the framework of Fermi liquid theory, the effect of tensor forces on the linear response of nuclear matter to long wavelength spin dependent perturbations.

The calculations performed using several sets of the spin dependent Fermi liquid parameters show that the effect of tensor forces may be very important. In particular, for the Fermi liquid parameters derived using variational methods from the Reid soft-core and Bethe Johnson potentials by Jackson et al. [8], tensor forces imply the instability of the ground state of nuclear matter near and above normal nuclear matter density. The instability occurs in the $J = 0$, $S = 1$, $T = 0$ channel and does not show up for the spin dependent Fermi liquid parameters derived from the Reid soft-core potential using perturbational methods [7].

Of course, it may well be that this instability is an artifact of the approximations made in the calculation. In particular, it may be due to the perturbative treatment of the state dependence of the two-body correlation. This is probably the case. The appearance of the instability in the $S = 1$, $T = 0$ channel may just reflect large uncertainty in our theoretical knowledge of the quasiparticle interaction in nuclear matter. In consequence of this instability, however, we were not able to study the linear response of nuclear matter in this channel for the sets of Fermi liquid parameters for which the instability appeared.

The calculation of the dynamic form factors for the spin and spin-isospin density fluctuations has been performed using the spin dependent Fermi liquid parameters derived from the Reid soft-core nucleon potential by Bäckman et al. [7]. In the presence of tensor forces dynamic form factor $S_{ij}(\mathbf{k}, \omega)$ in the spin and spin-isospin channels is no longer proportional to the unit matrix. Assuming k to be the z-axis, one has $S_{xx} = S_{yy} \neq S_{zz}$.

Fig. 1. The plots of the dynamic form factor $S_{\mu\mu}(\lambda)$ (in the units of $mp_F/\pi^2\hbar^2$) in the spin ($S = 1$, $T = 0$) and spin-isospin ($S = 1$, $T = 1$) channels calculated for the RSCI set of the Fermi liquid parameters. Dotted line: $\mu = -M = \pm 1$ channel, dash-dotted line: $\mu = M = 0$. The contribution from collective modes is shown using vertical lines. The heights of vertical lines are proportional to the pole contribution to the integrated form factor. For the sake of comparison, results for the case of no tensor forces (solid line) and those for an ideal Fermi gas of particles of mass m^* (short dashes) are also shown. The calculation has been performed at $k_F = 1.35\ \mathrm{fm}^{-1}$

Also, tensor force removes the degeneracy of both one quasiparticle-one quasihole and as well as collective excitations with respect to their magnetic quantum number, M. The effects of the tensor force is particularly large in the spin channel.

The methods elaborated in the present paper can be applied to astrophysically interesting case of neutron matter. The neutron matter dynamic form factor is relevant for the neutrino scattering in hot neutron star matter. In particular, dynamic form factor for spin density fluctuations enters the axial vector part of the scattering rate of neutrinos in neutron medium via weak neutral current. The knowledge of neutrino mean free path in dense hot matter is important for theoretical studies of cooling processes in newly born neutron stars as well for physics of supernova explosions. Recently, the calculations of the effect of the nucleon-nucleon interaction in neutrino mean free path in hot dense neutron matter has been done using the methods of the theory of Fermi liquids [21], assuming, however, purely central quasiparticle interaction. The formalism developed in the present paper will enable us to study this problem taking into account the full complexity of the nucleon--nucleon interaction.

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APPENDIX

In order to derive a useful formula for

$$\alpha^m_{ll'}(\lambda) = \frac{1}{\sqrt{(2l+1)(2l'+1)}} \int d\boldsymbol{n} \frac{z}{\lambda - z + i\eta} Y_{lm}(\boldsymbol{n}) Y^*_{l'm}(\boldsymbol{n}) \tag{A1}$$

we use the expansion [19]

$$\frac{1}{\lambda - z + i\eta} = \sum_{L=0}^{\infty} (2L+1) P_L(z) Q_L(\lambda + i\eta), \tag{A2}$$

where P_L and Q_L are Legendre functions of the first and second kind, respectively. This enables us to perform explicitly the integration over angular variables. For $\lambda > 1$ we get real function [9]

$$\alpha^m_{ll'}(\lambda) = -\delta_{ll'}/(2l+1) + \lambda(-)^m \sum_L (2L+1) \begin{pmatrix} l & l' & L \\ m & -m & 0 \end{pmatrix} \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix} Q_L(\lambda), \tag{A3}$$

where the values of $Q_L(\lambda)$ can be obtained putting $z = \lambda$ in the formulae (8.4) of Ref. [18] and using the recurrence relations. For $0 < \lambda < 1$ we obtain complex values of $\alpha^m_{ll'}$, with

$$\mathrm{Re}\, \alpha^m_{ll'}(\lambda) = -\delta_{ll'}/(2l+1) + \lambda(-)^m \sum_L (2L+1) \begin{pmatrix} l & l' & L \\ m & -m & 0 \end{pmatrix} \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix} \bar{Q}_L(\lambda)$$

$$\mathrm{Im}\, \alpha^m_{ll'}(\lambda) = -\tfrac{1}{2}\pi\lambda(-)^m \sum_L (2L+1) \begin{pmatrix} l & l' & L \\ m & -m & 0 \end{pmatrix} \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix} P_L(\lambda),$$

where the values of $\bar{Q}_L(\lambda)$ may be calculated using the formula 8.6.19 of Ref. [20].

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