HYPOTHESIS OF ZERO QUANTIZATION AND ITS CONSEQUENCES FOR FERMIONIC SPECIES

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(Received November 2, 1982)

The hypothesis is put forward that the world's three discrete spatial dimensions follow from some yet unknown quantum rule called the zero quantization. Then the next-quantization procedure (an analogue of the second quantization) is discussed, leading from the hypothetic zero-quantization level to the familiar first-quantization level. The Fermi-Dirac *or* Bose-Einstein version of this procedure is shown to originate the particle spin 1/2 *or* particle position and momentum, respectively. At the same time there appear additional "internal" degrees of freedom, both spin-1/2-like and orbital-like, the former implying two species of spin-1/2-particles, while the latter — their "internal" radial- and orbital-like excitations. The resulting group is SO(6,1) ⊃ SO(3,1) × SO(3), where SO(3,1) denotes the Lorentz group and SO(3) relates to these additional "internal" degrees of freedom, defining generators interpreted possibly as the weak isospin 1/2 plus its orbital-like analogue.

PACS numbers: 11.30.Ly, 12.90.+b

1. *Introduction: the zero quantization*

Living in the world of three spatial dimensions one can hardly realize that the fundamental empirical fact of discrete dimensionality of the physical space may be a consequence of quantum laws of nature. The main obstance to this realization is the lack of one's experience with a "classical world" having a continuum of spatial dimensions and approximating the real "quantum world" in some "classical limit". In spite of these unfavorable circumstances we dare make in this note the conjecture that the existence of world's three discrete spatial dimensions can be explained by some yet unknown quantum rule which we will call the *zero quantization*[1]. At the level of the zero quantization, as always

[1] An alternative version of the zero-quantization hypothesis may be the more *modest* conjecture that some yet unknown quantum rule establishes a correspondence between real "quantum world" of three spatial dimensions and an approximate "classical world" having *also* three spatial dimensions. Both versions of the hypothesis may lead effectively to the same theory at the zero-quantization level. Note, however, that in contrast to the second version, the first version may actually imply an infinite number of world's discrete spatial dimensions, all of which but three should be suppressed at "low energies".

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in the quantum theory, time t defining the fourth dimension of the relativistic space-time will be treated as a parameter [1][2].

Although not being based on a concrete physical model yet, the hypothesis of the zero quantization enables us to treat the index $k = 1, 2, 3$ numerating the three spatial dimensions as a quantum number (of the zero-quantization level). Then the *state space* of the zero-quantization level is defined as the Hilbert space spanned on the basis of three orthonormal kets $|k\rangle$ ($k = 1, 2, 3$),

$$\sum_k |k\rangle\langle k| = I, \quad \langle k|l\rangle = \delta_{kl}. \tag{1}$$

In consequence, the *wave function* in the corresponding representation [2] is

$$\Psi(k, t) = \langle k|\Psi(t)\rangle, \tag{2}$$

where $\Psi(t)$ denotes the abstract *state vector* of the zero-quantization level,

$$\Psi(t) = \sum_k |k\rangle\Psi(k, t), \quad \langle\Psi(t)|\Psi(t)\rangle = \sum_k |\Psi(k, t)|^2 = 1. \tag{3}$$

The wave function (2) can be interpreted as the probability amplitude of finding the dimension $k = 1, 2, 3$ in the physical space. An obvious deficiency of such a statement is that the phrase "finding the dimension $k = 1, 2, 3$ in the physical space" may gain its precise meaning only on the ground of a concrete physical model for the zero quantization, which should establish an observable related to the measurement of the dimension. Note that the state space of the zero-quantization level, being a three-dimensional Hilbert space, is equivalent to a six-dimensional real Euclidean space.

The hypothesis of the zero quantization becomes effective if we want to discuss the transition from the zero-quantization level of the theory to its first-quantization level where we meet a more familiar physical content. In this note we elaborate to some extent this transition, making use of the analogy with the well-known transition from the first- to the second-quantization level of the theory:

quantization level	physical object	quantized coordinates	wave function
zero	?	k	$\Psi(k, t)$
↓			
first	particle	x_k, σ_k	$\Psi(\vec{x}, \vec{\sigma}, t)$
↓			
second	field	$\psi(\vec{x}, \vec{\sigma})$	$\Psi(\psi, t)$

[2] This assumption on time is essential for the algebraic structure of the theory, which would become *richer* if *all* four discrete space-time dimensions could be explained by the zero quantization. Such a form of the hypothesis of the zero quantization was in fact suggested originally (W. Królikowski, *Acta Phys. Pol.* **B13**, 783 (1982)), but it seems to be not free of difficulties connected with its ignoring the usual parametric role of time in the quantum theory. The present form of the zero quantization makes an essential difference between three spatial dimensions explained by a quantum rule, and one time dimension related simply to time t being a parameter.

Here, σ_k and $\psi(\vec{x}, \vec{\sigma})$ (or $\psi(\vec{x})$ for short) denote spin 1/2 and a spinor field, respectively (the wave function $\Psi(\vec{x}, \vec{\sigma}, t)$ will be denoted for short by $\Psi(\vec{x}, t)$). An analogical (but somewhat simpler) scheme can illustrate the situation for a spinless particle corresponding to a scalar field $\phi(\vec{x})$. In these schemes $\Psi(\vec{x}) \to \psi(\vec{x})$ *or* $\Psi(\vec{x}) \to \phi(x)$ after the second quantization is carried out, obeying Fermi-Dirac *or* Bose-Einstein statistics, respectively. Similarly, from $\Psi(k)$ we shall get σ_k *or* x_k after the first quantization is performed. However, in addition to the familiar coordinates σ_k and x_k we shall obtain some new coordinates τ_k and y_k which, beside the spin σ_k, will play the role of "internal" particle coordinates.

2. *The first quantization*

When passing to the level of the first quantization, the wave function $\Psi(k)$ of the zero-quantization level becomes a *quantized "field"* defined on the discrete space of three spatial dimensions $k = 1, 2, 3$. In the case of Fermi-Dirac statistics we get $\Psi(k) \to \psi(k)$, where

$$\begin{cases} \{\psi(k), \psi^\dagger(l)\} = \delta_{kl}, \\ \text{others anticommuting,} \end{cases} \tag{4}$$

and in the case of Bose-Einstein statistics we obtain $\Psi(k) \to \phi(k)$, where

$$\begin{cases} [\phi(k), \Pi(l)] = i\delta_{kl}, \\ \text{others commuting,} \end{cases} \tag{5}$$

$\Pi(k)$ being a "field" canonically conjugate with $\phi(k)$. Here. two cases can be considered, when $\phi(k)$ is Hermitian or non-Hermitian. In the second case $[\phi^\dagger(k), \Pi(l)] = 0$.

In the case of the Fermi-Dirac quantization procedure (4) there appear the Hermitian operators

$$\alpha_k = \psi(k) + \psi^\dagger(k), \qquad \delta_k = \frac{1}{i}[\psi(k) - \psi^\dagger(k)] \tag{6}$$

satisfying the anticommutation relations

$$\{\alpha_k, \alpha_l\} = 2\delta_{kl}, \qquad \{\alpha_k, \delta_l\} = 0, \qquad \{\delta_k, \delta_l\} = 2\delta_{kl} \tag{7}$$

of a six-dimensional Clifford algebra. They can be represented minimally by 8×8 Hermitian matrices, e.g.

$$\begin{aligned} \alpha_k &= \sigma_k^{\mathrm{P}} \times \sigma_1^{\mathrm{P}} \times \mathbf{1}^{\mathrm{P}} = \varrho_1 \sigma_k, \\ \delta_k &= \mathbf{1}^{\mathrm{P}} \times \sigma_2^{\mathrm{P}} \times \sigma_k^{\mathrm{P}} = \varrho_2 \tau_k, \end{aligned} \tag{8}$$

where σ_k^P and $\mathbf{1}^P$ are 2×2 Pauli matrices and

$$\begin{aligned} \sigma_k &= \sigma_k^{\mathrm{P}} \times \mathbf{1}^{\mathrm{P}} \times \mathbf{1}^{\mathrm{P}}, \\ \varrho_m &= \mathbf{1}^{\mathrm{P}} \times \sigma_m^{\mathrm{P}} \times \mathbf{1}^{\mathrm{P}}, \\ \tau_k &= \mathbf{1}^{\mathrm{P}} \times \mathbf{1}^{\mathrm{P}} \times \sigma_k^{\mathrm{P}} \end{aligned} \tag{9}$$

are three commuting sets of spin-1/2-like matrices. The anticommutation relations (7) are covariant under the group SO(6) = SU(4) ⊃ SO(3) × SO(3) generated by

$$\sigma_k, \quad \varrho_3\sigma_k\tau_l, \quad \tau_l, \tag{10}$$

where ϱ_3 could be put ± 1 if only the group SO(6) were considered. It is important to notice that matrices introduced by relations (7) define *also* two noncommuting Lorentz groups SO(3, 1) generated by

$$\sigma_k, \quad i\varrho_1\sigma_k \tag{11}$$

and

$$\tau_k, \quad i\varrho_2\tau_k, \tag{12}$$

respectively. The group SO(6) as well as both groups SO(3, 1) are contained in the group SO(6,1) generated by

$$\sigma_k, \quad i\varrho_1\sigma_k, \quad \varrho_3\sigma_k\tau_l, \quad \tau_l, \quad i\varrho_2\tau_l \tag{13}$$

which, therefore, is *also* defined in terms of the matrices introduced by relations (7). Note that in terms of these matrices we can extend the anticommutation relations (7) to the form

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, \quad \{\gamma_\mu, \eta_l\} = 0, \quad \{\eta_k, \eta_l\} = 2g_{kl}, \tag{14}$$

where

$$\mu, \nu = 0, 1, 2, 3, \quad k, l = 1, 2, 3, \quad g_{00} = 1, \quad g_{0l} = 0, \quad g_{kl} = -\delta_{kl}$$

and

$$\begin{aligned} \gamma_0 &= \beta = \varrho_3, \\ \gamma_k &= -\varrho_3\alpha_k = -i\varrho_2\sigma_k, \\ \eta_k &= -\varrho_3\delta_k = i\varrho_1\tau_k. \end{aligned} \tag{15}$$

We will use also $\gamma^0 = \gamma_0$, $\gamma^k = -\gamma_k$ and $\eta^k = -\eta_k$. Notice that $\gamma_0 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \varrho_1$ and $\eta^1\eta^2\eta^3 = -\varrho_1$ (so that $\gamma^0\gamma^1\gamma^2\gamma^3\eta^1\eta^2\eta^3 = i$). The anticommutation relations (14) are covariant under the group SO(6,1) containing, as some its subgroups, the group SO(6) with generators (10) and two Lorentz groups SO(3,1) with generators (11) and (12). However, the present notation chooses the first of these two SO(3, 1) as an explicit Lorentz group: SO(6, 1) ⊃ SO(3, 1) × SO(3), where the group SO(3) is generated by τ_l.[3]

Since we can identify matrices σ_k and ϱ_m (and consequently also γ^μ) with the familiar Dirac matrices [3], we conclude that the *particle spin* $\frac{1}{2}\vec{\sigma}$ and *particle velocity* $\vec{\alpha} = \varrho_1\vec{\sigma}$,

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[3] The most general group containing SO(6,1), that can be defined in terms of the matrices introduced by relations (7), is the conformal extension of SO(6,1) given by SU(4,4) ⊃ SO(4,2) × SO(3), where SO(4.2) = SU(2,2) ⊃ SO(3,1) is the conformal group generated by σ_k, $i\varrho_1\sigma_k$, $i\varrho_2\sigma_k$, $\varrho_3\sigma_k$, $i\varrho_1$, $i\varrho_2$, ϱ_3.

which generate the familiar Lorentz group SO(3, 1), *follow* from the Fermi-Dirac quantization procedure applied to the zero quantization wave function. Then, however, there *appear* necessarily *additional* "internal" spin-1/2-like degrees of freedom described by the operator $\frac{1}{2}\vec{\tau}$ which we will call the *particle pseudospin* 1/2. Thus we get here two species of spin-1/2 particles corresponding to two eigenstates of $\frac{1}{2}\tau_3$. Obviously, it is tempting to interpret the pseudospin $\frac{1}{2}\vec{\tau}$ as the weak isospin 1/2, but *extended* to the whole Dirac bispinor containing both chiral components. Then $\frac{1}{2}\vec{\tau}$ generates the diagonal sum of the familiar chiral groups $SU_L(2)$ and $SU_R(2)$: $SO(3) = SU(2) = SU_L(2)+SU_R(2)$.

In the case of the Bose-Einstein quantization procedure (5) we can form the following Hermitian operators:

$$x_k = \lambda\phi(k), \quad p_k = \frac{1}{\lambda}\Pi(k) \tag{16a}$$

if $\phi(k) = \phi^\dagger(k)$, and

$$x_k = \frac{\lambda}{\sqrt{2}}[\phi(k)+\phi^\dagger(k)], \quad p_k = \frac{1}{\lambda\sqrt{2}}[\Pi(k)+\Pi^\dagger(k)],$$

$$y_k = \frac{\lambda}{i\sqrt{2}}[\phi(k)-\phi^\dagger(k)], \quad q_k = \frac{i}{\lambda\sqrt{2}}[\Pi(k)-\Pi^\dagger(k)] \tag{16b}$$

if $\phi(k) \neq \phi^\dagger(k)$.[4] Here, λ is a real constant of length dimension ($\hbar$ and later also c are put equal to 1). Operators (16a) and (16b) satisfy the canonical commutation relations

$$\begin{cases} [x_k, p_l] = i\delta_{kl}, \\ \text{others commuting} \end{cases} \tag{17a}$$

and

$$\begin{cases} [x_k, p_l] = i\delta_{kl}, \quad [y_k, q_l] = i\delta_{kl}, \\ \text{others commuting,} \end{cases} \tag{17b}$$

respectively.[5] Note that writing

$$x_0 = t \quad p_0 = i\frac{\partial}{\partial t} \tag{18}$$

we can extend Eq. (17a) and (17b) to the form

$$\begin{cases} [x_\mu, p_\nu] = -ig_{\mu\nu}, \\ \text{others commuting} \end{cases} \tag{19a}$$

[4] Notice that $z_k \equiv x_k+iy_k = \lambda\sqrt{2}\phi(k)$ and $-i\frac{\partial}{\partial z_k} = \frac{1}{2}(p_k-iq_k) = \frac{1}{\lambda\sqrt{2}}\Pi(k)$.

[5] It should be noticed that if (like in the usual canonical theory) the hamiltonian of the first quantization level contains λ through the canonical variables only, this constant does not appear explicitly and so cannot be measured. To be measurable it must emerge independently as a constant in the interaction term or mass term (or in the kinetic term as a coefficient).

and

$$\begin{cases}[x_\mu, p_\nu] = -ig_{\mu\nu}, \quad [y_k, q_l] = -ig_{kl}, \\ \text{others commuting,}\end{cases} \tag{19b}$$

respectively. Hence

$$p_\mu = i\frac{\partial}{\partial x^\mu}, \quad q_k = i\frac{\partial}{\partial y^k}, \tag{20}$$

where $x^0 = x_0$, $x^k = -x_k$ and $y^k = -y_k$. Making use of relations (19b) we can easily construct the orbital-like generators of the group SO(6, 1), which added to its spin-like generators (13) give in the case of $\phi(k) \neq \phi^\dagger(k)$ its total generators. In particular, $\vec{y}\times\vec{q}+\frac{1}{2}\vec{\tau}$ are three total generators of the subgroup SO(3).

Since we can identify operators x_k and p_k with the familiar canonical variables, we conclude that the *particle position* $\vec{x}$ and *particle momentum* $\vec{p}$ *follow* from the Bose-Einstein quantization procedure applied to the zero-quantization wave function. But then, in the case of $\phi(k) \neq \phi^\dagger(k)$, there *appear* necessarily *additional* "internal" orbital-like degrees of freedom described by the operators $\vec{y}$ and $\vec{q}$ which we shall call the *particle pseudoposition* and *particle pseudomomentum*, respectively. If the interpretation of the pseudospin $\frac{1}{2}\vec{\tau}$ as the weak isospin 1/2 is correct, then the *pseudoorbital angular momentum* $\vec{y}\times\vec{q}$ can be interpreted as the orbital-like weak isospin, and the *total pseudoangular momentum* $\frac{1}{2}\vec{\tau}+\vec{y}\times\vec{q}$ — as the total weak isospin whose third component should be related to the electric charge.

3. Wave equation for spin-1/2 particle

In the case of $\phi(k) = \phi^\dagger(k)$ there is no Dirac-like wave equation covariant under the group SO(6, 1) because no orbital-like degrees of freedom in the pseudospace, $\vec{y}$ and $\vec{q}$, exist in this case (though the spin-1/2-like degrees of freedom $\frac{1}{2}\vec{\tau}$ exist). Then the only Dirac-like wave equation is just the Dirac equation which in the free case has the form

$$i\frac{\partial}{\partial t}\Psi(\vec{x}, t) = (\vec{\alpha}\cdot\vec{p}+\beta m_0)\Psi(\vec{x}, t) \tag{21}$$

or

$$(\gamma^\mu p_\mu - m_0)\Psi(x) = 0 \tag{22}$$

and is covariant under $SO(3, 1)\times SO(3)$ subgroup of SO(6, 1). Here, due to Eqs. (15) and (9)

$$\gamma^\mu = \gamma^{\mu D}\times\mathbf{1}^P = \begin{pmatrix}\gamma^{\mu D}, & 0 \\ 0, & \gamma^{\mu D}\end{pmatrix}, \tag{23}$$

where $(\gamma^{\mu D}) = (\beta^D, \beta^D\vec{\alpha}^D) = (1^P \times \sigma_3^P, i\vec{\sigma}^P \times \sigma_2^P)$ are 4×4 Dirac matrices. Thus

$$\Psi(x) = \begin{pmatrix} \Psi^D_{up}(x) \\ \Psi^D_{down}(x) \end{pmatrix}, \tag{24}$$

where $\Psi^D_{up}(x)$ and $\Psi^D_{down}(x)$ are 4-component Dirac bispinors satisfying two free Dirac equations and being eigenstates of $\frac{1}{2}\tau_3$ with eigenvalues $\pm\frac{1}{2}$, respectively. It is so since due to Eq. (9)

$$\vec{\tau} = 1^D \times \vec{\sigma}^P, \tag{25}$$

where I^D denotes 4×4 Dirac unit matrix. We can see that two species of spin-1/2 particles, *up* and *down*, are here explicit and decoupled from each other as far as the free Dirac equation (22) is considered.

In the case of $\phi(k) \neq \phi^+(k)$ there exists the Dirac-like wave equation covariant under the group SO(6, 1). In the free case it has the form

$$i\frac{\partial}{\partial t}\Psi(\vec{x}, \vec{y}, t) = (\vec{\alpha}\cdot\vec{p} + \vec{\delta}\cdot\vec{q} + \beta m_0)\Psi(\vec{x}, \vec{y}, t) \tag{26}$$

or

$$(\gamma^\mu p_\mu + \eta^k q_k - m_0)\Psi(x, \vec{y}) = 0. \tag{27}$$

Due to the anticommutation relations (14) it leads to the Klein-Gordon-like equation

$$(p^2 - \vec{q}^2 - m_0^2)\Psi(x, \vec{y}) = 0, \tag{28}$$

where $p^2 = p^\mu p_\mu = p_0^2 - \vec{p}^2$. Equation (28) shows that also in the case of $\phi(k) \neq \phi^\dagger(k)$ the *up* and *down* species of spin-1/2 particles are decoupled from each other, although they are apparently coupled in Eq. (27) through the term $\eta^k q_k = -i\gamma_s\tau_k q_k$ violating conservation of $\frac{1}{2}\vec{\tau}$ (but conserving $\vec{y}\times\vec{q} + \frac{1}{2}\vec{\tau}$). This term has also another apparent effect in Eq. (27), absent from Eq. (28), namely it violates conservation of the parity defined by reflections in the familiar space, $\vec{x} \to -\vec{x}$ (through conservation of the combined parity defined by reflections in the space and pseudospace $\vec{x} \to -\vec{x}$, $\vec{y} \to -\vec{y}$ is not violated). Both these effects of nonconservation become real if appropriate interactions are introduced into Eq. (27). Note that non-zero interactions leaving the spin-1/2 particle free in the familiar space break necessarily SO(6) subsymmetry of SO(6, 1), though SO(3,1) × SO(3) subsymmetry may be preserved.

Interactions can be introduced into Eq. (27) by some substitutions. In order to give an example substitute in Eq. (27)

$$m_0 \to m_0 + S \tag{29}$$

where $S = S(\vec{y})$ is a scalar under the familiar Lorentz group SO(3, 1) (and may be a scalar also under SO(3) if $S = S(|\vec{y}|)$). Then the wave equation becomes

$$(\gamma^\mu p_\mu + \eta^k q_k - m_0 - S)\Psi(x, \vec{y}) = 0. \tag{30}$$

Equation (30) leads to the Klein-Gordon-like equation

$$(p^2 - M^2)\Phi(x, \vec{y}) = 0 \tag{31}$$

with the mass operator squared

$$M^2 = -\left(\frac{\partial}{\partial \vec{y}}\right)^2 + \gamma_5 \vec{\tau} \cdot \frac{\partial S}{\partial \vec{y}} + (m_0 + S)^2 \tag{32}$$

which mixes up and down components of $\Psi(x, \vec{y})$ and apparently violates the parity. The insertion $\Psi(x, \vec{y}) = \Psi(x)\chi(\vec{y})$ separates Eq. (31) into the pair of equations,

$$(p^2 - m^2)\Psi(x) = 0 \tag{33}$$

and

$$M^2\chi(\vec{y}) = m^2\chi(\vec{y}), \tag{34}$$

where m^2 is an eigenvalue of M^2. Denoting $r = |\vec{y}|$, $\hat{r} = \vec{y}/r$ and $\tilde{\vec{L}} = \vec{y} \times \vec{q}$ we get in the case of $S = S(r)$:

$$M^2 = -\frac{1}{r}\frac{\partial^2}{\partial r^2} r + \frac{\tilde{\vec{L}}^2 + \gamma_5 \vec{\tau} \cdot \hat{r}\alpha(r)}{r^2} + (m_0 + S)^2, \tag{35}$$

where

$$\alpha(r) = r^2 \frac{dS}{dr}. \tag{36}$$

In Eq. (35) we can use the identity

$$\tilde{\vec{L}}^2 + \gamma_5 \vec{\tau} \cdot \hat{r}\alpha(r) = \Lambda(\Lambda + 1) - \alpha^2(r), \tag{37}$$

where

$$-\Lambda = 1 + \vec{\tau} \cdot \tilde{\vec{L}} - \gamma_5 \vec{\tau} \cdot \hat{r}\alpha(r) \tag{38}$$

and

$$\Lambda^2 = (1 + \vec{\tau} \cdot \tilde{\vec{L}})^2 + \alpha^2(r). \tag{39}$$

Denoting $\tilde{\vec{J}} = \tilde{\vec{L}} + \frac{1}{2}\vec{\tau}$ and $\varepsilon = \pm 1$ we get

$$(1 + \vec{\tau} \cdot \tilde{\vec{L}}) \left| \tilde{j} = \tilde{l} + \frac{\varepsilon}{2} \right\rangle = \varepsilon(\tilde{j} + \tfrac{1}{2}) \left| \tilde{j} = \tilde{l} + \frac{\varepsilon}{2} \right\rangle \tag{40}$$

and $\left(\text{for } \left| \tilde{j} = \tilde{l} + \frac{\varepsilon}{2} \right\rangle \text{ being also eigenstates of } \Lambda\right)$

$$-\Lambda \left| \tilde{j} = \tilde{l} + \frac{\varepsilon}{2} \right\rangle = \varepsilon\lambda(r) \left| \tilde{j} = \tilde{l} + \frac{\varepsilon}{2} \right\rangle, \tag{41}$$

where

$$\lambda(r) = \sqrt{\tilde{j}(\tilde{j}+\tfrac{1}{2})+\alpha^2(r)}. \tag{42}$$

In the special case of $S = -\alpha/r$ we have $\alpha(r) = \text{const} \equiv \alpha$ and $\lambda(r) = \text{const} \equiv \lambda$. Then the operators M^2 and Λ commute and so we obtain from Eq. (34) the radial equation

$$\left[-\frac{d^2}{dr^2} + \frac{\lambda(\lambda-\varepsilon)-\alpha^2}{r^2} + \left(m_0 - \frac{\alpha}{r}\right)^2\right] r\chi(r) = m^2 r\chi(r) \tag{43}$$

whose discrete spectrum is

$$m = m_0\left[1-\left(\frac{\alpha}{\tilde{n}-\tilde{j}-\frac{1}{2}+\sqrt{(\tilde{j}+\frac{1}{2})^2+\alpha^2}}\right)^2\right]^{1/2}$$

$$= m_0\left[1-\frac{\alpha^2}{2\tilde{n}^2}+\frac{\alpha^4}{2\tilde{n}^4}\left(\frac{\tilde{n}}{\tilde{j}+\frac{1}{2}}-\tfrac{1}{4}\right)+O(\alpha^6)\right] \tag{44}$$

with $\tilde{n} = \tilde{n}_r+\frac{1}{2}(1-\varepsilon)+\tilde{j}+\frac{1}{2} = 1, 2, 3, \ldots$ (where $n_r = 0, 1, 2, \ldots$) and $\tilde{j} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$. Spectrum $m = m_{\tilde{n}\tilde{j}}$ given in Eq. (44) is degenerate with respect to $\tilde{m}_j = \tilde{j}, \ldots, -\tilde{j}$ (and also to $\varepsilon = \pm 1$ for $\tilde{j}+\frac{1}{2} \neq \tilde{n}$) and involves pseudoradial and pseudoorbital excitations, the latter displaying a fine structure of split pseudospin doublets with $\tilde{j} = \tilde{l}+\dfrac{\varepsilon}{2}$ $\Big($for $S(r) \neq -\alpha/r$ also the doublets $\varepsilon = \pm 1$ with $\tilde{l} = \tilde{j}-\dfrac{\varepsilon}{2}$ get split$\Big)$. Obviously, $\tilde{m}_j = \tilde{m}_l + \tilde{m}_\tau$ where $\tilde{m}_\tau = \pm\frac{1}{2}$ are eigenvalues of $\frac{1}{2}\tau_3$, so that $\tilde{m}_j = \pm\frac{1}{2}$ for the ground state $\tilde{n} = 1$, $\tilde{j} = \frac{1}{2}$ and for its pseudoradial excitations $\tilde{n} = 2, 3, \ldots, \tilde{l} = 0, \tilde{j} = \frac{1}{2}$.

Of course, we considered $S = -\alpha/r$ only for an illustration of a general pattern, where a spin-1/2 particle appearing in two species corresponding to two eigenstates of $\frac{1}{2}\tau_3$ gets pseudoradial and pseudoorbital excitations. If the weak-isospin-1/2 interpretation of the pseudospin $\frac{1}{2}\vec{\tau}$ is correct, these two species are up and down fermionic flavours, and then the pseudoradial excitations should be responsible for fermionic generations displaying in this case some structure due to pseudoorbital excitations. The latter should imply charge excitations when $|\tilde{m}_j| > \frac{1}{2}$. In this pattern, the familiar leptons and quarks would fit to separate weak-isospin doublets $\tilde{m}_j = \pm\frac{1}{2}$ with $\tilde{n}_r = 0, 1, 2, \tilde{l} = 0, \tilde{j} = \frac{1}{2}$ or, *alternatively*, with $\tilde{n}_r = 0, \tilde{l} = 0, 1, \tilde{j} = \frac{1}{2}$ and $\tilde{n}_r = 1, \tilde{l} = 0, \tilde{j} = \frac{1}{2}$, the second possibility suggesting the existence of the fourth fermionic generation consisting of weak-isospin quartets $\tilde{m}_j = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$ with $\tilde{n}_r = 0, \tilde{l} = 1, \tilde{j} = \frac{3}{2}$ (evidently, one member in each of these quartets should be charge excited, having charge ± 2 or $\pm\frac{5}{3}$ in the case of leptons and quarks, respectively). At this point we should stress that in our pattern leptons and quarks are *not* composite objects in the sense of preon models where radial and orbital excitations are related almost necessarily to a very high energy scale.

To give another example of interactions, substitute in Eq. (27)

$$\begin{cases} p_\mu \to p_\mu - \frac{1}{2} g\tau_k W_{\mu k} - \frac{1}{2} g'(Y_R + Y_L)B_\mu, \\ q_k \to q_k - \frac{1}{2} gi\gamma^\mu W_{\mu k}, \\ m_0 \to m_0 - \frac{1}{2} g'(Y_R - Y_L)\gamma_5\gamma^\mu B_\mu, \end{cases} \tag{45}$$

where $W_{\mu k}(x)$ and $B_\mu(x)$ are four-vectors under SO(3, 1) and a vector and a scalar under SO(3), respectively, whereas g, g', Y_R and Y_L denote real constants. Then the wave equation becomes

$$\{\gamma^\mu p_\mu + \eta^k q_k - m_0 - g\gamma^\mu \tfrac{1}{2}(1-\gamma_5)\tau_k W_{\mu k} - g'\gamma^\mu[\tfrac{1}{2}(1-\gamma_5)Y_L + \tfrac{1}{2}(1+\gamma_5)Y_R]B_\mu\}\Psi(x, \vec{y}) = 0, \tag{46}$$

displaying the Weinberg-Salam-type coupling of a spin-1/2 particle to the fields of pseudospin-1 and pseudospin-0 vector bosons $W_{\mu k}$ and B_μ. We can see that here only the left-handed component of the spin-1/2 particle can interact with $W_{\mu k}$. The right-handed component is, however, not decoupled from its left-handed partner because of the terms m_0 and $\eta^k q_k = -i\gamma_0\tau_k q_k$ in Eq. (46). Finally, we should mention that the substitution (45) is not gauging the global-symmetry group SO(3, 1) × SO(3) in Eq. (46).

4. *Oscillatory interpretation of the first quantization*

Needless to say that in order to understand better the hypothesis of zero quantization one should find a physically acceptable model of this quantization. Such a program is a challenge to one's imagination to recognize the physical objects which have to be quantized at the zero-quantization level. At present we can only say that (after the zero quantization is carried out) such an object has as its coordinate spatial dimension $k = 1, 2, 3$ (i.e. one of three basic spatial directions). We also know that there are two types of such objects, corresponding to the Fermi-Dirac and Bose-Einstein statistics. At the first-quantization level they can be described as *quanta* of the Fermi-Dirac and Bose-Einstein quantized "fields" $\psi(k, t)$ and $\phi(k, t)$ (the latter "field" appearing jointly with its canonical conjugate $\Pi(k, t)$), and may be conveniently called *fers* and *bos* (a *fer* and a *bo* in singular). The "fields" $\psi(k, t)$ and $\phi(k, t)$, collaborating mutually, construct physical systems which we recognize experimentally as elementary fermions (leptons and quarks).

In particular, in the case of $\phi(k, t) \neq \phi^\dagger(k, t)$ the fermion velocity, fermion position and fermion momentum are given by the following "field" superpositions:

$$v_k(t) = c\alpha_k(t) = c[\psi(k, t) + \psi^\dagger(k, t)],$$

$$x_k(t) = \frac{\lambda}{\sqrt{2}}[\phi(k, t) + \phi^\dagger(k, t)],$$

$$p_k(t) = \frac{\hbar}{\lambda\sqrt{2}}[\Pi(k, t) + \Pi^\dagger(k, t)], \tag{47}$$

implying the existence of the linearly independent superpositions that define the additional "internal" degrees of freedom:

$$c\delta_k(t) = \frac{c}{i}\,[\psi(k,t)-\psi^\dagger(k,t)],$$

$$y_k(t) = \frac{\lambda}{i\sqrt{2}}\,[\phi(k,t)-\phi^\dagger(k,t)],$$

$$q_k(t) = \frac{i\hbar}{\lambda\sqrt{2}}\,[\Pi(k,t)-\Pi^\dagger(k,t)]. \tag{48}$$

Note that in Eqs. (47) and (48) there appear three fundamental constants c, λ and $\hbar$. Although one can expect that the new constant of length dimension λ should play an important role in quantum dynamics of fers and bos, it does not appear explicitly in the free Dirac-like wave equation (26), being hidden in $\vec{x}$, $\vec{p} = -i\hbar\partial/\partial\vec{x}$ and $\vec{y}$, $\vec{q} = -i\hbar\partial/\partial\vec{y}$, and so it cannot be determined from this equation unless it appears in m_0. It may be a signal that a development of this wave equation should be expected.

The hamiltonian in the free Dirac-like wave equation (26) can be easily expressed by the Fermi-Dirac and Bose-Einstein quantized "fields" $\psi(k)$ and $\Pi(k)$ (cf. Eqs. (4) and (5)). Namely, making use of Eqs. (6) and (16b) (the latter with explicit $\hbar$ in p_k and q_k) and of the formula

$$\beta = i\alpha_1\alpha_2\alpha_3\delta_1\delta_2\delta_3 = [\psi^\dagger(1),\psi(1)]\,[\psi^\dagger(2),\psi(2)]\,[\psi^\dagger(3),\psi(3)], \tag{49}$$

we obtain

$$H = c(\vec{\alpha}\cdot\vec{p}+\vec{\delta}\cdot\vec{q})+\beta m_0c^2$$

$$= m_0c^2\beta+\frac{\hbar c}{\lambda}\sqrt{2}\sum_k [\psi(k)\Pi(k)+\psi^\dagger(k)\Pi^\dagger(k)], \tag{50}$$

where

$$m_0c^2\beta = m_0c^2\Bigg[-1+2\sum_k \psi^\dagger(k)\psi(k)-\frac{4}{2!}\sum_{\substack{k,l\\ \text{different}}}\psi^\dagger(k)\psi(k)\psi^\dagger(l)\psi(l)$$

$$+\frac{8}{3!}\sum_{\substack{k,l,m\\ \text{different}}}\psi^\dagger(k)\psi(k)\psi^\dagger(l)\psi(l)\psi^\dagger(m)\psi(m)\Bigg] = -m_0c^2\colon\exp\Bigg[-2\sum_k\psi^\dagger(k)\psi(k)\Bigg]\colon \tag{51}$$

Here, :( ): denotes the normal ordering of all $\psi^\dagger(k)$ and $\psi(k)$. In Eqs. (50) and (51) we can write

$$\psi(k) = \tfrac{1}{2}(\alpha_k+i\delta_k) = a_k \tag{52}$$

and

$$\phi(k) = \frac{1}{\lambda\sqrt{2}}(x_k + iy_k) = \frac{1}{\sqrt{2}}\left(\frac{c_k + c_k^\dagger}{\sqrt{2}} + i\,\frac{d_k + d_k^\dagger}{\sqrt{2}}\right),$$

$$\Pi(k) = \frac{\lambda}{\hbar\sqrt{2}}(p_k - iq_k) = \frac{1}{\sqrt{2}}\left(\frac{c_k - c_k^\dagger}{i\sqrt{2}} - i\,\frac{d_k - d_k^\dagger}{i\sqrt{2}}\right) = -i\,\frac{\partial}{\partial\phi(k)} \tag{53}$$

with a_k and c_k or d_k being respectively Fermi-Dirac and Bose-Einstein annihilation operators of the first quantization level. They annihilate fers and bos in the zero-quantization states $|k\rangle$ ($k = 1, 2, 3$). Of course, $a_k^\dagger$ and $c_k^\dagger$ or $d_k^\dagger$ are the respective creation operators. In the case of $\phi(k) \neq \phi(k)$ there are two kinds of bos since then $\phi^\dagger(k)$ (with $\Pi(k) \neq \phi^\dagger(k)$) describes two idependent Hermitian "fields" x_k and y_k.

We conclude from Eqs. (50) and (51) that the mass term $\beta m_0 c^2$ in the free Dirac-like hamiltonian (50), depending only on $\psi^\dagger(k)\psi(k)$, can be interpreted as energy of fers, and the kinetic term $c(\vec{\alpha}\cdot\vec{p} + \vec{\delta}\cdot\vec{q})$ which couples $\psi(k)$ with $\Pi(k)$ — as the interaction energy of fers with bos. So we come to an oscillatory interpretation of the first-quantization hamiltonian for an elementary fermion[6]. Note, however, that no term which could be interpreted as energy of bos is present in the hamiltonian (50). One may wonder as to whether such a term should not be introduced into the first-quantization hamiltonian *if* the existence of fers and bos as physical objects is taken seriously. For instance, introducing tentatively the term $(\hbar c/\lambda)\sum_k \Pi^\dagger(k)\Pi(k)$ (containing the same energy-dimensional coefficient $\hbar c/\lambda$ as the other $\Pi(k)$-dependent term in Eq. (50)) one gets the following free hamiltonian of an elementary fermion:

$$H = m_0 c^2 \beta + \frac{\hbar c}{\lambda}\sum_k \{\Pi^\dagger(k)\Pi(k) + \sqrt{2}\,[\psi(k)\Pi(k) + \psi^\dagger(k)\Pi^\dagger(k)]\}$$

$$= c(\vec{\alpha}\cdot\vec{p} + \vec{\delta}\cdot\vec{q}) + \beta m_0 c^2 + \frac{\lambda c}{\hbar}\,\tfrac{1}{2}(\vec{p}^{\,2} + \vec{q}^{\,2}). \tag{54}$$

Here, the constant λ appears explicitly and so can be measured.

However, it is easy to see that the term $O(\lambda)$ in Eq. (54) spoils the special-relativity covariance of the Dirac-like hamiltonian (50), so that SO(3, 1) symmetry is here violated (and, therefore, also SO(6, 1)) though SO(6) is still preserved. The origin of this effect can be related to the lack of the space-time unification at the fundamental level of the zero quantization (cf. footnote 2). Such a unification may *or* may not be established at

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[6] It is interesting to observe that if there were no additional "internal" degrees of freedom described by δ_k, the Fermi-Dirac annihilation operators a_k could not be constructed and then there would be no oscillatory interpretation of the first-quantization hamiltonian for a spin-1/2 particle. So, the requirement of such an interpretation (suggested by the analogy with the second quantization) implies necessarily the existence of the additional degrees of freedom δ_k (while the existence of y_k and q_k is not necessary since the case of $\phi(k) = \phi^\dagger(k)$ is not excluded). I am indebted to Stefan Pokorski for this remark.

the first-quantization level, where (beside the time parameter) three spatial continuous coordinates appear. Our example shows that a tiny *violation* of the special-relativity covariance may be natural at the first-quantization level *if* fers and bos represent a *physical reality* (possessing energy). At any rate, such an effect, if it exists, should be really tiny since λ is expected naturally as a very small length scale. E.g. the mass-dimensional constant $\hbar/\lambda c$ may be the Planck mass $M_{\mathrm{PL}} = \sqrt{\hbar c/G} \sim 10^{19}\ \mathrm{GeV}/c^2$, suggesting the gravitational affiliation of this effect.[7]

The energy spectrum evaluated from the tentatively considered free hamiltonian (54) is

$$E^2 = E_0^2 + \frac{\lambda}{\hbar c} E_0(E_0^2 - m_0^2c^4) + \left(\frac{\lambda}{\hbar c}\right)^2 \tfrac{1}{4}(E_0^2 - m_0^2c^4)^2, \tag{55}$$

where

$$E_0^2 = c\vec{p}^{\,2} + M^2c^4, \qquad M^2c^4 = c^2\vec{q}^{\,2} + m_0^2c^4. \tag{56}$$

Hence

$$E = E_0 + \frac{\lambda}{\hbar c}\,\tfrac{1}{2}(E_0^2 - m_0^2c^4) + O(\lambda^2). \tag{57}$$

Thus for $(Mc)^2 \ll \vec{p}^{\,2} \ll (\hbar/\lambda)^2$

$$E \simeq c|\vec{p}| + \tfrac{1}{2}\frac{\lambda c}{\hbar}\vec{p}^{\,2} \tag{58}$$

and for $\vec{p}^{\,2} \ll (Mc)^2 \ll (\hbar/\lambda)^2$

$$E \simeq \left(M + \frac{\lambda c}{\hbar}\,\frac{M^2 - m_0^2}{2}\right)c^2 + \tfrac{1}{2}\left(\frac{1}{M} + \frac{\lambda c}{\hbar}\right)\vec{p}^{\,2}. \tag{59}$$

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[7] For such λ hamiltonian (54) takes the form

$$H = m_0c^2\beta + M_{\mathrm{PL}}c^2 \sum_k \{\Pi^\dagger(k)\Pi(k) + \sqrt{2}[\psi(k)\Pi(k) + \psi^\dagger(ck)\Pi^\dagger(k)]\}$$

$$= c(\vec{\alpha}\cdot p + \vec{\delta}\cdot\vec{q}) + \beta m_0c^2 + \frac{1}{2M_{\mathrm{PL}}}(\vec{p}^{\,2} + \vec{q}^{\,2}).$$

This free one-body hamiltonian is *formally equivalent* to a free two-body hamiltonian of one Dirac-like particle of mass m_0 and one Schrödinger-like particle of mass M_{PL}, considered in their centre-of-mass frame where $\vec{p}_1 = -\vec{p}_2 \equiv \vec{p}$ and $\vec{q}_1 = -\vec{q}_2 \equiv \vec{q}$ (with $\vec{p} = -i\hbar\partial/\partial\vec{x}$, $\vec{x} \equiv \vec{x}_1 - \vec{x}_2$ and $\vec{q} = -i\hbar\partial/\partial\vec{y}$, $\vec{y} \equiv \vec{y}_1 - \vec{y}_2$). An external potential $V(\vec{x})$, if introduced into our one-body hamiltonian, is *formally equivalent* to the internal potential $V(\vec{x})$ acting between a Dirac-like particle of mass m_0 and a Schrödinger-like particle of mass M_{PL} in the two-body hamiltonian. The same can be said about a potential $V(\vec{y})$ in the pseudospace. So the term

$$M_{\mathrm{PL}}c^2 \sum_k \Pi^\dagger(k)\Pi(k) = \frac{1}{2M_{\mathrm{PL}}}(\vec{p}^{\,2} + \vec{q}^{\,2})$$

representing energy of bos in our one-body hamiltonian may be called the "vacuum recoil energy" associated with a Dirac-like particle (of mass m_0). The above formal analogy is, of course, true for any very small λ not necessarily equal to the Planck length $\hbar/M_{\mathrm{PL}}c \sim 10^{-33}$ cm. But for this Planck value of λ it is especially appealing.

We can see that in this case a tiny deviation $\sim \vec{p}^2$ from the special-relativistic behaviour $\sim |\vec{p}|$ should appear in the energy spectrum of a free elementary fermion at very high energies.

Finally, we should like to remark that (although important from the interpretative point of view) the oscillatory representation of the hamiltonian (50) or (54), being based on the annihilation and creation operators a_k, $a^\dagger$, c_k, $c_k^\dagger$ and d_k, $d_k^\dagger$, is of a meagre calculatory value. Instead, the "coherent" representation using the Dirac-like matrices α_k, δ_k and the continuous particle coordinates x_k, y_k and momenta p_k, q_k is adequate for all calculations at the first-quantization level thanks to the Newton's and Leibniz's discovery of differential and integral calculus. Similarly, the familiar oscillatory representation at the second--quantization level would be comparatively of little calculatory value if functional differential and integral calculus existed as a fully effective algorithm.

Similar conclusions to the above can be drawn in the case of $\phi(k) = \phi^\dagger(k)$ when d_k, $d_k^\dagger$ as well as y_k, q_k do not exist and when the starting point is simply the Dirac wave equation (21) instead of the Dirac-like wave equation (26).

5. *Conclusion and outlook*

In this paper we put forward the hypothesis of zero quantization and discussed the next quantization procedure leading from the yet unknown level of the zero quantization to the familiar level of the first quantization. We showed that the particle spin 1/2 and other Dirac degrees of freedom follow from the Fermi-Dirac quantization procedure applied to the wave function of the zero-quantization level. Similarly, the particle position and momentum follow from the Bose-Einstein quantization. Then, however, it turns out that for a spin-1/2 particle there appear some additional "internal" degrees of freedom both spin-1/2-like and orbital-like, the latter if the Bose-Einstein quantized "field" of the first--quantization level, $\phi(k)$ $(k = 1, 2, 3)$, is non-Hermitian. These additional spin-1/2-like degrees of freedom imply the existence of two species of spin-1/2 particles, interpreted possibly as two eigenstates of the weak isospin-1/2. On the other hand, the orbital-like degrees of freedom, if they exist, allow for radial- and orbital-like particle excitations interpreted possibly as giving fermionic generations. The emerging group is here SO(6, 1) $\supset$ SO(3, 1) $\times$ SO(3), where three generators of the second factor include generally both spin-1/2-like and orbital-like parts. The third of these generators should be related to the electric charge, if the weak-isospin-1/2 interpretation of their spin-1/2-like part is correct. If it is the case, our quantization procedure should provide a *geometrical* picture for the electric charge, implying the possibility of charge excitations.

To this end, however, the realistic electroweak interactions should be introduced into the free Dirac-like equation (27) valid in the case of the non-Hermitian $\phi(k)$. An ambitious version of this program would be to construct a realistic gauge theory in 6+1 dimensions based on SO(6, 1) group (or on some *extension* of it if we have not yet recognized properly *all* discrete coordinates appearing at the zero-quantization level). At any rate, to be consistent with the familiar phenomenology, the group SO(6, 1) $\supset$ SO(3, 1) $\times$ SO(3) should be multiplied by the Pati-Salam four-colour group $SO_C(6) = SU_C(4) \supset SU_C(3) \times U_{B-L}(1)$.

Then the electric charge

$$Q = (\tfrac{1}{2}\vec{\tau}+\vec{y}\times q)_3+\tfrac{1}{2}(\mathrm{B}-\mathrm{L}) \tag{60}$$

would link both groups. At "low energies" the weak-isospin group SO(3) = SU(2) = $SU_L(2)+SU_R(2)$ should lead effectively to the left-chiral group $SU_L(2)$ generated by $\frac{1}{2}(1-\gamma_5)(\frac{1}{2}\vec{\tau}+\vec{y}\times\vec{q})$ and, in collaboration with $U_{B-L}(1)$, to the group $U_Y(1) \subset SU_R(2) \times U_{B-L}(1)$ generated by

$$\tfrac{1}{2}Y = \tfrac{1}{2}(1+\gamma_5)(\tfrac{1}{2}\vec{\tau}+\vec{y}\times\vec{q})_3+\tfrac{1}{2}(\mathrm{B}-\mathrm{L}). \tag{61}$$

It would be the rearrangement process

$$SU(2)\times u_{B-L}(1) \to SU_L(2)\times U_Y(1) \tag{62}$$

justified if components 1 and 2 of the generator $\frac{1}{2}(1+\gamma_5)(\frac{1}{2}\vec{\tau}+\vec{y}\times\vec{q})$ of the right-chiral group $SU_R(2)$ disappear effectively.

In contrast, in the case of the Hermitian $\phi(k)$ there are no additional orbital-like degrees of freedom and, therefore, no radial- and no orbital-like (nor charge) particle excitations. In this case the problem of introducing the realistic interactions into the free Dirac equation (22) does not differ from that in the conventional theory.

Finally, we should like to emphasize that all first-quantization operators defining particle degrees of freedom involved in SO(6, 1) group can be constructed from Fermi-Dirac or Bose-Einstein annihilation and creation operators of the first-quantization level. The energy operator of the corresponding quanta (called fers and bos, respectively) can be identified with the free Dirac-like or Dirac hamiltonian of the first-quantization level plus possibly some tiny correction violating the special relativity. This correction may appear in a natural way if fers and bos are taken seriously as physical objects (possessing energy).

I want to thank Leszek Łukaszuk, Stefan Pokorski and Jacques Prentki for several helpful discussions. The major part of this paper has been completed during the author's happy stay at CERN (Geneva) in September 1982. The idea of a tiny violation of the special relativity in the world of fers and bos, which enables one to measure λ, occurred to the author in October 1982 in a discussion with Stefan Pokorski.

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