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CASCADE MULTIPLICITY INSIDE DEUTERON IN πd HIGH ENERGY COLLISIONS

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Multiplicity distribution of double scattering events is analysed using the additive quark model including the cascading effect. The mean multiplicity of particles produced in the process of cascading estimated for πd experiments at 100, 205 and 360 GeV/c is 1.15 ± 0.31. This value does not depend on the momentum of the incident pion. Some indications are found that the probability of cascading depends on multiplicity of the collision with the first nucleon and is smaller for low multiplicities.

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1. *Introduction*

It has been shown [1] that the additive quark model is able to explain the recent data on particle production in hadron-deuteron collisions provided one introduces cascading into the mechanism of hadronic interactions in deuterium. By cascading we mean here the interaction of the secondary particles created inside deuteron. This version of the additive quark model gives correct values of double scattering cross section in πd and pd collisions and also of the density of particles produced in double scattering processes in central rapidity region.

The effective cross section for interaction of secondaries produced in the deuteron with the remaining nucleon in the target turns out to be 20–25 mb which is a typical hadronic cross section.

Therefore further investigation of the nature of cascading is of interest. In particular we can ask how this effect modifies the multiplicity distribution or in which region of rapidity cascade produces particles.

In this paper we analyse the multiplicity distribution of double scattering events using the additive quark model including the cascading effect, as described in Ref. [1].

In this model we can obtain the mean multiplicity of particles produced in the process of cascading. If we assume that only slow particles from the first collision may possibly

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interact with the second nucleon, we can expect multiplicity distribution in cascade to be independent of the incident beam momentum.

It was found [2] that the probability of the second interaction does not depend on the multiplicity of particles in the first collision for all double scattering events. With this observation in mind we test an additional assumption that the probability of cascading is also independent of multiplicity of particles produced in the first collision.

Although, the model is able to describe both πd and pd interactions, in this paper we discuss only the πd processes. We use here the data from three πd experiments at 100, 205 and 360 GeV/c [2–4]. The value of the effective cascade cross section $\sigma^* = 20.2 \pm 1.7$ mb is taken from Ref. [1].

Our conclusions are:

1. The mean multiplicity of particles produced in the cascade is indeed independent of the incident beam momentum

$$\langle n \rangle = 1.15 \pm 0.31. \tag{1}$$

2. The probability of cascading depends on the multiplicity in the first collision and is smaller for low multiplicities.

The paper is organized as follows. In the next Section the multiplicity distribution for πd interactions derived from the model [1] is discussed. Section 3 contains a description of these experimental results for πd and πp, πn interactions which served as a basis for our calculation. In Section 4 we compare the experimental data with the model. The conclusions are presented in Section 5.

2. *Multiplicity distribution of double scattering events from the additive quark model*

According to the model described in Ref. [1] we can construct the multiplicity distribution of double scattering events from the known multiplicity distributions of elementary pion nucleon interactions. Table I shows contributions of different processes to the double scattering effects in deuterium for incident pion. In Table I we use the following notation: ϱ_N is the multiplicity distribution for different processes (ND — non-diffractive, ELEM — elementary π-nucleon, C — cascade, F — fragmentation of nucleon target, DIFF — diffractive), all normalized to unity. $\sigma^A, \sigma^B, \sigma^{DE}, \sigma^\kappa$ are partial cross sections for processes A, B, DE, κ. X_N are the multiplicity distributions for processes A, B, DE, κ, and finally

$$\varrho_k = (\varrho^1 \otimes \varrho^2) = \sum_{i=1}^{k} \varrho_i^1 \varrho_{k-i}^2 \tag{2}$$

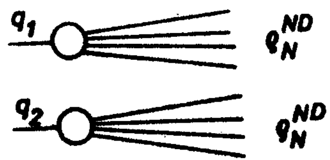
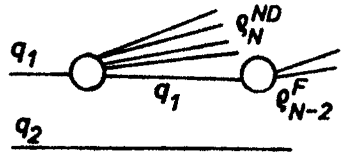
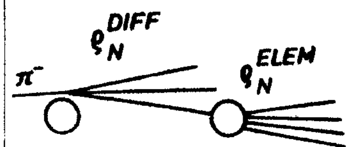
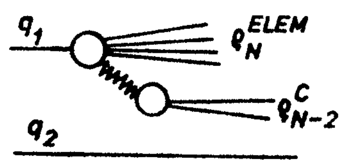
denotes a convolution of two multiplicity distributions. The formulae presented in column 3 of Table I are discussed in the Appendix.

To discuss the multiplicity distributions we split the cross section for double scattering into four terms which we identify as:

Process A — one of the quarks from the incident pion interacts with the proton, the other one with the neutron in the target. The multiplicity distribution for this kind of interac-

TABLE I

The contributions of different processes to the double scattering effects in deuterium for incident pion

Process	Type of process	Partial cross section from the model	Multiplicity distribution
A		$\sigma^A = 2 \int d^2S d^2b D(\vec{S}) \sigma^{q_1}(+) \sigma^{q_2}(-)$	$X_N^A = (\varrho_N^{ND} \otimes \varrho_{N-1}^{ND}) \sigma^A$
B		$\sigma^B = 2 \int d^2S d^2b D(\vec{S}) \sigma^{q_1}(+) \sigma^{q_1}(-)$	$X_N^B = (\varrho_N^{ND} \otimes \varrho_{N-2}^F) \sigma^B$
DE		$\sigma^{DE} = \sigma^{II} - \int d^2S d^2b D(\vec{S}) \sigma^{ND}(+) \sigma^{ND}(-)$	$X_N^{DE} \approx \varrho_N^{ELEM} \sigma^{DE}$
$\varkappa$		$\sigma^{\varkappa}$	$X_N^{\varkappa} = (\varrho_N^{ELEM} \otimes \varrho_{N-2}^C) \sigma^{\varkappa}$

tion is the convolution of multiplicity distributions for non-diffractive interactions of quark with proton ϱ_N^{ND} and of quark with neutron ϱ_{N-1}^{ND}.

Process B — one of the quarks from the incident pion interacts with both nucleons in the target. The other quark does not interact at all. The multiplicity distribution is the convolution of the distribution for non-diffractive quark nucleon interaction with the distribution for the fragmentation of one nucleon in the target. In the fragmentation only produced particles are taken into account.

Process DE — represents the class of processes which cannot be described in terms of quark model. It contains events in which interaction with one nucleon was diffractive or elastic and the interaction with the other nucleon of any type. The multiplicity distribution for this class is approximated by distribution in elementary π-nucleon collision.

Process κ — only of one incident quarks interacts with one of the nucleons in the target and the particles produced in this collision scatter from the second nucleon. This effect is called here cascade. The multiplicity distribution is constructed by the convolution of the multiplicity distribution in elementary interaction and of the multiplicity distribution of particles produced by cascade ϱ_{N-2}^{C}. Both ϱ_N^{ELEM} and ϱ_N^{ND} are the mean values of corresponding distributions for $\varrho_N^{\pi^- p}$ and $\varrho_{N+1}^{\pi^- n}$.

To construct the multiplicity distribution described in Table I we make the following assumptions:

1. The multiplicity distribution in the quark nucleon collision is the same as in the hadron-nucleon collision. (The additivity assumption).

2. The interaction of one quark with the two nucleons gives multiplicity distribution as in an elementary interaction with one of the nucleon. The collision with the second nucleon can add at most the particles produced in the fragmentation process of the second nucleon ϱ_{N-2}^{F}.

3. Secondary interaction of particles created in the first collision produces approximately the same multiplicity as in the nucleon fragmentation

$$\varrho_{N-2}^{F} = \varrho_{N-2}^{C} \tag{3}$$

4. The probability of cascading does not depend on the multiplicity in the first collision.

To extract the mean multiplicity of cascade we use the following technique.

1. From the experimental multiplicity distribution of double scattering sample we subtract the multiplicity distribution of processes A and DE

$$X_N^{B\kappa} = X_N^{DS} - X_N^{A} - X_N^{DE} \tag{4}$$

where X_N^{DS} is the multiplicity distribution of double scattering events. The distribution which we obtain in this way represents only that part of double scattering sample which contains the cascading process. Therefore according to the model this distribution should be a convolution of some known elementary distribution ϱ_N^{NC} and the cascade distribution ϱ_{N-2}^{C}

$$X_N^{B\kappa} = (\varrho_N^{NC} \otimes \varrho_{N-2}^{C})\sigma^{B\kappa}, \tag{5}$$

where

$$\varrho_N^{\rm NC} = \frac{\sigma^{\rm B}}{\sigma^{\rm B}+\sigma^{\kappa}}\,\varrho_N^{\rm ND} + \frac{\sigma^{\kappa}}{\sigma^{\rm B}+\sigma^{\kappa}}\,\varrho_N^{\rm ELEM} \tag{6}$$

and

$$\sigma^{\rm B\kappa} = \sigma^{\rm B}+\sigma^{\kappa}. \tag{7}$$

2. We calculate the moments: mean multiplicity $\langle N \rangle$ and dispersion squared D^2 for distributions $X_N^{\rm B}$ and $X_N^{\rm NC}$.

3. The following relations for convolution of the distribution of cascade, $X_N^{\rm B}$ and $X_N^{\rm NC}$ distribution are used

$$\langle N^{\rm B\kappa}\rangle = \langle N^{\rm C}\rangle + \langle N^{\rm NC}\rangle, \tag{8}$$

$$D_{\rm B\kappa}^2 = D_{\rm C}^2 + D_{\rm NC}^2. \tag{9}$$

If the model is correct and if we have reliable experimental data these two relations give us a possibility of finding the moments of the cascade multiplicity distribution.

3. *Experimental data*

In this section we discuss the experimental data which we used to calculate multiplicity distribution of the cascading process. The technique used to extract the multiplicity distribution of double scattering events was described in Ref. [5]. From this paper we take the partial cross sections for double scattering events for πd interactions at 100 205 and 360 GeV/*c* beam momentum. The corresponding multiplicity distributions for elementary interactions are taken from the following πp experiments: at 100 GeV/*c* Ref. [6], 205 GeV/*c* Ref. [7], 360 GeV/*c* Ref. [8] and from πn data Ref. [3], and of [9].

The separation of elementary interactions into diffractive and non-diffractive sample was done based on the results of Ref. [10]. Because of lack of such separation for corresponding πn experiments we have assumed that the multiplicity distribution for diffractive processes πn is the same as for πp.

Table II contains the moments of multiplicity distributions used in our calculations.

TABLE II

The moments of multiplicity distributions

Beam momentum	100 GeV/*c*	205 GeV/*c*	360 GeV/*c*
$\langle N^{\rm DS}\rangle$	9.64±0.54	10.28±0.32	11.66±0.38
$D^{\rm DS}$	2.94±0.34	4.00±0.22	4.38±0.26
$\langle N_{\rm INEL}^{\rm ELEM}\rangle$	7.38±0.08	8.43±0.08	9.16±0.10
$D_{\rm INEL}^{\rm ELEM}$	3.51±0.06	3.92±0.06	4.48±0.07
$\langle N_{N\geq 4}^{\rm ELEM}\rangle$	7.82±0.06	8.83±0.07	9.70±0.06
$D_{N\geq 4}^{\rm ELEM}$	3.01±0.05	3.71±0.07	4.21±0.07
$\langle N_{N\geq 4}^{\rm ND}\rangle$	8.22±0.11	9.43±0.10	10.39±0.10
$D_{N\geq 4}^{\rm ND}$	3.16±0.20	3.60±0.20	4.08±0.21

4. *Comparison with the model and discussion*

We have calculated the partial cross sections for the processes A, B, DE, and κ from the model using the formulae given in the Appendix. The measured values of the total and elastic cross section were taken from Ref. [11] The value of cascade cross section σ^* interpreted as an effective cross section for the interaction of products of the first collision with the second nucleon, was taken from Ref. [1] $\sigma^* = 20.2 \pm 1.7$ mb. The deuteron wave function and other detailed assumptions are the same as described in Ref. [1].

Table III presents contributions of the processes A, B, DE, and κ to the double scattering cross section. We do not observe the energy dependence of these fractions within the experimental errors.

The results of calculations of the average multiplicity of cascade $\langle N^C \rangle$ using the method described in Section 2 are given in Table IV.

TABLE III

The contributions of processes A, B, DE, and κ to the double scattering cross section

Beam momentum	100 GeV/*c*	205 GeV/*c*	360 GeV/*c*
σ^A/σ^{DS}%	12.4 ± 0.1	12.4 ± 0.1	12.6 ± 0.2
σ^B/σ^{DS}%	12.4 ± 0.1	12.4 ± 0.1	12.6 ± 0.2
σ^{DE}/σ^{DS}%	22.7 ± 0.3	23.0 ± 0.4	24.5 ± 0.5
$\sigma^{\kappa}/\sigma^{DS}$%	52.5 ± 0.5	51.8 ± 0.5	50.3 ± 0.6

TABLE IV

Average multiplicities and dispersions of various multiplicity distributions. $X_N^{B\kappa}$ is the experimental multiplicity distribution for double scattering events with cascade. X_N^{NC} is the distribution defined in equation (5). X_N^C is the distribution of cascade

Beam momentum	$\langle N^{B\kappa} \rangle$	$D^{B\kappa}$	$\langle N^{NC} \rangle$	D^{NC}	$\langle N^C \rangle$
100 GeV/*c*	9.37 ± 0.57	2.52 ± 0.32	7.90 ± 0.07	3.04 ± 0.07	1.47 ± 0.57
205 GeV/*c*	9.89 ± 0.47	3.24 ± 0.40	8.94 ± 0.07	3.70 ± 0.06	0.95 ± 0.47
360 GeV/*c*	11.08 ± 0.63	3.83 ± 0.47	9.83 ± 0.09	4.19 ± 0.07	1.25 ± 0.63

The values of $\langle N \rangle$ and D in Table IV correspond to the following multiplicity distributions:

— $\langle N^{B\kappa} \rangle$ and $D^{B\kappa}$ (columns 2, 3) describe the multiplicity distribution $X_N^{B\kappa}$ of only those double scattering events which contain cascade, obtained using formula (4).

— $\langle N^{NC} \rangle$ and D^{NC} (columns 4, 5) represent multiplicity distribution calculated according to the model [1] from elementary distributions which convoluted with cascade multiplicity distribution should reproduce experimental distribution $X_N^{B\kappa}$,

— $\langle N^C \rangle$ is the mean multiplicity of particles produced in cascade.

In the analysis of πd data only events with more than two prong were considered. Therefore lowest multiplicity in the distribution $X_N^{B\kappa}$ from which we determine $\langle N^{B\kappa} \rangle$

and $D^{B\kappa}$ is four. We have also decided to calculate $\langle N^{NC}\rangle$ and D^{NC} only for multiplicities higher than 2 because cascading for two-prongs events probably does not exist.

The average number of particles produced in cascade does not depend on the momentum of incident pions within experimental errors. This value averaged over the experiments is

$$\langle N^{C}\rangle^{PROD} = 1.15 \pm 0.31 \tag{10}$$

and corresponds to

$$\langle N\rangle^{\pi p} = \langle N^{C}\rangle^{PROD} + 2 = 3.15 \pm 0.31 \tag{11}$$

for π nucleon interactions. Such a number suggests that if the interaction of cascade does not differ from the elementary π-nucleon interactions, the average momentum of particles which initiate the cascade is about 3–4 GeV/c.

This result confirms our hypothesis that only slow particles produced in the first collision may interact with the second nucleon.

The comparison the dispersions $D^{B\kappa}$ and D^{NC} indicates that in the considered model we cannot obtain dispersion D^{C} of the multiplicity distribution of cascade. The dispersion D^{NC} is systematically higher than $D^{B\kappa}$ for all experiments at the level of one standard deviation. The possible interpretation of such a result is that the probability of cascading depends on the multiplicity in the first collision and is smaller for low multiplicities.

We may take into account this fact using formula (5)

$$X_N^{B\kappa} = (\varrho_N^{NC} \otimes \varrho_{N-2}^{C})\sigma^{B\kappa}$$

If the probability of cascading depends on the multiplicity in the first collision, we should use instead of ϱ_N^{NC} in formula (5) the distribution ϱ_N^{NC} multiplied by some damping factor for low multiplicities. Such a correction leads to the narrowing of the distribution $X_N^{B\kappa}$. On the basis of the available experimental data we were not able to determine the form of this damping factor.

A simple explanation of the fact that the probability of cascading depends on the multiplicity in the first collision may be the following. At first for low multiplicities the probability of second interaction is smaller than for higher ones. Furthermore, for elementary high energy interactions the momentum distribution of produced particles depends on the multiplicity. In particular for low multiplicities we observe less particles with low momenta than for higher multiplicities. If we assume that only slow particles created in the first collision can interact with the second nucleon in deuteron, we can also argue that cascading is smaller for low multiplicities in the first collision.

Another possible explanation of the fact that the experimental multiplicity distribution for double scattering events is too narrow when compared with the multiplicity distribution obtained from the model is that in elementary interactions there exist a small fraction of collisions in which both beam quarks interact with the target nucleon. This effect is non-additive in the standard quark model. If we do not include the non-additive effects we overestimate the mean value of multiplicity and dispersion of distribution in process A (see Table 1). This leads to the broadening of the multiplicity distribution X_N^{NC} calculated from the model.

5. *Conclusions*

We have analysed the multiplicity distribution of double scattering events using the additive quark model including the cascading effect [1]. The mean multiplicity of particles produced in the process of cascading was estimated as

$$\langle N^C \rangle = 1.15 \pm 0.31. \tag{12}$$

This value is independent of momentum of the incident hadron in πd interactions and indirectly confirms the hypothesis that in the process of cascading only slow particles produced in the first collision may interact.

The observation that only slow particles initiate the cascade can also explain the dependence of cascading on multiplicity in the first collision and in particular the smaller probability of cascading for lower multiplicities in first step of interaction.

The effect that the dispersion in the experimental distribution of double scattering sample is narrower than obtained from the model can also be explained by the dependence of the cascading process on the multiplicity in collision with the first nucleon.

Our general conclusion is that the model of Ref. [1] can well describe the multiplicity distribution of double scattering events. However there are some discrepancies e.g. the predicted dispersion of multiplicity distribution is too small. This, if confirmed by data of better accuracy, may lead to a revision of the model.

If on the other hand the model is correct, the more precise measurements of double scattering events should allow us to find the quantitative dependence of cascading effect on the multiplicity in the first step of interaction.

Another analysis of the double scattering process in terms of the additive quark model was done by Nikolaev et al. [12]. After assuming that the secondaries are pions, they attempt to determine the length of the formation zone. They also have found that only slow particles ($\lesssim 5$ GeV/c) created in the first collision in deuterium can interact with the second nucleon.

Finally, we would like to emphasize that the value of $\langle N^C \rangle$ obtained in a relatively simple way for the deuteron can be used for the investigation of multiple scattering processes in hadron-nucleus interactions.

Useful discussions with Prof. A. Białas and Prof. A. Eskreys are gratefully acknowledged.

APPENDIX

Cross section formulae

Using the basic assumptions of the additive quark model including the cascading effect described in Ref. [1] we can construct the multiplicity distribution of double scattering events from the known multiplicity distributions for elementary hadron-nucleon interactions.

The purpose of this Appendix is to present the method of calculating of the partial cross sections for the processes A, B, DE, and κ defined in Table I. The multiplicity distributions for this class of processes can be obtained in a relatively simple way from the elementary interactions. We use the same notation as in Ref. [1].

The total double scattering cross section σ_2 splits into two terms

$$\sigma_2 = \sigma^{II} + \sigma^{\kappa}, \tag{A1}$$

where

$$\sigma_{II} = \int d^2Sd^2bD(\vec{S})\left[\sigma_{\rm eff}(-)\sigma_{\rm eff}(+) - \tilde{\sigma}_{\rm E}(-)\tilde{\sigma}_{\rm E}(+)\right] \tag{A2}$$

corresponds to the class of processes without cascade and

$$\sigma^{\kappa} = 2\int d^2Sd^2bD(\vec{S})\kappa(\vec{S})\sigma_{\rm in}(-)\left[1-\sigma_{\rm eff}(+)\right] \tag{A3}$$

represents the fraction of double scattering with cascade. The notation is as follows. The profile of the effective hadron-nucleon cross section $\sigma_{\rm eff}$ (b) consists of three terms

$$\sigma_{\rm eff}(b) = \sigma_{\rm ND}(b) + \sigma_{\rm D}(b) + \tilde{\sigma}(b) \tag{A4}$$

where b is the impact parameter. Here $\sigma_{\rm ND}(b)$ is the non-diffractive cross section, $\sigma_{\rm D}(b)$ is the diffractive cross section and $\tilde{\sigma}_{\rm E}(b)$ is this part of the elastic cross section which is responsible for knocking the nucleons out of the target deuterium with momentum transfer large enough ($q \gtrsim 100$ MeV/c) to be observed. Further

$$\sigma_{\rm in} = \sigma_{\rm ND}(b) + \sigma_{\rm D}(b) \tag{A5}$$

is the elastic cross section, and

$$D(S) = \int_{-\infty}^{\infty} dZ\psi_{\rm D}^*(\vec{S}, Z)\psi_{\rm D}(\vec{S}, Z) \tag{A6}$$

is the projection of the probability density on the plane perpendicular to the incident beam. Here $\psi_{\rm D}(\vec{s}, z)$ is the deuteron wave function [13] and $\vec{r}(\vec{s}, z)$ is the distance between the neutron and the proton in the target deuteron.

We also use abbreviation $\sigma(\pm) = \sigma(\vec{b} \pm \frac{1}{2}\vec{S})$. The profile

$$\kappa(\vec{S}) = \frac{\sigma^*}{\pi a^2}\exp\left(-S^2/a^2\right), \tag{A7}$$

where the value of $\sigma^* = 20.2 \pm 1.7$ mb, was obtained in Ref. [1]. This value can be interpreted as the effective cross section for scattering of the products of the first collision on the second nucleon. The constant a determines the spatial extension of this cascade. Following Ref. [1] we also assume:

$$\sigma_{\rm D} = \sigma_{\rm E} \tag{A8}$$

$$a^2 = \sigma^*/\pi \tag{A9}$$

$$\tilde{\sigma}_{\rm E}(b) = \sigma_{\rm E}(b) - \delta_{\rm E}(b) \tag{A10}$$

$$\delta_{\rm E}(b) = 0.08\sigma_{\rm E}(b). \tag{A11}$$

The total double scattering cross section σ_2 splits into four parts presented in Table I. These parts correspond to the contributions of some known elementary multiplicity distributions.

Process κ is simply represented by cross section as in Ref. [1].

Processes A and B have the cross sections

$$\sigma^{A} = 2 \int d^2Sd^2bD(\vec{S})\sigma^{q_1}(+)\sigma^{q_2}(-),$$
$$\sigma^{B} = 2 \int d^2Sd^2bD(\vec{S})\sigma^{q_1}(+)\sigma^{q_1}(-). \tag{A12}$$

When we take into account the additive quark model relation

$$\sigma^{ND}(b) = \sigma^{q_1}(b)+\sigma^{q_2}(b) \cong 2\sigma^{q}(b) \tag{A13}$$

the cross sections σ^{A} and σ^{B} become equal.

$$\sigma^{A} = \sigma^{B} = 1/2 \int d^2Sd^2bD(\vec{S})\sigma^{ND}(+)\sigma^{ND}(-). \tag{A14}$$

The process DE participates in double scattering interactions with the cross section which is obtained by subtracting $(\sigma^{A}+\sigma^{B})$ from σ^{II}

$$\sigma^{DE} = \sigma^{II} - \int d^2Sd^2bD(\vec{S})\sigma^{ND}(+)\sigma^{ND}(-). \tag{A15}$$

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