HADRON MASS CORRECTIONS TO THE DRELL-YAN PROCESS

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The total cross-section for the Drell-Yan process is calculated with the hadron mass corrections included. We show that these corrections are important for the collision energy less than 10 GeV and they become negligible for higher energies.

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Introduction and conclusions

The total cross-section $d\sigma/dQ^2$ in the Drell-Yan process, as derived from the leading log QCD calculations, is 2-3 times smaller than its experimental value. It was expected that the above mentioned discrepancy could be removed by taking mass of hadrons into considerations.

In this paper we present calculations for the total cross-section in Drell-Yan process with mass corrections.

The main conclusion of our work is that the mass corrections are important in the low energy region and are negligible for the center of mass energy $s > 100 \text{ GeV}^2$. This result is intuitively expected because, for $s > 100 \text{ GeV}^2$, hardon mass is small in comparison with the collision energy.

The method for including the hadron mass effects is discussed in Section 1. We recall the known results for the deep inelastic scattering and we then apply the same technique to the Drell-Yan process. Numerical results are presented in Section 2.

1. Deep inelastic scattering and Drell-Yan process

The total cross-section in deep inelastic scattering can be presented as a product of leptonic and hadronic parts

$$\sigma = \sigma^{\mu\nu}(q, p)L_{\mu\nu} \tag{1}$$

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where $L_{\mu\nu}$ is the amplitude for the emission of the virtual photon with the four momentum q, and $\sigma^{\mu\nu}(q, p)$ describes the photon-hadron scattering.

The cross-section $\sigma^{\mu\nu}(q, p)$ is expressed as follows (Fig. 1):

$$\sigma^{\mu\nu}(q, p) = \int d^4kM^{\mu\nu}(q, k)H(k, p) \tag{1'}$$

where $M^{\mu\nu}(q, k)$ is the cross-section for the hard process and $H(k, p)$ is the hadronic structure function.

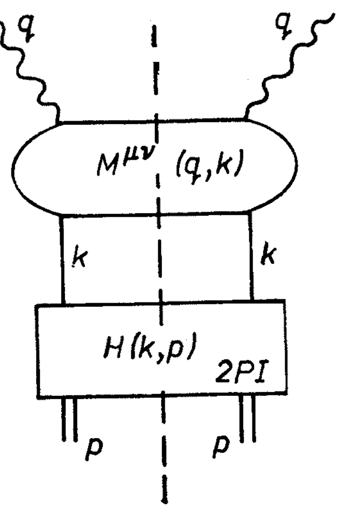

Fig. 1. Diagrammatic representations of the deep inelastic scattering process

The parton model formula for $\sigma^{\mu\nu}$ in the limit of the massless target reads

$$\sigma^{\mu\nu} = \int dy \int dx M^{\mu\nu}(y, p, q)F(x, Q^2)\delta\left(\frac{x_B}{x} - y\right),$$

$$x_B = \frac{q^2}{2pq}, \quad x = \frac{kq}{pq}, \quad Q^2 = -q^2. \tag{2}$$

The target mass effects can be included by using the Nachtmann moments formalism [4] or, equivalently, the scaling technique [8].

However, none of these methods allows for a natural extension to other hard processes. Recently, the target mass effects have been reanalyzed in the context of the complete analysis of the twist 4 sector in the deep inelastic scattering [7]. The prescription given therein consists in replacing the general hadronic blob $H(k, p)$ in Eq. (1′) by

$$H(k, p) \to \frac{1}{\pi M^2}\delta(k^2)\Phi\left(\frac{2kp}{M^2}\right)\theta\left(\frac{2kp}{M^2}\right) \tag{2'}$$

where the function $\Phi(x)$ is related to the leading twist parton density $F(x)$ (Eq. (2)) as follows

$$F(x) = \frac{1}{2(2\pi)^3} \int_x^1 dy \Phi(y).$$

Including the QCD scaling violation effects we get the following expression for the deep inelastic cross-section:

$$\sigma^{\mu\nu}(q, p) = \int dy \int \frac{d^4k}{\pi M^2} M^{\mu\nu}(y, k, p) \Phi\left(\frac{2kp}{M^2}, Q^2\right) \delta(k^2) \delta\left(x_B \frac{pq}{kq} - y\right). \tag{3}$$

The function $\delta(k^2)$ in Eq. (3) forces the interacting parton to be on-shell, as in the standard parton model formula, Eq. (2). However, the parton momentum in Eq. (3) is not collinear to the hadron momentum since the parametrization in Eq. (2′) allows for a non-zero transverse momentum. In the case of the deep inelastic scattering, the $k_\perp$ integration can be performed analytically and one gets the standard Nachtmann ξ scaling expressions.

We use the given above prescription for including the target mass effects in the Drell--Yan process. In this case it is difficult to obtain exact analytical expression for $d\sigma/d\tau$ (as for deep-inelastic cross-section), so we calculate it numerically.

The cross-section $d\sigma/d\tau$ (Fig. 2) for the Drell-Yan process can be expressed [2] as follows:

$$\frac{d\sigma}{d\tau} = \int_\tau^1 dx_1 \int_\tau^1 dx_2 \int_\tau^1 d\hat{\tau} \frac{d\sigma}{d\hat{\tau}} \delta\left(\hat{\tau} - \frac{\tau}{x_1 x_2}\right) F_1(x_1, Q^2) F_2(x_2, Q^2) \tag{4}$$

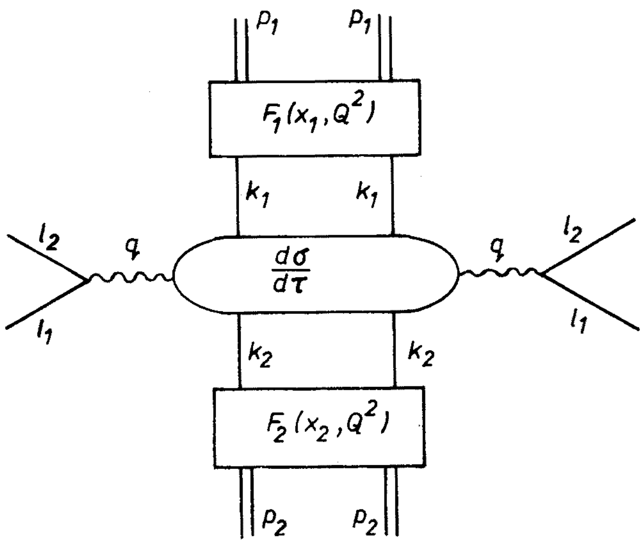

Fig. 2. Drell-Yan process

where $d\sigma/d\hat{\tau}$ is the cross-section for the hard process, $F_1(x_1, Q^2)$, $F_2(x_2, Q^2)$ are the hadronic structure functions and $\tau = Q^2/s$, $s = (p_1+p_2)^2$, $Q^2 = (l_1+l_2)^2$. Mass effects, according to Ref. [6], are included if (4) is changed to

$$\frac{d\sigma}{d\tau} = \frac{1}{(2\pi)^8}\int d\hat{\tau}\int \frac{d^4k_1}{M^2}\,\frac{d^4k_2}{M^2}\,\delta\left(\hat{\tau}-\frac{Q^2}{2k_1k_2}\right)\delta(k_1^2)\delta(k_2^2)$$

$$\frac{d\sigma}{d\hat{\tau}}\,\Phi_1\left(\frac{2k_1p_1}{M^2}, Q^2\right)\Phi_2\left(\frac{2k_2p_2}{M^2}, Q^2\right). \tag{5}$$

We parametrize k_1 and k_2 by Sudakov variables:

$$k_1^\mu = x_1p_1^\mu + \frac{k_1^2+k_{1\perp}^2-x_1^2M^2}{2x_1}\,n_1^\mu + k_{1\perp}^\mu, \qquad n_1^2 = 0, \qquad p_1n_1 = 1,$$

$$k_2^\mu = x_2p_2^\mu + \frac{k_2^2+k_{2\perp}^2-x_2^2M^2}{2x_2}\,n_2^\mu + k_{2\perp}^\mu, \qquad n_2^2 = 0, \qquad p_2n_2 = 1, \tag{6}$$

$$p_1^2 = p_2^2 = M^2. \tag{7}$$

From Eq. (6) and (7) we obtain:

$$n_2^\mu = \frac{p_1^\mu}{M^2}\left(1-\frac{p_1p_2}{\sqrt{(p_1p_2)^2-M^4}}\right)+\frac{p_2^\mu}{\sqrt{(p_1p_2)^2-M^4}}, \tag{8}$$

$$n_2^\mu = \frac{p_2^\mu}{M^2}\left(1-\frac{p_1p_2}{\sqrt{(p_1p_2)^2-M^4}}\right)+\frac{p_1^\mu}{\sqrt{(p_1p_2)^2-M^4}}. \tag{9}$$

The particles produced in the final state are on mass-shell then $(p_1-k_1)^2 > 0$, $(p_2-k_2)^2 > 0$, which can be reduced to:

$$0 < x_1 < 1,\ k_{1\perp}^2 < x_1(1-x_1)M^2,$$

$$0 < x_2 < 1,\ k_{2\perp}^2 < x_2(1-x_2)M^2. \tag{10}$$

After some algebraic manipulations we have:

$$\frac{d\sigma}{d\tau} = \frac{1}{4\pi(2\pi)^6}\int_A^1 d\hat{\tau}\,\frac{d\sigma}{d\hat{\tau}}\int_B^1 dx_1\int_{x_1}^1 dz_1\int_B^1 dx_2\int_{x_2}^1 dz_2$$

$$\int_0^\pi d\alpha\,\delta\left(\hat{\tau}-\frac{Q^2}{\varrho}\right)\Phi_1(z_1, Q^2)\Phi_2(z_2, Q^2), \tag{11}$$

where

$$A = \frac{Q^2}{s_1}, \qquad s_1 = (p_1+p_2)^2-2M^2,$$

$$\varrho = 2x_1x_2(p_1p_2)+(p_1p_2)\zeta(z_2-2x_2)+(p_1p_2)\zeta(z_1-2x_1)$$
$$+(p_1p_2)\zeta(z_1-2x_1)(z_2-2x_2)+2M^2\cos\alpha\sqrt{x_1x_2(z_1-x_1)(z_2-x_2)}. \quad (12)$$

B was calculated numerically from condition $\delta\left(\hat{\tau}-\frac{Q^2}{\varrho}\right)$.

$$\zeta = 1-\sqrt{1-M^4/(p_1p_2)^2}.$$

By taking the limit $M \to 0$ in Eq. (12) we recover the standard parton model formula. Then:

$$\varrho = 2x_1x_2(p_1p_2) = x_1x_2s, \quad s_1 = s, \quad A = \frac{Q^2}{s}, \quad \zeta = 0, \quad B = A,$$

$$\frac{d\sigma}{d\tau} = \int_A^1 d\hat{\tau}\frac{d\sigma}{d\hat{\tau}}\int_A^1 dx_1\int_A^1 dx_2\delta\left(\hat{\tau}-\frac{\tau}{x_1x_2}\right)\int_{x_1}^1 dz_1\frac{\Phi_1(z_1,Q^2)}{2(2\pi)^3}\int_{x_2}^1 dz_2\frac{\Phi_2(z_2,Q^2)}{2(2\pi)^3}. \quad (13)$$

Eqs. (13) and (4) are identical if we identify:

$$F_1(x_1,Q^2) = \frac{1}{2(2\pi)^3}\int_{x_1}^1 dz_1\Phi_1(z_1,Q^2),$$

$$F_2(x_2,Q^2) = \frac{1}{2(2\pi)^3}\int_{x_2}^1 dz_2\Phi_2(z_2,Q^2).$$

In the next Section we present the results of the numerical analysis of Eq. (11).

2. *Results*

Numerical calculations were made for $p+p \to \mu^+\mu^-+X$ scattering with the center of mass energy squared $s = 25$ GeV^2 and $s = 36$ GeV^2. We used the proton structure functions as in Ref. [5]. Figs 3 and 4 show results for 25 GeV^2 and 36 GeV^2 respectively. Curves *1*, *3* illustrate $d\sigma/dQ^2$ as a function of Q^2 with proton mass $M = 0$. Curve *2* corresponds to proton mass of 1 GeV.

Taking proton mass into account leads to two effects. First, the cross-section decreases due to the decrease of the energy available for quarks (A and B in Eq. (12) increase). Secondly, we can expect intuitively that integrals over $k_{\perp 1}$ and $k_{\perp 2}$ in Eq. (12) should enlarge the cross-section. We have estimated the values of these integrals by calculating the cross--section for $s = 23$ GeV^2 and $s = 34$ GeV^2 with $M = 0$ GeV. The energy available for quarks is identical as in case $s = 25$ GeV^2, $s = 36$ GeV^2 with $M = 1$ GeV. The results do not differ within the range of Monte Carlo errors 2.5%. Therefore only the first effect is important.

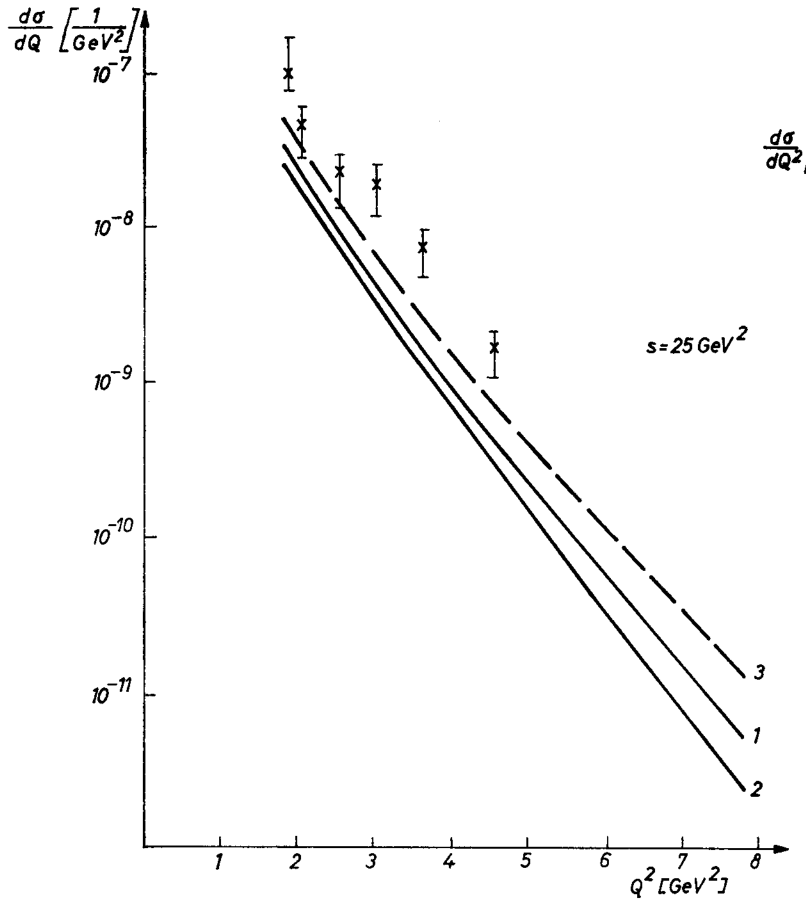

Fig. 3

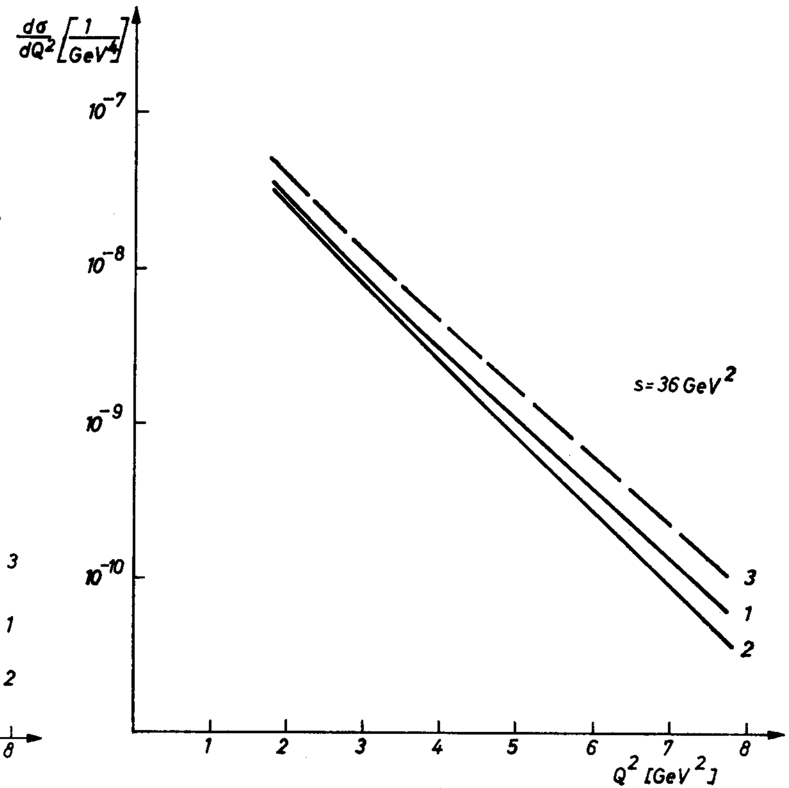

Fig. 4

Fig. 3. Q^2 dependence of $d\sigma/dQ^2$ in D-Y process for $s = 25\ \text{GeV}^2$. Curve *1*: without mass corrections, curve *2*: with mass corrections, curve *3*: without mass corrections and with next to leading corrections

Fig. 4. Q^2 dependence of $d\sigma/dQ^2$ in D-Y process for $s = 36\ \text{GeV}^2$. Curve *1*: without mass corrections, curve *2*: with mass corrections, curve *3*: without mass corrections and with next to leading corrections

From comparison of Figs. 3 and 4 it is evident that the mass corrections decrease with s increasing, and for $s = 100\ \mathrm{GeV}^2$ they are already within the range of our Monte Carlo errors.

It is clear that the proton mass corrections are important for energies smaller than 10 GeV. However, they do not improve the consistency between the QCD calculations and the experimental data.

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