COSMOLOGICAL CHARACTER OF THE HIGGS FIELD

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(Received May 19, 1982)

The Higgs-boson mass in the Weinberg-Salam electroweak theory $M_H = 6.6$ GeV is obtained from the experimental limits on the value of the cosmological constant.

PACS numbers: 98.80 12.20.-h

It is commonly believed that elementary particles interactions can be described in terms of gauge theories with spontaneous symmetry breaking. The only teoretically known way to produce such a symmetry breaking is the Higgs mechanism. It consists in introducing a suitable set of scalar fields (Higgs fields) and in arranging the parameters in the Lagrangian in such a way that some of the Higgs fields acquire non-zero vacuum expectation values.

Elementary particles theories are renormalized by dividing vacuum-vacuum amplitudes. Therefore, the vacuum self-energy $\langle T_{\mu\nu}\rangle_{vac}$ can be made arbitrary by adding a suitable constant K in the Lagrangian. However, the vacuum energy density ε

$$\varepsilon g_{\mu\nu} = \langle T_{\mu\nu}\rangle_{vac} \tag{1}$$

can be observed gravitationally, and it must be interpreted [1] as a cosmological constant

$$\Lambda = \frac{8\pi G}{c^4}\varepsilon \tag{2}$$

in the Einstein gravitational field equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} - g_{\mu\nu}\Lambda. \tag{3}$$

Astronomical observations impose stringent limits on the value of the cosmological constant [2]:

$$|\Lambda| < 4 \cdot 10^{-84} m_p^2,$$

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therefore

$$|\varepsilon| < 2.5 \cdot 10^{-47}\ \mathrm{GeV}^4. \tag{4}$$

In gauge theories with spontaneous symmetry breaking the cosmological term is temperature dependent [3]. Above the critical temperature T_c the symmetry is restored [4]. Therefore, in the Big Bang cosmology the vacuum energy density in the early Universe $\varepsilon(T > T_c) = K$ can be interpreted as a primordial cosmological term. The vacuum energy density difference between the hot ($T > T_c$) Universe, and the present cold ($T = 0$) one (from which any possible constants cancel out) is calculable for definite gauge symmetry group.

In the Weinberg-Salam SU(2) × U(1) model of electroweak interactions with minimal coupling, the effective potential at the tree level is given by:

$$V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + K. \tag{5}$$

Therefore, the vacuum energy density difference becomes:

$$\varepsilon(T > T_c) - \varepsilon(T = 0) = V(\sigma(T_c) = 0) - V(\sigma(0) = \sigma) = \frac{\lambda\sigma^4}{4}. \tag{6}$$

The vacuum expectation value of the Higgs field $\sigma(T)$ diminishes with the growth of the temperature and vanishes above T_c. If, for example, $\lambda = O(10^{-2})$ this difference is of the order of 10^5 GeV^4. It means that the primordial cosmological term was at least 10^{52} times greater than it is now [3]. Usually this tremendeous vacuum energy density is negligible compared with the thermal energy of particles [2], but in certain cases it can dominate the cosmological expansion and complicate their usual scenarios.

More serious problem is the necessity of a fine tuning of the primordial cosmological constant. In principle, the primordial cosmological term and the constant induced by the spontaneous symmetry breaking are unrelated and there is no known reason for them to cancel to one part in 10^{52}.

An alternative view, avoiding the problem of the fine tuning, was presented by Dreitlein [6]. He assumed that the self-energy of the symmetric vacuum must vanish, leading to vanishing primordial cosmological term. The experimental limits (4) on the present vacuum energy density in the Weinberg-Salam model:

$$|\varepsilon(T = 0)| = \frac{\lambda\sigma^4}{4} = \frac{M_H^2\sigma^2}{8} < 2.5 \cdot 10^{-47}\ \mathrm{GeV}^4 \tag{7}$$

can be interpreted as a limit on the Higgs-boson mass M_H

$$M_H < 5.5 \cdot 10^{-26}\ \mathrm{GeV}. \tag{8}$$

Such a light particle is ruled out experimentally because it would mediate forces of macroscopic range (10^{12} cm), much stronger then the gravity [7].

It was shown that one can avoid such unrealistic limit on the physical Higgs-boson mass at the price of rejecting the minimal coupling condition in the SU(2)×U(1) electroweak theory (e.g. introducing two Higgs doublets) [8], what strongly diminishes the predictive power of the model.

We will show that one can obtain a realistic Higgs-boson mass in the conventional Weinberg-Salam model with the minimal coupling. If the mass of the Higgs-boson is sufficiently small, the ϕ^4 self coupling λ can be weak enough ($\lambda = O(g^4)$) for the vector boson loops to compete with tree graphs, while the perturbation theory remains valid, because higher order effects are negligible. In the one loop approximation the effective potential takes the form [9]

$$V(\phi) = \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + B\phi^4 \ln\frac{\phi^2}{M^2}, \tag{10}$$

where

$$B = \frac{1}{1024\pi^2}\Big(\sum_{\text{gauge bosons}} 3g_i^4 - \sum_{\text{fermions}} f_i^4\Big), \tag{11}$$

and M is an arbitrary renormalization mass. We have omitted higher order effects (λ^2 terms). Assuming that there are no heavy fermions and using the experimental value of the Weinberg mixing angle $\sin^2\theta_W = 0.23$ we obtain

$$B = \frac{3\alpha^2}{64}\left(\frac{2+\sec^2\theta_W}{\sin^4\theta_W}\right) = 1.74\cdot 10^{-4}. \tag{12}$$

The spontaneous symmetry breaking occurs when the (potential has a local minimum at $\phi = \sigma = 247$ GeV, such that:

$$\left.\frac{dV}{d\phi}\right|_{\phi=\sigma} = 0, \qquad \left.\frac{d^2V}{d\phi^2}\right|_{\phi=\sigma} = M_H^2 \geqslant 0. \tag{13}$$

Eliminating the parameters μ^2 and M^2 appearing in the effective potential $V(\phi)$, in favour of the physical quantities σ and Higgs-boson mass M_H, we can rewrite (10) in the form:

$$V(\phi) = \left(-\frac{M_H^2}{4} + 2B\sigma^2\right)\phi^2 + \left(\frac{M_H^2}{8} - \frac{3B}{2}\right)\phi^4 + B\phi^4 \ln\frac{\phi^2}{\sigma^2}. \tag{14}$$

Therefore, the present vacuum energy density in the one loop approximation is equal to

$$\varepsilon(T=0) = -\frac{M_H^2}{8}\sigma^2 + \frac{B\sigma^4}{2}, \tag{15}$$

and the experimental estimates (4) set limits on the Higgs-boson mass:

$$|M_H^2 - 4B\sigma^2| < 3\cdot 10^{-51}\ \text{GeV}^2. \tag{16}$$

In the Weinberg-Salam model we obtain:

$$4B\sigma^2 = (6.6 \text{ GeV})^2, \tag{17}$$

therefore it is fair to accept:

$$M_{\rm H} = 2B^{1/2}\sigma = 6.6 \text{ GeV}. \tag{18}$$

As stated by Weinberg [10], the phase transition into the broken symmetry is energetically favourable only if $V(0) \geqslant V(\sigma)$, what yields the lower bound on the Higgs-boson mass:

$$M_{\rm H} \geqslant 2B^{1/2}\sigma. \tag{19}$$

Therefore, the experimental fact that the present vacuum energy density is negligible can be interpreted that the Higgs-boson mass is equal to its lower cosmological limit $M_{\rm H} = 6.6$ GeV.

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