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FAILURE OF A RECENTLY PROPOSED $Q\bar{Q}$ POTENTIAL TO REPRODUCE HEAVY QUARKONIUM SPECTROSCOPY

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Flory has recently found support for the Shifman, Vainshtein and Zakharov estimate of G (the density of gluon condensate in Vacuum) by deriving a q-$\bar{q}$ potential which uses G as an input and correctly predicts the string tension. Here we show that Flory's potential gives wrong predictions for energy levels and leptonic widths of quarkonia. Consequently agreement found for the string tension is not conclusive.

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1. *Introduction*

Successful potential models for heavy quark-antiquark systems give quarkonium mass spectra agreeing with experimental data within some 20 MeV and leptonic widths correct within a factor of two [1]. The non-relativistic treatment can be justified because the heavy quark masses are much greater than their kinetic energies in the $q\bar{q}$ centre of mass system. In the present work an effective quarkonium potential derived by Flory [2, 3] is applied to compute the ψ and Υ mass spectra and the corresponding leptonic widths. A comparison of this theoretically calculated potential with experiment is interesting, because of its plausible short and long distance behaviour consistent with other theoretical anticipations and results of fitting phenomenological potentials to experimental data. What is more — Flory's agreement supports the value found by Shifman, Vainshtein and Zakharov [4] for the parameter M_0^4 (see the next Section). According to other authors the value of this important parameter may be underestimated by a factor of 3 [5] or even 5 [6]. Therefore Flory's support is very interesting.

2. *Flory's potential*

Non perturbative methods of Shifman, Vainshtein and Zakharov [4] show that the vacuum state has a nonvanishing average value of the operator $g^2 \cdot G^a_{\mu\nu}G^{\mu\nu a}$, where $G^a_{\mu\nu}$ is the gluon field. They found

$$G \equiv \langle 0|\,(\alpha_S/\pi)G^a_{\mu\nu}G^{\mu\nu a}|0\rangle \equiv M_0^4 = (330\ \text{MeV})^4. \tag{1}$$

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This information was used by Flory to calculate the q-$\bar{q}$ colour singlet potential. He included gluon exchange between the two quarks and an additional interaction with the vacuum gluon condensate. The problem is analogous to that of calculating the energy levels of positronium in a spatially constant but fluctuating electric field. Using the multipole expansion Flory found for the effective q-$\bar{q}$ potential.

$$V(r) = -\frac{4\alpha_S}{3r} + \frac{\pi^2 M_0^4 r^2}{18[(2\alpha_S/3r) - \sqrt{5/288}\,\pi M_0^2 r]}, \tag{2}$$

where α_S is the strong coupling constant, M_0 is given by formula (1) and r is the q-$\bar{q}$ separation. The encouraging properties of this potential are: the proper Coulomb like behaviour at $r \to 0$ and the linear rise for large r, which can give rise to quark confinement. From formula (2)

$$V(r) \xrightarrow[r\to\infty]{} 0.144\ \mathrm{GeV}^2 r. \tag{3}$$

For comparison fitting a phenomenological potential to the spectrum gives [1]

$$V(r) \xrightarrow[r\to\infty]{} 0.155\ \mathrm{GeV}^2 r, \tag{4}$$

while trying to reproduce the Regge slope $\alpha' = 0.9\ \mathrm{GeV}^{-2}$ one has

$$V(r) \xrightarrow[r\to\infty]{} 0.143\ \mathrm{GeV}^2 r. \tag{5}$$

This surprising agreement and the proper short distance behaviour makes it interesting to find how this potential works at medium distances relevant for all today's experimental data. This was the motivation for the present work.

3. Fitting of free parameters to experimental data

Writing the Schrödinger equation with potential (2) one sees that the mass of given q-$\bar{q}$ state depends on two parameters: m_q and α_S. The relevant formula is

$$m_{q\bar{q}} = 2m_q + E(m_q, \alpha_S), \tag{6}$$

where E is an eigenvalue of the Schrödinger equation. These parameters, the coupling constant α_S and the quark mass m_q, are free and can be fitted to experimental data. Choosing the masses of J/ψ and Υ as input, we find after numerical computations m_c and m_b as functions of α_S. The results (see Table I) imply

$$1250 < m_c < 1788\ \mathrm{MeV}$$

$$4535 < m_b < 5512\ \mathrm{MeV}$$

TABLE I

Dependence on α_S of the quark masses and of the ψ and Υ spectra

α_S	0.0	0.2	0.4	0.6	0.8
m_c [MeV]	1250	1416	1529	1645	1788
m_b [MeV]	4535	4724	4892	5137	5512
$\Delta\psi$ [MeV]	447	494	508	545	637
$\Delta\Upsilon$ [MeV]	291	351	446	714	1219
$\Delta'\Upsilon$ [MeV]	238	276	297	321	366

for α_S between 0.0 and 0.8. This is consistent with the masses calculated for other potential models fitting the experimental data.

4. *Meson mass spectra and leptonic widths calculated from Flory's potential*

Now it is possible to compute mass spectra for ψ and Υ resonances. In order to avoid coupled channel problems we limit our calculation to the stable resonances. With fitted m_q, chosen so as to reproduce correctly the mass of the lowest ^3S state, numerical calculations for various α_S give the results collected in Table I and presented in Fig. 1, where we denote

$$\Delta\psi = m_{\psi'} - m_{\psi} \tag{7}$$

$$\Delta\Upsilon = m_{\Upsilon'} - m_{\Upsilon} \tag{8}$$

$$\Delta'\Upsilon = m_{\Upsilon''} - m_{\Upsilon'}. \tag{9}$$

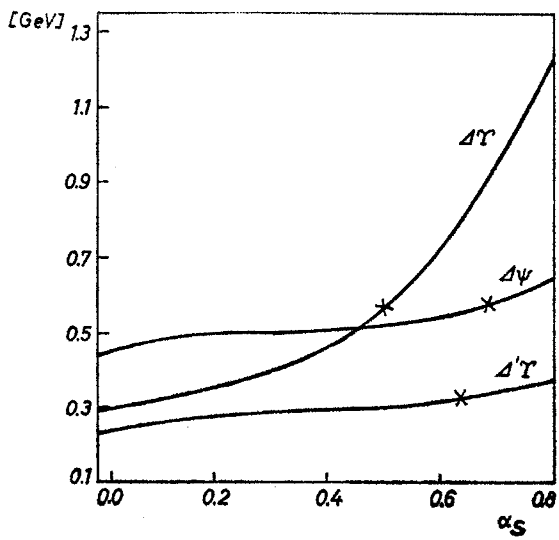

Fig 1. The mass differences of the ψ and Υ resonances as a function of α_S, × — experimental value

The experimental values are

$$\Delta\psi = 588.2 \pm 0.9 \text{ MeV},$$
$$\Delta\Upsilon = 560 \pm 5 \text{ MeV},$$
$$\Delta'\Upsilon = 330 \pm 8 \text{ MeV}.$$

Each of these values separately can be reproduced by a suitable choice of α_S (the corresponding points are shown in Fig. 1) but looking at Fig. 1 one can easily see that it is impossible to find one value of α_S which would reproduce correctly all the mass differences. Fitting each mass difference separately one finds that the best values of α_S are:

$$\alpha_S = 0.693 \quad \text{for} \quad \Delta\psi, \quad \alpha_S = 0.484 \quad \text{for} \quad \Delta\Upsilon, \quad \alpha_S = 0.593 \quad \text{for} \quad \Delta'\Upsilon.$$

Using the least square fit for all mass differences one finds

$$\alpha_S = 0.512$$

which implies $\Delta\psi = 523.6$ MeV, $\Delta\Upsilon = 570.6$ MeV, $\Delta'\Upsilon = 308.9$ MeV roughly consistent with experiment for $\Delta\Upsilon$ and $\Delta'\Upsilon$, but completely wrong for $\Delta\psi$. This is the first signal that Flory's potential is not as universal as it was expected. Maybe the charm quark is too light for such a non-relativistic model but for Υ it may be good.

Fitting α_S to $\Delta\Upsilon$ and $\Delta'\Upsilon$ only gives

$$\alpha_S = 0.506; \quad m_b = 5008 \text{ MeV}$$

and mass spectra

$$\Delta\Upsilon = 562 \text{ MeV}; \quad \Delta'\Upsilon = 308 \text{ MeV}$$

which roughly agrees with experimental values. The value of α_S is however much greater than usually considered plausible. In order to check whether it is not too big we compute the leptonic width for Υ. We use the Van Royen–Weisskopf formula [7].

$$\Gamma(^3S_1 \to e^+e^-) = [16\pi\alpha^2 Q_q^2/M_R^2] \cdot |\psi(0)|^2, \tag{10}$$

where $\alpha = 1/137$ is the electromagnetic coupling constant, Q_q is the quark charge in units of the electron charge and M_R is the mezon mass. The result for $\alpha_S = 0.506$ is $\Gamma_e(\Upsilon) = 7.96$ keV that is about 8 times greater than experimental value $\Gamma_e(\Upsilon)_{exp} = 1.07 \pm 0.24$ keV. Thus we see that a good fit to masses has been obtained at the price of having a nonrealistic wave function.

5. *Conclusions*

The final conclusion is that Flory's potential is not a good effective potential for heavy quarkonia. It can give acceptable masses for $\Upsilon, \Upsilon', \Upsilon''$ but not for ψ and ψ'. This could be explained by assuming that the charm quarks are too light, but the leptonic width for Υ is also completely wrong. Thus, the model seems excluded. This invalidates the argument in favour of estimation (1) for M_0^4.

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