UNIFIED GAUGE THEORIES FROM HIGHER DIMENSIONS*

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We describe how the unification of gauge interactions with gravitational interactions can be obtained from pure higher dimensional gravity. The possibility of such a theory having some predictive power for the low-energy parameters is discussed.

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In how many dimensions do we live? The usual answer is four: three space dimensions and one time dimension. This answer is based on the successful predictions of Einstein's relativity, so far confirmed by all experiments. However, let us recall that the number of dimensions which can be tested by experiment may well depend on the distance which is probed. Imagine that the space-time characterizing our universe has two very different length scales. The characteristic length for the four known dimensions is given by the inverse curvature of our present universe. The supplementary dimensions form a compact space with characteristic length of the order of the Planck length. At energies much below the Planck mass the supplementary dimensions could not be seen directly and we would observe an effective four dimensional theory. Only when the wavelength of a particle becomes comparable or smaller than Planck's length, the modes corresponding to the supplementary dimensions could be excited and the higher dimensions could be explored.

In this talk we advocate that our world has indeed more than four dimensions. We argue that the assumption of supplementary dimensions is not just a useless game for theoreticians. It may help us to understand the outcome of past and future experiments. We will show that the energy scale where these supplementary dimensions can be seen directly is of the order of the Planck mass. Even if the supplementary dimensions cannot be observed directly, they may reflect themselves in the effective four dimensional theory. This is very similar to the idea of unification of weak electromagnetic and strong interactions at a scale above 10^{14} GeV. This unification may never be tested directly. However, it predicts relations between parameters of the effective low energy theory which can be observed.

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The theory we will discuss is very simple: pure gravity in more than four dimensions. (Later we have to add a higher dimensional spinor in order to account for the four dimensional spinor degrees of freedom.) What are the four dimensional consequences of this enlarged gravity theory?

First we observe that the d-dimensional ($d > 4$) metric tensor describes spin 2, spin 1 and spin 0 fields in four dimensions, thus unifying all bosonic degrees of freedom of the four dimensional theory. This can be seen easily by splitting the d-dimensional indices (we denote them by $\hat{\mu}, \hat{\nu}$ ...) into Minkowski indices for the usual four dimensions (μ, ν ...) and indices for the supplementary dimensions (α, β ...). The d dimensional metric tensor $g_{\hat{\mu}\hat{\nu}}$ reads

$$g_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} g_{\mu\nu}, & A_{\mu\beta} \\ A_{\alpha\nu}, & S_{\alpha\beta} \end{pmatrix}. \tag{1}$$

Under four dimensional general coordinate transformations the fields $g_{\mu\nu}$, $A_{\mu\beta}$ and $S_{\alpha\beta}$ transform as second rank symmetric tensor, vector and scalar, respectively.

Since d dimensional gravity unites all spin 1 and spin 2 fields, it must unite the gauge bosons with the graviton. It therefore should give a unified description [1–3] of the corresponding fundamental symmetries, the usual gauge symmetry and the symmetry under four dimensional general coordinate transformations. There is only one symmetry in d dimensional gravity: the symmetry under general coordinate transformations in d dimensions. (Including spinors, we have to add the d-dimensional local Lorentz transformations.) This symmetry contains the four dimensional general coordinate transformations. It also contains gauge transformations which correspond to a special class of coordinate transformations among the supplementary d-4 coordinates. To make this more explicit, consider as an example d dimensional gravity and assume that the ground state is a direct product of four dimensional Minkowski space and a d-4 dimensional hypersphere. This ground state is invariant under all transformations among the d-4 "internal" coordinates leaving the hypersphere invariant. These transformations form a SO(d-3) group. Since they can be made independently at every point of Minkowski spacetime, SO(d-3) is a local gauge symmetry of this ground state. This example is easily generalized to any compact "internal" manifold admitting a symmetry. It becomes clear that very simple ground states can account for an acceptable unified gauge group (for example SO(10)).

What can we learn about scalar fields? Four dimensional unified gauge theories have many free parameters, most of them from the scalar sector of the theory where the couplings are not completely determined by the gauge principle. There are scalar mass terms and selfinteractions and Yukawa couplings between scalars and fermions. Is there a possibility that higher dimensional gravity predicts some of these parameters? In principle, all couplings can be calculated from the parameters of d-dimensional gravity.

What are these parameters? In general, d dimensional gravity has one free parameter for every invariant under general coordinate transformations included in the action. There is certainly the d dimensional scalar curvature R and a d dimensional cosmological constant ε. There may be many more invariants contributing to the action. Possible invariants involve higher powers of R, terms like $R_{\hat{\mu}\hat{\nu}}R^{\hat{\mu}\hat{\nu}}$ or $R_{\hat{\mu}\hat{\nu}\hat{\sigma}\hat{\lambda}}R^{\hat{\mu}\hat{\nu}\hat{\sigma}\hat{\lambda}}$, etc. ($R_{\hat{\mu}\hat{\nu}\hat{\sigma}\hat{\lambda}}$ and $R_{\hat{\mu}\hat{\nu}}$ are the d dimensional curvature tensor and Ricci tensor). There is *a priori* no reason why these invariants should not be included. As we will see later, they are even needed to generate an acceptable ground state. And they are also expected to be induced by any form of quantum gravity. The couplings corresponding to these invariants have dimension (mass)^N, where N depends on the number of derivatives appearing in such an invariant. (The coupling of the curvature scalar has dimension M^{d-2}, the couplings of terms like $R_{\hat{\mu}\hat{\nu}}R^{\hat{\mu}\hat{\nu}}$ have dimension M^{d-4}, etc.). The number of invariants with couplings of positive mass dimension ($N \geqslant 0$) grows very rapidly with the dimension d. If all these couplings are relevant for the low energy theory, there seems to be almost no chance of making predictions for the pure scalar sector.

However, the quadratic, cubic and quartic terms of the scalar potential only depend on a much more limited number of parameters of the d dimensional theory. Consider as an example an action which is only a function of the curvature scalar R. The ground state will be characterized by a certain value R_0 and we can expand the action in powers of $R-R_0$. The expansion coefficients are considered as a new set of free parameters. Excitations above the ground state are described by scalar fields ϕ which vanish for the ground state. Then $R-R_0$ is at least linear in ϕ and only terms up to $(R-R_0)^4$ are relevant for the calculation of the scalar potential up to ϕ^4 terms. At low energies, higher powers of ϕ in the scalar potential correspond to irrelevant operators and are believed to be negligible. These arguments are easily generalized to more complicated actions involving $R_{\hat{\mu}\hat{\nu}}R^{\hat{\mu}\hat{\nu}}$ terms, etc. The number of relevant parameters remains limited. Especially, for $d \geqslant 8$, it does not grow with d anymore. This gives some hope that higher dimensional gravity is flexible enough to account for rather complicated ground states admitting different steps of spontaneous symmetry breaking and still is predictive enough to explain some of the relevant features of the scalar potential.

The most important source of predictions from higher dimensional gravity may be the fermionic sector. Indeed, the inclusion of a spinor coupled to d dimensional gravity does not introduce any new free parameter in the theory. Once the ground state has been fixed, the number of light fermion generations and their Yukawa couplings are in principle calculable and may therefore be predicted. Unfortunately, not much is known about the fermionic sector in Kaluza Klein theories. We still lack convincing examples of non-trivial ground states admitting fermions with masses much below the Planck mass. This remains the most serious problem for the construction of a realistic model of higher dimensional gravity. In the remainder of this talk we will discuss why we hope that a realistic Kaluza Klein theory can be constructed at least for the bosonic sector.

Before discussing in some more detail how gauge interactions come out of higher dimensional gravity, let us briefly describe the theoretical status of such a theory. Pure d dimensional gravity with spinors is presumably not a renormalizable theory. Inclusion of invariants containing more than two derivatives of the metric tensor may also lead to problems with unitarity. However, these problems appear only at length scales smaller or comparable with a characteristic length of the theory given by the couplings with dimension (mass)^N. We do not believe this theory to be valid beyond this length scale. It should rather be considered as an effective "infrared" theory for length scales large compared with this characteristic length. (It may be compared with the Fermi theory for weak interactions.) At large length scales such an effective theory contains renormalizable interactions for the massless or light particles. Renormalizability is here a *consequence* of the existence of interacting massless particles. If there are non-suppressed interactions between massless particles at very large distances (such as gauge interactions), these interactions must be renormalizable since they can be rescaled from these very large distances to distances of the order of the characteristic length scale. Furthermore, there are non-renormalizable interactions suppressed by powers of the small characteristic length.

What can (in principle) be calculated in such a theory? First one can use the classical approximation to calculate the structure of the ground state and the particle spectrum. This calculation should be reliable if the length scale characterizing the ground state is larger than the characteristic length where the effective theory breaks down. In a second step, we neglect the effects of all particles with mass of the order or larger than the inverse characteristic length scale, M_G. The contribution of an individual particle with mass $\sim M_G$ to the effective four dimensional theory is suppressed by powers of M_G^{-1}. But there are infinitely many such particles and it is difficult to estimate the total correction coming from the inclusion of all these particles. However, experience with effective lower dimensional theories indicates that at least for the long distance interactions between massless (and light) particles their contribution should be negligible. Finally, quantum fluctuations with length scales large compared with the characteristic length can be included for the renormalizable sector of the four dimensional theory. The characteristic length acts as an effective cut-off.

No assumption has been made on what happens at the characteristic length scale and beyond. The effective theory may be the infrared theory of a renormalizable theory extending to even shorter length scales. It is also possible that at the scale M_G our usual field theoretical concepts break down and that we have to find an even more complete theory. In this case, M_G is expected to be an intrinsic scale of the more complete theory and the concept of distance may loose sense beyond this characteristic length. Note that for both cases there is no need that the effective theory is renormalizable.

To illustrate these ideas, let us discuss d dimensional gravity with an action given by

$$S = -\int d^d x \cdot e \cdot (\alpha R^2 + \beta R_{\hat{\mu}\hat{\nu}} R^{\hat{\mu}\hat{\nu}} + \gamma R_{\hat{\mu}\hat{\nu}\hat{\sigma}\hat{\lambda}} R^{\hat{\mu}\hat{\nu}\hat{\sigma}\hat{\lambda}} + \delta R + \varepsilon), \tag{2}$$

where e is the determinant of the d dimensional vielbein. What is the ground state of this theory? The ground state does not necessarily minimize the Euclidean action. This is not the case in usual four dimensional gravity. Therefore the ground state of d dimensional gravity cannot correspond to a minimum of the action if the theory contains four dimensional gravity. We rather require four weaker conditions for an acceptable ground state:

a) The ground state obeys the classical field equations.
b) Four dimensional gravity is governed by a term linear in the four dimensional curvature scalar with positive Newton's constant.
c) The effective four dimensional cosmological constant vanishes. This may be achieved by an unnatural adjustment of one parameter.
d) The non-gravitational excitations in four dimensions have positive Euclidean action.

Especially, the ground state corresponds to the minimum of the scalar potential of the effective four dimensional theory.

Such a ground state has the same problems as Minkowski space as ground state of ordinary four dimensional gravity, but at least the problems are not worse. With these conditions it cannot be explained why the effective theory we observe is four dimensional.

The scalar potential in four dimensions is given by $-S$ of Eq. (2), where all quantities now refer to the $D = d$-4 dimensions of the "internal" manifold. (Note that for an action including only the curvature scalar the scalar potential is necessarily unbounded from below.) For

$$\alpha + \frac{1}{D}\beta + \frac{2}{D(D-1)}\gamma > 0, \qquad \beta + \frac{4}{D-2}\gamma > 0, \qquad \gamma > 0, \qquad \delta > 0 \tag{3}$$

one finds [4] that the minimum of the scalar potential corresponds to a D dimensional hypersphere S^D with radius L given by

$$L^2 = [2\alpha D(D-1) + 2\beta(D-1) + 4\gamma]/\delta. \tag{4}$$

To have a vanishing four dimensional cosmological constant, the parameter ε has to be adjusted:

$$\varepsilon = \frac{\delta^2}{4}\,\frac{D(D-1)}{\alpha D(D-1) + \beta(D-1) + 2\gamma}\,. \tag{5}$$

It is easy to check that the field equations are fulfilled. One also finds the right sign for the kinetic terms of the vector and scalar fields and for Planck's constant. To see this, we have to discuss excitations above this ground state.

The ground state symmetry is lower than the full symmetry of d dimensional general coordinate transformations — we have spontaneous compactification. In our case the ground state symmetry is a direct product of a local SO($D+1$) gauge group with global four dimensional Poincaré symmetry. Since the characteristic length of Minkowski space is infinite (including matter, our universe has a finite, but very large characteristic length scale today) the effective four dimensional action has not only global Poincaré invariance, but is invariant under four dimensional general coordinate transformations.

To discuss the spectrum of excitations above the ground state, we first have to reduce the d dimensional metric tensor (or equivalently the vielbein) with respect to SO($D+1$) and four dimensional general coordinate transformations. The metric tensor contains infinitely many representations of this symmetry. Its dependence on the S^D coordinates gives rise to different representations of SO($D+1$). (This can be compared to a field depending on the coordinates of the usual three space dimensions: it contains different values of angular momentum). To find the effective four dimensional action for these representations, we have to integrate the d dimensional action over the coordinates of S^D. Every representation will then correspond to a field in the four dimensional theory. Most of these infinitely many fields will have masses of the order M_G and we neglect them.

Still, we know that at least two sorts of fields must remain massless: a massless graviton is guaranteed from general coordinate invariance in four dimensions and massless gauge fields follow from local SO($D+1$) symmetry. The four dimensional gravitational interactions are obtained from the term in the action linear in the four dimensional curvature scalar R_4, putting all excitations above the ground state except the graviton ($g_{\mu\nu} \equiv g_{\mu\nu}(x)$, x: four dimensional coordinates) to zero. One finds for Newton's constant

$$\frac{1}{4\kappa^2} = \frac{\beta(D-1)+2\gamma}{\alpha D(D-1)+\beta(D-1)+2\gamma}\,\delta V_D \tag{6}$$

where V_D is the volume of S^D. Note that without the inclusion of the $R_{\hat{\mu}\hat{\nu}}R^{\hat{\mu}\hat{\nu}}$ and $R_{\hat{\mu}\hat{\nu}\hat{\sigma}\hat{\lambda}}R^{\hat{\mu}\hat{\nu}\hat{\sigma}\hat{\lambda}}$ terms in the action the Planck mass would vanish. A positive value for κ^2 requires

$$\beta+\frac{2}{D-1}\gamma > 0. \tag{7}$$

The gauge bosons correspond to the following ansatz for the vielbein

$$\begin{aligned} e_\mu^m &= e_\mu^m(x), \\ e_\alpha^m &= 0, \\ e_\mu^a &= gA_\mu^z(x)K_z^\alpha(y)\mathring{e}_\alpha^a(y), \\ e_\alpha^a &= \mathring{e}_\alpha^a(y). \end{aligned} \tag{8}$$

Here m and a are Lorentz indices for the four dimensional and the D dimensional space, respectively and x and y denote the coordinates of these spaces. The vielbein corresponding to the ground state is given by $\mathring{e}_\alpha^a(y)$. The four dimensional vielbein is $e_\mu^m(x)$ and the $D(D+1)/2$ gauge fields are given by $A_\mu(x)$. $K_z^\alpha(y)$ are the Killing vectors of S^D and g denotes the gauge coupling. With this ansatz one finds for the d dimensional curvature scalar

$$R = -D(D-1)/L^2+R_4-\tfrac{1}{4}\,g^2\mathring{g}_{\alpha\beta}K_z^\alpha K_y^\beta F_{\mu\nu}^z F^{y\mu\nu}, \tag{9}$$

where $F_{\mu\nu}^z$ is the usual field strength for non-abelian gauge fields. The gauge coupling can be calculated from the normalization of the kinetic term for the gauge fields. Neglecting corrections due to the $R_{\hat{\mu}\hat{\nu}}R^{\hat{\mu}\hat{\nu}}$ and $R_{\hat{\mu}\hat{\nu}\hat{\sigma}\hat{\lambda}}R^{\hat{\mu}\hat{\nu}\hat{\sigma}\hat{\lambda}}$ terms, one finds

$$g^2 = \frac{D+1}{2}\frac{4\kappa^2}{L^2} = \tfrac{1}{4}\frac{D+1}{D-1}\frac{1}{\beta+2\gamma/(D-1)}\frac{1}{V_D}. \tag{10}$$

The relation that g is proportional to the ratio of Planck's length over the typical length of the internal space is very general. (The proportionality factor depends on the specific ground state). This implies an important constraint: whereas experiments so far do not exclude the appearance of further dimensions at length scales as large as 10^{-16} cm, realistic Kaluza Klein theories with a gauge coupling not extremely small compared with unity predict that further dimensions will not be directly observable up to length scales of the order of the Planck length.

To conclude, higher dimensional gravity is the simplest way to unify gauge interactions with gravity: one keeps the structure of gravitational interactions and only enhances the number of dimensions to account for the gauge interactions and scalar fields as well. It may even have some predictive power for the observable parameters of the low energy theory. But the development of this theory is only at the beginning. There is still a wide gap to fill before making contact with experiment and deciding whether the theory can be tested or not.

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