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ATTENUATION OF HADRONS LEPTOPRODUCED FROM NUCLEAR TARGETS

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We estimate the inclusive quark-nucleon cross section in the lowest order of QCD and apply it to the description of the spectrum of final hadrons leptoproduced from nuclear targets. The numerical calculations in the case of neutrino-nucleus scattering are performed.

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1. Introduction

It has been suggested by many authors [1] that the high-energy experiments with nuclear targets can be used to investigate the hadronic interactions at very short times. In these processes the collisions inside the nucleon can serve as a detector of the short-lived objects produced in the first collision. According to these arguments one can expect that analysis of similar processes provides information about interactions of the high-energy quarks in nuclear matter. These ideas were used to study the spectra of hadrons produced in the deep-inelastic scattering of leptons from nuclear targets [2], i.e. in the process:

$$l+A \rightarrow l'+\text{anything}. \tag{1.1}$$

In accordance with the quark-parton model the deep inelastic scattering of the lepton from a nucleon inside the nucleus leads predominantly to the production of a single quark with momentum determined by the momentum transfer of a lepton. The produced quark moves through the nuclear matter interacting with other nucleons (Fig. 1). Its momentum distribution depends on:

a) the presence of the nuclear matter and its density distribution (A-dependence);

b) the cross section for the process:

$$\text{quark} + \text{nucleon} \rightarrow \text{quark} + \text{anything}. \tag{1.2}$$

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The final hadron spectrum is sensitive to the momentum distribution of the quark which scatters inside the nucleus, therefore it depends on A and quark-nucleon cross section. This dependence, in general fairly complicated, becomes considerably simpler when the quark-nucleon cross section is so small that the probability of the multiple scattering is negligible. In the following we consider just this case.

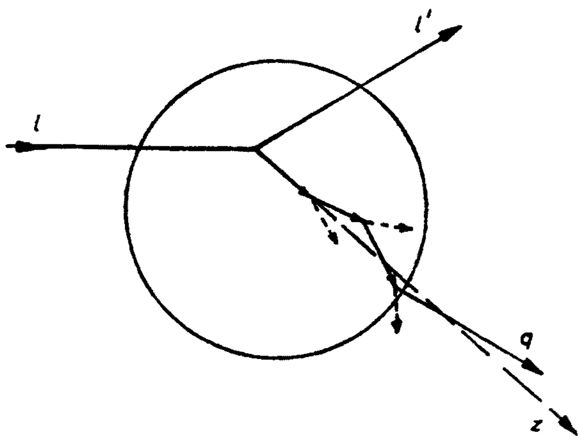

Fig. 1. Final-state interactions of quark in the nucleus

The purpose of this paper is to estimate the inclusive quark-nucleon cross section in the lowest order of QCD and to use it for the study of the final hadron spectra in the reaction (1.1). In the next Section the formulae for the inclusive quark-nucleon cross section and final hadron spectra are discussed. Results of the numerical calculations are given n Section 3. Our conclusions are summarized in the last section.

2. *Basic formulae*

A. Quark-nucleon cross section

To evaluate the inclusive quark-nucleon cross section we adapt the formalism used previously to the description of the hadron-hadron collisions with high $p_\perp$ particles production [3, 4]. In this approach one treats quark-nucleon interaction as a collision between a quark and a parton from the nucleon with dynamics (i.e. elementary parton-parton cross sections $d\hat{\sigma}/d\hat{t}$, effective strong coupling constant $\alpha_s(Q^2)$, Q^2 — evolution of the quark and gluon distributions $G_n^p(x, Q^2)$ given by QCD (Fig. 2). Thus the quark-nucleon cross section is equal to the sum of the elementary quark-parton subprocess cross sections weighted by the number of partons inside the nucleon:

$$\sigma(\vec{P}, \vec{p}) \equiv \frac{Ed\sigma(s, t, u; q_i+n \rightarrow q_i+X)}{d^3p} = \sum_p N_p(x) \frac{Ed\hat{\sigma}(\hat{s}, \hat{t}, \hat{u}; q_i+p \rightarrow q_i+p)}{d^3p}, \quad (2.1)$$

where s, t, u — usual invariants for the quark-nucleon system

$$s = (p_a+p_n)^2, \quad t = (p_c-p_a)^2, \quad u = (p_n-p_c)^2; \tag{2.2}$$

$\hat{s}, \hat{t}, \hat{u}$ — these invariants for two-body parton reaction

$$\hat{s} = (p_a+p_b)^2 = xs, \quad \hat{t} = (p_c-p_a)^2 = t, \quad u = (p_n-p_c)^2 = xu; \tag{2.3}$$

x — momentum fraction of nucleon carried by the parton; $Ed\hat{\sigma}/d^3p$ — two-body quark--parton cross section; $N_p(x)$ — number of p-type partons from nucleon with momentum fractions between x and $x+dx$.

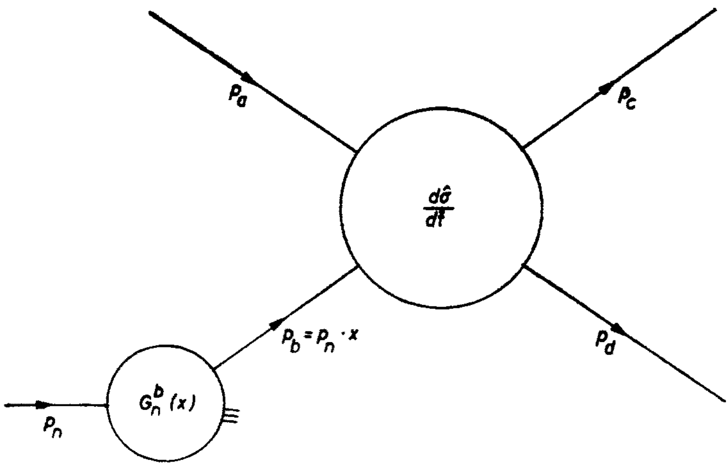

Fig. 2. Schematic representation of quark-nucleon scattering

We sum over all types of partons. Masses of partons and of nucleon are neglected, so we have:

$$\hat{s}+\hat{t}+\hat{u} = 0. \tag{2.4}$$

From (2.3) and (2.4) we obtain:

$$x = -\frac{t}{s+u}. \tag{2.5}$$

Expressing the number of partons $N_p(x)$ in terms of quark and gluon distributions we have:

$$\sigma(\vec{P}, \vec{p}) = \sum_p \frac{2x}{\pi} G_n^p(x, Q^2) \frac{d\hat{\sigma}(\hat{s}, \hat{t}, \hat{u}; q_i+p \to q_i+p)}{d\hat{t}}. \tag{2.6}$$

In our calculations we use Buras and Gaemers analytic parametrisation of the parton distribution [5]. As two-body parton cross section $d\hat{\sigma}/d\hat{t}$ we employ the formulae given previously by Combridge, Kripfganz and Ranft [6]. For Q^2 we choose [4]:

$$Q^2 = \frac{2\hat{s}\hat{t}\hat{u}}{\hat{s}^2+\hat{t}^2+\hat{u}^2}. \tag{2.7}$$

There are, however, some problems: QCD formulae are valid for the hard processes (i.e. processes with $Q^2 \gtrsim 1\text{Gev}^2$) only, but we have to use them for *hard* and *soft* processes as well, since we want to obtain an estimate of the quark-nucleon cross section. In order to do this we make the following modifications:

a) hard process cross section diverge at $\hat{s}$, $\hat{t}$ or $\hat{u}$ equal zero. To remove this unwanted singularity, we replace [4]:

$$\hat{s} \to \hat{s}+M_0^2, \quad \hat{t} \to \hat{t}-M_0^2, \quad \hat{u} \to \hat{u}-M_0^2, \tag{2.8}$$

with $M_0^2 \sim 1\ \text{GeV}^2$;

b) for soft processes ($Q^2 \leqslant 1.8\ \text{GeV}^2$) we use QCD formulae for $G_n^p(x, Q^2)$ and $\alpha_s(Q^2)$ evaluated for $Q^2 = 1.8\ \text{GeV}^2$;

c) we multiply the overall cross section (2.4) by the factor

$$\left(\frac{Q^2}{Q^2+M_0^2}\right)^N. \tag{2.9}$$

So, in our model there are two parameters: M_0 and N. Their numerical values should be extracted from the data.

B. Final hadron spectrum

In the case of small quark-nucleon cross section the spectrum of the observed hadrons is particularly simple since it consists only of two terms: hadrons may arise from fragmentation of the quarks which either did or did not scatter inside nucleus. Using the normalization to one deep inelastic trigger we obtain [2]:

$$\left.\frac{E_h dN_h}{d^3p_h}\right|_A = [1-\sigma_T^* W_1^A]D_{p\to h}(\vec{P}, \vec{p}_h)+W_1^A \int \frac{d^3p}{E}\sigma(\vec{P}, \vec{p})D_{p\to h}(\vec{p}, \vec{p}_h), \tag{2.10}$$

where σ_T^* — quark-nucleon cross section integrated over final-quark momentum; W_1^A — the factor describing the effect of the size of the nucleus and nuclear matter density distribution on the probability of the quark-nucleon collision. Following [2] it reads:

$$W_1^A = \frac{A-1}{2}\int d^2b\left[\int_{-\infty}^{+\infty} \varrho_A(\vec{b}, z)dz\right]^2; \tag{2.11}$$

$\varrho_A(\vec{b}, z)$ — nuclear density normalized to unity (the z axis direction is defined with respect to the momentum transferred between initial and final lepton). We use usual Saxon-Woods density [7]; $D_{p\to h}(\vec{p}_1, \vec{p}_2)$ — quark fragmentation function can be expressed in terms of usual scaling quark fragmentation functions $D_p^h(z)$ [3]

$$D_{p\to h}(\vec{p}_1, \vec{p}_2) = zD_p^h(z)\delta^{(2)}\left(\vec{p}_2 - \vec{p}_2 \cdot \frac{\vec{p}_1}{|\vec{p}_1|}\right), \tag{2.12}$$

$$z = \frac{|\vec{p}_2|}{|\vec{p}_1|}. \tag{2.13}$$

In Eq. (2.10) we neglect hadrons which are not produced from the fragmentation of the leading quark (e.g. by fragmentation of the diquark arising in the first collision). They are expected to be rather slow (in lab. system), so Eq. (2.10) is a good description of the fast hadrons leptoproduced from a nuclear target.

In order to investigate the nuclear effects (i.e. attenuation) we define the ratio:

$$R_A(z) = \left(\frac{dN_h}{dz}\right)_A \cdot \left(\frac{dN_h}{dz}\right)_H^{-1}, \tag{2.14}$$

where $z = p_{\parallel h}/p_0$, p_0 — initial quark momentum. From Eq. (2.10) we obtain:

$$R_A(z) = 1 - \sigma_T^* W_1^A + R_A^1(z), \tag{2.15}$$

where

$$R_A^1(z) = \left(\frac{dN_h^1}{dz}\right)_A \cdot \left(\frac{dN_h}{dz}\right)_H^{-1}, \tag{2.16}$$

dN_h^1/dz — distribution of h-type hadrons, which arise from fragmentation of the quarks, which scatter once inside the nucleus.

3. *Results*

We have performed the numerical calculations for the case of hadron production in neutrino-nucleus interactions. We have investigated final π^+ spectra and attenuation in nuclei.

A. Quark-nucleon cross section

We have investigated the influence of parameters M_0, N (Eqs. (2.8), (2.9)) on the quark-nucleon cross section. In Fig. 3 σ_T^* is plotted versus M_0^2 for different N. One can see that if M_0, or N increases σ_T^* decreases. One can say however, that in no case σ_T^* was so large as to make our single-scattering model to fail (if $M_0 \geqslant 0.5$ GeV, $\sigma_T^* \leqslant 7.5$ mb). Correct choice of M_0, N requires comparison with the data. In further calculations we have chosen:

$$N = 1, M_0 = 1 \text{ GeV},$$

which gives $\sigma_T^* = 1.51$ mb.

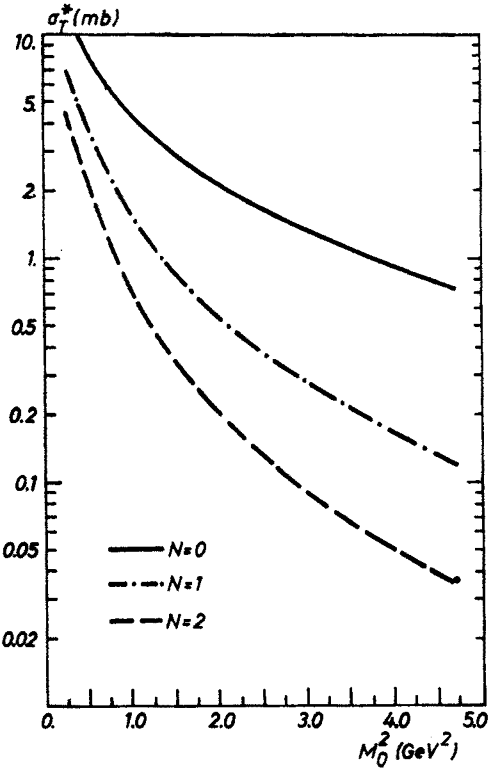

Fig. 3. Total quark-nucleon cross section σ_T^* plotted versus cut-off parameter M_0^2 for different values of N

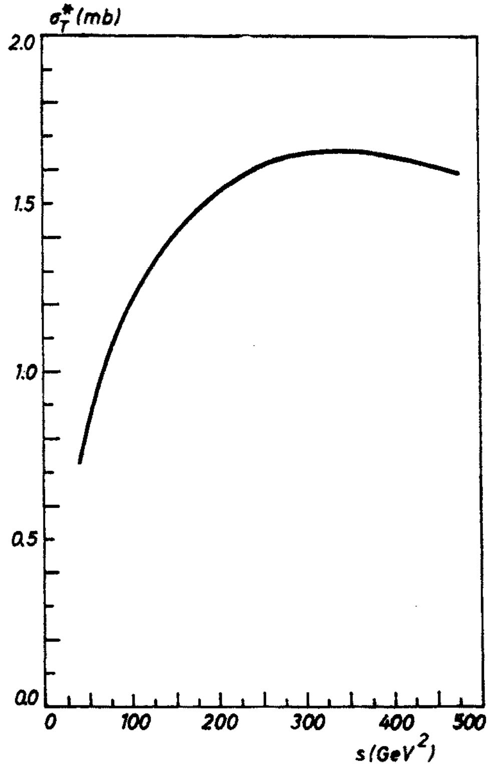

Fig. 4. Energy-dependence of the total quark-nucleon cross section σ_T^*

Energy dependence of σ_T^* is shown in Fig. 4. The nature of this dependence is the same for all M, N — as s increases σ_T^* initially increases, realizes a maximum (for $M_0 = 1$ GeV, $N = 1$ it is at $s = 320$ GeV^2, $\sigma_T^* = 1.67$ mb), then slowly decreases.

Inclusive quark-nucleon cross section integrated over $\vec{p}_\perp$

$$\frac{E d\sigma(\vec{P}, \vec{p})}{dp_\parallel} = \int d^2 p_\perp \sigma(\vec{P}, \vec{p}) \tag{3.1}$$

versus $p_\parallel$ is given in Fig. 5. One can see that the quark-nucleon collision with small energy loss of the quark (quasi-elastic scattering) is rather unlike — quarks are slowing down. This effect should give the depression in hadronic spectra for fast particles.

B. Final π^+ spectra. Attenuation in nuclear matter

Typical final π^+ spectrum (Eq. (2.10)) is plotted in Fig. 6. These spectra are dominated by the quark fragmentation function $D_u^{\pi^+}(z)$, because in our model above 90% π^+ are produced by simple fragmentation of the leading quark.

In Fig. 7 the ratio $R_A(z)$ (Eq. (2.14)) for different A is plotted. One can see the attenuation of the final pions which increases with A. The attenuation is smaller for lower momen-

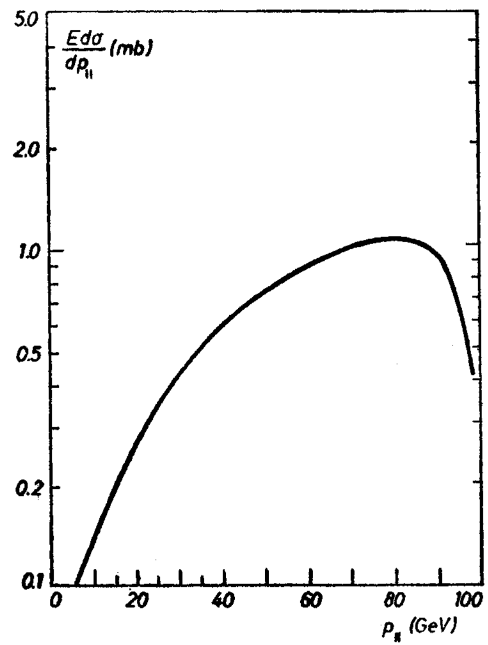

Fig. 5

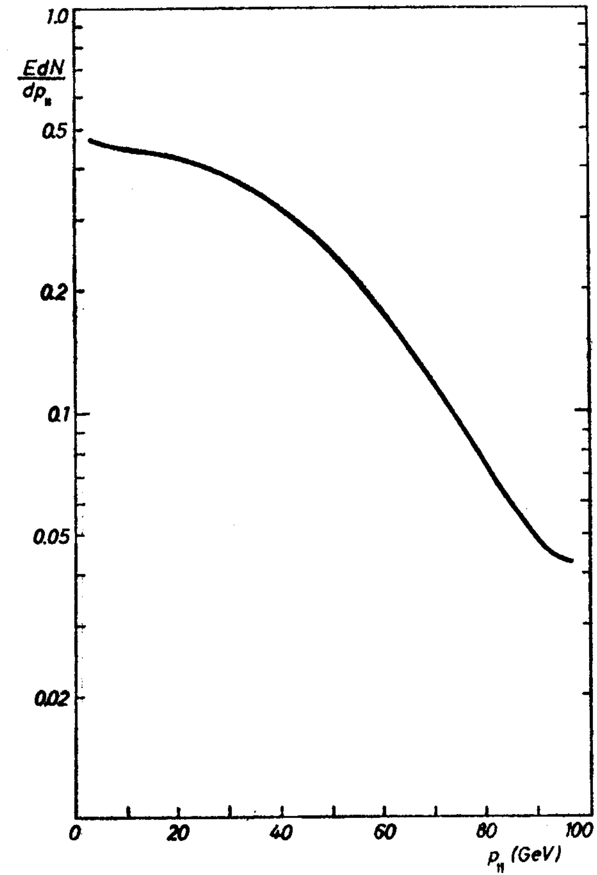

Fig. 6

Fig. 5. Inclusive quark-nucleon cross section (integrated over transversal momentum $\vec{p}_T$) plotted versus longitudinal momentum of the quark $p_{\parallel}$

Fig. 6. Invariant longitudinal-momentum distribution of π^+ in νPb interactions following from single-scattering formula (2.10)

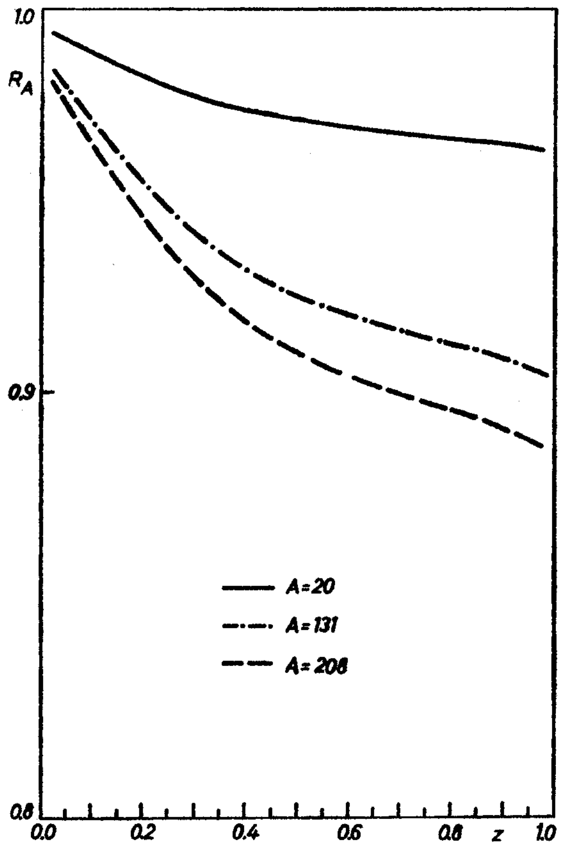

Fig. 7

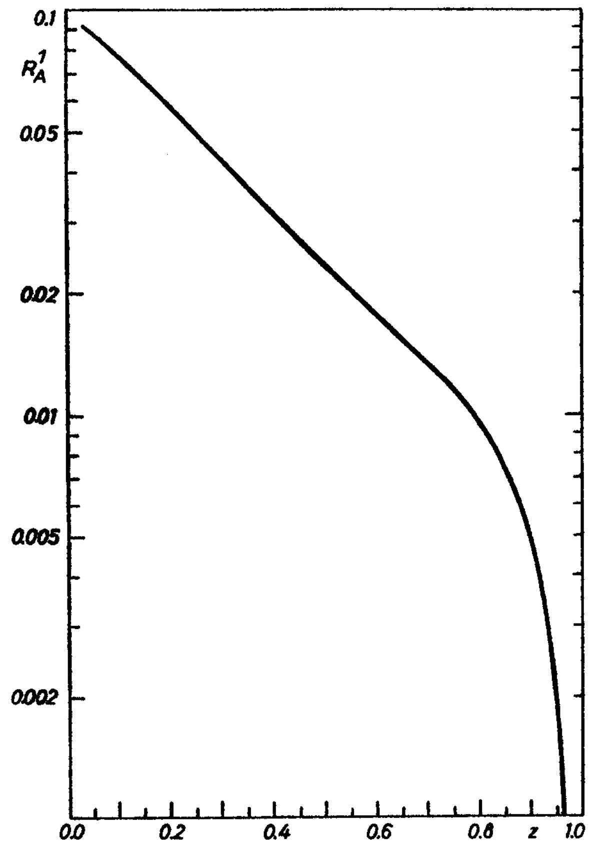

Fig. 8

Fig. 7. Ratio R_A versus z plotted for different nuclei

Fig. 8. R_A^1 versus z plot for νPb interact ons following from single-scattering model

ta ($z \to 0$) and depends very weekly on z for higher momenta (z-dependent part of $R_A(z)$; $R_A^1(z)$ falls very strongly as $z \to 1$, see Fig. 8), so the measurement of $R_A(z)$ gives information about magnitude of σ_T^*.

We have also performed numerical calculations to estimate higher order corrections (i.e. π^+ production by fragmentation of a quark, which scatter two times inside nuclear matter) and found them negligible.

Our results agree qualitatively with existing data [8], but if we attempt to estimate σ_T^* from [8] we obtain the result higher than we predicted in our model. It is possible that at energies at which the experiment [8] was performed the „formation time" (i.e. the time scale of hadronization) is short, so that high-energy quark can fragment into observed hadrons inside the nucleus [9]. These hadrons interact with nuclear matter, hence attenuation increases. To verify this possibility and correctness of our model the high-energy experiments are needed.

5. *Conclusions*

We have estimated the quark-nucleon cross section in the lowest order of QCD and used it to the description of the lepton-nucleus collisions. Our conclusions can be summarized as follows:

a) for the standard choice the cut-off parameters M_0, N the QCD-predicted magnitude of the quark-nucleon cross section σ_T^* is about few mb, so the single-scattering model is a reasonable approximation;

b) quasi-elastic quark-nucleon scattering is rather unlikely — a quark slows down in a collision with a nucleon;

c) interaction of quark in the nuclear matter implies attenuation of final hadrons. The magnitude of attenuation was estimated using this QCD-suggested quark-nucleon cross section. The attenuation increases with increasing A and reaches value of 10% already for $A \approx 150$, so it should be readily observable.

d) the attenuation is smaller for lower momenta ($z \to 0$) and increases for higher momenta ($z \to 1$). For higher momenta ($z \geqslant 0.7$) $R_A(z)$ depends very weakly on z and the measurement of $R_A(z)$ in this region would give information about magnitude of σ_T^*.

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