TIME-DEPENDENT HARTREE-FOCK-BOGOLYUBOV STUDY OF THE $^{16}O+^{16}O$ REACTION

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The two-body dissipation mechanism in the heavy-ion reaction is investigated in the framework of the time-dependent Hartree-Fock-Bogolyubov theory with a Skyrme II force and a constant gap monopole pairing interaction. The numerical calculations for the central collision of ^{16}O ions show that the two-body dissipation amounts to as much as 40% of the total dissipation of the kinetic energy of colliding ions. This leads to a significant increase of the limiting energy for nonfusion events in central collisions ($E_{CM}/A > 1.75$ MeV) when short range nucleon-nucleon correlations are included. Consequently, the low-L fusion window is predicted at much higher energies than expected on a basis of time-dependent Hartree-Fock calculations.

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It is generally believed that the time-dependent mean field approach provides a natural framework for the description of various dynamical phenomena which appear in the heavy-ion (HI) collisions. Results of extensive calculations performed in this approach suggest that several inclusive properties of the HI reactions can be described qualitatively and in some cases also quantitatively assuming the wave function to be a Slater determinant at each instant of time and using the density-dependent Skyrme interaction which has a correct saturation properties [1, 2]. Probably the largest success of the time-dependent Hartree–Fock (TDHF) calculations was the reproduction of experimental fusion excitation functions for different reactions. This feature of the theory shows that the one-body dissipation mechanism contained in TDHF approach dominates and, consequently, the TDHF theory provides a good basis for systematic refinements of the description of nuclear dynamics by including the two-body collision terms [3] and (or) the residual nucleon--nucleon correlations. These correlations are responsible for a two-body nuclear dissipation which leads to a frequent pair scattering between different conjugate single particle (s.p.) orbits and is expected to be most effective at various level crossings in the time-dependent average field [4, 5]. Yet it is not clear how important is this two-body dissipation mecha-

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nism for the description of various inclusive reaction properties. TDHF theory predicts the existence of a low angular momentum cutoff which limits the complete-fusion cross section at high energy [1]. The precise value of the kinetic energy of ions above which the nonfusion events at low-L appear is expected to d pendent rather sensitively on the detailed assumptions about the nuclear dissipation mechanism. This fact has stimulated numerous investigations of the low-L cutoff in the TDHF evolution for different reactions [1] as well as a number of experimental investigations attempting to isolate and detect nonfusion events [6, 7]. In case of $^{16}O+^{16}O$ reaction the three dimensional TDHF calculations using a zero-rang density dependent force supplemented by contributions from the direct parts of the Yukawa and Coulomb interactions predict the existence of a dynamical low-L limit to fusion for $\varepsilon_{CM}^{(f)} = E_{CM}/A \gtrsim 0.84$ MeV [2]. Improving the force by using the density dependent effective mass one obtains a further increase of the limiting energy $\varepsilon_{CM}^{(f)}$ ($\varepsilon_{CM}^{(f)} \gtrsim 1$ MeV) [8]. The experimental search in the $^{16}O-^{16}O$ system does not confirm the existence of a fusion L-window and puts the lowest limit for $\varepsilon_{CM}^{(f)}$ equal 1.06 MeV or possibly even higher [6]. Studies of the shapes of angular distributions for the mass asymmetric reaction ^{12}C (^{16}O, α) ^{24}Mg (in the ground state) also do not show evidence for the fusion L-window until $\varepsilon_{CM}^{(f)} \simeq 1.53$ MeV, though this experimental method seems to have an unsatisfactory resolution for low-L values at $\varepsilon_{CM}^{(f)} \gtrsim 1.35$ MeV [7]. Thus, it seems that all available experimental data contradict the existence of the low-L cutoff and put the lower limit for $\varepsilon_{CM}^{(f)}$ significantly above the TDHF estimate of $\varepsilon_{CM}^{(f)}$. It is plausible that the dissipation mechanism in HI reactions is by far more complicated than the one-body dissipation mechanism contained in TDHF approximation. A straightforward extension of the TDHF approach which allows to treat also the two-body residual interaction in the mean field approximation is provided by the time-dependent Hartree–Fock–Bogolyubov (TDHFB) approximation. In this approximation one can investigate this part of the two-body dissipation which is contained in the average quasi-particle field. Here we use the TDHFB approach to describe the central collision of ^{16}O - ions. The complicated nucleon-nucleon residual interaction is approximated by the state-independent monopole pairing field. This simple form of the residual two-body force gives a reasonable account of various static nuclear properties. There is however a considerable uncertainty in choosing values of the strength parameter G for various configurations of the dinucleus. Since the average value of G is inversely proportional to the number of particles then it can change by factor ~ 2 going from the initial state of two separated ^{16}O ions to the dinucleus ^{32}S close to the ground state. Corresponding variations of the average value of the gap parameter Δ are smaller since $\Delta \sim A^{-1/2}$ [9]. The range of possible variations of $G(t)$ (or $\Delta(t)$) going from the initial state to the final state determines also uncertainty of the constant-G (or constant-Δ) TDHFB calculations. Moreover, even if all pairing matrix elements $G_{\nu\nu'}$ for various s.p. configurations remain unchanged during the collision, then the appropriately defined average pairing matrix element $G(t) = \sum_{\nu\nu'} G_{\nu\nu'} U_\nu(t) V_\nu(t) U_{\nu'}(t) V_{\nu'}(t) / (\sum_\nu U_\nu(t) V_\nu(t))^2$ depends on time due to the explicit time-dependence of occupation amplitudes U_ν, V_ν. Thus, we prefer to solve TDHFB equations for a constant value of the deformation Δ in the gauge space rather than for a constant average pairing matrix element G. However, one should stress that it remains

unclear to what extend the deformation parameter in the gauge space can be treated as being independent from properties of the mean field evolving in time [10].

TDHFB eqs. can be derived by varying the action integral:

$$\delta \int_{t_1}^{t_2} \langle \phi | H - i\hbar \frac{\partial}{\partial t} | \phi \rangle dt = 0, \tag{1}$$

where H is the sum of a Skyrme Hamiltonian and a monopole pairing field $-\sum_\tau \Delta_\tau \sum_{\nu>0} (c_\nu^+ c_{\bar\nu}^+ + c_{\bar\nu} c_\nu)$. For a central collision the most general Bogoliubov trial wave function can be written in the form of a BCS wave functions $|\phi\rangle = \prod_{k>0} (U_k + V_k a_k^+ a_{\tilde k}^+)|0\rangle$, where a_k^+, $a_{\tilde k}^+$ are creation operators for conjugate canonical s.p. states k, $\tilde k$. In this canonical representation the density matrix $\hat\varrho_{kk'}$ and the pairing tensor $\hat\kappa_{kk'}$ take the form:

$$\varrho_k \equiv \hat\varrho_{kk} = \hat\varrho_{\tilde k\tilde k} = |V_k|^2,$$

$$\kappa_k = \hat\kappa_{k\tilde k} = U_{\tilde k} V_k = -\hat\kappa_{\tilde k k}, \quad (V_{\tilde k} = -V_k;\ U_{\tilde k} = U_k), \tag{2}$$

where amplitudes U_k, V_k satisfy the relation $|V_k|^2 + U_k^2 = 1$. Therefore, as independent variational quantities in eq. (1) one can choose the occupation amplitudes $V_k\,(V_k^*)$ of conjugate canonical states $|k\rangle$, $|\tilde k\rangle$ and the amplitudes of canonical s.p. wave functions $\psi_k(\boldsymbol{r}) = \langle \boldsymbol{r}|a_k^+|0\rangle$, $\psi_{\tilde k}(\boldsymbol{r}) = \langle \boldsymbol{r}|a_{\tilde k}^+|0\rangle$ $(\psi_k^*(\boldsymbol{r}), \psi_{\tilde k}^*(\boldsymbol{r}))$. The detailed discussion of TDHFB eqs. resulting from the variational principle (1) can be found in Refs. [4, 5]. Also the final form of TDHFB eqs. for a constant Δ can be found in Ref. [5] (Eqs. (21a)–(21c)).

Parameters of the Skyrme force (Skyrme II parameters) Hamiltonian are same as used by Dhar and Nilsson [8]. This choice allows for a direct comparison of our TDHFB results with results of earlier TDHF calculations in which the largest one-body dissipation in $^{16}O-^{16}O$ dinuclear system was found for the Skyrme II force. S.p. wave functions $\psi_k(\boldsymbol{r}, t)$ entering TDHFB eqs. are expanded in a static, two-center basis [11] which is obtained as a product of one-dimensional basis $\varphi_n^{(z_0)} = \sqrt{G_{z_0}(z)}\, P_n^{(z_0)}(z)$ in the z-direction with the set of eigenfunctions of the cylindric oscillator potential in $x-y$ plane. $P_n^{(z_0)}(z)$ form a set of orthogonal polynomials for the two-center weight function $G_{z_0}(z) = [(0.5 + z_0^2)/(1+z_0^2)]\,\{\exp[-(z-z_0)^2 + \exp[-(z+z_0)^2]\}^{1/2}$, where $2z_0$ is the separation distance between centers of the basis. In this way the equation of motion for $\psi_k\,(\psi_k^*)$ is reduced to a system of differential equations for the expansion coefficients. Those equations can be solved using the Crank–Nicholson method as described in Ref. [12]. It was found by Flocard [11] that in the static constrained HF calculations for the reaction $^{16}O+^{16}O$ the total energy and the quadrupole moment of the dinucleus converge fast with increasing shell number N for each separation distance between oxygen ions and reach their asymptotic values already for small N. In our calculations both the separation distance between centers of the basis $2z_0$ and the deformation $q \equiv \omega_\perp/\omega_z$ of the basis are adjustable parameters and have to be chosen to yield an optimal description of the dynamics of the $^{16}O + ^{16}O$ collision as well as static properties of ^{16}O. For $2z_0 = 5$ fm, $q = 4$ and $N = 13$

we achieve an excellent agreement with results of TDHF calculations of Ref. [8] where the evolution of $^{16}O-^{16}O$ system is studied on the discrete spatial mesh (compare for $E_{CM}/A = 1.25$ MeV ($E_{LAB} = 80$ MeV) the curve drawn with a dashed line in Fig. 1 with the corresponding curve in Fig. 2 of Ref. [8]). TDHFB calculations reported in this letter are performed for a constant gap parameter ($\Delta = 3$ MeV) which for ^{16}O corresponds to the value of an average pairing gap parameter $\tilde{\Delta} = 12/A^{1/2}$ MeV [9]. The time variation of the fragment separation coordinate $2R = \frac{2}{A}\int d^3\tau |z|\varrho(r)$ is displayed in Fig. 1 for various kinetic energies per particle in the center of mass. The solid lines denote the TDHFB ($\Delta = \text{const}$) evolutions whereas the dashed line presents results for $\Delta = 0$ (the TDHF evolution). The central collision of ^{16}O -ions at $E_{CM}/A = 1$ MeV described in the TDHF limit leads to the fusion (see Fig. 2 of Ref. [8] whereas already at $E_{CM}/A = 1.25$ MeV the dinuclear system splits into fragments. Additional dissipation induced by pairing correlations is sufficiently large to fuse the two ^{16}O -ions up to $E_{CM}/A \lesssim 1.75$ MeV. For $E_{CM}/A \gtrsim 1.75$ MeV the two ions fly apart after a few giant shape oscillations ($t = 2-14.5 \cdot 10^{-22}$ s). It is interesting to notice that the separation distance of two ions at the point of a closest approach $2R_{min}$ decreases extremely slowly with increasing the kinetic energy of incoming ions, whereas it increases rather fast with Δ. Therefore, the value of R_{min} for HI collisions characterizes not only the size of reacting ions but also properties of the nuclear medium and hence the dissipation mechanism. Table I shows the calculated values of $R_{min}(E_{CM}/A = 1.25\text{ MeV})$ for various Δ. Variations of R_{min} with Δ can be fitted very well by the expression quadratic in Δ: $R_{min}(\Delta) - R_{min}(\Delta = 0) = \alpha \cdot \Delta^2$ with

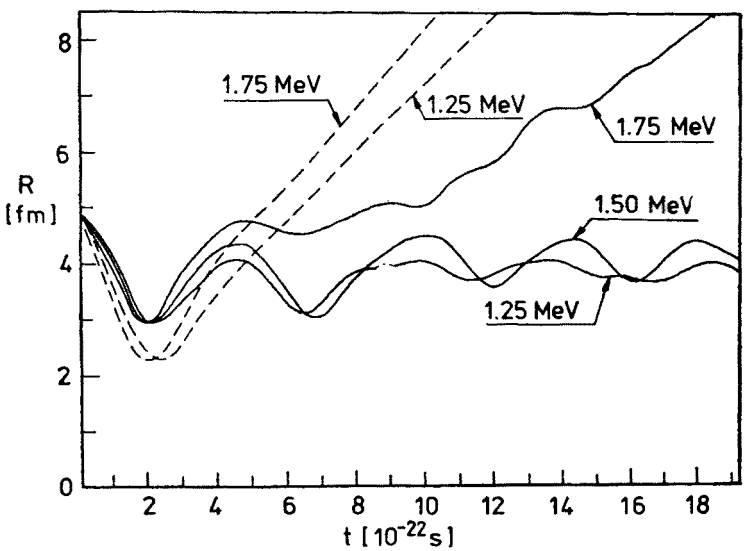

Fig. 1. The fragment separation coordinate ($2R$) is plotted at various instants of time for the $^{16}O+^{16}O$ reaction at $E_{CM}/A = 1.25$, 1.50 and 1.75 MeV. The solid lines depict the TDHFB ($\Delta = 3$ MeV) trajectory whereas the dashed lines show $R(t)$ for the TDHF evolution. (Note the difference between definitions of the fragment separation coordinate in this letter and in Ref. [8])

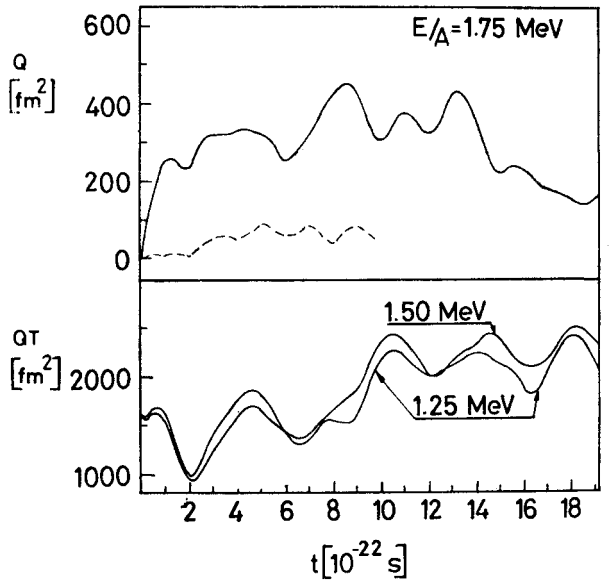

Fig. 2. The total quadrupole moment QT of the $^{16}O-^{16}O$ system and the fragment quadrupole moment Q are plotted as a function of time for various incident kinetic energies of ions. Results for the TDHFB ($\Delta = 3$ MeV) evolution are plotted with a solid line, whereas the dashed line denotes $Q(t)$ in the TDHF approximation. For definitions of QT, Q and other details see the description in the text

a parameter α ($\alpha = 0.07$ fm/MeV^2) which decreases slowly with increasing kinetic energy of incoming ions. It is instructive to compare various features of the collective motion described by TDHFB approximation at $E_{CM}/A = 1.25$, 1.50 and 1.75 MeV. At $E_{CM}/A = 1.25$ MeV and for $t = 2-6.5 \cdot 10^{-22}$ s the separation coordinate R exhibits half of the oscillation period. For $t > 6.5 \cdot 10^{-22}$ s the collective shape vibrations are strongly damped. One sees only the small irregular variations of R around $R_0 \simeq 3.9$ fm with am-

TABLE I

The dependence of the separation coordinate $2R_{min}$ between two ions at the point of the closest approach on the value of the pairing gap parameter

Δ [MeV]	0	1	2	3
R_{min} [fm]	2.27	2.34	2.54	2.92

plitude ~ 0.15 fm. An increase of the kinetic energy from 1.25 to 1.50 MeV per nucleon in the center of mass leads to the appearance of regular slowly damped oscillations of R with an amplitude ~ 0.4 fm. These oscillations are centered at around $R_0 = 4.1$ fm and R_0 increases slowly in time. This characteristic increase is strongest for the nonfusion TDHFB evolution at $E_{CM}/A = 1.75$ MeV. Shape variations of ^{16}O -ions can be studied by looking at the changes in time of the total quadrupole moment $QT \equiv Q_{20} = 2\sqrt{(4\pi/5)} \int r^2 Y_{20} \varrho(r) d\tau$

which is defined with respect to the center of mass of the dinucleus. The lower part of Fig. 2 exhibits $Q_{20}(t)$ at energies $E_{CM}/A = 1.25, 1.50$ MeV for which the $^{16}O-^{16}O$ system fuses. For both energies the quadrupole moment of the fused system becomes larger than the quadrupole moment of two spherical ^{16}O -ions at the distance of 10 fm (the initial condition). This fact emphasizes an extremely large spreading of the nuclear matter in the fused system when pairing correlations are included. The quadrupole moment at $E_{CM}/A = 1.25$ and 1.5 MeV shows oscillations which are closely related to the oscillations of R (see Fig. 1). For $t < 1 \cdot 10^{-21}$s changes of Q_{20} in time are irregular and the value of a quadrupole moment shows a tendency towards increasing. Finally, the fused system approaches its equilibrium deformation at $t \simeq 1.10^{-21}$ s for both energies considered. The upper part of Fig. 2 exhibits variations in time of the quadrupole moment of ^{16}O-like fragment for $E_{CM}/A = 1.75$ MeV. Since at this energy both the TDHF and TDHFB ($\Delta = 3$ MeV) evolutions do not lead to the fusion, therefore we show the time-dependence of the fragment quadrupole moment only. This moment equals $Q \equiv q_{20}(t) = Q_{20}(t) - \bar{q}_{20}(R(t))$, where $Q_{20}(t)$ is the total quadrupole moment of the dinuclear system and $\bar{q}_{20}$ denotes the quadrupole moment of the point mass oxygen ion with respect to the center of mass of the dinuclear system. For comparison the fragment quadrupole moment for $\Delta = 0$ (the (TDHF evolution) is presented in Fig. 2 with a dashed line. In the latter case the nucleus reach its equilibrium quadrupole moment ($q_{20} \simeq 70$ fm^2) in $\Delta t \simeq 3 \cdot 10^{-22}$ s after the collision and afterwards exhibits small amplitude oscillations around the equilibrium configuration. Similarly as in the case of a fused system, the fragment quadrupole moment depends sensitively on the residual pairing interaction reflecting markedly different properties of the nuclear matter in presence of short range attractive correlations.

The role of short range correlations contained in the mean quasi-particle field is expected to decrease while increasing the excitation energy of the system. Fig. 3 shows the fragment quadrupole moment and the fragment separation coordinate R as functions of time as well as the nucleon density at various coordinates z ($x = y = 0.0$) for the separation distance between centers of fragments equal $2R = 7.24$ fm. Results plotted with the dashed line correspond to the TDHF evolution whereas the solid line exhibits the evolution as obtained from TDFHB approximation for $\Delta = 3$ MeV. Comparing $q_{20}(t)$ and $R(t)$ for TDHF and TDHFB evolutions at $E_{CM}/A = 8$ MeV with those presented in Figs. 1,2 for $E_{CM}/A = 1.75$ MeV one concludes that apparently both evolutions become more similar at high energy. This observation is also confirmed by comparing the amount of dissipated kinetic energy of the relative motion. Whereas at $E_{CM}/A = 1.75$ MeV the initial energy is dissipated in 87% and 98% in case of TDHF and TDHFB evolutions respectively, for $E_{CM}/A = 8$ MeV one finds that the dissipation amounts for both kinds of evolutions to ~50% of the initial kinetic energy. One should notice however, that even at $E_{CM}/A = 8$ MeV the differences between paired and upaired systems remain ($R_{min}(\Delta) - R_{min}(\Delta = 0) \simeq 0.15$ fm and $q_{20}(\Delta) - q_{20}(\Delta = 0) \simeq 35$ fm^2 at $t > 2.2 \cdot 10^{-22}$ s) and can be seen in a markedly different succeptibility towards shape changes. In the lower part of Fig. 3 the values of the nuclear density along the z-axis ($x = y = 0.0$) are plotted the paired ($\Delta = 3$ MeV) and unpaired systems. The picture presents half of the density profile $\varrho(z)$ which is symmetric with respect to the reflection in the $x-y$ plane at $z = 0$.

TDHFB and TDHF nucleon densities in Fig. 3 are shown here for the same fragment separation coordinate $2R$. This allows to compare properties of both systems at the similar stage of the reaction. For the unpaired system the nucleon density in the middle of each fragment is approximately 20% larger than in the paired system. On the contrary, the spreading of the nucleon density in the z-direction is larger for the paired system. Consequently, when pairing correlations are present then the surface thickness of the nucleon density after the collision increases largely and influences directly a value of the mass quadrupole moment which becomes significantly larger in highly excited nuclei interacting via the residual short range correlations.

In conclusion, the two-body dissipation mechanism which is contained in the time-

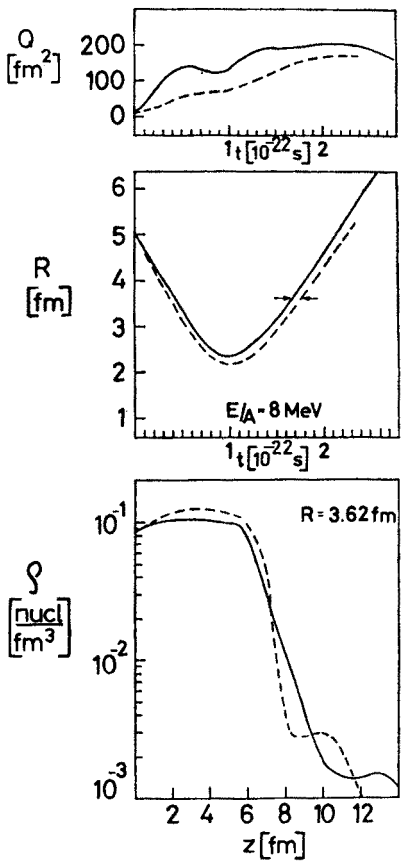

Fig. 3. The fragment quadrupole moment and the fragment separation coordinate are plotted as a function of time for the $^{16}O+^{16}O$ collision at $E_{CM}/A = 8$ MeV. The solid line denotes results for the TDHFB evolution at $\Delta = 3$ MeV whereas the dashed line corresponds to the TDHF evolution. The arrows in the plot $R(t)$ denote a value of the separation coordinate of oxygen ions for which the density profile $\varrho(z)$ at $x = y = 0$ is plotted for TDHFB and TDHF evolutions respectively (the lowest figure). At this bombarding energies $R(t)$ takes the same value for TDHFB and TDHF evolutions for approximately the same instants of time

-dependent average quasi-particle field seems to be significant for explaining the energy loss in HI collisions at small impact parameters. For the $^{16}O-^{16}O$ system and for a reasonable value of the Δ-parameter in the pairing field the two-body dissipation mechanism amounts to more than 40% of the total energy dissipated in the central collision. Consequently, the limiting kinetic energy $\varepsilon_{CM}^{(f)}$ for nonfusion at low L is shifted up significantly ($\varepsilon_{CM}^{(f)} \gtrsim 1.7$ MeV) as compared with values obtained in the TDHF approximation by Dhar and Nilsson [8]. The two-body dissipation contained in the TDHFB field becomes less important at higher energies for which the one-body dissipation mechanism dominates. In the whole range of energies, discussed in this letter, we find a large influence of pair correlations on values of the quadrupole moment, R_{min} and on the density distribution in post-scission configurations. This latter feature confirm again an importance of the dynamical residual nucleon-nucleon correlations for a correct description of final reaction channels. It is expected that also at peripheral collisions the two-body dissipation mechanism plays an important role increasing a value of the maximum fusion angular momentum $L_>$. The larger interaction radius of nuclei in the presence of pairing force can lead to the significant enlargement of $L_>$ and thus to the increase of a fusion cross section $\sigma_{fus} \sim (L_>+1)^2-(L_<+1)^2$. This happens mainly at low bombarding energies when the collision time is significantly long to permit a frequent scattering of nucleons from s.p. levels of each fragment separately to the s.p. levels of a dinucleus. It is plausible that the absence of dynamical pair correlations in TDHF approximation is responsible for systematically too low values of calculated σ_{fus} as compared with experimental data [2] for low bombarding energies.

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