# QED RADIATIVE CORRECTIONS TO ELECTRON-POSITRON ANNIHILATION INTO HEAVY FERMIONS*

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We reexamine the $O(\alpha^3)$ corrections to the process $e^+e^- \to \tau^+\tau^-$ (or any other heavy fermion pair) taking into account the effects of the masses of the final-state particles. The relevant analytic formulae are presented as well as some Monte Carlo results.

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## 1. *Introduction*

In $e^+e^-$ annihilation at currently available energies, the one-photon exchange channel can produce a variety of fermion pairs with masses ranging from 0.1 to 5 GeV. Higher order QED corrections to these processes are most conveniently studied using Monte Carlo simulation techniques [1]. The existing numerical programs, however, assume the validity of the ultrarelativistic approximation, where the masses are negligible with respect to the relevant energies. It is clear that for heavy fermions this assumption is not always justified. The effects of a finite mass will show up in various circumstances. For example, near the production threshold the angular dependence will be more isotropic and the energy dependence of the cross section will be affected by the mass. This threshold behaviour also shows up in the bremsstrahlung spectrum at the hard photon end, which may be of relevance in excited lepton searches.

The purpose of this paper is to present analytical and numerical results for QED radiative corrections in which the mass of the produced fermions is kept throughout the calculation. The incoming electron-positron pair is of course treated ultra-relativistically. Thus, this paper is a finite fermion mass extension of Ref. [1].

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It should be remarked at this point that the material of the present paper deserves extension into two directions. At high energies and for even higher fermion masses (like the — still hypothetical — top quark with $m > 18$ GeV) one certainly reaches a point, where the $Z_0$ exchange should be included. Another extension is the treatment of the polarization of the produced fermions. The decay of femions like the $\tau$-lepton into other particles depends on the polarization of the decaying fermion, which is a quantity one likes to measure eventually. The present paper is a necessary first step to these two extensions.

The outline of the paper is as follows. Section 2 contains virtual and soft bremsstrahlung corrections to the lowest order cross section. Hard bremsstrahlung is discussed in Section 3, together with the photon spectrum. Some numerical examples resulting from a Monte Carlo simulation program are presented in Section 4. An Appendix deals with the evaluation of the box diagrams, which is somewhat different from the treatment in Ref. [2].

## 2. *Virtual and soft photon bremsstrahlung corrections*

In this section we present formulae for the radiative corrections to the differential cross section at beam energy $E$ (c.m.s. energy $= 2E$) in the limit $m = m_e/E \to 0$, but keeping the mass $m_f$ of the produced fermions.

The lowest order cross section for the process

$$e^+(p_1)+e^-(p_2) \to \bar{f}(q_1)+f(q_2) \tag{2.1}$$

reads

$$\frac{d\sigma^0}{d\Omega} = \frac{3\sigma_p}{16\pi} Q^2 Q'^2 N_c \beta'(1+c^2+\mu^2), \tag{2.2}$$

where $Q$ and $Q'$ are the charges of the positron and antifermion $\bar{f}$ respectively. The number of colours $N_c$ equals 1 or 3 for lepton or quark pair production. Furthermore, the following quantities are used

$$\sigma_p = \frac{\pi\alpha^2}{3E^2}, \quad \mu = m_f/E, \quad \beta' = (1-\mu^2)^{1/2}, \quad c = \beta' \cos\theta, \tag{2.3}$$

where $\theta$ is the angle between $\vec{p}_1$ and $\vec{q}_1$. As usual $\alpha$ denotes the fine structure constant.

In the following we will be interested in the differential cross section $d\sigma/d\Omega$ up to order $\alpha^3$, that is the lowest order cross section including first order virtual corrections and taking into account emission of a soft photon with an energy less than $E_0$ ($E_0 \ll E$). The complete set of formulae for $d\sigma/d\Omega$ can be found in Refs. [2, 3] for the case when both $m_e$ and $m_f$ are nowhere neglected. It is possible to recombine those results in order to simplify the final expression, which then also gives a reasonably compact formula for the $m = m_e/E \to 0$ limit. For the present considerations where we shall have to compute $d\sigma/d\Omega$ many times in a Monte Carlo simulation program a simple form is of importance.

The complete expression for the differential cross section $d\sigma/d\Omega$ containing virtual and soft bremsstrahlung corrections is given by

$$\frac{d\sigma}{d\Omega}(c, k_0) = \frac{3\sigma_p}{16\pi} Q^2 Q'^2 N_c \beta' \{(1+c^2+\mu^2)\,[1+\delta^{SX}(m^2, k_0)$$

$$+\delta^{SX}(\mu^2, k_0) - 2\,\mathrm{Re}\,\Pi] + 4\,\mathrm{Re}\,F_2(\mu^2) + \delta^{AS}(c, k_0) - \delta^{AS}(-c, k_0)\} \tag{2.4}$$

where $k_0 = E_0/E$. In this expression the last two terms represent the $c$-odd part, the others form the $c$-even part. The latter are easily obtained from the expressions of Ref. [3] by taking the $m \to 0$ limit. Thus one obtains for the sum of the vertex correction and the $c$-even bremsstrahlung part

$$\delta^{SX}(\mu^2, k_0) = \beta_{\mathrm{fin}} \ln \frac{2k_0}{\mu} + \frac{\alpha}{\pi} Q'^2 \left\{ \frac{3+2\beta'^2}{2\beta'} Y \right.$$

$$\left. -2 - \frac{1+\beta'^2}{2\beta'} \left[ Y^2 + 4\,\mathrm{Li}_2\left(\frac{2\beta'}{1+\beta'}\right) - \pi^2 \right] \right\}, \tag{2.5}$$

where

$$Y = \ln \left[ \frac{(1+\beta')^2}{\mu^2} \right] \tag{2.6}$$

$$\beta_{\mathrm{fin}} = \frac{2\alpha}{\pi} Q'^2 \left( \frac{1+\beta'^2}{2\beta'} Y - 1 \right) \tag{2.7}$$

and

$$\delta^{SX}(m^2, k_0) = \beta_{\mathrm{ini}} \ln k_0 + \frac{\alpha}{\pi} Q^2 \left( \tfrac{3}{2} \ln \frac{4}{m^2} - 2 + \frac{\pi^2}{3} \right), \tag{2.8}$$

with

$$\beta_{\mathrm{ini}} = \frac{2\alpha}{\pi} Q^2 \left( \ln \frac{4}{m^2} - 1 \right). \tag{2.9}$$

The vacuum polarization $\mathrm{Re}\,\Pi$ contains contributions from quarks and leptons. The explicit form can be found elsewhere [1]. The magnetic part of the vertex correction vanishes in the ultrarelativistic limit, so only the part arising from the produced fermion pair should be considered:

$$\mathrm{Re}\,F_2(\mu^2) = -\frac{\alpha}{\pi} Q'^2 \frac{\mu^2}{4\beta'} Y. \tag{2.10}$$

The $c$-odd part of $d\sigma/d\Omega$ consists of two infrared diverging contributions, the bremsstrahlung part $d\sigma^{\mathrm{B}}/d\Omega$ and the box diagram part $d\sigma^{\mathrm{Box}}/d\Omega$. In the $m^2 \to 0$ limit we have

$$\frac{d\sigma_{\mathrm{B}}}{d\Omega} = \frac{3\sigma_p}{16\pi} Q^2 Q'^2 N_c \beta' (1+c^2+\mu^2) Q Q' \frac{\alpha}{\pi}$$

$$\times \left[ 4 \ln(1-c) \ln \frac{2k_0}{\lambda} + D(c) - (c \to -c) \right], \tag{2.11}$$

where $\lambda$ is a small photon mass divided by $E$ and

$$D(c) = 2\,\mathrm{Re}\left\{\mathrm{Li}_2\left(\frac{-x}{1-y}\right) - \mathrm{Li}_2\left(\frac{1-x}{1-y}\right) - \mathrm{Li}_2\left(\frac{1+x}{y}\right) + \mathrm{Li}_2\left(\frac{x}{y}\right)\right\}$$

$$+\ln^2\left|\frac{y}{1-y}\right| - \mathrm{Re}\,\mathrm{Li}_2(x^2) + \tfrac{1}{2}\ln^2(x^2) - \ln(x^2)\ln(1-x^2), \tag{2.12}$$

with

$$x = (1-c)/\sqrt{\omega}, \quad y = \tfrac{1}{2}\left[\sqrt{\omega} + \frac{\mu^2}{1+\beta'}\right], \quad \omega = 2-2c-\mu^2. \tag{2.13}$$

The box diagram contribution can be written in a relatively compact form, as is indicated in the Appendix

$$\frac{d\sigma^{\mathrm{Box}}}{d\Omega} = \frac{3\sigma_p}{16\pi} Q^2 Q'^2 N_c \beta' Q Q' \frac{\alpha}{\pi}$$

$$\times\left[(1+c^2+\mu^2) 4\ln(1-c)\ln\frac{\lambda}{2} + \tfrac{1}{2}B(c) - (c \to -c)\right], \tag{2.14}$$

where

$$B(c) = \tilde{A}(c)\left[1+c-\frac{\mu^2}{\omega}(3-c)\right] - \tilde{B}[2c+\mu^2] - 2\tilde{F}_\Delta c$$

$$-2\tilde{F}_Q c\left[1-\frac{\mu^2}{2}-\frac{\mu^2}{2\beta'^2}\right] + \ln\frac{4}{m^2}\left[-c+\frac{\mu^2}{\omega}(3-c)\right]$$

$$+\ln\frac{4}{\mu^2}\left[-c-\frac{\mu^2}{\omega}(3-c)-\frac{2\mu^2 c}{\beta'^2}\right], \tag{2.15}$$

with

$$\tilde{A}(c) = 2\ln\frac{1-c}{2} + \ln\frac{4}{m^2} + \ln\frac{4}{\mu^2},$$

$$\tilde{B}(c) = \ln^2\frac{1-c}{2} - 2\,\mathrm{Li}_2\left(\frac{\omega}{2(1-c)}\right) - \tfrac{1}{2}\ln^2\frac{4}{m^2} - \tfrac{1}{2}\ln^2\frac{4}{\mu^2},$$

$$\tilde{F}_\Delta = \tfrac{1}{2}\left(\ln^2\frac{4}{m^2} + \frac{\pi^2}{3}\right),$$

$$\tilde{F}_Q = \frac{1}{2\beta'}\left[Y^2 + 4\,\mathrm{Li}_2\left(\frac{-\mu^2}{(1+\beta')^2}\right) + \frac{\pi^2}{3}\right]. \tag{2.16}$$

Combining (2.11) and (2.14) we find for the $c$-odd contribution to $d\sigma/d\Omega$ (cf Eq. (2.4))

$$\delta^{AS}(c) = QQ'\frac{\alpha}{\pi}\{(1+c^2+\mu^2)[4\ln(1-c)\ln k_0 + D(c) + \tfrac{1}{2}B(c)\}. \tag{2.17}$$

The $m^2 \to 0$ limit of formula (2.4) and the formulae related to it reduce to the results of Ref. [1]. As the illustration of the above, we give in Fig. 1 the $\cos\theta$ distribution for $\tau$ leptons ($m_\tau = 1.782$ GeV) including virtual and soft bremsstrahlung corrections. Both the exact result, and the one obtained by taking the ultrarelativistic limit are shown, for a beam energy $E = 2m_\tau = 3.564$ GeV. As remarked before, the effect of a finite final--state mass is a flatter angular distribution.

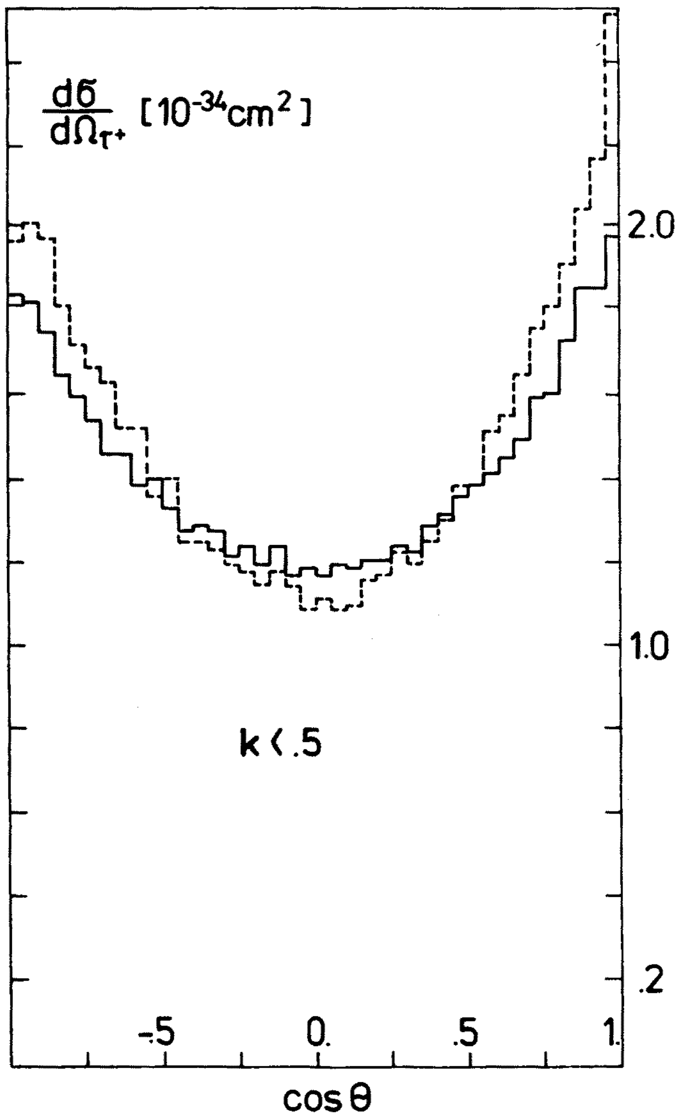


Fig. 1. Angular distribution of $\tau^+$ with virtual and soft bremsstrahlung corrections, both exact (solid line) and in the ultrarelativistic approximation (dashed line). Only leptonic contribution to $\delta_{VP}$ is included

For completeness we also give the total cross section up to order $\alpha^3$, obtained by integration of Eq. (2.4):

$$\sigma(k < k_0) = \sigma_p Q^2 Q'^2 N_c \beta' \left\{ \left(1+\frac{\mu^2}{2}\right) [1+\delta^{SX}(m^2, k_0) + \delta^{SX}(\mu^2, k_0) - 2 \operatorname{Re} \Pi] + 3 \operatorname{Re} F_2(\mu^2) \right\}. \tag{2.18}$$

## 3. Hard bremsstrahlung

The multidifferential cross section for the process

$$e^+(p_1)+e^-(p_2) \to \bar{f}(q_1)+f(q_2)+\gamma(k) \tag{3.1}$$

is given by

$$d\sigma = \frac{\alpha^3 Q^2 Q'^2 N_c}{16\pi^2 E^2} [A_{\text{ini}}+A_{\text{fin}}+A_{\text{int}}] d\tau \tag{3.2}$$

with

$$d\tau = \frac{d^3 q_1}{q_1^0} \frac{d^3 q_2}{q_2^0} \frac{d^3 k}{k^0} \delta^4(p_1+p_2-q_1-q_2-k). \tag{3.3}$$

The three terms in (3.2), representing initial, final state radiation and their interference are given by

$$A_{\text{ini}} = \frac{Q^2}{\tilde{s}_1 x_1 x_2} \left\{ \left[ \tilde{t}^2+\tilde{u}^2+\mu^2 \frac{(\tilde{t}+\tilde{u})^2}{\tilde{s}_1} \right] \left(1-\frac{m^2 x_1}{\tilde{s}_1 x_2}\right) + \left[ \tilde{t}_1^2+\tilde{u}_1^2+\mu^2 \frac{(\tilde{t}_1+\tilde{u}_1)^2}{\tilde{s}_1} \right] \left(1-\frac{m^2 x_2}{\tilde{s}_1 x_1}\right) \right\}, \tag{3.4}$$

$$A_{\text{fin}} = \frac{Q'^2}{\tilde{s} y_1 y_2} \left\{ [\tilde{t}^2+\tilde{u}_1^2+\mu^2 \tilde{s}] \left[1-\frac{\mu^2}{\tilde{s}}\left(1+\frac{y_1}{y_2}\right)\right] + [\tilde{u}^2+\tilde{t}_1^2+\mu^2 \tilde{s}] \left[1-\frac{\mu^2}{\tilde{s}}\left(1+\frac{y_2}{y_1}\right)\right] + \frac{\mu^2}{\tilde{s}}(x_1^2+x_2^2) - 4\mu^2(\tilde{s}-\tilde{s}_1) \right\}, \tag{3.5}$$

$$A_{\text{int}} = \frac{QQ'}{\tilde{s}\tilde{s}_1 x_1 x_2 y_1 y_2} \{ (\tilde{t} x_2 y_2 + \tilde{t}_1 x_1 y_1 - \tilde{u} x_2 y_1 - \tilde{u}_1 x_1 y_2)(\tilde{t}^2+\tilde{t}_1^2 + \tilde{u}^2+\tilde{u}_1^2+\mu^2(\tilde{s}+\tilde{s}_1)) + \mu^2 x_1 x_2 [(\tilde{s}-\tilde{s}_1)(\tilde{t}+\tilde{t}_1-\tilde{u}-\tilde{u}_1) - (x_1-x_2)(y_1-y_2)] \}, \tag{3.6}$$

where the following invariants are introduced[1]

$$\begin{aligned} &\tilde{s} = p_1 p_2, \quad \tilde{s}_1 = q_1 q_2 + \mu, \quad \tilde{t} = p_1 q_1, \quad \tilde{t}_1 = p_2 q_2, \\ &\tilde{u} = p_1 q_2, \quad \tilde{u}_1 = p_2 q_1, \quad x_1 = p_1 k, \quad x_2 = p_2 k, \\ &y_1 = q_1 k, \quad y_2 = q_2 k. \end{aligned} \tag{3.7}$$

[1] The quantities used in this paper differ somewhat from those used in the massless case [1].

Here, all four-momenta and masses have been made dimensionless (in units of $E$), so that both the quantities $A$ and $d\tau$ are dimensionless. Both the infrared and ultrarelativistic limits of (3.2) agree with previous results. However, the factorization property of the squared matrix element into a generalized lowest order squared matrix element and a bremsstrahlung factor [4] does not hold anymore for this case of massive fermion production.

By making various choices of phase space variables, different for initial and final state radiation Eq. (3.2) can be integrated, leading to the photon spectra

$$\frac{d\sigma_{\rm ini}}{dk} = \sigma_p Q^2 Q'^2 N_c \beta_{\rm ini} \varrho_{\rm ini}, \tag{3.8}$$

$$\frac{d\sigma_{\rm fin}}{dk} = \sigma_p Q^2 Q'^2 N_c \frac{\alpha Q'^2}{\pi} \varrho_{\rm fin}, \tag{3.9}$$

where

$$\varrho_{\rm ini} = \frac{1+(1-k)^2}{4k(1-k)} v(k)\,[3-v^2(k)], \tag{3.10}$$

$$\varrho_{\rm fin} = \frac{1}{k}\left\{\left[1+(1-k)^2-\mu^2 k-\frac{\mu^4}{2}\right]\ln\left(\frac{1+v(k)}{1-v(k)}\right)-v(k)\,[k^2+(2+\mu^2)(1-k)]\right\}, \tag{3.11}$$

with

$$v(k) = \left(1-\frac{\mu^2}{1-k}\right)^{1/2}. \tag{3.12}$$

Upon integration in the range $(k_0, 1-\mu^2)$, with $k_0 \ll 1$ we find for the total cross section for hard radiation

$$\sigma_{\rm ini} = \sigma_p Q^2 Q'^2 N_c \beta_{\rm ini}\left\{\tfrac{1}{2}\beta'(3-\beta'^2)\left[\ln\frac{1}{k_0}-\tfrac{4}{3}+\ln\frac{4\beta'^2}{\mu^2}\right]+\tfrac{1}{2}\ln\frac{\mu^2}{(1+\beta')^2}\right\}, \tag{3.13}$$

$$\begin{aligned}\sigma_{\rm fin} = \sigma_p Q^2 Q'^2 N_c &\left\{\tfrac{1}{2}\beta'(3-\beta'^2)\beta_{\rm fin}\ln\frac{1}{k_0}\right.\\
&+\frac{\alpha Q'^2}{\pi}\left[(3-\beta'^2)(1+\beta'^2)\left[\mathrm{Li}_2\left(\frac{1-\beta'}{1+\beta'}\right)-\mathrm{Li}_2\left(\frac{1-\beta'}{2}\right)-\frac{\pi^2}{12}\right.\right.\\
&\left.-\tfrac{1}{2}\ln\left(\frac{1+\beta'}{2}\right)\ln\frac{(1-\beta')^2}{2(1+\beta')}\right]+\beta'(3-\beta'^2)\ln\left(\frac{1-\beta'^2}{4\beta'^2}\right)\\
&\left.\left.+\tfrac{1}{16}(9-2\beta'^2+\beta'^4)\ln\frac{1+\beta'}{1-\beta'}+\tfrac{39}{8}\beta'-\tfrac{17}{8}\beta'^3\right]\right\}.\end{aligned} \tag{3.14}$$

In the expressions (3.13) and (3.14) the $\beta' \to 1$ limit is easily taken and the expressions of Ref. [1] are obtained. By adding (3.13) and (3.14) to (2.18) one obtains the total cross section for $e^+e^- \to \bar{f}f\gamma$ up to order $\alpha^3$:

$$\sigma^{\rm TOT} = \sigma_0^{\rm TOT}(1+\delta_{\rm ini}+\delta_{\rm fin}+\delta_{\rm VP}) \tag{3.15}$$

where

$$\sigma_0^{\rm TOT} = \sigma_p Q^2 Q'^2 N_c \tfrac{1}{2}\beta'(3-\beta'^2), \tag{3.16}$$

$$\delta_{\rm ini} = \delta^{SX}(m^2, k_0)+\sigma_{\rm ini}/\sigma_0^{\rm TOT} = \beta_{\rm ini}\left[-\tfrac{4}{3}+\ln\frac{4\beta'^2}{\mu^2}+\frac{1}{\beta'(3-\beta'^2)}\ln\frac{\mu^2}{(1+\beta')^2}\right]$$
$$+\frac{\alpha}{\pi}Q^2\left(\tfrac{3}{2}\ln\frac{4}{m^2}-2+\frac{\pi^2}{3}\right), \tag{3.17}$$

$$\delta_{\rm fin} = \delta^{SX}(\mu^2, k_0)+\sigma_{\rm fin}/\sigma_0^{\rm TOT}+6\,{\rm Re}\,F_2/(\beta'(3-\beta'^2))$$
$$= \frac{\alpha}{\pi}Q'^2\left\{\frac{2(\beta'^2+1)}{\beta'}\left[-\ln\frac{1+\beta'}{\mu}\ln\frac{2}{1+\beta'}-\tfrac{1}{2}\ln^2\left(\frac{1+\beta'}{2}\right)\right.\right.$$
$$\left.-{\rm Li}_2\left(\frac{2\beta'}{1+\beta'}\right)+\frac{\pi^2}{6}-{\rm Li}_2\left(\frac{1-\beta'}{2}\right)+{\rm Li}_2\left(\frac{1-\beta'}{1+\beta'}\right)\right]$$
$$-2(1+2\ln\beta')+\frac{39-17\beta'^2}{4(3-\beta'^2)}-\frac{3\mu^2}{2\beta'(3-\beta'^2)}\ln\frac{(1+\beta')^2}{\mu^2}$$
$$\left.+\frac{-7\beta'^4+24\beta'^3+10\beta'^2-72\beta'+45}{8(3-\beta'^2)\beta'}\ln\frac{4}{\mu^2}+\frac{-7\beta'^4+10\beta'^2+45}{4(3-\beta'^2)\beta'}\ln\frac{1+\beta'}{2}\right\}, \tag{3.18}$$

$$\delta_{\rm VP} = -2\,{\rm Re}\,\Pi. \tag{3.19}$$

We like to remark that the lowest order cross section $\sigma_0^{\rm TOT}$ vanishes for $\beta' \to 0$, reflecting that the $\bar{f}f$ pair is produced in a P-wave state. When we consider $\sigma_0^{\rm TOT}\delta_{\rm ini}$, for which quantity the fermion pair is again produced in a P-wave state we find also a vanishing behaviour in the $\beta' \to 0$ limit. For $\sigma_0^{\rm TOT}\delta_{\rm fin}$, where the fermion pair is not produced in a P-wave state, we obtain for $\beta' \to 0$ the value $\frac{\alpha}{\pi}Q'^2\frac{3}{4}\pi^2$.

The $\beta' \to 1$ ($\mu^2 \to 0$) limit of $\delta_{\rm fin}$ yields $\frac{3}{4}\frac{\alpha}{\pi}Q'^2$. Apart from a colour factor this gives the QCD correction $\delta_{\rm QCD}$ to $R$, which is $\alpha_S/\pi$. Thus from (3.18) the analogous formula for massive quark pair production can be obtained

$$\delta_{\rm QCD} = \tfrac{4}{3}\frac{\alpha_S}{\alpha Q'^2}\delta_{\rm fin}. \tag{3.20}$$

## 4. *Monte Carlo simulation*

In this section we shall present some results of the computer program developed to simulate the production of heavy fermions with radiative corrections. The structure of this program is as follows. First a value $k$ for the photon energy is generated according to an approximate expression. If $k < k_0$ the event is taken to be nonradiative, with an angular distribution as given in Section 2. This distribution is obtained by modifying a flat distribution using a weighing-rejection procedure (WRP) as described in Refs. [5, 6]. If $k > k_0$ a hard bremsstrahlung event is generated, according to the cross section either for initial-state radiation or final-state radiation. All variables except the photon direction are generated using WRP. The photon direction is generated by analytic inversion [6] which is necessary because the cross section depends very strongly on this direction. Finally, the interference between initial- and final-state radiation is imposed on the generated events by WRP.

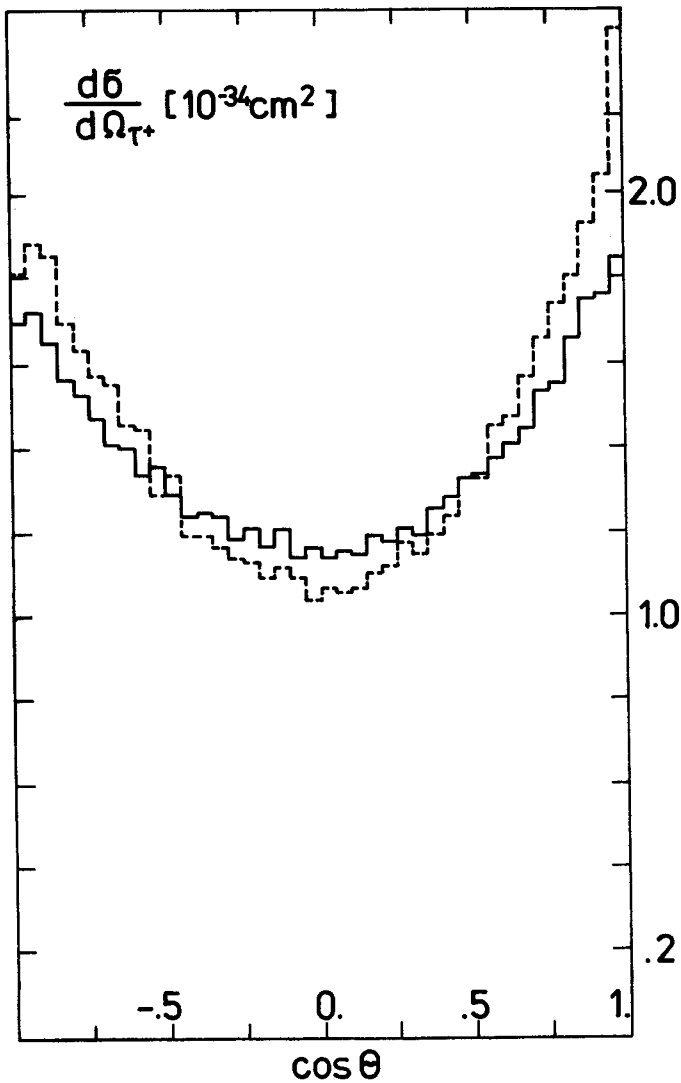


Fig. 2. Same as Fig. 1, but now including hard bremsstrahlung up to $E_\gamma/E_{\text{beam}} = 0.50$

Using the above approach all experimentally interesting distributions can be generated, with arbitrary cuts. As an example we present in Fig. 2 the distribution of the polar production angle of the $\tau^+$, and in Fig. 3 the distribution of the acollinearity angle of the produced $\tau^+\tau^-$ pair. For comparison, we also generated the same number of events for massless particles, using the program presented in [5]. In both cases, the allowed range of photon energies was taken to be the same, namely $(0, 1-\mu^2)$. It is seen that the distributions of

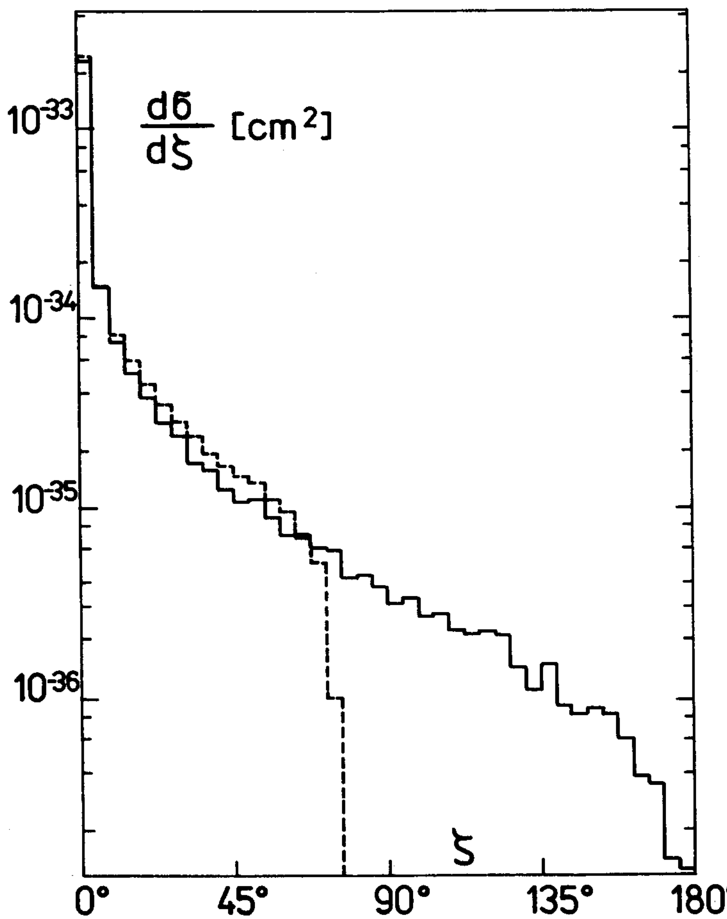


Fig. 3. Acollinearity of the produced $\tau$ pair

Fig. 2 are comparable to those of Fig. 1. The acollinearity distribution shows a dramatic difference between the massive and massless case. Although this effect is mainly kinematical in nature, we want to stress the point that there is no sensible way to adopt massive particle kinematics on the one hand, and on the other hand use the ultrarelativistic approximation for the cross section.

## APPENDIX

In this appendix we give a derivation of the box diagram contribution, which is somewhat different from the one in Ref. [2]. The interference of one box diagram with the lowest order amplitude is given by Eq. (12) in the former reference. Using the explicit forms of $X$, $X_\mu$ and $X_{\mu\nu}$ (to be obtained from Eq. (11)) one is lead to the expression

$$\frac{d\sigma^a}{d\Omega} = \frac{3\sigma_p}{16\pi} Q^2 Q'^2 N_c \beta' Q Q' \frac{\alpha}{\pi}$$
$$\times \{[1+c^2-2c^3+(1-2c)(m^2+\mu^2)]\bar{\int}(1)+[2c(1-c)-m^2-\mu^2 c]\bar{\int}(k\Delta)$$
$$+[2c(1-c)-\mu^2-m^2c]\bar{\int}(kQ)+[2-c+c^2+m^2+\mu^2]\bar{\int}(k^2)-(1+c)\bar{\int}(Pk)^2$$
$$+\bar{\int}((kQ)^2+(k\Delta)^2)+(1-c)\bar{\int}(kQ)(k\Delta)\}, \tag{A.1}$$

where in units of $E$

$$P = \tfrac{1}{2}(p_1+p_2), \quad \Delta = \tfrac{1}{2}(p_1-p_2), \quad Q = \tfrac{1}{2}(q_1-q_2) \tag{A.2}$$

and

$$\bar{\int} f(k) = \frac{4}{\pi^2} \mathrm{Im} \int \frac{f(k) d^4k}{(\Delta)(Q)(+)(-)}, \tag{A.3}$$

with

$$(\Delta) = k^2-2k\Delta-1+i\varepsilon$$
$$(Q) = k^2-2kQ-1+i\varepsilon$$
$$(\pm) = k^2 \pm 2kP+1-\lambda^2+i\varepsilon. \tag{A.4}$$

The various integrals (A.3) can be expressed just like in Ref. [2] in terms of four functions $A$, $B$, $F_\Delta$ and $F_Q$

$$\bar{\int}(1) = A \ln \frac{\lambda^2}{4},$$

$$\bar{\int}(k\Delta) = -B - \frac{2}{\pi^2} \mathrm{Im}\, F_Q,$$

$$\bar{\int}(kQ) = -B - \frac{2}{\pi^2} \mathrm{Im}\, F_\Delta,$$

$$\bar{\int}(k^2) = A \ln \frac{\lambda^2}{4} - 2B,$$

$$\bar{\int}(kP)^2 = A \ln \frac{\lambda^2}{4} + 2A - B,$$

$$\bar{\int}((k\Delta)^2+(kQ)^2) = \frac{\beta^2\beta'^2-c^2}{\omega}4A+2B+\left(1+\frac{c}{\beta^2}\right)\frac{2}{\pi^2}\,\mathrm{Im}\,F_\Delta$$

$$+\left(1+\frac{c}{\beta'^2}\right)\frac{2}{\pi^2}\,\mathrm{Im}\,F_Q+\frac{\beta^2-\beta'^2}{\omega}\left(\ln\frac{4}{m^2}-\ln\frac{4}{\mu^2}\right)-\frac{c}{\beta^2}\ln\frac{4}{m^2}-\frac{c}{\beta'^2}\ln\frac{4}{\mu^2},$$

$$\bar{\int}(k\Delta)(kQ) = \frac{\beta^2\beta'^2-c^2}{\omega}2A+B+\frac{2}{\pi^2}\,\mathrm{Im}\,F_\Delta+\frac{2}{\pi^2}\,\mathrm{Im}\,F_Q$$

$$-\frac{\beta'^2-c}{\omega}\ln\frac{4}{m^2}-\frac{\beta^2-c}{\omega}\ln\frac{4}{\mu^2}, \qquad (A.5)$$

with

$$\beta = \sqrt{1-m^2}.$$

In order to obtain Eqs (2.14) and (2.15) we have to insert Eqs (A.5) into (A.1), take the $m^2 \to 0$ limit and find the coefficients of $A$, $B$, $F_\Delta$ and $F_Q$. It is then convenient to introduce

$$\tilde{A} = 2(1-c)A, \qquad \tilde{B} = 2(1-c)B,$$

$$\tilde{F}_\Delta = \frac{4}{\pi^2}\,\mathrm{Im}\,F_\Delta, \qquad \tilde{F}_Q = \frac{4}{\pi^2}\,\mathrm{Im}\,F_Q. \qquad (A.6)$$

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