LETTERS TO THE EDITOR

VISCOUS COSMOLOGICAL MODELS WITH MATTER AND RADIATION

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(*Received December 22, 1982*)

Robertson-Walker world models filled with interacting matter and radiation, including the bulk viscosity dissipation, are constructed. It is shown that there exist stationary solutions in which the bulk viscosity term can be interpreted as phenomenologically describing a creation of matter and radiation.

PACS numbers: 98.80.Dr

In this work we construct a class of relativistic world models filled with interacting matter and radiation. One could expect that within such two-fluid models the bulk viscosity dissipation might be physically relevant [1], we include it, therefore, into our equations. It turns out that, in general, this leads to a time rescaling. This result was anticipated in [2].

In the previous work [3], it has been shown that, from the formal point of view, Hoyle's steady-state solution is equivalent to a special case of stationary solutions to Einstein's field equations with bulk viscosity. Now, we demonstrate that the relativistic cosmology with dissipation also admits stationary solutions in which the bulk viscosity term can be interpreted as phenomenologically describing a creation of matter and radiation at just the rate necessary to preserve the stationary character of the model.

1. The standard form of equations describing the Robertson-Walker cosmological models with bulk viscosity (e.g. see, [4])) may be reduced to the following form

$$\frac{du}{dR} = \frac{2+aR}{2R+2aR^2}(u+u^3k)-\alpha u^2, \qquad \frac{dt}{dR} = u, \tag{1}$$

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where one looks for a solution $t = \varphi(R)$; $\alpha = \frac{2}{3}\zeta$, ζ being the bulk viscosity coefficient and the equation of state $p = \varepsilon/3(1+aR)$ has been assumed. Here a may be a function of R, and it models the interaction of matter and radiation. If $\alpha = 0$, and $aR = \varepsilon_m/\varepsilon_r$, where indices m and r refer to matter and radiation, respectively, one can easily find solutions describing a non-interacting mixture of matter and radiation, and, in fact, such solutions have been already found ([5], [6, Proposition 6.4.5]).

By the direct computation, one can check the following

Proposition: If $t = \varphi(R)$ is a solution of a system (1) with $k = 0$, $a = a(R)$, and $\alpha = 0$ then the functions: *(i)* $\bar{t} = \bar{\varphi} = \alpha^{-1}\ln(\varphi+C)$, *(ii)* $\bar{t} = \bar{\varphi} = \alpha^{-1}\ln C\varphi$, and *(iii)* $\bar{t} = \bar{\varphi} = C_1e^{\alpha t}-C_2$ are solutions of the same system (1) with $\alpha \neq 0$.

Let us notice that for $\alpha \to 0$, the above viscous solutions have no perfect fluid limits which is not surprising in the contexts of cosmological models with bulk viscosity where quite often we meet such a situation. This effect means simply that the solutions with $\alpha = 0$ are structurally unstable with respect to viscous perturbations.

2. In the literature, one can find some proposals to replace the "proper time" t of the universe by its "physical time" $\tau \sim \ln t$, which removes the initial singularity to minus infinity, with the motivation that "the universe is meaningfully infinitely old because infinitely many things have happened since the beginning" [7]. A modification of this philosophy [8] suggests the foliating the space-time with spacelike hypersurfaces of constant mean curvature, and measuring the "physical time" by the minus extrinsic curvature of these hypersurfaces (the minus sign is needed in order to guarantee that the initial singularity be in the infinitely distant past; see also [9]). The above proposition shows that one must be very careful with such procedures because what one would like to do just for esthetic reasons may have precise physical meaning (logarithmic time meaning the inclusion of bulk viscosity effects).

From the mathematical point of view, solutions *(i)*–*(iii)* of the above proposition are perfectly regular (singularity free); whether they are also physically realistic, it remains to be seen. This problem, for viscous solutions in general, has been discussed in [10].

3. Robertson–Walker cosmological models with the equation of state $p = (\gamma-1)\varepsilon-3\zeta H$ where $1 \leqslant \gamma \leqslant 2$, and $H \equiv \dot{R}/R$, may be written in the form of the dynamical system (see [11])

$$\dot{H} = -H^2-\tfrac{1}{6}(\varepsilon+3p), \quad \dot{\varepsilon} = -3H(\varepsilon+p). \tag{2}$$

We will look for stationary solutions of system (2); they correspond to its critical points (see [3])

$$\varepsilon = 3H^2, \tag{3a}$$

$$\varepsilon+p = 0. \tag{3b}$$

(3a) represents the equation of the flat model trajectory ($k = 0$), (3b) is the equation of the boundary of the energy condition $\varepsilon+p \geqslant 0$ [12].

Let us consider n fluids with the equations of state $p_i = (\gamma_i - 1)\varepsilon_i$, $1 \leqslant \gamma_i \leqslant 2$ $(i = 1, 2, \dots n)$. By using (3) we get

$$\sum_i \gamma_i \varepsilon_i = 3\zeta H \quad \text{or} \quad \zeta = \frac{\sum_i \gamma_i \varepsilon_i}{\sqrt{3\sum_i \varepsilon_i}}. \tag{4}$$

Let us notice that from the stationarity condition (3) it follows that $\sum_i \varepsilon_i = \text{const.}$ Knowing $H = \sum_i \gamma_i \varepsilon_i / 3\zeta$ we can easily compute

$$R = R_0 e^{(\sum_i \gamma_i \varepsilon_i / 3\zeta)t}. \tag{5}$$

In special cases, we have

(A) For dust $(p = 0)$

$$\zeta = \sqrt{\varepsilon_{m/3}}, \quad R = R_0 e^{\zeta t} \quad \text{(see, [3])};$$

(B) For radiation $(p = \frac{1}{3}\varepsilon_r)$

$$\zeta = \tfrac{4}{3}\sqrt{3\varepsilon_r}, \quad R = R_0 e^{\frac{3}{4}\zeta t} \quad \text{(see, [4])};$$

(C) For matter and radiation there is ζ such that

$$\zeta = \frac{\varepsilon_m + \frac{4}{3}\varepsilon_r}{\sqrt{3(\varepsilon_m + \varepsilon_r)}}.$$

4. Let us assume

$$\sum_i \gamma_i \varepsilon_i = \bar{\gamma} \sum_i \varepsilon_i$$

where $\bar{\gamma}$ is an average value of γ_is. Then we have

$$\zeta = \frac{\bar{\gamma}}{\sqrt{3}} \sqrt{\sum_i \varepsilon_i} \quad \text{and} \quad H = \zeta / \bar{\gamma}.$$

Hence

$$R = R_0 e^{(\zeta/\bar{\gamma})t} \tag{6}$$

where $\bar{\gamma} = \sum_i \gamma_i \varepsilon_i / \sum_i \varepsilon_i$.

Let us notice that if we assume $\Lambda = 3\zeta^2/\bar{\gamma}^2$, we recover Hoyle's original steady-state solution $R = R_0 e^{\sqrt{\Lambda/3}\, t}$. Therefore, we can see that stationary two-fluid models can be treated as solutions with dissipation in which bulk viscosity creates matter and radiation, i.e. one always can choose ζ in (4) in such a way that the solutions have the form (6).

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