LEPTON PAIR PRODUCTION IN QUANTUM CHROMODYNAMICS*

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After a brief review of experimental data on lepton pair production, I describe the perturbative calculation of this process in QCD and the exponentiation of soft gluon effects. Results of numerical calculations for the "K-factor" are compared with experiment.

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1. Introduction

In the past few years, a considerable amount of work, both theoretical and experimental [1–3], has been devoted to the study of lepton pair (l^+l^-) production in hadronic collisions.

$$H_1+H_2 \to (l^+l^-)+X, \tag{1}$$

where H_1 and H_2 are two hadrons. As I will be interested in what follows in reaction (1) only, I'll call for the sake of brevity $\sigma(H_1H_2)$ the cross-section for this reaction.

One of the main interests of lepton pair production is of course that its cross-section can be computed in perturbative quantum chromodynamics (QCD), provided effects to be mentioned below are not too important. The topic I'll be interested in is the theoretical computation and the comparison with experiment of the cross-section $d\sigma/dQ^2dy$ where Q^2 is the mass of the pair squared and y its rapidity[1]. Instead of the cross-section, one often uses the so-called "K-factor", which is the ratio of $d\sigma/dQ^2dy$ to the Drell-Yan (DY) cross-section (to be defined in the next section):

$$K(Q^2, y) = \frac{d\sigma/dQ^2dy}{d\sigma^{\mathrm{DY}}/dQ^2dy}. \tag{2}$$

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[1] Instead of the rapidity, one could also use the Feynman variable x_F, which is however technically less convenient.

I'll not discuss the transverse momentum dependence of the cross-section, which is an important, but technically somewhat different, subject [4]. I'll also ignore possible difficulties with factorization which were pointed out by Bodwin et al. [5]. The theoretical status of this problem is at present unclear, and it is quite possible that these difficulties do not appear at all, because of Sudakov [6] or other effects [7].

The plan of these lectures is as follows: in Section 2, I recall the so called naive Drell-Yan (parton) model [8], and in Section 3 I give a brief (and biased) review of experimental data. Then in Section 4, I examine the first order perturbative calculation in QCD, while in Section 5, I turn to the problem of soft gluons. A summary of the theoretical results and a discussion will be found in Section 6.

2. *The naive Drell-Yan model*

More than ten years ago, Drell and Yan (DY) proposed a parton model for the description of lepton pair production [8]. In this model, as is well known, the virtual photon γ^*, whose decay gives the lepton pair, is produced via the annihilation of a quark-antiquark pair (Fig. 1). It is clear from Fig. 1 that an absolute prediction for the cross-section is

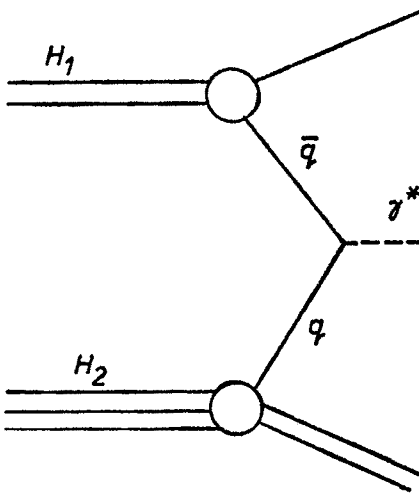

Fig. 1. The Drell-Yan mechanism

obtained once one knows from deep inelastic electron and neutrino scattering the probabilities $q_H(x)$ and $\bar{q}_H(x)$ (which I call quark distributions in what follows) of finding a quark or an antiquark of fractional momentum x in a hadron H. These quark distributions are available if one considers $\bar{p}p$ or pp collisions. Conversely, in the case of $\pi^{\pm}p$ collisions, since there are no deep inelastic scattering experiments on pions, the Drell-Yan model allows to extract the pion structure functions from the cross-section for reaction (1).

To be more precise, let me define the kinematics of reaction (1). Since one does not observe the final hadronic system (X), the kinematics is completely defined by:

$$\left.\begin{aligned} \sqrt{s} &= \text{center of mass energy of the } H_1H_2 \text{ system} \\ Q^2 &= (\text{mass})^2 \\ y &= \text{rapidity} \\ q_\perp &= \text{transverse momentum} \end{aligned}\right\} \text{ of the } (l^+l^-) \text{ pair.}$$

In addition, useful information is provided by the decay of the virtual photon. This decay is described by the angles $\Omega = (\theta, \varphi)$ of emission of the l^+ (for example) in the rest frame of the pair, with respect to some axis.

Instead of s, Q^2, y, it is convenient to introduce variables x_1, x_2 and τ:

$$x_1 = \sqrt{\tau}\, e^{y}, \quad x_2 = \sqrt{\tau}\, e^{-y}, \quad \tau = \frac{Q^2}{s} = x_1 x_2. \tag{3}$$

It is easy to convince oneself that in the case of the Drell-Yan mechanism, x_2 and x_1 are nothing but the fractional momenta of the quark and the antiquark (Fig. 1), but the definition (3) will be used in the general case. Notice the kinematical limitation:

$$(1-x_1)(1-x_2) \geqslant \frac{m_X^2 \tau}{Q^2}, \tag{4}$$

where m_X is the mass of the final hadronic system (X), which leads to:

$$\tau \leqslant x_1, x_2 \leqslant 1. \tag{5}$$

The Drell-Yan cross-section[2]:

$$\frac{d\sigma_H^{DY}}{dQ^2 dy} = \frac{1}{s}\, \sigma_H^{DY}(x_1, x_2) \tag{6}$$

is then given by:

$$\sigma_H^{DY}(x_1, x_2) = \frac{4\pi\alpha^2}{9Q^2} \sum_i (e_q^i)^2 \left(\bar{q}_{H_1}^i(x_1) q_{H_2}^i(x_2) + q_{H_1}^i(x_1) \bar{q}_{H_2}^i(x_2)\right), \tag{7}$$

where e_q^i is the charge of the quark of flavor (i) and α the fine structure constant. In order to simplify the notations, I will consider the case where there is only one type of antiquark in hadron H_1 and one type of quark in hadron H_2: this is almost realized for $\pi^- p$ collisions, where the dominant mechanism is the annihilation of an anti-up quark in the π^- with an up quark in the proton. It will be also convenient to define the constant A:

$$A = \frac{4\pi\alpha^2}{9Q^2}\, e_q^2. \tag{8}$$

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[2] In order to maintain a clear distinction between hadronic and partonic cross-sections, all hadronic cross-sections will receive a subscript H.

The predictions of the Drell-Yan model are well known:

(*i*) Scaling: $Q^2\sigma_H^{DY}(x_1, x_2)$ is a function of the scaling variables x_1 and x_2 only.

(*ii*) The average transverse momentum of the pair is of the order of the average transverse momenta of partons in a hadron: $\langle q_\perp \rangle \sim 300$ MeV/c.

(*iii*) In the absence of transverse momenta of the partons, the angular distribution of the l^+ follows a $(1+\cos^2\theta)$ law, where the z-axis is chosen parallel to the collision axis.

Notice also that, in the case of a π^-p collision for example, the cross-section decreases very rapidly when $\tau \to 1$. In fact, taking into account the dominant mechanism, and assuming a parametrization of the form:

$$V_\pi(x) \sim (1-x)^{\beta_\pi}, \quad u(x) \sim (1-x)^{\beta_u} \tag{9}$$

for the $\bar{u}$-distribution in the pion and the u-distribution in the proton respectively, we get at fixed s and for $\tau \to 1$:

$$\left.\frac{d\sigma_H^{DY}}{dQ^2 dy}\right|_{y=0} \simeq \frac{4\pi\alpha^2}{9s^2\tau} \tfrac{4}{9} V_\pi(\sqrt{\tau})u(\sqrt{\tau}) \sim (1-\sqrt{\tau})^{\beta_\pi+\beta_u} \sim (1-\sqrt{\tau})^4. \tag{10}$$

Equation (10) shows that the statistics will be poor at large values of Q^2. Indeed most of the statistics is contained in the range:

$$M_\psi^2 \lesssim Q^2 \lesssim M_\Upsilon^2$$

(for $Q^2 \lesssim M_\psi^2$, it is likely that the Drell-Yan mechanism is in competition with other effects).

3. *Review of experimental data*

A summary of experimental results available at present is given in Table I. This table is borrowed from the review of Burgun [3] and will need updating when the new results from CERN (NA3, NA10) and Fermi-lab (E-537, E-615 and E-605) are released (probably at the Paris Conference).

The main comments on Table I are that data are available for a variety of incident particles (p, $\bar{p}$, $\pi^\pm$, $K^\pm$), energies, and targets (heavy targets being preferred because of statistics). The cross-sections are measured essentially around $y = 0$ (ISR, CFS) or for positive rapidity. The main domain of Q^2 which has been explored lies between the ψ and the Υ, for reasons explained previously.

The conclusions, on which experiments must agree, are [1–3]:

(*i*) The cross-section depends linearly on the atomic number A, in agreement with a partonic interpretation.

(*ii*) The average transverse momentum $\langle q_\perp \rangle$ is much larger than the ~ 300 MeV/c of the parton model. In the case of π^-p collisions it is even much larger than that predicted by the first order QCD calculation [4].

(*iii*) The data are consistent with scaling. However, due to the uncertainties in absolute normalization and lack of overlap between different experiments, one does not really

TABLE I

Experiments on lepton pair production

Collaboration		Inc. part	$\sqrt{s}$ (ISR) or p_{lab} (GeV)	Target	x_F	Q	Number of events
ISR	ABCS	p	28, 53, 62	p	$-.2 \to .2$	4–18	10^3
	CHFMNP	p	62	p	$-.1 \to .5$	5–25	2.5×10^3
Fermi L.	CFS	p	200, 300, 400	Be, Cu, Pt	$-.1 \to .1$	5–20	2×10^5
	CIP	$\pi^\pm$	225	C, Cu, W	$0 \to 1.$	4–8.5	$\pi^-: 2\times 10^3$
	MNTW	p	400	Fe	$-.2 \to 1.$	4–18	10^5
CERN	SISI	π^-	150	Be	$-.2 \to .8$	4–8	1.5×10^3
	Ω	$K^\pm, \pi^\pm, p^\pm$	40	W		2–3	$\sim 10^4$
	NA3	$K^\pm, \pi^\pm, p^\pm$	150–400	H, Pt	$-.3 \to 1.$	4–14	$\pi^-: 5\times 10^4$
							$\pi^+: 2\times 10^3$
							$p: 10^5 \bar{p}: 300$
	NA10	π^-	280	C, W, Cu		4–14	2×10^3

Other experiments in progress: E 537 ($\bar{p}$, 125 GeV/c), E-615 (π^-, 225 GeV/c), E. 605 (p, 400 GeV/c) etc...

expect to see scaling violations. It is possible that high statistics experiments at large Q^2 will bring new information on this subject.

(*iv*) The angular distribution of leptons has been studied in many experiments. However the errors are still large, and no clear-cut conclusion has emerged yet.

(*v*) The K-factor (2) is found to be around 2. The cleanest way to measure K is to look at the cross-section difference

$$\sigma(\bar{p}\,\mathrm{Pt}) - \sigma(p\,\mathrm{Pt})$$

as in the NA3 experiment, because this difference depends on valence quarks only, whose distribution is rather well measured in deep inelastic scattering. Taking the CDHS parametrization of structure functions [9], the NA3 collaboration finds:

$$K = 2.3 \pm 0.4.$$

This result has been confirmed at the 82-Moriond meeting by the preliminary results of the E537 collaboration (Fig. 2). A summary of data on the K-factor is given in Table II (borrowed from D. Decamp, Ref. [3]). One can see that there is a rather good (too good?) agreement between various experiments.

One can also remark that the data on an hydrogen target are similar to those obtained on a nuclear target, thus excluding a nuclear effect. Of course sea distributions are needed when one does not take cross-section differences, and for this reason the results for pp-collisions are less conclusive. In the case of πp-collisions, the pion structure functions are of course unknown, but Adler's sum rule gives a strong constraint on the normalization of the dominant valence quark contribution.

(*vi*) Assuming that the naive Drell-Yan cross-section can be simply rescaled with

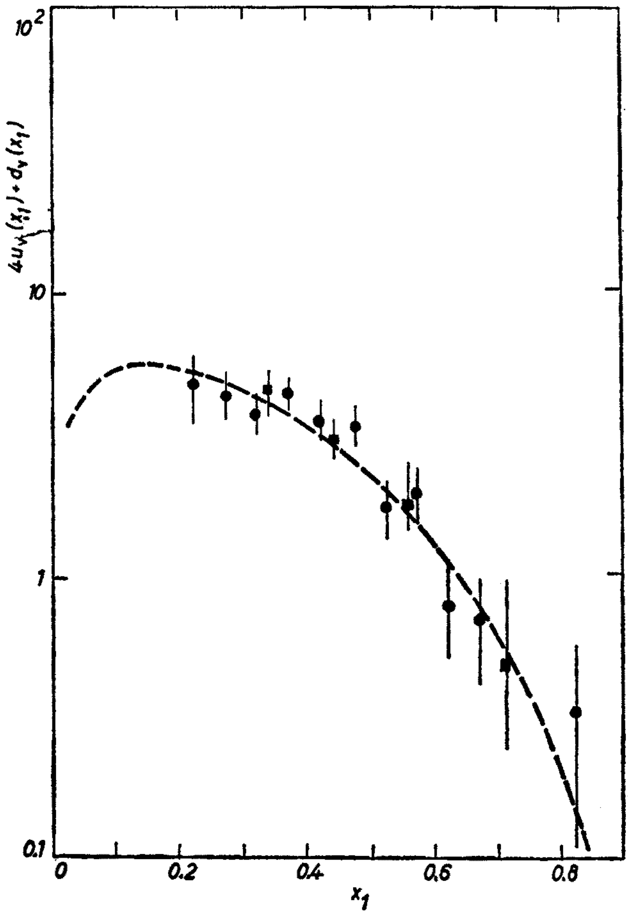

Fig. 2. Proton structure function from Drell-Yan. Dotted line: prediction from CDHS structure functions multiplied by 2.3. Circles: NA3 collaboration. Squares: E 537 collaboration

TABLE II

Experimental values of the *K*-factor

Collaboration	Reaction	p_{lab} (GeV)	*K*-factor
NA3	$(\bar{p}-p)$ Pt	150	$2.3\pm.4$
NA3	$(\pi^--\pi^+)$ Pt	200	$2.2\pm.4$
NA3	π^-H_2	200	$2.4\pm.4$
NA3	p Pt	200	$2.3\pm.3$
Ω	π^- W	40	$2.4\pm.4$
Ω	$(\pi^--\pi^+)$ W	40	$2.2\pm.25$
SISI	π^- C	150	$2.8\pm.6$
CFS	p W	200–400	$1.9\pm.3$
MNTW	p Fe	400	$1.6\pm.3$
ISR	pp	$\sqrt{s}=62$	~ 1.8

a constant (Q^2 and y-independent) K-factor, one can determine the pion and kaon structure functions. As already mentioned, the most reliable results are obtained by looking at the cross-section difference:

$$\sigma(\pi^- \text{ Pt}) - \sigma(\pi^+ \text{ Pt}) \sim V_\pi(x_1) u_v^A(x_2), \tag{11}$$

where in the case of a Platinum target $u_v^A(x)$ is given from the valence quark distributions $u_v(x)$ and $d_v(x)$ in the proton by:

$$u_v^A(x) = 0.4 u_v(x) + 0.6 d_v(x). \tag{12}$$

The valence structure function $V_\pi(x)$ is constrained by Adler's sum rule:

$$\int_0^1 dx V_\pi(x) = 1. \tag{13}$$

One often uses the Buras-Gaemers parametrization for valence quarks:

$$V_\pi(x) = A_\pi x^{\alpha_\pi}(1-x)^{\beta_\pi} \tag{14}$$

and for sea quarks:

$$S_\pi(x) = A'_\pi(1-x)^{\beta'_\pi}. \tag{15}$$

At an average value of Q^2, $Q^2 = \bar{Q}^2 = 25\,(\text{GeV})^2$, the NA3 collaboration finds [3]:

$$\alpha_\pi = 0.45 \pm 0.10, \quad \beta_\pi = 1.05 \pm 0.10, \quad \beta'_\pi = 5.4 \pm 0.2.$$

The corresponding curves are given on Fig. 3.

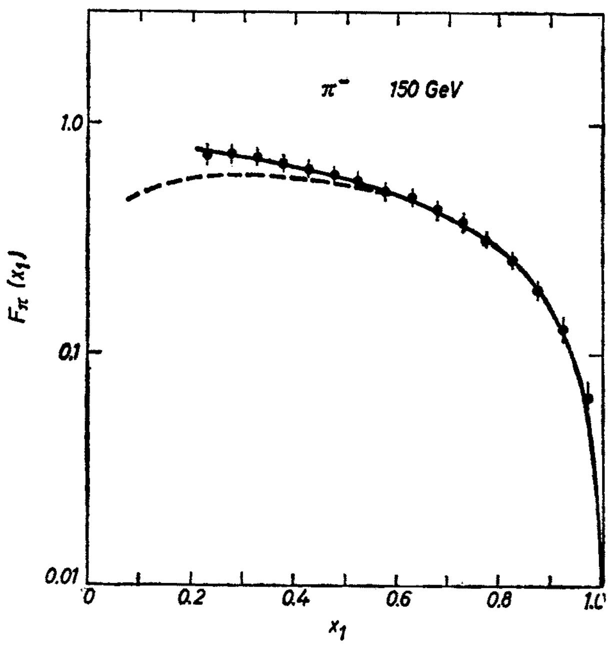

Fig. 3. The pion structure function $F_\pi(x)$, as determined by the NA3 collaboration. Full line: valence + sea. Dotted line: valence only

4. *The first order calculation*

4.1. General structure of the cross-section

Before proceeding to the calculation of the first order correction to the Drell-Yan process, I first rewrite the naive D-Y cross-section (7) in a form which is suitable for generalization:

$$\sigma_{\mathrm{H}}^{\mathrm{DY}}(x_1, x_2) = \int_{x_1}^{1} \frac{dt_1}{t_1} \int_{x_2}^{1} \frac{dt_2}{t_2}\, \bar{q}_{\mathrm{H}_1}(t_1) q_{\mathrm{H}_2}(t_2)$$

$$\times \left\{ A\delta\left(1-\frac{x_1}{t_1}\right)\delta\left(1-\frac{x_2}{t_2}\right)\right\}, \tag{16}$$

where the quantity between brackets is the partonic cross-section for the process $\bar{q}+q \to \gamma^*(y, Q^2)$:

$$\sigma^{\mathrm{DY}}(z_1, z_2) = A\delta(1-z_1)\delta(1-z_2) \tag{17}$$

with $z_i = x_i/t_i$. The hadronic cross-section appears as a double convolution of the partonic cross-section with quark distributions $\bar{q}_{\mathrm{H}_1}$ and q_{H_2}:

$$\sigma_{\mathrm{H}}^{\mathrm{DY}}(x_1, x_2) = [(\bar{q}_{\mathrm{H}_1} q_{\mathrm{H}_2}) * \sigma^{\mathrm{DY}}]\,(x_1, x_2). \tag{18}$$

As we shall see in what follows, the generalization of equation (16) including QCD corrections will be given by:

$$\sigma_{\mathrm{H}}(x_1, x_2) = \int_{x_1}^{1} \frac{dt_1}{t_1} \int_{x_1}^{1} \frac{dt_2}{t_2}\, \bar{q}_{\mathrm{H}_1}(t_1, Q^2) q_{\mathrm{H}_2}(t_2, Q^2)\bar{\sigma}(z_1, z_2)$$

$$= [(\bar{q}_{\mathrm{H}_1}(Q^2) q_{\mathrm{H}_2}(Q^2)) * \bar{\sigma}]\,(x_1, x_2). \tag{19}$$

Notice that in equation (19) the quark distributions depend on Q^2. The convolution in equation (19) can be transformed into a product by taking double moments[3]:

$$\sigma_{\mathrm{H}}(N_1, N_2) = \iint dx_1 dx_2 x_1^{N_1-1} x_2^{N_2-1} \sigma_{\mathrm{H}}(x_1, x_2) \tag{20}$$

and similarly for $\bar{\sigma}(z_1, z_2)$. Then equation (19) gives:

$$\sigma_{\mathrm{H}}(N_1, N_2) = \bar{q}_{\mathrm{H}_1}(N_1, Q^2) q_{\mathrm{H}_2}(N_2, Q^2)\bar{\sigma}(N_1, N_2), \tag{21}$$

where the moments of the quark distributions are defined as usual by:

$$q_{\mathrm{H}}(N, Q^2) = \int_0^1 dx x^{N-1} q_{\mathrm{H}}(x, Q^2). \tag{22}$$

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[3] σ_{H} and $\bar{\sigma}$ depend of course on Q^2, but I have suppressed the dependence with respect to this variable in order to simplify the writing.

4.2. Factorization of mass singularities in deep inelastic scattering[4]

The essential tool for the derivation of the QCD formula (19) is the theorem on factorization of mass singularities. Let me first review the use of this theorem in the case of deep inelastic scattering [14]. According to it, the structure function $q_H(x, Q^2)$ can be written as (see Fig. 4):

$$q_H(x, Q^2) = \left[q_H^0 * \Gamma\left(\frac{\mu^2}{\lambda^2}\right) * \hat{q}\left(\alpha_S(\mu^2); \frac{Q^2}{\mu^2}\right)\right](x). \tag{23}$$

In equation (23), μ is a "factorization mass" which is taken to be equal to the renormalization mass μ (it is quite possible to make a different choice, but nothing is gained by doing so); q_H^0 is the "bare" structure function, and the factorization of the partonic

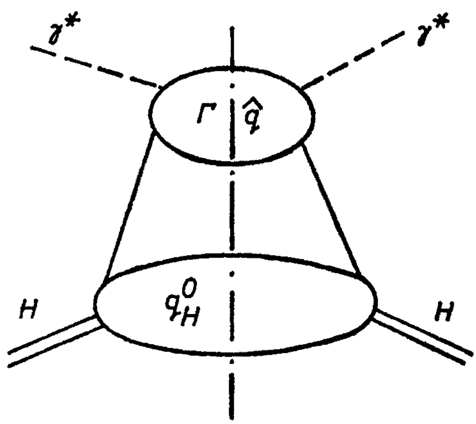

Fig. 4. Deep inelastic scattering. The dotted-dashed line is a unitarity cut

cross-section allows to write it as the convolution of two pieces Γ and $\hat{q}$. Γ depends on the regularization mass λ, and contains the mass singularities when $\lambda \to 0$ (if one uses dimensional regularization, Γ contains $1/\varepsilon$ poles [15]. Finally $\hat{q}$, which is independent of λ, can be expanded perturbatively into powers of the strong coupling constant α_S. As already mentioned, the convolution in equation (23) can be transformed into a product by taking moments:

$$q_H(N, Q^2) = q_H^0(N)\Gamma\left(N, \frac{\mu^2}{\lambda^2}\right)\hat{q}\left(N; \alpha_S(\mu^2), \frac{Q^2}{\mu^2}\right). \tag{24}$$

Since the hadronic cross-section is regular in the limit $\lambda^2 \to 0$, the mass singularities of Γ must be absorbed into q_H^0, and one defines a function $F(N, \mu^2)$:

$$F(N, \mu^2) = q_H^0(N)\Gamma\left(N, \frac{\mu^2}{\lambda^2}\right) \tag{25}$$

[4] For the sake of simplicity, I consider only non singlet distributions.

which is free of mass singularities and obeys a renormalization group (RG) equation:

$$\mu^2 \frac{dF(N,\mu^2)}{d\mu^2} = \frac{\alpha_S(\mu^2)}{2\pi}\left[P_{qq}(N) + \frac{\alpha_S(\mu^2)}{2\pi} P^{(1)}_{qq}(N) + O(\alpha_S^2)\right]. \tag{26}$$

$P_{qq}(N)$ is the N^{th} moment of the Altarelli-Parisi function $C_F\left(\frac{1+x^2}{1-x}\right)_+$ ($C_F = 4/3$ in $SU(3)_c$), while $P^{(1)}_{qq}(N)$ is given by the calculation of the anomalous dimension to second order [15].

The perturbative expansion of $\hat{q}$ reads:

$$\hat{q}(N, Q^2) = 1 + \frac{\alpha_S(\mu^2)}{2\pi}\left[P_{qq}(N)\ln\frac{Q^2}{\mu^2} + C(N)\right] + O(\alpha_S^2). \tag{27}$$

It is clear that one should choose $Q^2 \sim \mu^2$: otherwise the coefficient of α_S would be large and the perturbative expansion would not have any meaning. The "coefficient function" $C(N)$ depends on the regularization and factorization scheme. However in all reasonable schemes one finds an infrared singular factor which is dominant when $N \to \infty$:

$$C(N) \underset{N\to\infty}{\simeq} C_F \ln N. \tag{28}$$

The preceding discussion is summarized by writing the moment $q_H(N, Q^2)$ of the quark distribution as:

$$q_H(N, Q^2) = F(N, \mu^2)\hat{q}\left(N; \alpha_S(\mu^2), \frac{Q^2}{\mu^2}\right), \tag{29}$$

where $F(N, \mu^2)$ obeys the R. G. equation (26), while $\hat{q}$ has the perturbative expansion (27).

Finally I must be more accurate in the definition of $q_H(x, Q^2)$, since there are many structure functions, which are equivalent to leading order, but differ by terms of order α_S. It will be convenient to choose (with standard notations):

$$q_H(x, Q^2) = \nu W_2(x, Q^2)/x \tag{30}$$

so that q_H (for valence distributions) obeys Adler's sum rule:

$$\int_0^1 dx\, q_H(x, Q^2) = 1. \tag{31}$$

As already mentioned, this sum rule is very useful to constrain the pion structure function.

4.3. Lepton pair production

If I wanted to compute naively the QCD corrections to the parton model, I would start from the expression (see Fig. 5):

$$\sigma_H(x_1, x_2) = [(\bar{q}^0_{H_1} q^0_{H_2}) * \sigma](x_1, x_2), \tag{32}$$

or in terms of moments:

$$\sigma_H(N_1, N_2) = \bar{q}^0_{H_1}(N_1) q^0_{H_2}(N_2)\sigma(N_1, N_2), \tag{33}$$

where $\sigma(N_1, N_2)$ is computable in perturbation theory with an infrared regulator λ^2 (Fig. 5). However σ contains mass singularities, and we have to use again the factorization theorem which allows to rewrite equation (33) as:

$$\sigma_H(N_1, N_2) = \bar{q}^0_{H_1}(N_1) q^0_{H_2}(N_2) \bar{\Gamma}\left(N_1, \frac{\mu^2}{\lambda^2}\right) \times \Gamma\left(N_2, \frac{\mu^2}{\lambda^2}\right) \hat{\sigma}\left(N_1, N_2; \alpha_S, \frac{Q^2}{\mu^2}\right) \quad (34)$$

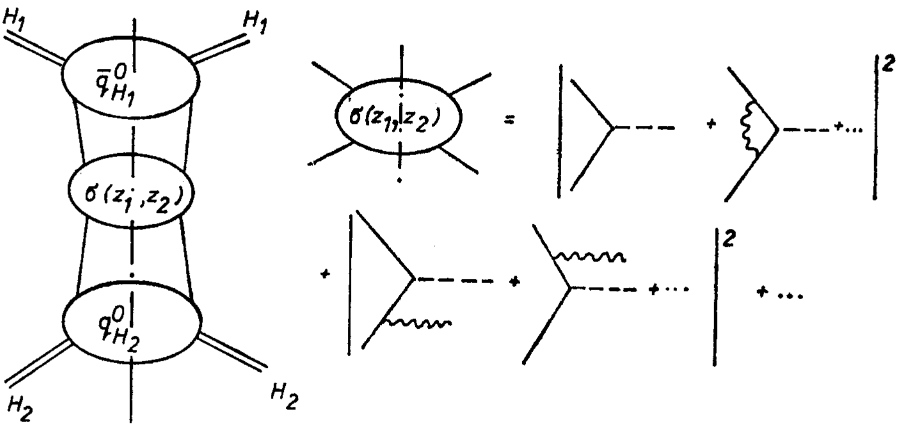

Fig. 5. Lepton pair production. The dotted-dashed line is a unitarity cut. Wiggly lines represent gluons and dashed lines photons

where the functions q^0_H and Γ are the *same* as in deep inelastic scattering. We then obtain, using (29)

$$\sigma_H(N_1, N_2) = \bar{F}(N_1, \mu^2) F(N_2, \mu^2) \hat{\sigma}\left(N_1, N_2; \alpha_S, \frac{Q^2}{\mu^2}\right). \quad (35)$$

The cross-section $\hat{\sigma}$ has a perturbative expansion similar to that of $\hat{q}$ (Eq. (27)).

$$\hat{\sigma} = A\left\{1 + \frac{\alpha_S(\mu^2)}{2\pi}\left[(P_{qq}(N_1) + P_{qq}(N_2)) \ln \frac{Q^2}{\mu^2} + C(N_1, N_2)\right] + O(\alpha_S^2)\right\}. \quad (36)$$

(Note again that $Q^2 \sim \mu^2$, since otherwise the perturbative expansion does not make sense.) The last step is to express F and $\bar{\text{F}}$ as functions of q_H and $\bar{q}_H$ using equations (27) and (29), which lead immediately to:

$$\sigma_H(N_1, N_2) = A\bar{q}_{H_1}(N_1, Q^2) q_{H_2}(N_2, Q^2) \left\{1 + \frac{\alpha_S(\mu^2)}{2\pi} \times [C(N_1, N_2) - C(N_1) - C(N_2)] + O(\alpha_S^2)\right\}. \quad (37)$$

Coming back to x-space, equation (37) can be written as:

$$\sigma_H(x_1, x_2) = [(\bar{q}_{H_1}(Q^2)q_{H_2}(Q^2)) * \bar{C}](x_1, x_2), \tag{38}$$

where:

$$\bar{C}(N_1, N_2) = C(N_1, N_2) - C(N_1) - C(N_2), \tag{39}$$

and [13]:

$$\bar{C}(z_1, z_z) = \delta(1-z_1)\delta(1-z_2)\left[1 + \frac{\alpha_S C_F}{2\pi}\left(1+\pi^2+\frac{2\pi^2}{3}\right)\right]$$
$$+ \frac{\alpha_S C_F}{2\pi}\delta(1-z_1)\left[\frac{1+z_2^2}{1-z_2}\ln\frac{2z_2}{1+z_2} + \frac{3}{2(1-z_2)_+} - 2 - 3z_2\right] + (1 \leftrightarrow 2)$$
$$+ \frac{\alpha_S C_F}{\pi}(1+z_1z_2)\left[\frac{1+z_1^2z_2^2}{(1+z_1)(1+z_2)[(1-z_1)(1-z_2)]_{++}} - \frac{2z_1z_2}{(z_1+z_2)^2}\right]. \tag{40}$$

The principal value distribution $1/(1-z)_+$ is defined as usual by:

$$\int_0^1 \frac{f(z)dz}{(1-z)_+} = \int_0^1 \frac{f(z)-f(1)}{1-z}\,dz \tag{41}$$

and similarly in the case of two variables.

4.4. Comments

The long expression for $\bar{C}(z_1, z_2)$ needs some explanations.

1) The term of order zero in $\alpha_S(\delta(1-z_1)\delta(1-z_2))$ corresponds to the naive D.Y. model, with Q^2-dependent structure functions: this is the leading logarithmic approximation.

2) Integrating over y, one gets the cross-section $d\sigma_H/dQ^2$:

$$s\frac{d\sigma_H}{dQ^2} = \iint \frac{dt_1}{t_1}\frac{dt_2}{t_2}\bar{q}_{H_1}(t_1, Q^2)q_{H_2}(t_2, Q^2)$$
$$\bar{C}\left(\frac{\tau}{t_1t_2}\right), \tag{42}$$

$$\bar{C}(z) = \delta(1-z)\left[1 + \frac{\alpha_S C_F}{2\pi}\left(\pi^2 + \left(\frac{\pi^2}{3} - \tfrac{7}{2}\right)\right)\right]$$
$$+ \frac{\alpha_S C_F}{\pi}\left[\frac{1+z^2}{1-z}\ln(1-z) + \frac{3}{2(1-z)} - 3 - 2z\right]_+ \tag{43}$$

3) It turns out that numerically $\bar{C}(z_1, z_2)$ is well approximated by:

$$\bar{C}(z_1, z_2) = \delta(1-z_1)\delta(1-z_2)\left(1 + \frac{\alpha_S C_F}{2\pi}\pi^2\right) + \frac{\alpha_S C_F}{\pi}\frac{1}{[(1-z_1)(1-z_2)]_{++}} \tag{44}$$

where the last term (infrared singular) is important only when $\tau \to 1$. The "π^2-term" in equation (44) comes from the analytic continuation of a Dirac form-factor from space-like (deep inelastic) to time-like (l^+l^-) values of q^2. One may wonder about the $\frac{2\pi^2}{3}$ term in equation (40) which looks important, but is compensated by other terms, so that the correction to the approximate formula (44) does not exceed 10%. Notice that in equation (43), the last term contains a (+) prescription, so that its z-integral is zero. Then the first term contains in addition to π^2, a term $\left(\frac{\pi^2}{3} - \frac{7}{2}\right)$ which however is very small.

4) In terms of double moments, the infrared singular term gives, as expected, a doubly logarithmic contribution:

$$\frac{\alpha_S C_F}{\pi} \ln N_1 \ln N_2 \tag{45}$$

while, when convoluted with the quark distribution, it leads in the region $x_1, x_2 \to 1$ to:

$$\frac{\alpha_S C_F}{\pi} \ln(1-x_1) \ln(1-x_2) \bar{q}_{H_1}(x_1, Q^2) q_{H_2}(x_2, Q^2) \tag{46}$$

5) The argument of α_S is $\mu^2 \sim Q^2$, but it is not possible to be more precise at this stage of the calculation. The standard choice is $\mu^2 = Q^2$, but there are no compelling arguments for it.

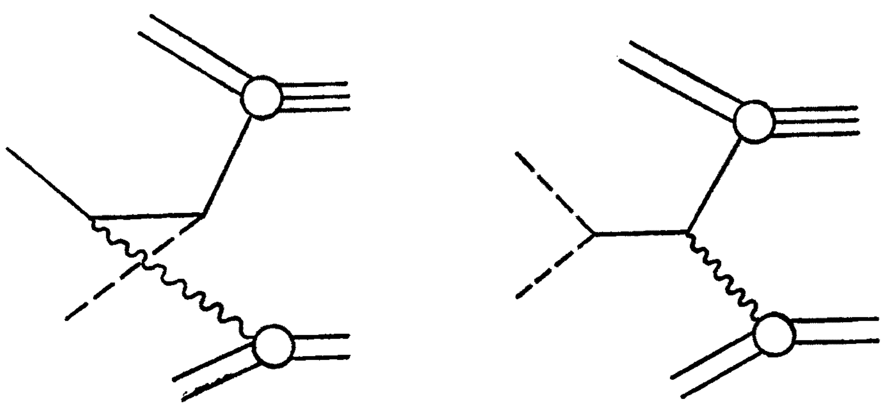

Fig. 6. Initial gluons and Compton graphs

6) If one does not consider cross-section differences, such as $\sigma(\pi^- p) - \sigma(\pi^+ p)$, it is necessary to take into account not only sea quarks, but also initial gluons (Fig. 6), which leads to the so-called "Compton graphs". There is another contribution to $\sigma_H(x_1, x_2)$ which reads:

$$\sigma'_H(x_1, x_2) = A[(g_{H_1}(Q^2)(q_{H_2}(Q^2) + \bar{q}_{H_2}(Q^2)) * \tilde{C}'](x_1, x_2) + (1 \leftrightarrow 2) \tag{47}$$

with $\bar{C}'(z_1, z_2)$ given by [13]:

$$\bar{C}'(z_1, z_2) = \delta(1-z_2)\frac{\alpha_S T_F}{2\pi}\left[\left(z_1^2+(1-z_1)^2 \ln\frac{2z_1}{1+z_1} +1+6z_1(1-z_1\right)\right]$$

$$+\frac{\alpha_S T_F}{\pi}\frac{z_2(1+z_1z_2)}{(z_1+z_2)}\left[\frac{z_1z_2+(1-z_1z_2)^2}{(1+z_2)(1-z\ )_+}+\frac{z_1^2(1+z_2^2+2z_1z_2)}{(z_1+z_2)^2}\right] \quad (48)$$

and $g_H(x, Q^2)$ is the gluon distribution in hadron H.

7) I give in Fig. 7 numerical results obtained for $K(Q^2, y)$ in π^-p collisions. These results are obtained with the following parametrization of $\alpha_S(Q^2)$:

$$\alpha_S(Q^2) = \frac{1}{b \ln Q^2/\Lambda^2}, \quad b = \frac{25}{12\pi} \quad (49)$$

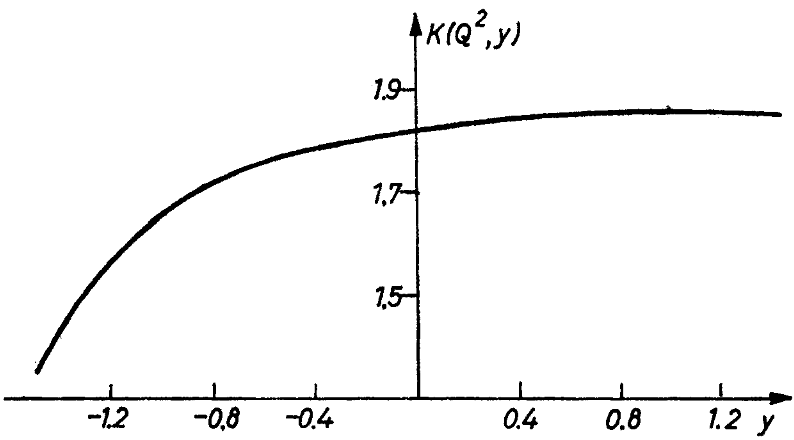

Fig. 7. $K(Q^2, y)$ to first order in perturbation theory for π^- p collisions, at $Q^2 = 25$ (GeV)^2, $s = 563$ (GeV)^2

and the "old" value of Λ : $\Lambda = 500$ MeV. The nucleon structure functions are taken from the CDHS collaboration [9] while those of the pion are assumed to follow counting rules at $Q^2 = 25$ (GeV)^2:

$$xV_\pi(x) = 0.75x^{1/2}(1-x)$$

$$xS_\pi(x) = 0.1(1-x)^5$$

$$xg_\pi(x) = 2(1-x)^3$$

One notices that there is an important effect of initial gluons for negative rapidities: with initial quarks only, the K-factor is almost flat in y.

8) Finally I give in Fig. 8 some results on angular distribution of the leptons, integrated over $q_\perp$, in the Gottfried-Jackson frame [16]. The angular distribution is parametrized as:

$$\frac{d\sigma}{dQ^2 dy d\Omega} = \frac{d\sigma}{dy dQ^2} \frac{1}{4\pi(1+\lambda/3)}$$

$$\{1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\varphi+\tfrac{1}{2}\nu\sin^2\theta\cos 2\varphi\}. \tag{50}$$

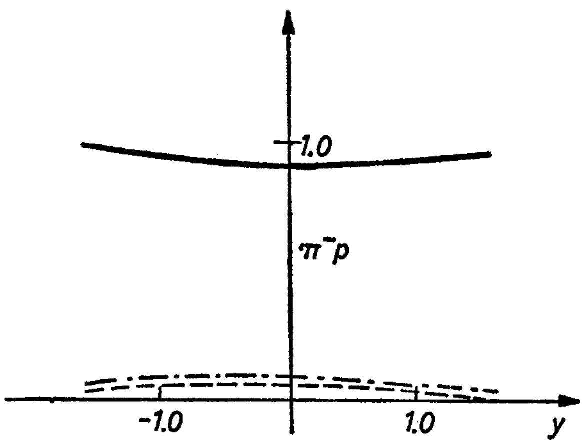

Fig. 8. Coefficients λ, μ, ν as defined in the text (equation (50)) for π^- p collisions, $Q^2 = 25$ (GeV)^2, $s = 735$ (GeV)^2. Solid line : λ; dotted dashed line: μ; dashed line: ν

One can see that, contrary to the case of the K-factor, the first order QCD correction is small: λ is of order 1 and μ and ν do not exceed 10%.

5. *Exponentiation of soft gluons*

The theoretical prediction of the first order calculation $K(y, Q^2) \simeq 2$ is in qualitative agreement with experiment and is often considered as a success of perturbative QCD. However one can ask two questions:

1) Since the $O(\alpha_S)$ term is of the same magnitude as the Born term, can we really trust the first order calculation?

2) Assuming that one is able to control the higher order terms of the perturbative expansion, will the agreement between theory and experiment persist in a quantitative way? In particular one should be aware that the predictions in references [10] to [13] were computed with a value of the QCD parameter $\Lambda = 0.5$ GeV which is certainly too large by present standards.

The answer to the first question has been given by Parisi [17] and Curci and Greco [18], and in more detail by Chiappetta et al. [19]. The large terms in the first order calculation are, as we have seen in the previous section:

(*i*) an $\left(\frac{\alpha_S C_F}{2\pi}\right)\pi^2$ term which comes from the analytical continuation of a form factor from space-like to time-like q^2;

(*ii*) an infrared singular term which is important when $\tau \to 1$.

Many authors [20] have given convincing arguments that the factor of π^2 in (*i*) exponentiates, so that I'll restrict my discussion to the exponentiation of the infrared singular term in equation (44). I'll compute the cross-section for the emission of n soft gluons in a quark-antiquark collision (Fig. 9):

$$\bar{q}(p_1)+q(p_2) \to n \text{ soft gluons } (k_i)+\gamma^*(q) \tag{51}$$

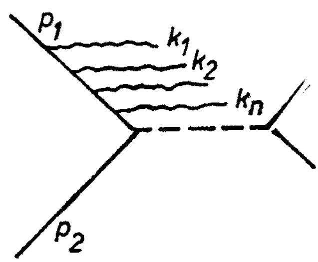

Fig. 9. Soft gluon emission in a $\bar{q}q$ collision

where p_1, p_2, k_i and q are the 4-momenta of the anti-quark, quark, soft gluons and virtual photon respectively. The kinematical variables $\hat{s}$, $\hat{\tau}$ and $\hat{y}$ will refer to the $q\bar{q}$ reaction:

$$\hat{s} = (p_1+p_2)^2, \quad \hat{\tau} = Q^2/\hat{s}, \quad z_1 = \sqrt{\hat{\tau}}\, e^{\hat{y}}, \quad z_2 = \sqrt{\hat{\tau}}\, e^{-\hat{y}} \tag{52}$$

and I will use a Sudakov parametrization for the k_i's:

$$k_i = \alpha_i p_1 + \beta_i p_2 + k_{i\perp} \tag{53}$$

which leads to:

$$q^2 = \hat{s}(1-\sum_i \alpha_i)(1-\sum_i \beta_i)-(\sum_i \vec{k}_{i\perp})^2 \simeq \hat{s}(1-\sum_i \alpha_i)(1-\sum_i \beta_i), \tag{54}$$

since $\vec{k}_{i\perp}^2 = \alpha_i\beta_i\hat{s}$ and α_i, $\beta_i \ll 1$ in the soft gluon limit. Moreover if I call q_+ and q_- the light cone coordinates of q in a reference frame in which $p_1(p_2)$ has only a $+$ $(-)$ component, I have:

$$\frac{q_+}{q_-} = \frac{1-\sum_i \alpha_i}{1-\sum_i \beta_i} = e^{2\hat{y}}. \tag{55}$$

The kinematical δ-functions in the calculation of $\sigma(z_1, z_2)$ are thus:

$$\delta\left(z_1-\left(\frac{q^2}{\hat{s}}\frac{q_+}{q_-}\right)^{1/2}\right)\delta\left(z_2-\left(\frac{q^2}{\hat{s}}\frac{q_-}{q_+}\right)^{1/2}\right)$$
$$\simeq \delta(1-z_1-\sum_i \alpha_i)\delta(1-z_2-\sum_i \beta_i). \tag{56}$$

It has been shown recently [21] that the matrix element squared, $|M|^2$, for the emission of n soft gluons is given by:

$$|M|^2 = \prod_{i=1}^{n} \frac{\alpha_S C_F}{\pi} \frac{1}{\alpha_i \beta_i} \tag{57}$$

in the region strongly ordered in angles and momenta:

$$\alpha_i \ll \alpha_{i+1}$$

$$\theta_i \ll \theta_{i+1} \quad \text{or} \quad \theta_{i+1} \ll \theta_i$$

where θ_i is the emission angle of the gluon with respect to the quark[5]. Collinear singularities occur when $\theta_i \sim \theta_{i+1}$, but they are cancelled by similar singularities of virtual graphs (vertices and propagators), and, as a final result, one builds the running couplig constant $\alpha_S(\vec{k}_{i\perp}^2) = \alpha_S(Q^2\alpha_i\beta_i)$ [21]. Taking (56) and (57) into account, one obtains the cross-section for real gluon emission:

$$\sigma_R(z_1, z_2) = A \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} \left[\frac{d\alpha_i d\beta_i}{\alpha_i \beta_i} \right.$$

$$\left. \times \frac{\alpha_S(Q^2\alpha_i\beta_i)C_F}{\pi} \theta\left(\alpha_i\beta_i - \frac{Q_0^2}{Q^2}\right)\right] \delta\left(1 - z_1 - \sum_i \alpha_i\right) \delta\left(1 - z_2 - \sum_i \beta_i\right), \tag{58}$$

$$A = \frac{4\pi\alpha^2 e_q^2}{9Q^2},$$

where the $1/n!$ takes care of the ordering. I have introduced an infrared cut-off Q_0^2 (for example a gluon mass) which leads to a θ-function of kinematical origin in equation (19). Adding the virtual contribution which is given by the Sudakov form factor and taking the (N_1, N_2) moments I get[6]:

$$\sigma(N_1, N_2) = A \exp\left[\int \frac{d\alpha d\beta}{\alpha\beta} (e^{-N_1\alpha} e^{-N_2\beta} - 1) \frac{\alpha_S(Q^2\alpha\beta)C_F}{\pi} \theta\left(\alpha\beta - \frac{Q_0^2}{Q^2}\right)\right]. \tag{59}$$

It is convenient to rewrite the expression of the Sudakov form factor F^2 (given by the (-1) in equation (59)) as [22]:

$$F^2 = \Delta(Q^2, Q_0^2) = \exp\left[-\int \frac{d\alpha d\beta}{\alpha\beta} \frac{\alpha_S(Q^2\alpha\beta)C_F}{\pi} \theta\left(\alpha\beta - \frac{Q_0^2}{Q^2}\right)\right]. \tag{60}$$

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[5] This result is established in an axial gauge where the gauge vector n_μ is parallel to $p_{2\mu}$: $n_\mu = p_{2\mu}$, so that there is no radiation from the quark line in the soft limit. The final result is of course gauge invariant [22].

[6] Equation (59) has been obtained independently by Ciafaloni (private communication).

We can then rewrite equation (59) in a convenient form:

$$\sigma(N_1, N_2) = \frac{\Delta(Q^2, Q_0^2)}{\Delta\left(\dfrac{Q^2}{N_1 N_2}, Q_0^2\right)}. \tag{61}$$

An analogous calculation in the case of deep inelastic scattering on a quark target gives for the quark distribution inside a quark [22]:

$$q(N_1, Q^2) = \frac{\Delta(Q^2, Q_0^2)}{\Delta\left(\dfrac{Q^2}{N}, Q_0^2\right)}. \tag{62}$$

Equation (62) is equivalent to the modified Altarelli-Parisi equations proposed by Amati et al. [23]. Using equation (62), we can eliminate all dependence with respect to Q_0^2 by expressing $\sigma(N_1, N_2)$ in terms of $q(N, Q^2)$:

$$\sigma(N_1, N_2) = A \frac{\Delta(Q^2, Q_0^2)}{\Delta\left(\dfrac{Q^2}{N_1 N_2}, Q_0^2\right)} \frac{\Delta\left(\dfrac{Q^2}{N_1}, Q_0^2\right)}{\Delta(Q^2, Q_0^2)} \times \frac{\Delta\left(\dfrac{Q^2}{N_2}, Q_0^2\right)}{\Delta(Q^2, Q_0^2)} \bar{q}(N_1, Q^2) q(N_2, Q^2). \tag{63}$$

This formula is valid in the large (N_1, N_2) limit. The double moments $\sigma_{\mathrm{H}}(N_1, N_2)$ are also given in the same limit by equation (63), provided one replaces the quark distributions inside a quark by those inside a hadron, $q_{\mathrm{H}}(N, Q^2)$.

5.1. Final results

The final formula, which is studied numerically in Section 6, is obtained in the following way from the first order calculation (equation (40)):

(*i*) There is an overall

$$\exp\left(\frac{\alpha_S(Q^2) C_F}{2\pi} \pi^2\right)$$

factor, corresponding to the exponentiation of the first order result;

(*ii*) The term (see equation (46))

$$\left[1 + \frac{\alpha_S C_F}{\pi} \ln(1 - x_1) \ln(1 - x_2)\right] \bar{q}_{\mathrm{H}_1}(x_1, Q^2) q_{\mathrm{H}_2}(x_2, Q^2)$$

is exponentiated following (63):

$$\exp\left[-\frac{C_F}{\pi b}\left(\ln\frac{Q^2(1-x_1)}{\Lambda^2}\ln\frac{\alpha(Q^2)}{\alpha(Q^2(1-x_1))}+\ln\frac{Q^2(1-x_2)}{\Lambda^2}\right.\right.$$
$$\left.\left.\times\ln\frac{\alpha(Q^2)}{\alpha(Q^2(1-x_2))}-\ln\frac{Q^2(1-x_1)(1-x_2)}{\Lambda^2}\ln\frac{\alpha(Q^2)}{\alpha(Q^2(1-x_1)(1-x_2))}\right)\right]$$
$$\times\,\bar{q}_{H_1}(x_1, Q^2)q_{H_2}(x_2, Q^2) = S^{(2)}(x_1, x_2; Q^2)\bar{q}_{H_1}(x_1, Q^2)q_{H_2}(x_2, Q^2), \tag{64}$$

where I have used the leading logarithm approximation for the running coupling constant:

$$\alpha_S(Q^2) = \frac{1}{b\ln Q^2/\Lambda^2} \tag{65}$$

and I have made in the double moment formula (24) the substitutions:

$$\frac{1}{N_1}\to 1-x_1,\quad \frac{1}{N_2}\to 1-x_2 \tag{66}$$

which are valid in the limit $x_1, x_2 \to 1$. Finally I keep all terms of order α_S which have not been exponentiated, and I choose $\mu^2 = Q^2$ for the scale of the coupling constant. The final formula thus obeys the following criteria:

1) if expanded into powers of α_S, it reproduces correctly the first order calculation (7);
2) the leading infrared behaviour is correctly given in the limit $x_1, x_2 \to 1$.

I conclude with some remarks on the coupling constant. Since I have a well defined prescription for higher order terms in the perturbative expansion, there is in principle no ambiguity about the choice of the coupling constant. In particular in the exponentiated terms, the argument of the coupling constant is unambiguously determined to be Q^2 (of course the ambiguity persists for those terms which are not exponentiated, but this is not really a problem since they give a small ($\sim$ 10%) contribution).

However, in order to choose Λ, some amount of guesswork is needed. If we compare our approximations to those which are used in deep inelastic scattering, we see that we are very close to a leading logarithm (LL) approximation for the evolution of structure functions, except of course in the region $x \to 1$ where we take into account the $\left[\frac{1+x^2}{1-x}\ln(1-x)\right]_+$ term. Notice that we are using the LL approximation (26) of the strong coupling constant. The guess for the choice of Λ is then [24]:

$$\Lambda \simeq \Lambda_{LL} \simeq 100-200\text{ MeV}.$$

6. *Discussion*

The exponentiation of soft gluons which is described in the previous section is not unique: only the infrared singular part is completely determined. In reference [19], an alternative method of exponentiation, which treats in a different way infrared non-singular terms has been given. Both methods agree numerically within 10%, and in fact the difference between the two methods can be reduced to almost nothing, provided one uses suitable values of Λ [19]. In what follows, I give only the numerical results of the method described in the previous section, and refer to [19] for details on the other method.

The numerical computations have been performed for the cross-section difference

$$\sigma(\pi^- \mathrm{Pt}) - \sigma(\pi^+ \mathrm{Pt})$$

at an incident momentum $p_{\mathrm{lab}} = 280$ GeV/c. The structure functions at $Q^2 = \bar{Q}^2 = 25$ (GeV)^2 are parametrized as follows:

$$xV_\pi(x) = A_\pi x^{0.40}(1-x)^{1.03}, \tag{67.a}$$

$$xu_v(x) = A_u x^{0.52}(1-x)^{1.79}, \tag{67.b}$$

$$xd_v(x) = A_d x^{0.52}(1-x)^{3.79}. \tag{67.c}$$

Let us recall that experimentalists define the K-factor as:

$$K_{\mathrm{exp}}(x_1, x_2) = \frac{\sigma_{\mathrm{H}}(x_1, x_2)}{A\bar{q}_{\mathrm{H}_1}(x_1, \bar{Q}^2) q_{\mathrm{H}_2}(x_2, \bar{Q}^2)} \tag{68}$$

namely they do not take into account the evolution of structure functions, while theorists would probably prefer:

$$K_{\mathrm{th}}(x_1, x_2) = \frac{\sigma_{\mathrm{H}}(x_1, x_2)}{A\bar{q}_{\mathrm{H}_1}(x_1, Q^2) q_{\mathrm{H}_2}(x_2, Q^2)}\,. \tag{69}$$

I have plotted in Figs. 10 and 11 $K(y, Q^2)$ at $Q^2 = 25$ (GeV)^2 and $Q^2 = 80$ (GeV)^2 respectively for 3 values of Λ : $\Lambda = 0.1$, 0.3 and 0.5 GeV. One can see that at $Q^2 = 25$

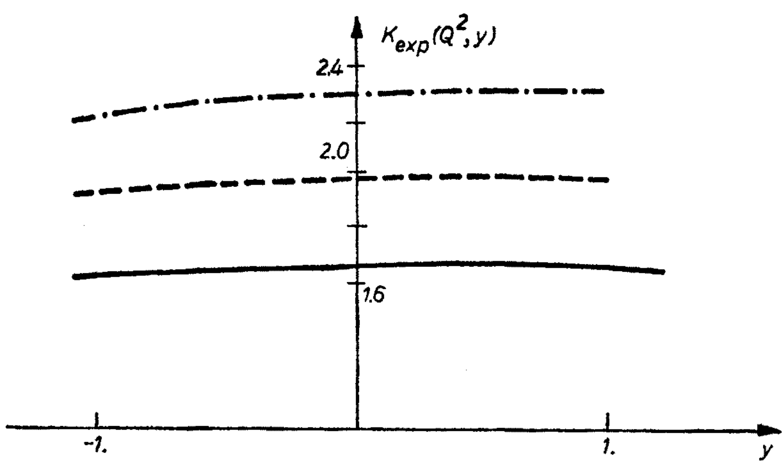

Fig. 10. $K_{\mathrm{exp}}(Q^2, y)$ for $[\sigma(\pi^- \mathrm{p}) - \sigma(\pi^+ \mathrm{p})]$ at $Q^2 = 25$ (GeV)^2, $s = 525$ (GeV)^2. Full line: $\Lambda = 0.1$ GeV. Dotted line: $\Lambda = 0.3$ GeV. Dotted-dashed line: $\Lambda = 0.5$ GeV

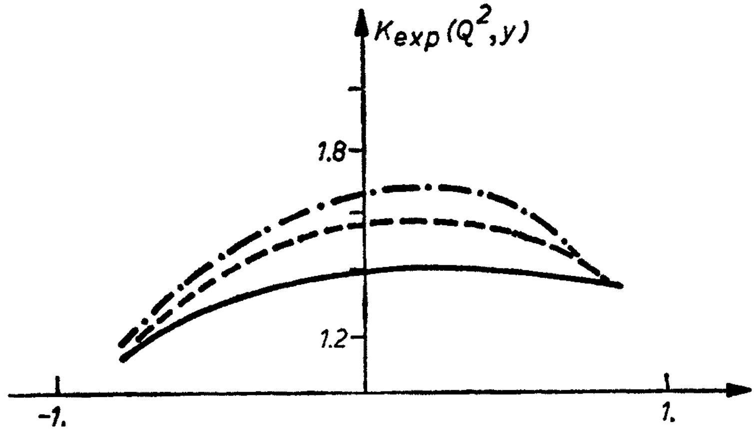

Fig. 11. Same as in Fig. 10 except that $Q^2 = 80\,(\mathrm{GeV})^2$

$(\mathrm{GeV})^2$ the K-factor is almost flat in y, while there is some curvature at $Q^2 = 80\,(\mathrm{GeV})^2$. This curvature is largely due to the definition (68) and reflects scaling violations. As discussed in some detail in reference [19], such scaling violations have implications on the determination of the pion structure functions.

The infrared singular term dominates when $\tau \to 1$ (Fig. 12), and there is a dramatic rise of K_{th} in this region[7]. Unfortunately the statistics is very poor there, and there is little hope to see this rise experimentally in the near future.

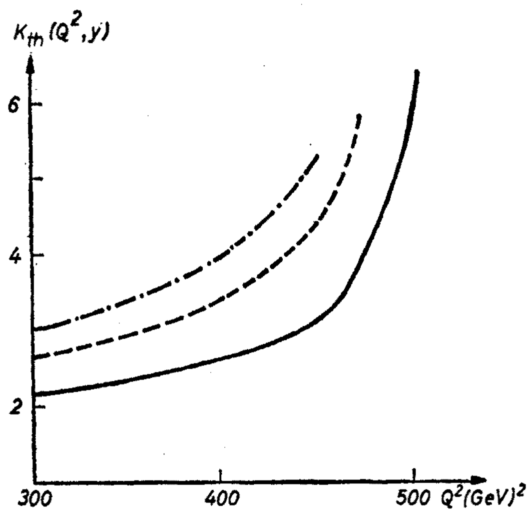

Fig. 12. $K_{th}(Q^2, y)$ as a function of Q^2 at $y = 0$, for $[\sigma(\pi^- p) - \sigma(\pi^+ p)]$ and $s = 525\,(\mathrm{GeV})^2$. Full line: $\Lambda = 0.1$ GeV. Dotted line: $\Lambda = 0.3$ GeV. Dotted-dashed line: $\Lambda = 0.5$ GeV

[7] This rise was first predicted by Curci and Greco, Ref. [18].

To conclude, let me say that the large terms in the first order calculation are well under control, and that we can be reasonably confident in the theoretical predictions.

The numerical calculations show that, with the values of Λ which are suggested at present by deep inelastic scattering experiments, the predictions seem to be somewhat too low ($K \simeq 1.8$ instead of 2.4 ± 0.6 experimentally). It would be very interesting to reduce the experimental errors to see whether the discrepancy is real, but this depends of course on difficult normalization problems.

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