RELATIVISTIC TWO-BODY EQUATION FOR ONE DIRAC AND ONE DUFFIN-KEMMER PARTICLE

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(*Received August 2, 1982*)

A new relativistic two-body wave equation is proposed for one spin-1/2 and one spin-0 or spin-1 particle which, if isolated from each other, are described by the Dirac and the Duffin-Kemmer equation, respectively. For a static mutual interaction this equation splits into two equations: a two-body wave equation for one Dirac and one Klein-Gordon particle (which was introduced by the author previously) and a new two-body wave equation for one Dirac and one Proca particle. The proposed equation may be applied in particular to the quark-diquark system. In Appendix, however, an alternative approach is sketched, where the diquark is described as the point limit of a very close Breit system rather than a Duffin-Kemmer particle.

PACS numbers: 11.10.Qr

1. *Introduction*

A need for establishing a relativistic two-body wave equation for one spin-1/2 particle and one spin-0 or spin-1 particle arises from some recent models, where such particles are constituents of close bound states. It is the case both in the composite models where leptons and quarks are built of spin-1/2 and spin-0 preons [1], and in the composite models operating with quarks and diquarks as effective building blocks of hadrons [2]. But the new equation may be also useful in considering relativistic effects in some more conventional bound states as e.g. the helium ion or deuterium atom or various mesoatoms.

Such a relativistic two-body wave equation has been already found for one spin-1/2 and one spin-0 particle which, if isolated from each other, are described by the Dirac and the Klein-Gordon equation, respectively [3]. The Coulomb fine-structure formula has been derived from the new equation and verified to reproduce in one-body limits the well-known formulae [4].

In the present paper we propose a relativistic two-body wave equation for one spin-1/2 and one spin-0 *or* spin-1 particle which, if isolated, are described by the Dirac and the

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Duffin-Kemmer equation, respectively. It turns out that the new equation implies the previously found equation for one Dirac and one Klein-Gordon particle [3], if the spin-0 states of the Duffin-Kemmer particle can be separated out of its spin-1 states. Then the latter states satisfy a new relativistic two-body wave equation for one spin-1/2 and one spin-1 particle which, if isolated, correspond to the Dirac and the Proca equation, respectively. This spin separation takes place in general for a static interaction between the Dirac and the Duffin-Kemmer particle.

2. *Concise form*

To start with consider the free Duffin-Kemmer equation describing a free spin-0 *or* spin-1 particle:

$$(\beta^\mu p_\mu - M)\psi = 0, \tag{1}$$

where

$$\beta_\lambda\beta_\mu\beta_\nu + \beta_\nu\beta_\mu\beta_\lambda = g_{\lambda\mu}\beta_\nu + g_{\nu\mu}\beta_\lambda \; (\lambda, \mu, \nu = 0, 1, 2, 3), \tag{2}$$

$(\beta^\mu) = (\beta^0, \vec{\beta})$ and $(p_\mu) = (p_0, -\vec{p})$ with $p_0 = E$ [5]. We will use for the Duffin-Kemmer matrices the convenient representation

$$\beta^\mu = \tfrac{1}{2}(\gamma_1^\mu + \gamma_2^\mu), \tag{3}$$

where $(\gamma_i^\mu) = (\beta_i, \beta_i\vec{\alpha}_i)$, $\vec{\alpha}_i = \gamma_i^5\vec{\sigma}_i$ and $\gamma_i^5 = i\gamma_i^0\gamma_i^1\gamma_i^2\gamma_i^3$ $(i = 1, 2)$ are two commuting sets of the Dirac matrices. So the spin of the Duffin-Kemmer particle is

$$\vec{S} = \tfrac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2) \tag{4}$$

in consistency with the relations $S_{kl} = \varepsilon_{klm}S_m$ and

$$S^{\mu\nu} = i[\beta^\mu, \beta^\nu] = \tfrac{1}{2}(\sigma_1^{\mu\nu} + \sigma_2^{\mu\nu}), \tag{5}$$

where $\sigma_i^{\mu\nu} = \dfrac{i}{2}[\gamma_i^\mu, \gamma_i^\nu]$. The wave-function components corresponding to spin $s = 0$ and $s = 1$ of the Duffin-Kemmer particle are $\psi_s = P_s\psi$ $(s = 0, 1)$, where the spin projection operators are given by

$$P_0 = \tfrac{1}{4}(1 - \vec{\sigma}_1\cdot\vec{\sigma}_2), \quad P_1 = \tfrac{1}{4}(3 + \vec{\sigma}_1\cdot\vec{\sigma}_2) = \tfrac{1}{2}\vec{S}^2. \tag{6}$$

Note that $[P_s, \vec{S}] = 0$, whilst $P_0(\vec{\sigma}_1 - \vec{\sigma}_2) = (\vec{\sigma}_1 - \vec{\sigma}_2)P_1$ and $P_1(\vec{\sigma}_1 - \vec{\sigma}_2) = (\vec{\sigma}_1 - \vec{\sigma}_2)P_0$. The mass M in Eq. (1) has generally the operator form

$$M = P_0M_0 + P_1M_1, \tag{7}$$

where masses M_s may be different.

Now, we go over to the system of one Dirac and one Duffin-Kemmer particle. In the free case we introduce the relativistic two-body wave equation that arises from the

Duffin-Kemmer equation by the substitution

$$E \to E_{\mathrm{DK}} \to E-(\vec{\alpha}\cdot\vec{p}_{\mathrm{D}}+\beta m), \qquad \vec{p} \to \vec{p}_{\mathrm{DK}} \tag{8}$$

providing the correct energy eigenvalues for our free system (the subscripts D and DK refer here to the Dirac and the Duffin-Kemmer particle, respectively). The Dirac matrices $\vec{\alpha}$ and β appearing in Eq. (8) commute with the Duffin-Kemmer matrices β^μ and, therefore, also with their component Dirac matrices γ_i^μ $(i = 1, 2)$. In this way we get the following free equation:

$$\{\beta^0[E-(\vec{\alpha}\cdot\vec{p}_{\mathrm{D}}+\beta m)]-\vec{\beta}\cdot\vec{p}_{\mathrm{DK}}-M\}\vec{\psi}^{(0)}(\vec{r}_{\mathrm{D}}, \vec{r}_{\mathrm{DK}}) = 0. \tag{9}$$

In the centre-of-mass frame where $\vec{p}_{\mathrm{D}} = -\vec{p}_{\mathrm{DK}} \equiv \vec{p}$ and $\vec{r}_{\mathrm{D}}-\vec{r}_{\mathrm{DK}} \equiv \vec{r}$, this equation takes the form

$$\{\beta^0[E-(\vec{\alpha}\cdot\vec{p}+\beta m)]+\vec{\beta}\cdot\vec{p}-M\}\psi^{(0)}(\vec{r}) = 0 \tag{10}$$

which will be our basic kinematical relationship.

The interaction can be introduced into Eq. (10) by some substitutions. If the particles interact mutually by the vector potential $V = V(\vec{r})$ (being the time component of a four--vector) and the scalar potential $S = S(\vec{r})$, we substitute

$$E \to E-V, \qquad M \to M+\tfrac{1}{2}S, \qquad m \to m+\tfrac{1}{2}S, \tag{11}$$

where in general

$$V = P_0V_0+P_1V_1, \qquad S = P_0S_0+P_1S_1 \tag{12}$$

with P_s given in Eq. (6) (we stress that $s = 0, 1$ denotes spins of the Duffin-Kemmer particle). So, in such a case we obtain the following new relativistic two-body wave equation:

$$\{\beta^0[E-V-(\vec{\alpha}\cdot\vec{p}+\beta m+\beta\,\tfrac{1}{2}\,S)]+\vec{\beta}\cdot\vec{p}-M-\tfrac{1}{2}\,S\}\psi(\vec{r}) = 0. \tag{13}$$

In this equation we will use the abbreviations.

$$D = \vec{\alpha}\cdot\vec{p}+\beta(m+\tfrac{1}{2}S)+V, \qquad I = M+\tfrac{1}{2}S. \tag{14}$$

3. Spinor components

In order to make Eq. (13) more explicit we write it down as a system of equations in the Dirac representation of matrices γ_i^μ $(i = 1, 2)$. In this representation, $\beta_i (i = 1, 2)$ are diagonal and so can be replaced by their eigenvalues $\beta_i = \pm 1$. Then

$$\begin{cases} \langle\beta_1, \beta_2|\beta^0 = \tfrac{1}{2}(\beta_1+\beta_2)\langle\beta_1, \beta_2|. \\ \langle\beta_1, \beta_2|\vec{\beta} = \tfrac{1}{2}\,\beta_1\vec{\sigma}_1\langle-\beta_1, \beta_2|+\tfrac{1}{2}\,\beta_2\vec{\sigma}_2\langle\beta_1, -\beta_2| \end{cases} \tag{15}$$

since $\vec{\gamma}_i = \beta_i\gamma_i^5\vec{\sigma}_i$ (and $\vec{\sigma}_i$ commute with β_i and γ_i^5). Thus, having written down Eq. (13) for $\psi_{\beta_1\beta_2} = \langle\beta_1, \beta_2|\psi\rangle$ $(\beta_i = \pm 1)$ and then combining the wave-function components

$\psi_{\beta_1\beta_2}$ into the new components

$$f^{\pm} = \frac{\psi_{++} \pm \psi_{--}}{\sqrt{2}}, \quad g^{\pm} = \frac{\psi_{+-} \pm \psi_{-+}}{\sqrt{2}}, \tag{16}$$

we arrive at the following system of equations in the case of potentials V_s and S_s *not* containing the matrices γ_i^{μ}[1]:

$$\begin{cases} (E-D)f^{\pm} - If^{\mp} = \mp\frac{1}{2}(\vec{\sigma}_1 \pm \vec{\sigma}_2)\cdot\vec{p}g^{\pm}, \\ \qquad\qquad -Ig^{\mp} = \mp\frac{1}{2}(\vec{\sigma}_1 \mp \vec{\sigma}_2)\cdot\vec{p}f^{\pm}. \end{cases} \tag{17}$$

Note that the components $f^{\pm}$ include the "large-large" and "small-small" components of ψ, whereas $g^{\pm}$ include the "large-small" and "small-large" components of ψ (all the notions referring to the Duffin-Kemmer particle).

Applying to Eqs. (17) the spin projection operators (6) for the Duffin-Kemmer particle we obtain the system of equations for the wave function components $f_s^{\pm} = P_s f^{\pm}$ and $g_s^{\pm} = P_s g^{\pm}$ $(s = 0, 1)$. This system splits into two independent subsystems involving the "large-large" components for spin $s = 0$ and $s = 1$, respectively. In this way we get for $s = 0$:

$$\begin{cases} (E-D_0)f_0^+ - I_0 f_0^- = 0, \\ (E-D_0)f_0^- - I_0 f_0^+ = \frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\vec{p}g_1^-, \\ \qquad\qquad -I_1 g_1^- = -\frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\vec{p}f_0^+ \end{cases} \tag{18}$$

and for $s = 1$:

$$\begin{cases} (E-D_1)f_1^+ - I_1 f_1^- = -\frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\vec{p}g_1^+, \\ (E-D_1)f_1^- - I_1 f_1^+ = \frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\vec{p}g_0^-, \\ \qquad\qquad -I_1 g_1^+ = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\vec{p}f_1^-, \\ \qquad\qquad -I_0 g_0^- = -\frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\vec{p}f_1^+, \end{cases} \tag{19}$$

while $g_0^+ \equiv 0$ is a trivial component. Here, $D_s = \vec{\alpha}\cdot\vec{p} + \beta(m + \frac{1}{2}S_s) + V_s$ and $I_s = M_s + \frac{1}{2}S_s$. We can see that the splitting (17) → (18)+(19) follows for static potentials (more generally, an analogical splitting holds for some non-static potentials.

From the system of equations (18) we can easily derive the following equation for f_0^+ in the case of $I_1 \equiv I_0 \equiv M_0$ (i.e. $S \equiv 0$ and $M_1 = M_0$):

$$[(E-D_0)^2 - \vec{p}^2 - M_0^2]f_0^+ = 0. \tag{20}$$

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[1] It is the case of *static* potentials which arise from the general potentials through the substitution $\vec{\alpha}_i \to 0$, $\beta_i \to 1$ and $\gamma_i^5 \to 0$ (and also $\vec{\alpha} \to 0$, $\beta \to 1$ and $\gamma^5 \to 0$).

It is the previously found relativistic two-body wave equation for one Dirac and one Klein-Gordon particle interacting mutually via a static vector potential [3].

In the general case we can conclude that one Dirac and one Duffin-Kemmer particle interacting mutually via static potentials are described by the new wave equation (18) *or* (19), depending on whether the spin of the latter particle is $s = 0$ *or* $s = 1$.

4. Scalar and vector components

In the systems of equations (18) and (19) the dependence on the matrices $\frac{1}{2}(\vec{\sigma}_1 \pm \vec{\sigma}_2)$ can be absorbed into the wave-function components. To this end we define for the system (18) the following new components:

$$\phi = f_0^+,$$
$$(\phi^\mu) = (\phi^0, \vec{\phi}) = \left(f_0^-, \frac{\vec{\sigma}_1 - \vec{\sigma}_2}{2} g_1^- \right). \tag{21}$$

Then after a simple calculation we obtain the following system of equations equivalent to the system (18):

$$\begin{cases} (E - D_0)\phi = I_0 \phi_0, \\ \vec{p}\phi = I_1 \vec{\phi} \end{cases} \tag{22a}$$

and

$$(E - D_0)\phi_0 - \vec{p} \cdot \vec{\phi} = I_0 \phi. \tag{22b}$$

Putting $D_0 \equiv 0$ and $I_1 \equiv I_0 \equiv M_0$ we identify Eqs. (22) with the familiar equations

$$p_\mu \phi = M_0 \phi_\mu \tag{23a}$$

and

$$p_\mu \phi^\mu = M_0 \phi \tag{23b}$$

leading to the free Klein-Gordon equation

$$(p^2 - M_0^2)\phi = 0. \tag{24}$$

In the case of $I_1 \equiv I_0 \equiv M_0$ (i.e. $S \equiv 0$ and $M_1 = M_0$) but $D_0 \not\equiv 0$, Eqs. (22) lead to Eq. (20) with $f_0^+ = \phi$. Putting $D_0 \equiv 0$ in Eq. (20) we come to Eq. (24).

Analogically, for the system (19) we define the following new wave-function components:

$$(U^\mu) = (U^0, \vec{U}) = \left(g_0^-, \frac{\vec{\sigma}_1 - \vec{\sigma}_2}{2} f_1^- \right),$$

$$U^{\mu\nu} = \begin{cases} U^{0l} = -U^{l0} = \left(\dfrac{\vec{\sigma}_1 - \vec{\sigma}_2}{2}\right)_l f_1^+, \\ U^{kl} = \left[\left(\dfrac{\vec{\sigma}_1 - \vec{\sigma}_2}{2}\right)_k \left(\dfrac{\vec{\sigma}_1 + \vec{\sigma}_2}{2}\right)_l - \left(\dfrac{\vec{\sigma}_1 + \vec{\sigma}_2}{2}\right)_k \left(\dfrac{\vec{\sigma}_1 - \vec{\sigma}_2}{2}\right)_l\right] g_1^+ \\ \quad = \frac{1}{2}(\sigma_{1k}\sigma_{2l} - \sigma_{2k}\sigma_{1l}) g_1^+, \end{cases} \tag{25}$$

where $U^{\mu\nu} = -U^{\nu\mu}$. Then after some calculations we get the following system of equations equivalent to the system (19):

$$\begin{cases} (E-D_1)U_l - p_l U_0 = I_1 U_{0l}, \\ p_k U_l - p_l U_k = I_1 U_{kl} \end{cases} \tag{26a}$$

and

$$\begin{cases} (E-D_1)U_{0l} - p_k U_{kl} = I_1 U_l, \\ -p_k U_{k0} = I_0 U_0. \end{cases} \tag{26b}$$

Again putting $D_1 \equiv 0$ and $I_0 \equiv I_1 \equiv M_1$ we recognize Eqs. (26) as the free Proca equations

$$p_\mu U_\nu - p_\nu U_\mu = M_1 U_{\mu\nu} \tag{27a}$$

and

$$p_\mu U^{\mu\nu} = M_1 U^\nu \tag{27b}$$

leading to the familiar equations

$$(p^2 - M_1^2)U_\nu = 0, \qquad p_\mu U^\mu = 0. \tag{28}$$

In the case of $I_0 \equiv I_1 \equiv M_1$ (i.e. $S \equiv 0$ and $M_0 = M_1$) but $D_1 \not\equiv 0$, Eqs. (26) lead to the new equations

$$\begin{cases} [(E-D_1)^2 - (\vec{p}^2 + M_1^2)]U_l + p_k p_l U_k = (E-D_1)p_l U_0, \\ -(\vec{p}^2 + M_1^2)U_0 + p_k(E-D_1)U_k = 0 \end{cases} \tag{29a}$$

and

$$(E-D_1)U_0 - p_k U_k = -\frac{1}{M_1}[p_k, V_1]U_{k0}. \tag{29b}$$

Eliminating U_0 from Eqs. (29a) we obtain

$$[(E-D_1)^2 - \vec{p}^2 - M_1^2]U_l + \left[p_k p_l - (E-D_1)\frac{p_k p_l}{\vec{p}^2 + M_1^2}(E-D_1)\right]U_k = 0. \tag{30}$$

It is a new relativistic two-body wave equation for one Dirac and one Proca particle interacting mutually via a static vector potential[2].

Note that in the two-body wave equations (20) and (30) for spin $s = 0$ and $s = 1$, respectively, we can write

$$(E-D_s)^2-\vec{p}^2-M_s^2$$
$$= (E-V_s)^2-2(E-V_s)(\vec{\alpha}\cdot\vec{p}+\beta m)+\vec{\alpha}\cdot[\vec{p}, V_s]+m^2-M_s^2$$
$$= \sqrt{E-V_s}\left[E-V_s-2(\vec{\alpha}\cdot\vec{p}+\beta m)+\frac{m^2-M_s^2}{E-V_s}\right]\sqrt{E-V_s}, \tag{31}$$

so that in the case of Eq. (20) the factor $\sqrt{E-V_0}$ can be absorbed into the wave function $f_0^- = \phi$, what gives the equation

$$\left[E-V_0-2(\vec{\alpha}\cdot\vec{p}+\beta m)+\frac{m^2-M_0^2}{E-V_0}\right](\sqrt{E-V_0}\,\phi) = 0 \tag{32}$$

that was handled before [3, 4].

In the general case we conclude that one Dirac and one Duffin-Kemmer particle interacting mutually via static potentials can be described by the new two-body wave equation (22) *or* (26) depending on spin $s = 0$ *or* $s = 1$ of the latter particle. The systems of equations (22) and (26) are perhaps more convenient in application than their equivalent systems (18) and (19).

APPENDIX

Alternative approach: diquark as a very close Breit system

The two-body wave equations (20) and (30) may be applied to the quark-diquark system q (qq), if the diquark (qq) can be treated as a Duffin-Kemmer particle with spin $s = 0$ and $s = 1$, respectively. One can wonder, however, if the better description of the

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[2] Eqs. (26) give also the equations for U_l and U_{0l}:

$$\begin{cases} (E-D_1)U_l-\dfrac{1}{M_1}p_kp_lU_{0k} = M_1U_{0l}, \\ (E-D_1)U_{0l}-\dfrac{1}{M_1}p_k(p_kU_l-p_lU_k) = M_1U_l. \end{cases}$$

Eliminating U_{0l} we get after some calculations:

$$[(E-D_1)^2-\vec{p}^2-M_1^2]U_l$$
$$-[p_kp_l, V_1]\frac{1}{E-D_1}\left(\frac{\vec{p}_1^2+M_1^2}{M_1^2}U_k-\frac{p_kp_m}{M_1^2}U_m\right) = 0.$$

This equation must be equivalent to Eq. (30).

system q (qq) is not provided by the triple-Dirac equation where two of three quarks, being bound very closely, are put together *kinematically*:

$$\vec{r}_1 = \vec{r}_2 \equiv \vec{r}_B, \quad \vec{p}_1 = \vec{p}_2 \equiv \tfrac{1}{2}\vec{p}_B, \quad m_1 = m_2 \equiv \tfrac{1}{2}M. \tag{A1}$$

In this alternative approach the diquark (qq) corresponds to the point limit of a very close system described by the Breit or double-Dirac equation. In such an approach the free triple-Dirac equation

$$[E - \sum_{i=1}^{3} (\vec{\alpha}_i \cdot \vec{p}_i + \beta_i m_i)]\psi^{(0)}(\vec{r}_1, \vec{r}_2, \vec{r}_3) = 0 \tag{A2}$$

transits into the equation

$$[E - (\vec{\alpha} \cdot \vec{p}_D + \beta m) - \tfrac{1}{2}(\vec{\alpha}_1 + \vec{\alpha}_2) \cdot \vec{p}_B - \tfrac{1}{2}(\beta_1 + \beta_2)M]\psi^{(0)}(\vec{r}_D, \vec{r}_B) = 0, \tag{A3}$$

where $\vec{\alpha} = \vec{\alpha}_3$, $\beta = \beta_3$, $\vec{r}_D = \vec{r}_3$, $\vec{p}_D = \vec{p}_3$ and $m = m_3$. In the centre-of-mass frame, where $\vec{p}_D = -\vec{p}_B \equiv \vec{p}$ and $\vec{r}_D - \vec{r}_B \equiv \vec{r}$, this equation assumes the form

$$[E - (\vec{\alpha} \cdot \vec{p} + \beta m) + \tfrac{1}{2}(\vec{\alpha}_1 + \vec{\alpha}_2) \cdot \vec{p} - \tfrac{1}{2}(\beta_1 + \beta_2)M]\psi^{(0)}(\vec{r}) = 0 \tag{A4}$$

and has to be compared with Eq. (10) of our previous approach. Though $\beta^0 = \frac{1}{2}(\beta_1 + \beta_2)$ and $\vec{\beta} = \frac{1}{2}(\vec{\gamma}_1 + \vec{\gamma}_2)$ both the equations differ evidently because

$$\tfrac{1}{2}(\vec{\gamma}_1 + \vec{\gamma}_2) \neq \tfrac{1}{2}(\beta_1 + \beta_2)\,\tfrac{1}{2}(\vec{\alpha}_1 + \vec{\alpha}_2) \tag{A5}$$

and, moreover, there is no inverse of $\frac{1}{2}(\beta_1 + \beta_2)$.[3]

Introducing the interaction into Eq. (A4) through the substitution (11) one gets the following relativistic two-body wave equation (instead of Eq. (13)):

$$[E - V - (\vec{\alpha} \cdot \vec{p} + \beta m + \beta\,\tfrac{1}{2}S) + \tfrac{1}{2}(\vec{\alpha}_1 + \vec{\alpha}_2) \cdot \vec{p} - \tfrac{1}{2}(\beta_1 + \beta_2)(M + \tfrac{1}{2}S)]\psi(\vec{r}) = 0. \tag{A6}$$

Here, the abbreviations (14) may be also used.

Writing down Eq. (A6) as a system of equations in the Dirac representation of matrices $\vec{\alpha}_i = \gamma_i^5 \vec{\sigma}_i$ $(i = 1, 2)$ and then combining the wave-function components $\psi_{\beta_1\beta_2}$ $(\beta_i = \pm 1)$ into $f^{\pm}$ and $g^{\pm}$ as given in Eq. (16), one obtains in the case of static potentials V_s and S_s the following system of equations (instead of Eqs. (17)):

$$\begin{cases} (E-D)f^{\pm} - If^{\mp} = \mp\tfrac{1}{2}(\vec{\sigma}_1 \pm \vec{\sigma}_2) \cdot \vec{p}g^{\pm}, \\ (E-D)g^{\pm} \qquad\quad = \mp\tfrac{1}{2}(\vec{\sigma}_1 \pm \vec{\sigma}_2) \cdot \vec{p}f^{\pm}. \end{cases} \tag{A7}$$

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[3] It follows that Eq. (A4) even with the term $\vec{\alpha} \cdot \vec{p} + \beta m$ put zero is not evidently relativistic covariant, what is in contrast to Eq. (10). It is a consequence of the lack of evident relativistic covariance for the two-body Breit equation. Note that in the case of $m_1 \neq m_2$ the operator $(\beta_1 m_1 + \beta_2 m_2) \times (m_1 + m_2)^{-1}$ gets its inverse equal to $(\beta_1 m_1 - \beta_2 m_2)(m_1 - m_2)^{-1}$.

Note the differences between Eqs. (A7) and Eqs. (17):

$$(E-D)g^{\pm} \rightleftarrows -Ig^{\mp}, \quad (\vec{\sigma}_1 \pm \vec{\sigma}_2) \cdot \vec{p}f^{\pm} \rightleftarrows (\vec{\sigma}_1 \mp \vec{\sigma}_2) \cdot \vec{p}f^{\pm}. \tag{A8}$$

The spin projection operators (6) applied to Eqs. (A7) give the system of equations for $f_s^{\pm} = P_s f^{\pm}$ and $g_s^{\pm} = P_s g^{\pm}$ ($s = 1, 2$), which splits into two independent subsystems involving the "large-large" components for spin $s = 0$ and $s = 1$, respectively. In this way one gets for $s = 0$ (instead of Eqs. (18)):

$$\begin{cases} (E-D_0)f_0^+ - I_0 f_0^- = 0, \\ (E-D_0)f_0^- - I_0 f_0^+ = \frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{p}g_1^-, \\ (E-D_1)g_1^- = \frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{p}f_0^- \end{cases} \tag{A9}$$

and for $s = 1$ (instead of Eqs. (19)):

$$\begin{cases} (E-D_1)f_1^+ - I_1 f_1^- = -\frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{p}g_1^+, \\ (E-D_1)f_1^- - I_1 f_1^+ = \frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{p}g_0^-, \\ (E-D_1)g_1^+ = -\frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{p}f_1^+, \\ (E-D_0)g_0^- = \frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{p}f_1^-, \end{cases} \tag{A10}$$

while $(E-D_0)g_0^+ = 0$ so that g_0^+ is a trivial component which can be put zero: $g_0^+ \equiv 0$.

From the system of equations (A9) one can deduce in the case of $D_1 \equiv D_0$ and $I_0 \equiv M_0$ (i.e. $V_1 \equiv V_0$ and $S \equiv 0$) the following equation for f_0^+ (instead of Eq. (20)):

$$\left\{(E-D_0)^2 - \vec{p}^2 - M_0^2 + \vec{p}\,\frac{1}{E-D_0} \cdot [\vec{p}, V_0]\right\} f_0^+ = 0. \tag{A11}$$

It is a more involved equation than the two-body wave equation (20) for one Dirac and one Klein-Gordon particle [3]. Note that the resulting equation for f_0^- is a bit simpler than Eq. (A11):

$$\left\{(E-D_0)^2 - \vec{p}^2 - M_0^2 - [\vec{p}, V_0]\,\frac{1}{E-D_0} \cdot \vec{p}\right\} f_0^- = 0. \tag{A12}$$

Also for the systems of equations (A9) and (A10) one can absorb the matrices $\frac{1}{2}(\vec{\sigma}_1 \pm \vec{\sigma}_2)$ into the wave-function components. In fact, in terms of the components defined in Eq. (21) the system (A9) assumed the form (to be compared with Eqs. (22)):

$$\begin{cases} (E-D_0)\phi = I_0 \phi_0, \\ (E-D_1)\vec{\phi} - \vec{p}\phi_0 = 0, \\ (E-D_0)\phi_0 - \vec{p} \cdot \vec{\phi} = I_0 \phi. \end{cases} \tag{A13}$$

If $D_1 \equiv D_0 \equiv 0$ and $I_0 \equiv M_0$, Eqs. (A13) reduce to the relations

$$\begin{cases} p_0\phi = M_0\phi_0, \\ p_0\phi_l - p_l\phi_0 = 0, \\ p_\mu\phi^\mu = M_0\phi \end{cases} \tag{A14}$$

implying the free Klein-Gordon equation (24). In the case of $D_1 \equiv D_0 \not\equiv 0$ and $I_0 \equiv M_0$ (i.e. $V_1 \equiv V_0$ and $S \equiv 0$), Eqs. (A13) lead to Eq. (A11) for $f_0^+ = \phi$ and to Eq. (A12) for $f_0^- = \phi_0$.

Similarly, in terms of the wave-function components given in Eq. (25) the system (A10) takes the form (to be compared with Eqs. (26)):

$$\begin{cases} (E-D_1)U_l - p_l U_0 = I_1 U_{0l}, \\ (E-D_1)U_{kl} - (p_k U_{0l} - p_l U_{0k}) = 0, \\ (E-D_1)U_{0l} - p_k U_{kl} = I_1 U_l, \\ (E-D_0)U_0 - p_k U_k = 0. \end{cases} \tag{A15}$$

If again $D_0 \equiv D_1 \equiv 0$ and $I_1 \equiv M_1$, Eqs. (A15) transit into the relations

$$\begin{cases} p_0 U_l - p_l U_0 = M_1 U_{0l}, \\ p_0 U_{kl} - (p_k U_{0l} - p_l U_{0k}) = 0, \\ p_\mu U^{\mu l} = M_1 U^l, \\ p_\mu U^\mu = 0 \end{cases} \tag{A16}$$

giving the familiar equations (28). In the case of $D_0 \equiv D_1 \not\equiv 0$ and $I_1 \equiv M_1$ (i.e. $V_0 \equiv V_1$ and $S \equiv 0$), Eqs. (A15) lead to the following equation for $\vec{U}$:

$$\left\{(E-D_1)^2 - \vec{p}^2 - M_1^2 + \vec{p}\,\frac{1}{E-D_1}\cdot[\vec{p}, V_1]\right\} U_l - \left(p_k \frac{1}{E-D_1}[p_l, V_1] + [p_l, V_1]\frac{1}{E-D_1} p_k\right) U_k = 0 \tag{A17}$$

that has to be compared with Eq. (30) of our previous approach. The resulting equation for U_{0l} is a bit simpler:

$$\left\{(E-D_1)^2 - \vec{p}^2 - M_1^2 - [\vec{p}, V_1]\frac{1}{E-D_1}\cdot\vec{p}\right\} U_{0l} + \left([p_k, V_1]\frac{1}{E-D_1} p_l + p_l \frac{1}{E-D_1}[p_k, V_1]\right) U_{0k} = 0. \tag{A18}$$

In the general case, the two-body wave equation (A13) *or* (A15) is proper for spin $s = 0$ *or* $s = 1$, respectively.

While it is likely that our previous approach fits better a preon system, the alternative approach sketched in this Appendix seems to be more proper for a quark-diquark system since the diquark is *really* a composite state of two quarks. The essential difference between both approaches consists in the fact that the Duffin-Kemmer equation arises from formal adding of two operations

$$\frac{1}{2}(\gamma_i^\mu p_\mu - M) \quad (i = 1, 2) \tag{A19}$$

so that *mass* is there formally additive. In contrast, taking the point limit of the Breit equation one adds two operations

$$\frac{1}{2}(p_0 - \vec{\alpha}_i \cdot \vec{p} - \beta_i M) \quad (i = 1, 2) \tag{A20}$$

in consistency with the physical additivity of *energy* which always holds when some constituents form a composite system.

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