MEAN FIELD THEORY WITH RADIATIVE CORRECTIONS FOR LATTICE GAUGE THEORIES*

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I present recent results for lattice gauge theories obtained by mean field theory with radiative corrections (MFT). After a quick survey of the strong and weak sides of other methods on the lattice, an introduction to MFT is given. The utility of MFT is discussed. I report on some work in progress, and motivate some future projects.

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1. Results

Let me show you some results for a start. Some of you may not know what they are all about; do not worry, I will explain soon. All I ask you to understand now is that *dots* are results of lengthy computer calculations, *lines* are results of analytic calculations, so-called mean field theory (MFT) plus radiative corrections. See Figs. 1, 2, 3 and 4.

Now the explanation: Consider U(1) lattice gauge theory [7]; instead of d-dimensional continuous Euclidean space-time, we have a d-dimensional hypercubical *lattice* with lattice spacing a. A function $l \to U_l \in \mathrm{U}(1)$ from the links of the lattice and into the group U(1) is a *field*. As in continuum field theory, there is a *classical action* S mapping any field configuration into a real, possibly infinite, number: $\{U_l\} \equiv U \to S(\beta; U)$. This defines a *classical* field theory. This classical field theory may be *quantized* using *path integrals*. The simplest path integral encountered in the resulting quantum field theory is the vacuum-to-vacuum amplitude

$$Z(\beta) \equiv \int_{\mathrm{U}(1)} \prod_{\text{all } l} dU_l \exp S(\beta; U). \tag{1.1}$$

dU_l is the normalized invariant measure of integration over the group. For U(1) with $U_l = e^{i\phi_l}$, $\phi_l \in [-\pi, \pi]$ we have $dU_l = \dfrac{d\phi_l}{2\pi}$. $\beta = g_0^{-2}$, where g_0 is the bare coupling con-

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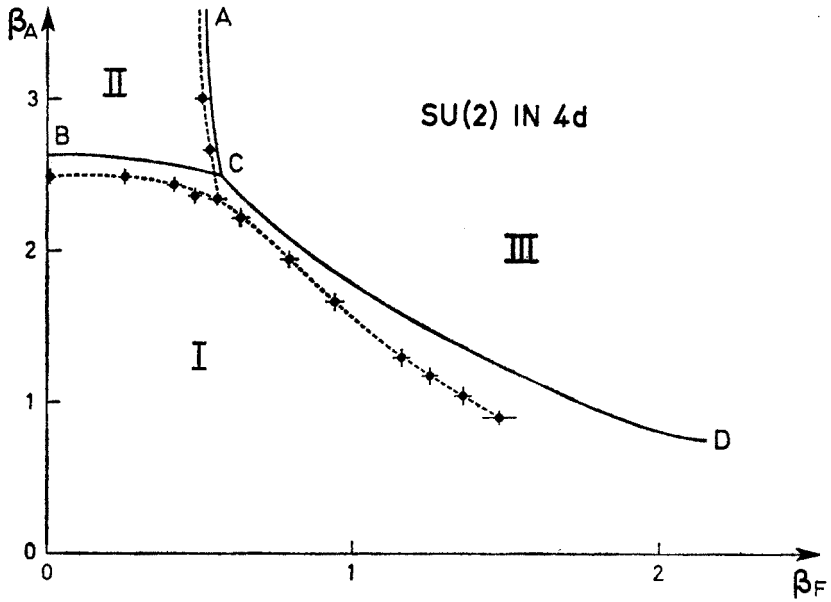

Fig. 1. Phase plane of SU(2) fundamental-adjoint mixed action theory in 4*d*. *AC*, *BC* and *CD* are lines of first order transitions found by mean field theory with corrections in Ref. [1]. Crosses are Monte Carlo results with error bars from Ref. [2]

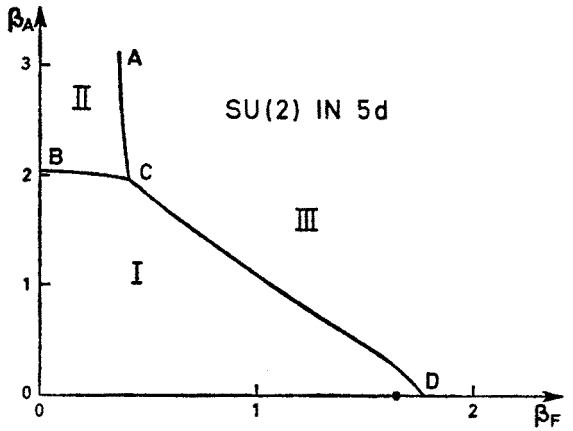

Fig. 2. Same as Fig. 1, but in 5*d*. Only one M.C. result, on the β_F-axis, is available for comparison [3]

stant. Here we are considering U(1) in order to avoid a lot of indices below. We shall make no use of its abelian nature, and generalization to non-abelian groups is straightforward. For classical action we shall use Wilson's form [7]:

$$S(\beta; U) \equiv \beta \sum_{\text{all } P} \operatorname{Re}\left(\prod_{l \in \partial P} U_l\right), \tag{1.2}$$

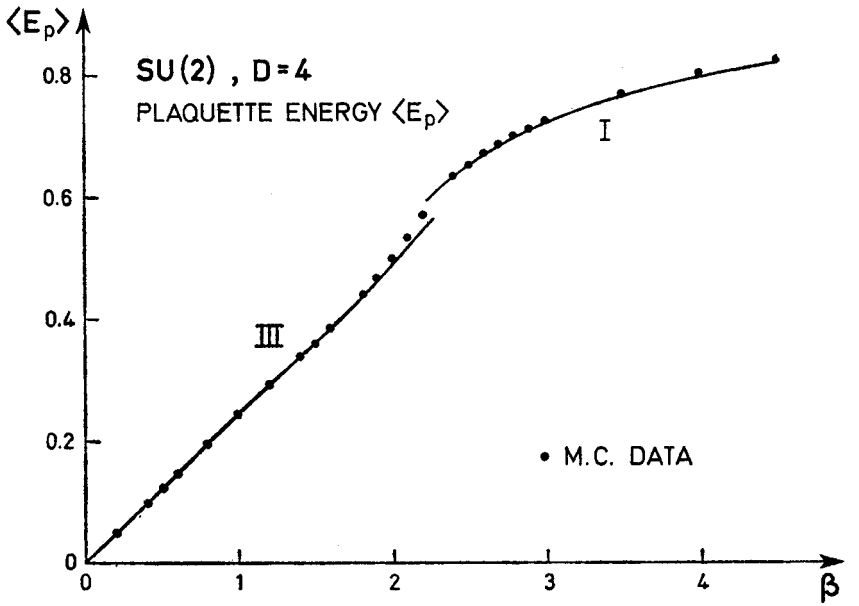

Fig. 3. Internal energy of SU(2) LGT in 4*d*, as predicted by MFT [4], and as obtained by M.C. calculation [5]. Graph labelled I is the tree level mean field result for the strong and weak coupling phases. Graph labelled III is the result of correcting the mean field approximation in the strong coupling phase with the first three terms in the strong coupling expansion

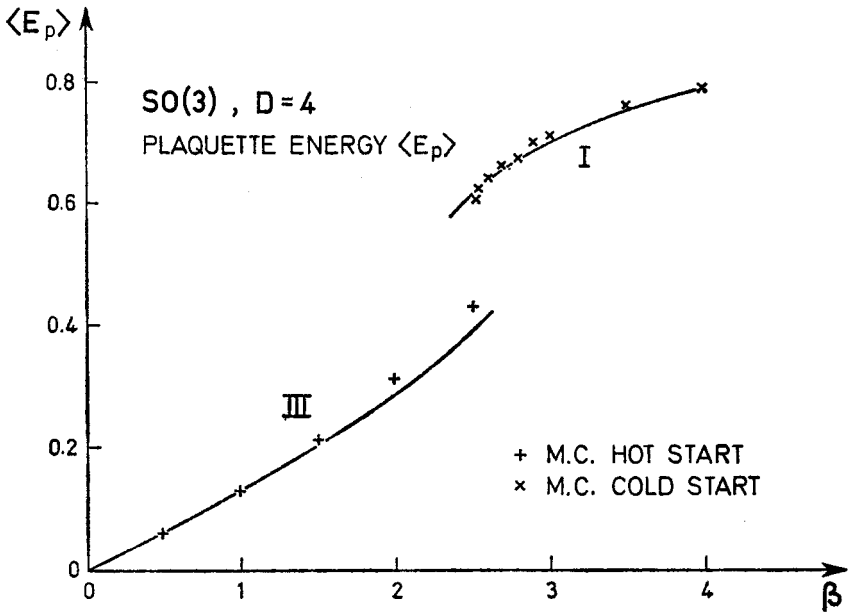

Fig. 4. Same as Fig. 3, but for SO(3) in 4*d* [4, 6]

where P stands for *plaquette*, i.e. an elementary square formed by four links of the lattice, $\prod_{l \in \delta P} \ldots$ is the product over four such links $l \in \delta P$ forming the boundary δP of the plaquette P, and $\sum_P$ is the sum over all plaquettes in the lattice. The Wilson action (1.2) is just one out of several interesting actions. It is particularly well-suited for MFT, since it is linear in all link variables U_l. What is said below may be generalized to other actions, however. With reference to statistical mechanics, the vacuum-to-vacuum amplitude (1.1) is often called the *partition function*, since $\int_{U(1)} \prod_l dU_l$ can be read as a sum over all states, i.e. field configurations, and $\exp S(\beta; U)$ can be read as the Boltzmann factor $\exp\left(-\frac{1}{kT} E(U)\right)$.

Why do we do lattice gauge theory? Because it is a nice, regularized version of *continuum* gauge theory, which for *its* part is believed to describe the real world of the strong interactions, when quarks are included and the gauge group is SU(3), as opposed to U(1) considered here. A particularly nice feature of the *lattice* regularization is that it is defined without reference to perturbation theory, as opposed to other regularization schemes. That was *why* we do lattice gauge theory. Now, *how* do we do it? I.e. how is the vacuum--to-vacuum amplitude (1.1) and other path integrals calculated? There are several methods, all of which have their own strong and weak points:

A. Strong coupling expansion

Taylor-expand $Z(\beta)$ and other path integrals around $\beta = 0$:

$$Z(\beta) = \sum_{n=0}^{\infty} \frac{1}{n!} Z^{(n)}(0) \beta^n. \tag{1.3}$$

This is a highly precise method for low values of β. The various Taylor series obtained have finite radii of convergence, however. This is a serious problem, since information about the *continuum* theory is obtained from the behaviour of the path integrals for β approaching infinity. Advanced methods exist that will identify and circumvent the singularities in the complex β-plane which are responsible for the finite radius of convergence of the Taylor series. The utility of such methods is limited by the rapidly increasing amount of work required to calculate $Z^{(n)}(0)$ for increasing values of n. $Z^{(n)}(0)$, or rather the coefficients in the Taylor series for $\log Z(\beta)$, have been calculated for some groups up to $n = 16$ [8, 9].

B. Numerical methods (M.C.)

Put your lattice in a box, and get rid of the surface of the box by identifying opposite faces. Then you have a *finite* lattice, and consequently *a finite number of variables.* This is a problem suited for your computer. You can calculate $Z(\beta)$ and other path integrals numerically using Monte Carlo (M.C.) techniques. You do not have to calculate expansion coefficients, you do not have to worry about finite radii of convergence. This is the strong

side of M.C. calculations. Instead you worry about the finite size and speed of your computer and calculate budgets. That also gives an upper limit to the β-values accessible!

The largest $4d$-lattice that has been used in M.C.-calculations is to my knowledge one with 16^4 sites [10]. The string tension (a non-local quantity) of SU(2) has been measured on this lattice, using data up to $\beta = 2.75$ [10]. Local quantities can be calculated with precision on much smaller lattices up to much larger β-values, for example the internal energy for SU(2) on a 4^4 lattice to β larger than 5 [5].

C. Weak coupling expansion

When β is large, $\beta^{-1} \sim g_0^2$ is small. On every link $l = (x, \mu)$ expand

$$U_l = \exp(iag_0 A_\mu(x)) = 1 + iag_0 A_\mu(x) + \ldots \tag{1.4}$$

Plug it into the (gauge fixed) lattice action and expand again in g_0. Plug the resulting action into the Boltzmann factor and expand for the third time in g_0. This is ordinary perturbation theory in g_0, but on the lattice. As opposed to the strong coupling expansion and M.C. methods, the weak coupling expansion *thrives* on large β. It takes a lot of work, however, and yields only perturbative results [11].

D. Variational method

Variational methods apply to lattice gauge theories. The quality of the results depends on the sophistication of the variational ansatz for the wave functional. One rather simple ansatz for the vacuum wave functional gives results identical to those of mean field theory (MFT) without radiative corrections (see below), when applied to the Wilson action [12]. A more advanced ansatz has been applied and yields the average plaquette energy, the string tension, and the mass gap for a few groups in $3d$ with good numerical precision [13, 14]. $4d$ and more sophisticated ansatzes are too complicated to handle analytically. For this reason, weak and strong coupling expansions, MFT, and duality arguments have been applied to expressions based upon variational ansatzes [15, 16, 17]. Yet another variational crossbreed occurs when a variational family for the wave functional of the first excited state is formed by applying appropriate operators to the "exact" ground state wave functional obtained numerically by M.C. methods, see Ref. [18] and references therein.

E. Mean field theory with radiative corrections

This method seems to be the natural perturbation theory on the lattice. It works equally well for β small, where it reproduces the strong coupling expansion with nothing new to add, and for $2(d-1)\beta$ large, where it has simpler Feynman rules than the weak coupling expansion, and gives much better results [1, 4, 19, 20]. Actually, its results for $2(d-1)\beta$ large may be thought of as a partial summation of the weak coupling series. No nonlocal quantities have been calculated yet at large β using this method, and it may very well be the weak point of this method that this is impossible.

2. How to do mean field theory with radiative corrections

Consider the partition function (1.1). We cannot calculate it by integrating over the link variables U_l, since these couple to each other in a complicated way through $S(\beta; U)$. It is also ill-suited for a perturbative evaluation due to the constraint $U_l \in U(1)$ on the integration variables. Using Lagrange multipliers, we rid ourselves of this constraint as follows: rewrite the Boltzmann factor as

$$\exp S(\beta; U) = \int_{-\infty}^{\infty} \prod_l dV_l^{(1)} dV_l^{(2)} \delta(\mathrm{Re}\, U_l - V_l^{(1)}) \delta(\mathrm{Im}\, U_l - V_l^{(2)}) \exp S(\beta; U) \qquad (2.1)$$

with $V_l \equiv V_l^{(1)} + iV_l^{(2)}$. Now it is the real integration variables $\{V_l^{(1)}, V_l^{(2)}\}$ that couple through $S(\beta; V)$. The group elements U_l do not couple any longer, and in order to integrate over them we write the delta functions

$$\delta(\mathrm{Re}\, U_l - V_l^{(1)}) \delta(\mathrm{Im}\, U_l - V_l^{(2)})$$

$$= \int_{-\infty}^{\infty} \frac{da_l^{(1)}}{2\pi} \frac{da_l^{(2)}}{2\pi} \exp\left(ia_l^{(1)}(\mathrm{Re}\, U_l - V_l^{(1)}) + ia_l^{(2)}(\mathrm{Im}\, U_l - V_l^{(2)})\right) \qquad (2.2)$$

and introduce the function

$$w(a^{(1)}, a^{(2)}) \equiv \ln \int_{U(1)} dU \exp\left(ia^{(1)}\, \mathrm{Re}\, U + ia^{(2)}\, \mathrm{Im}\, U\right)$$

$$= \ln \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} \exp\left(ia^{(1)} \cos\theta + ia^{(2)} \sin\theta\right) = \ln I_0\left(\sqrt{-(a^{(1)2} + a^{(2)2})}\right). \qquad (2.3)$$

Then the partition function (1.1) can be written

$$Z(\beta) = \int_{-\infty}^{\infty} \prod_{\text{all } l} dV_l^{(1)} dV_l^{(2)} \frac{da_l^{(1)}}{2\pi} \frac{da_l^{(2)}}{2\pi} \exp \tilde{S}(\beta; V, \alpha) \qquad (2.4)$$

with

$$\tilde{S}(\beta; V, \alpha) \equiv S(\beta; V) + \sum_l \left(w(a_l^{(1)}, a_l^{(2)}) - iV_l^{(1)} a_l^{(1)} - iV_l^{(2)} a_l^{(2)}\right). \qquad (2.5)$$

Eq. (2.4) is an exact rewriting of Eq. (1.1). It may, however, be thought of as the vacuum-to-vacuum amplitude of a field theory in its own right, defined by the classical, albeit complex, action $\tilde{S}$. A field is in *this* theory *four* real numbers $(V_l^{(1)}, V_l^{(2)}, a_l^{(1)}, a_l^{(2)})$ attached to every link in the lattice, whereas a field in the *original* theory was *two* real numbers (Re U_l, Im U_l) attached to every link, and with the constraint $(\mathrm{Re}\, U_l)^2 + (\mathrm{Im}\, U_l)^2 = 1$. We see that in passing from S to $\tilde{S}$, at the cost of doubling the number of variables, we have arrived at a field theory with unconstrained, i.e. independent, variables. The integral in Eq. (2.4)

is evaluated perturbatively, using the stationary phase or saddle point method. $\tilde{S}$ is stationary at field configurations satisfying

$$V_l^{(j)} = -i \frac{\partial w}{\partial a^{(j)}}(a_l^{(1)}, a_l^{(2)}), \quad j = 1, 2 \tag{2.6}$$

$$ia_l^{(j)} = \frac{\partial S}{\partial V_l^{(j)}}(\beta; V), \quad j = 1, 2. \tag{2.7}$$

Due to the local gauge invariance of S and consequently $\tilde{S}$, any local gauge transformation applied to a non-zero solution of Eqs. (2.6)–(2.7) leads to another solution, i.e. the non-trivial saddle points of $\tilde{S}$ are gauge degenerate. Consequently, before applying the saddle point method to a non-zero saddle point of $\tilde{S}$, we must *either* fix the gauge *or* perform an *exact* integration in Eq. (2.4) over the gauge degenerate saddle points. The latter strategy has been employed by Alessandrini, Hakim and Krzywicki in a very recent paper [21], and boils down to path integral quantization of $\tilde{S}$ in background field gauge. This is a very nice approach, since results are explicitly gauge invariant functions of the saddle point $\{V_l^{(1)}, V_l^{(2)}, a_l^{(1)}, a_l^{(2)}\}$. We shall use the former strategy, however, and fix the gauge. This is technically simpler, at least for the low order considered here, of the expansion around the saddle point. In Eq. (1.1) we fix U_l to equal one for all links in a given direction. Call this direction the time-direction (though we are really considering a euclidean theory) and this gauge the temporal gauge. Then all formulas given in this section up to this point are valid, also in temporal gauge, as long as "link" is read everywhere as "spatial link". We assume Eqs. (2.6)–(2.7) are solved by a constant field on the spatial links. Then global gauge invariance and Eqs. (2.6)–(2.7) allow, respectively require, this field to have the form

$$(V_l^{(1)}, V_l^{(2)}, a_l^{(1)}, a_l^{(2)}) = (V, 0, -i\alpha, 0) \tag{2.8}$$

with V and α real and satisfying

$$V = -i \frac{\partial w}{\partial a^{(1)}}(-i\alpha, 0) = I_1(\alpha)/I_0(\alpha), \tag{2.9}$$

$$\alpha = 2\beta V(1+(d-2)V^2). \tag{2.10}$$

Since α is real, $a_l^{(1)}$ is imaginary, i.e. the contour of integration for $a_l^{(1)}$ in Eq. (2.4) must be shifted off the real axis in order to pass through the saddle point of $\tilde{S}$ found from Eqs. (2.8–10). The qualitative behaviour of the solutions $(V(\beta), \alpha(\beta))$ to Eqs. (2.9)–(2.10) may be read in Fig. 5. Eq. (2.9) gives V as a function of α. Eq. (2.10) gives α as a function of V with an overall factor β. For small values of β the graph (a) of $\alpha(V)$ intersects the graph $V(\alpha)$ only for $(V, \alpha) = (0, 0)$. For intermediate values of β the graph (b) of $\alpha(V)$ inters cts that of $V(\alpha)$ at three points, and for larger values of β, (c) only at two points. $V(\beta)$ of these solutions to Eqs. (2.9)-(2.10) is sketched in Fig. 6. The field configurations (2.8) with $(V(\beta), \alpha(\beta))$ perturbatively stable solutions to Eqs. (2.9)–(2.10) are *classical* ground states, or vacua, of the field theory given by $\tilde{S}$ and quantized in Eq. (2.4). The classical, or tree

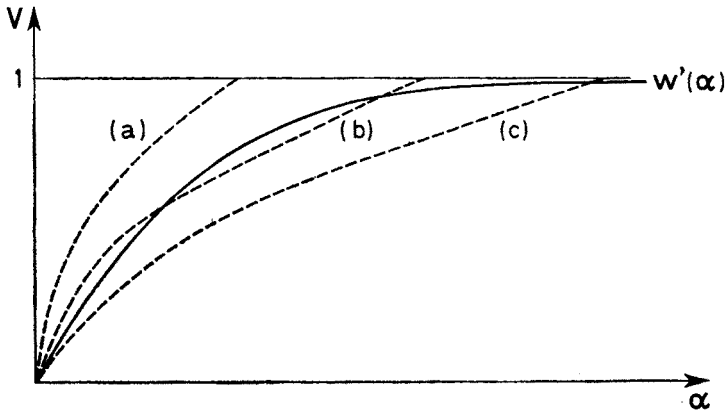

Fig. 5. Graphical representation of Eq. (2.9) (full line) and Eq. (2.10) (dashed line) for small (*a*), intermediate (*b*), and large (*c*) value of β

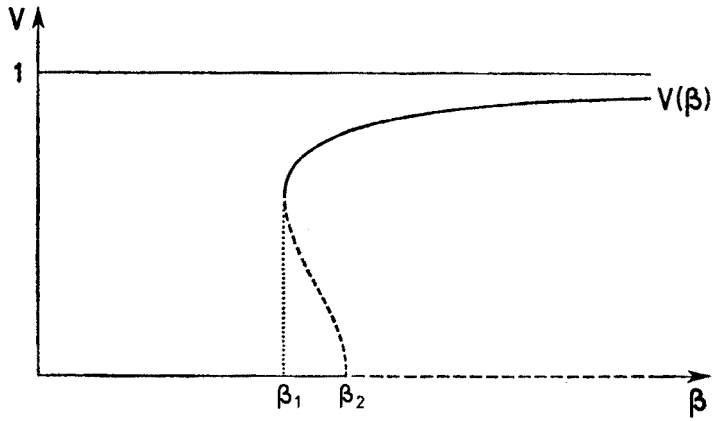

Fig. 6. Qualitative behaviour of solutions $V(\beta)$ to Eqs. (2.9–10). Full lines are perturbatively stable solutions, dashed lines perturbatively unstable solutions

approximation, to its vacuum energy equals the classical action $\tilde{S}$ of the vacuum configuration (2.8) with $(V(\beta), \alpha(\beta))$ determined from (2.9)–(2.10):

$$F^{\text{tree}}(\beta) = \log Z^{\text{tree}}(\beta) = \tilde{S}(\beta; V(\beta), \alpha(\beta)). \tag{2.11}$$

The qualitative behaviour of this vacuum energy is shown in Fig. 7. This tree approximation is identical to mean field theory. It predicts a first order phase transition at β_t, and not just for the gauge group U(1), but also for SU(2), SU(3), SO(3), U(N), probably for all compact semi-simple groups, provided $d \geqslant 3$ and we use the Wilson action. We should not trust this prediction, however, since it is based upon the tree approximation. It is an almost trivial comment that the tree, or classical, approximation to a quantum theory is not necessarily a good approximation to that quantum theory. It certainly is not the case in question here, as we see by comparing our result $F^{\text{tree}}(\beta) = 0$ for low values of β with the

result of the strong coupling expansion: $F(\beta) \propto \beta^2 + O(\beta^3)$. We conclude that in order to make reliable predictions we must take into account radiative corrections to the tree, classical, or mean field result.

The radiative corrections are given by the machinery of standard perturbation theory as a loop expansion. We still have to find out which diagrams to evaluate and which to neglect when a given approximation is desired. When calculating radiative corrections in

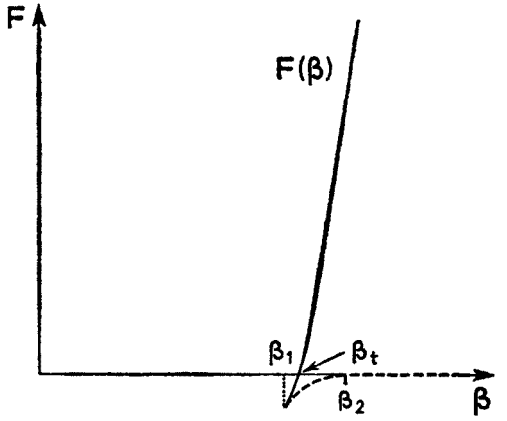

Fig. 7. Qualitative behaviour of vacuum energy density/free energy density given in Eq. (2.11)

the vacuum characterized by $(V(\beta), \alpha(\beta)) = (0, 0)$ the natural way of doing this is by counting powers of β in a given diagram. In this way we recover the strong coupling expansion, which is gratifying since it is a check of MFT, but it does not add to known methods. When we calculate radiative corrections in the vacuum characterized by $(V(\beta), \alpha(\beta)) \neq (0, 0)$ we count powers of α^{-1}, where α is given in (2.10), $\alpha \sim 2(d-1)\beta$. Notice that in the interval $[\beta_1, \beta_2]$ (see Fig. 6), both vacua exist and are perturbatively stable. We can have both β small and α^{-1} small in this interval and therefore calculate perturbatively the free energy starting from *both* vacua, and compare the results. Here is what we find [4]: The tree result (Figs. 8 and 9, graph I) for the free energy of the vacuum characterized by $(V(\beta), \alpha(\beta)) = (0, 0)$ is shifted by radiative corrections to graph III. The tree result for the vacuum characterized by $(V(\beta), \alpha(\beta)) \neq (0, 0)$ is shifted by radiative corrections to graph II. For the gauge group SO(3) and a $4d$-lattice we find a first order transition at $\beta_t = 2.62$, where graphs II and III intersect. This is 6% above the M.C. result $\beta_t^{\text{M.C.}} = 2.48$ [6, 2, 22]. The error on graph III in Fig. 8 is negligible, so graph II must lie below the exact free energy of the phase it represents. Graph II is the result of a one-loop approximation, and, counting power of α^{-1}, we believe only the two-loop term can shift graph II sufficiently upwards to shift β_t down 6%. For SU(2) in $4d$ (Fig. 9), graphs II and III do not intersect. Moreover, if graph II is shifted upwards by two-loop corrections by approximately the same amount as we have just learned it should in the case of SO(3), then graphs II and III in the case of SU(2) lie practically on top of each other in the interval $[\beta_1, \beta_2]$, where both are defined. This indicates that the free energy is *one*

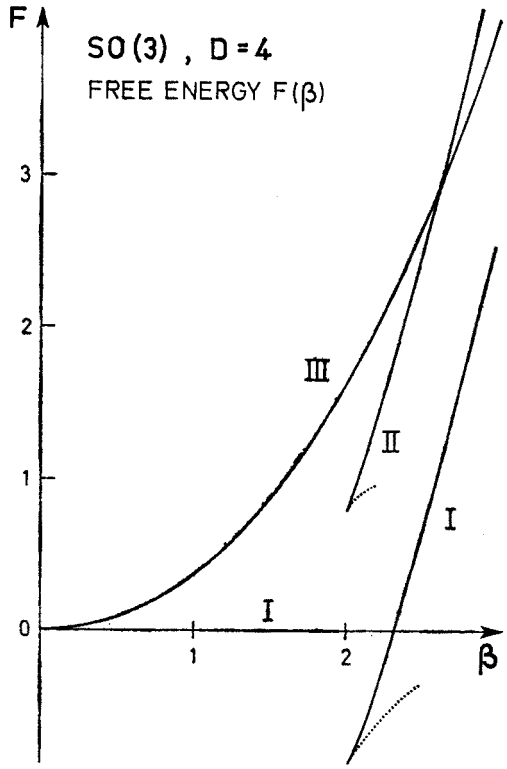

Fig. 8. Free energy for SO(3). Graph labelled I is the zeroth order mean field results for the strong and weak coupling phases. Graph labelled II is mean field approximation plus one-loop corrections from the isospin 1-channel for the weak coupling phase. Graph labelled III is the result of correcting the mean field approximation in the strong coupling phase with the first three terms in the strong coupling expansion [4]

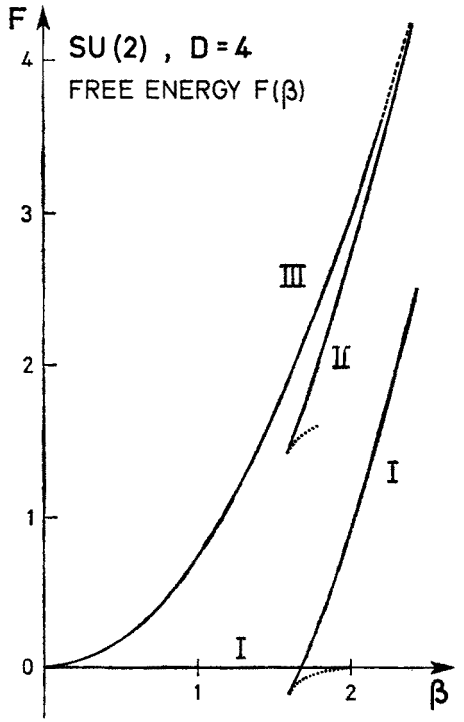

Fig. 9. Same as Fig. 8, but for SU(2)

analytic function, which we have approached perturbatively from two different starting points. Consequently, MFT with radiative corrections included predicts absence of a phase transition for SU(2) in $4d$, in full agreement with M.C. results. Predictions by MFT with radiative corrections for lattice gauge theories with Wilson action and various groups have been collected in the table (Table I).

The following variant action for the gauge group SU(2) has received much attention during the last year [2, 27–34, 43] because it interpolates between the Wilson actions for Z(2), SO(3), and, most interesting, SU(2):

$$S(\beta; U) = \sum_P (\beta_F \tfrac{1}{2} \operatorname{Tr} U_P + \beta_A \tfrac{1}{3} ((\operatorname{Tr} U_P)^2 - 1)),$$

$$U_P \equiv \prod_{l \in \partial P} U_l, \quad U_l \in \mathrm{SU}(2). \tag{2.12}$$

TABLE I

Transition points β_t for various Wilson lattice gauge theories

	4d		5d	
	MC	MFT	MC	MFT
SU(2)	none	none	1.642±.015 [24]	1.77
SO(3)	2.48 [6, 22, 2]	2.62	not available	2.03
Z(2)	0.4407 [23] (analytical result)	0.440	not available	0.329
U(1)	1.01 (second order) [25]	1.03 [21]		
U(∞)	0.75±0.05 [26] (extrapolation of MC beyond U(6))	0.74 [26]		

For U(∞) it is $\lim_{N \to \infty} N^{-2}\beta_t(N)$ that is given.

This action has also been studied by mean field methods with radiative corrections. The main results were shown in Figs. 1 and 2. For details on how these results were obtained, see Ref. [1].

3. *What is so good about these results?*

We saw that MFT with some radiative corrections will predict and locate first order phase transitions for gauge groups and actions that have such transitions. It will also describe the internal energy as a function of β. The accuracy is agreeable, and we believe it can be improved to fine by taking into account two-loop vacuum bubbles in the vacuum $(V, \alpha) \neq (0, 0)$. This is interesting because:

(I) It is worth money: As Korthals Altes has told you in his lecture at this school, M.C. investigations of lattice gauge theories with Wilson and other one-plaquette actions may not have reached the scaling region, i.e. may not represent the physics of continuum gauge theory [35]. A straightforward way to settle this and other issues of the continuum limit is to repeat the M.C. calculations for larger values of β. This is not possible with computers and budgets available today. So, since you cannot bring β to the scaling region, you must bring the scaling region to β. For this reason, people these days design actions supposed to scale at lower values of β than one-plaquette actions do [36, 37]. Once you have written down such an action, or a family of actions, you do not have to spend your computing budget investigating it. MFT will put you out of business faster and cheaper by locating the thing that *will* put you out of business: a first order transition in, or pointing to, the β-region where you hoped for scaling behaviour[1].

(II) Reproduction of M.C. results is a *check* on MFT. First order transitions and crossovers do not really interest us. We want to work in the large β limit, far away from such reminiscents of strong coupling. Neither does the internal energy truely interest us. It is large Wilson loops, glueball masses, and eventually hadron masses, we would like

[1] I thank R. B. Pearson for drawing this application to my attention.

to evaluate by MFT. We have attempted nothing of that sort, and such projects will certainly require a creative application of the method to succeed. What we *have* done is to show that the method passes the natural first small test one should submit it to before attempting anything more ambitious with it.

(III) It confirms that MFT with radiative corrections is the natural perturbation theory on the lattice. The plaquette energy of SU(2) provides a convincing illustration: MFT to tree level gives graph I in Fig. 3. This graph fits the M.C. data better than weak coupling perturbation theory to order β^{-2} does [44], and is considerably easier to obtain.

4. *What is next?*

(*I*) The contribution to the free energy from *two loop diagrams* in the vacuum $(V, \alpha) \neq (0, 0)$ should be evaluated to order α^{-1}, since ignorance of this contribution is the major source of error in MFT for the moment. This is a tedious, but straightforward calculation, which has been started.

(*II*) Bachas and Dashen have recently suggested that lattice gauge theories in $4d$ having one-plaquette actions with non-trivial, local maxima, as a consequence hereof have first order phase transitions [38]. Theories with Wilson action and gauge group SU(2) in a representation with isospin $I > 1$ are examples of such theories. And indeed, they all seem to have first order transitions [39]. Moreover, it seems possible to check the role of the non-trivial maxima of the one-plaquette action directly. It seems possible to isolate and remove by hand the contributions to the partition function from the non-trivial local maxima of the one-plaquette action. The resulting partition function is close to the one obtained for $I = \frac{1}{2}$, so it probably has no phase transition. If this is so, we have *confirmed the Bachas-Dashen hypothesis* by direct inspection and manipulation of a set of lattice gauge theories.

(*III*) Application of the machinery of MFT with radiative corrections to lattice gauge theories with *finite physical temperature* seems straightforward, though technically more complicated, and should enable one to locate first order phase transitions, when they are there [40].

(*IV*) The Wilson loop and the string tension are the next things to consider, if the two-loop calculation mentioned in (*I*) really proves MFT a high precision method for evaluation of *local* densities. It is not at all straightforward to apply MFT to this, nor any other, nonlocal object. It nevertheless seems to be the natural next step to take, if possible, on the way to calculations with dynamical fermions in the continuum limit.

(*V*) Mean field theory has already been applied to calculations with dynamical fermions for zero and finite β values [41, 42]. See Zinn-Justin's lectures at this school. At large values of β, only MFT *without* radiative corrections has been applied [42]. Considering the striking agreement found between experimental hadron masses and those calculated with the rather crude approximations of Refs. [41] and [42], I am convinced that we will hear more about this project in the future.

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