ABSORPTION OF Σ HYPERONS IN NUCLEAR MATTER, Σ HYPERNUCLEI AND Σ ATOMS*

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The width $\Gamma_{NM}(\varrho, k_\Sigma)$ of the state of Σ hyperon moving with momentum k_Σ in nuclear matter of density ϱ is expressed through the ΣN scattering cross sections. Exclusion principle and dispersive effects are taken into account. Results obtained for $\Gamma_{NM}(\varrho, k_\Sigma)$ are used in estimating the width of Σ bound states in Σ hypernuclei and in Σ atoms, and the absorptive potential for Σ-nucleus scattering.

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1. Introduction

In the present paper, we present a simple theory of the observed narrow width of Σ hypernuclear states [1], of the width measured in Σ atoms [2], and of the strong absorption of Σ hyperons produced in Σ^- absorption in nuclei [3]. The only input are the experimental cross sections for ΣN scattering. The theory consists of two steps. First, we calculate the absorption of a Σ hyperon moving in symmetric ($N = Z$) nuclear matter (NM). Second, we apply the results obtained in NM to estimate the width of Σ bound states in finite systems: in Σ hypernuclei and in Σ atoms, and to estimate the absorptive potential for Σ-nucleus scattering.

In 1979 the Heidelberg-Saclay group (Bertini et al. [1]) at CERN reported the first observation of hypernuclei with Σ particles (${}^{9}_{\Sigma}$Be) in the (K, π) reaction. The shape of the observed pion spectrum suggests that the width of the Σ states $\Gamma \lesssim 8$ MeV. The existence of narrow Σ hypernuclear states was confirmed at Brookhaven (Piekarz et al. [4]) where hypernuclei ${}^{6}_{\Sigma}$H and ${}^{16}_{\Sigma}$C were observed.

At first glance, the narrowness of the Σ hypernuclear states seems surprising. In nuclear matter (NM) the Σ hyperon decays to the Λ hyperon via the strong conversion process ΣN → ΛN. The large energy and momentum releases in this process ($\Delta E \simeq 80$ MeV) suggest the use of semiclassical arguments in estimating Γ (see, e.g., Gal, Toker, and Alexan-

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der [5]). Let us consider, e.g., a Σ^- hyperon moving through NM of density ϱ. The probability per unit time, ω, of its decay into Λ is given by $\omega = \frac{1}{2}\varrho v \sigma^c$, where σ^c is the total cross section for the conversion process $\Sigma^- p \rightarrow \Lambda n$, v is the relative $\Sigma^- p$ velocity, and $\frac{1}{2}\varrho$ is the proton density. The Σ lifetime $\tau = 1/\omega$ is connected with Γ by $\tau\Gamma = \hbar$, and we obtain

$$\Gamma = \tfrac{1}{2}\varrho\hbar\langle v\sigma^c\rangle_{AV} = \tfrac{1}{2}\varrho\frac{\hbar^2}{\mu_{\Sigma N}}\langle k_{\Sigma N}\sigma^c\rangle_{AV}, \tag{1.1}$$

where $\langle\rangle_{AV}$ indicates averaging over nucleon momenta, $k_{\Sigma N}$ is the relative ΣN momentum (in units of $\hbar$), and $\mu_{\Sigma N}$ is the ΣN reduced mass ($\hbar k_{\Sigma N}/\mu_{\Sigma N} = v$).

If we use in (1) for ϱ the empirical equilibrium density of NM, $\varrho_0 = 0.166\ \text{fm}^{-3}$ (and the corresponding value of k_F (the Fermi momentum of NM), $k_{F0} = 1.35\ \text{fm}^{-1}$, in calculating $\langle\rangle_{AV}$) and the experimental cross section σ^c, we get for zero momentum of Σ (i.e., for the ground state of Σ in NM) the result: $\Gamma \simeq 24$ MeV, which is much bigger than the width measured in the observed Σ hypernuclei.

The main shortcoming of expression (1.1) is that it disregards the exclusion principle and dispersive effects (momentum dependence of the nucleon and hyperon single particle (s.p.) potentials in NM). It was shown in [6] that the two effects suppress strongly the $\Sigma\Lambda$ conversion in NM. Because of the exclusion principle the $\Sigma N \rightarrow \Lambda N$ processes, in which the final nucleon momenta are smaller than k_F, are forbidden. This suppression is further enhanced by dispersive effects which diminish the final nucleon momenta. Both effects were included in the calculation of Γ performed in [6] in the frame of the Brueckner reaction matrix theory with model D of the Nijmegen hyperon-nucleon interaction [7], with the resulting values of Γ ranging from 0 to 12 MeV (depending on the choice of the s.p. potentials of nucleons and hyperons in NM), in agreement with values suggested by the (K, π) experiments.

It was shown in [6] that an approximation in the Brueckner theory scheme enables one to express Γ directly in terms of σ^c. The resulting expression for Γ is a modification of semiclassical expression (1.1), which takes into account the exclusion principle and dispersive effects. Results obtained for Γ with this modified expression reproduce quite accurately the results of the tedious Brueckner theory calculations. The principle advantage of this procedure (apart from its computational simplicity) is that it bypasses the problem of determining the hyperon-nucleon interaction.

In the present paper, we use this modification of (1.1) to calculate the Σ width in NM. The simplicity of this procedure enables us to discuss more carefully the proper choice of the s.p. potentials in NM. Furthermore, without much effort, we obtain results for the Σ width in NM for different values of NM density ϱ, and Σ momentum k_Σ, $\Gamma_{NM}(\varrho, k_\Sigma)$.

To estimate the width of Σ bound in a finite system, we approximate it by $\Gamma_{NM}(\bar{\varrho}, \bar{k}_\Sigma)$, where $\bar{\varrho}$ and $\bar{k}_\Sigma$ are average values of ϱ and k_Σ in this system. We apply this procedure to Σ hypernuclei, and to Σ atoms, and obtain very reasonable results.

To estimate the absorptive potential W for Σ-nucleus scattering, we start with W for NM, $W_{NM}(\varrho, k_\Sigma) = -\Gamma_{tot,NM}(\varrho, k_\Sigma)/2$. The subscript "tot" indicates that here we have to consider the total cross section for ΣN scattering (for the $\Sigma\Lambda$ conversion, for elastic scattering, and for charge exchange scattering). To calculate W for finite systems, we use $W_{NM}(\varrho, k_\Sigma)$

and apply a local density approximation. For increasing energy of the scattered Σ hyperons, we obtain a strong absorption.

The paper is organized as follows. In Section 2.1, we formulate the theory of Σ width in NM, in Section 2.2, we specify the s.p. energies in NM, in Section 2.3, we discuss the energy conservation in ΣN scattering in NM, and in Section 2.4, we present our results for the conversion and elastic parts of Γ, Γ^c and Γ^e, in NM. In Section 3, we apply our NM results in calculating Γ for Σ bound states in finite systems: in Σ hypernuclei (Sec. 3.1), and in Σ atoms (Sect. 3.2). In Sect. 4 we apply our NM results to calculate the imaginary part of the Σ-nucleus optical potential. In Sect. 5, we discuss our results, compare them with experiment, and mention other approaches to the problem of the Σ width.

The main results of the present paper were reported in [8].

2. Σ *width in nuclear matter*

2.1. Formalism

To describe the YN interaction (Y = Σ, N), we use the two-channel approach with a 2×2 potential matrix:

$$\hat{v} = \begin{pmatrix} v(\Sigma N \to \Sigma N) & v(\Lambda N \to \Sigma N) \\ v(\Sigma N \to \Lambda N) & v(\Lambda N \to \Lambda N) \end{pmatrix} = \begin{pmatrix} v_{\Sigma\Sigma} & v_{\Sigma\Lambda} \\ v_{\Lambda\Sigma} & v_{\Lambda\Lambda} \end{pmatrix}. \tag{2.1}$$

The ΣΛ conversion occurs only in the isotopic spin $T = \frac{1}{2}$ state, and only in this state is the two-channel approach necessary. In the $T = \frac{3}{2}$ state, the only nonvanishing component of $\hat{v}$ is $v_{\Sigma\Sigma}$, and only the ΣN channel exists. To simplify the presentation, we suppress spin and isospin in most of our equations.

According to the Brueckner theory (see [6], and references quoted therein), the energy E_Σ of a Σ hyperon (with momentum $\boldsymbol{k}_\Sigma$) in NM is:

$$E_\Sigma = \hbar^2 k_\Sigma^2/2M_\Sigma + 4\int^{<k_F} \frac{d\boldsymbol{k}_N}{(2\pi)^3} \langle \boldsymbol{k}_{\Sigma N} | \mathscr{K}_{\Sigma\Sigma} | \boldsymbol{k}_{\Sigma N} \rangle, \tag{2.2}$$

where $\boldsymbol{k}_N$ is the nucleon momentum, and $\boldsymbol{k}_{\Sigma N}$ is the relative ΣN momentum,

$$\boldsymbol{k}_{\Sigma N} = \mu_{\Sigma N}(\boldsymbol{k}_N/M_N - \boldsymbol{k}_\Sigma/M_\Sigma). \tag{2.3}$$

The factor 4 takes care of the four nucleons (neutron and proton, each with spin up and down) occupying each momentum state in NM.

The reaction matrix $\mathscr{K}_{\Sigma\Sigma}$ in the ΣN channel, together with the reaction matrix $\mathscr{K}_{\Lambda\Lambda}$ in the ΛN channel and the transition matrix $\mathscr{K}_{\Lambda\Sigma}$, satisfies the coupled equations:

$$\mathscr{K}_{\Sigma\Sigma} = v_{\Sigma\Sigma} + v_{\Sigma\Sigma} \frac{Q_\Sigma}{\alpha_\Sigma + i\varepsilon} \mathscr{K}_{\Sigma\Sigma} + v_{\Sigma\Lambda} \frac{Q_\Lambda}{\alpha_\Lambda + i\varepsilon} \mathscr{K}_{\Lambda\Sigma},$$

$$\mathscr{K}_{\Lambda\Sigma} = v_{\Lambda\Sigma} + v_{\Lambda\Sigma} \frac{Q_\Sigma}{\alpha_\Sigma + i\varepsilon} \mathscr{K}_{\Sigma\Sigma} + v_{\Lambda\Lambda} \frac{Q_\Lambda}{\alpha_\Lambda + i\varepsilon} \mathscr{K}_{\Lambda\Sigma}, \tag{2.4}$$

where

$$\alpha_\Sigma = e_N(k_N)+e_\Sigma(k_\Sigma)-e_N(k'_N)-e_\Sigma(k'_\Sigma),$$

$$\alpha_\Lambda = e_N(k_N)+e_\Sigma(k_\Sigma)-e_N(k'_N)-e_\Lambda(k'_\Lambda)+\Delta, \tag{2.5}$$

where $\Delta = (M_\Sigma - M_\Lambda)c^2$, e_N and e_Y are nucleon and hyperon s.p. energies in NM (see Sect. 2.2), and k'_N, k'_Y are momenta of the particles in the intermediate states (summation over these states is implied in Eqs (2.4)). By Q_Y we denote the exclusion principle operator in the YN channel (a projection operator onto nucleon states above the Fermi sea).

Singularities are expected to appear in both Q_Y/α_Y (Y = Σ,Λ). In other words, real energy conserving transitions ΣN → ΣN and ΣN → ΛN are expected to occur. Only for $k_\Sigma = 0$ (the case considered in [6]) the first possibility, ΣN → ΣN, is excluded. The infinitesimal parameter $+i\varepsilon$ guarantees that only outgoing waves appear in states degenerate with our initial state (ground state of NM+Σ with momentum $\boldsymbol{k}_\Sigma$). This means, we consider the decay of our initial state, whose Γ is:

$$\Gamma = -2\,\mathrm{Im}\,E_\Sigma = -4\int \frac{d\boldsymbol{k}_N}{(2\pi)^3}\, 2\,\mathrm{Im}\,\langle \boldsymbol{k}_{\Sigma N}|\mathscr{K}_{\Sigma\Sigma}|\boldsymbol{k}_{\Sigma N}\rangle. \tag{2.6}$$

Now, Eq. (2.4) imply the optical theorem:

$$-2\,\mathrm{Im}\,\langle \boldsymbol{k}_{\Sigma N}|\mathscr{K}_{\Sigma\Sigma}|\boldsymbol{k}_{\Sigma N}\rangle = (2\pi)^{-2}\int d\boldsymbol{k}'_{\Lambda N} Q_\Lambda(\boldsymbol{K}, \boldsymbol{k}'_{\Lambda N})\delta(\alpha_\Lambda)\, |\langle \boldsymbol{k}'_{\Lambda N}|\mathscr{K}_{\Lambda\Sigma}|\boldsymbol{k}_{\Sigma N}\rangle|^2$$

$$+(2\pi)^{-2}\int d\boldsymbol{k}'_{\Sigma N} Q_\Sigma(\boldsymbol{K}, \boldsymbol{k}'_{\Sigma N})\delta(\alpha_\Sigma)\, |\langle \boldsymbol{k}'_{\Sigma N}|\mathscr{K}_{\Sigma\Sigma}|\boldsymbol{k}_{\Sigma N}\rangle|^2, \tag{2.7}$$

where $\boldsymbol{K}$ is the conserved total YN momentum,

$$\boldsymbol{K} = \boldsymbol{k}_N+\boldsymbol{k}_\Sigma = \boldsymbol{k}'_N+\boldsymbol{k}'_Y \tag{2.8}$$

and $\boldsymbol{k}'_{YN}$ are the relative YN momenta,

$$\boldsymbol{k}'_{YN} = \mu_{YN}(\boldsymbol{k}'_N/M_N - \boldsymbol{k}_Y/M_N), \tag{2.9}$$

where μ_{YN} is the reduced YN mass. Since

$$\boldsymbol{k}'_N = (\mu_{YN}/M_Y)\boldsymbol{K}+\boldsymbol{k}'_{YN}, \tag{2.10}$$

the exclusion principle operators Q_Y in the total and relative momentum representation are:

$$Q_Y(\boldsymbol{K}, \boldsymbol{k}'_{\Sigma N}) = \begin{cases} 1 & \text{for} \quad |(\mu_{YN}/M_Y)\boldsymbol{K}+\boldsymbol{k}'_{YN}| > k_F, \\ 0 & \text{otherwise.} \end{cases} \tag{2.11}$$

The δ functions in (2.7) lead to energy conservation equations, $\alpha_Y = 0$, which fix the value of k'_{YN} (see Sect. 2.2). Notice that the s.p. energies in expression (2.5) for α_Y contain momentum dependent s.p. potentials, and consequently, in Eq. (2.7), we have in general $k'_{\Sigma N} \neq k_{\Sigma N}$.

Let us introduce the total cross section for the conversion process, $\Sigma N \to \Lambda N$, in NM:

$$\sigma^c_{NM}(\Sigma N) = \frac{k'_{\Lambda N}\mu_{\Sigma N}}{k_{\Sigma N}\mu_{\Lambda N}}\left(\frac{\mu_{\Lambda N}}{2\pi\hbar^2}\right)^2 \int d\hat{k}'_{\Lambda N}|\langle k'_{\Lambda N}|\mathscr{K}_{\Sigma\Sigma}|k_{\Sigma N}\rangle|^2 \tag{2.12}$$

and the total cross section for the elastic scattering, $\Sigma N \to \Sigma N$, in NM:

$$\sigma^e_{NM}(\Sigma N) = \frac{k'_{\Sigma N}}{k_{\Sigma N}}\left(\frac{\mu_{\Sigma N}}{2\pi\hbar^2}\right)^2 \int d\hat{k}'_{\Sigma N}|\langle k'_{\Sigma N}|\mathscr{K}_{\Sigma\Sigma}|k_{\Sigma N}\rangle|^2. \tag{2.13}$$

If we put $Q_Y = 1$ in Eq. (2.4), and use for the s.p. energies in Eqs (2.5) pure kinetic energies, then Eq. (2.4) become equations for free ΣN scattering, matrices, $\mathscr{K}^0_{Y\Sigma}$, and expressions (2.12) and (2.13) become expressions for the total cross sections for the conversion process, $\sigma^c(\Sigma N)$, and for the elastic scattering, $\sigma^e(\Sigma N)$, for an isolated ΣN pair.

With the help of Eqs (2.12), (2.13), and (2.7), we may write expression (2.6) in the form:

$$\Gamma = \Gamma^c + \Gamma^e, \tag{2.14}$$

where

$$\Gamma^c = \hbar^4 4\int \frac{dk_N}{(2\pi)^3}\frac{k_{\Sigma N}}{\mu_{\Sigma N}}\int dk'_{\Lambda N}\frac{k'_{\Lambda N}}{\mu_{\Lambda N}} Q_\Lambda(K, k'_{\Lambda N})\delta(\alpha_\Lambda)\sigma^c_{NM}(\Sigma N), \tag{2.15}$$

$$\Gamma^e = \hbar^2 4\int \frac{dk_N}{(2\pi)^3}\frac{k_{\Sigma N}}{\mu_{\Sigma N}}\int dk'_{\Sigma N}\frac{k'_{\Sigma N}}{\mu_{\Sigma N}} Q_\Sigma(K, k'_{\Sigma N})\delta(\alpha_\Sigma)\sigma^e_{NM}(\Sigma N). \tag{2.16}$$

In deriving expressions (2.15) and (2.16), we have approximated the exclusion principle operators $Q_Y(\boldsymbol{K}, \boldsymbol{k}'_{YN})$ by their angle averages $Q_Y(K, k'_{YN})$ (or equivalently, neglected the angular dependence of the cross sections),

$$Q_Y(K, k) = \begin{cases} 1 & \text{for} \quad |k-\mu_{YN}K/M_Y| > k_F, \\ 0 & \text{for} \quad k+\mu_{YN}K/M_Y < k_F, \\ [(k+\mu_{YN}K/M_Y)^2-k_F^2]/(4\mu_{YN}Kk/M_N) & \text{otherwise.} \end{cases} \tag{2.17}$$

With the sign of the absolute value in the definition of the range for which $Q_Y = 1$, expression (2.17) is valid for any magnitude of k_Σ. (The sign is not present in the expression for Q_Y given in [6], where only the case of $k_\Sigma = 0$ was considered.)

In the effective mass approximation, explained in Sect. 2.2,

$$\alpha_Y = -\hbar^2 k'^2_{YN}/(2\mu_{YN}\tilde{\nu}) + \text{terms indep. of } k'_{YN}, \tag{2.18}$$

where $\tilde{\nu}$ is the ratio of the effective to the real mass, assumed to be equal for Σ, Λ, and N (see Eqs. (2.40) and (2.43). With this form of α_Y, we may perform the k'_{YN} integrations in (2.15) and (2.16), and we obtain

$$\Gamma^c = \hbar^2\tilde{\nu}4/(2\pi)^3 \int dk_N(k_{\Sigma N}/\mu_{\Sigma N})Q_\Lambda(K, k'_{\Sigma N})\sigma^c_{NM}(\Sigma N), \tag{2.19}$$

$$\Gamma^e = \hbar^2\tilde{\nu}4/(2\pi)^3 \int dk_N(k_{\Sigma N}/\mu_{\Sigma N})Q_\Sigma(K, k'_{\Sigma N})\sigma^e_{NM}(\Sigma N), \tag{2.20}$$

where $k'_{\Lambda N}$ and $k'_{\Sigma N}$ are determined by the respective energy conservation equations, $\alpha_\Lambda = 0$ and $\alpha_\Sigma = 0$.

Now, we make the crucial approximation,

$$\sigma^x_{NM}(\Sigma N) \cong \sigma^x(\Sigma N), \qquad x = c, e, \tag{2.21}$$

which inserted into Eqs. (2.19) and (2.20) enables us to express Γ^c and Γ^e through the experimental cross sections, σ^c and σ^e. One might argue that $\mathscr{K}^0_{Y\Sigma}$ differs from $\mathscr{K}_{Y\Sigma}$ by terms of at least quadratic order in $\mathscr{K}^0_{Y\Sigma}$. Since expressions (2.19) and (2.20) are already of second order in $\mathscr{K}_{YN}$, approximations (2.21) introduce an error of third order in $\mathscr{K}^0_{Y\Sigma}$. In [6], approximation $\sigma^c_{NM} \cong \sigma^c$ turned out to work remarkably well. As far as approximation $\sigma^e_{NM} \cong \sigma^e$ is concerned, an analogical approximation applied by Lane and Wandel [9] in their calculation of the imaginary part of the low energy nucleon-nucleus optical potential led to surprisingly good results. Still, it is hard to present a fully convincing justification of approximation (2.21), especially at low energies which we consider. Being aware of this difficulty, Lane and Wandel called their approach "frivolous". (Actually, our expression (2.24) with $\sigma^e_{NM} \cong \sigma^e$ takes into account not only the Pauli principle but also dispersive effects (the factor $\tilde{\nu}$), and corresponds to the "modified frivolous model" of the nuclear optical potential, discussed in [10].)

So far, the two isospin states of a nucleon in NM, proton and neutron, were taken into account by a factor 2 which together with the two spin states produced the factor 4 multiplying the k_N integration. However, the reaction matrices which describe the interaction, and the corresponding cross sections $\sigma(NM)$, are different for Σp and Σn pairs. This means that we should substitute for $2\sigma^c_{NM}(\Sigma N) \cong 2\sigma^c(\Sigma N)$ in (2.19), and for $2\sigma^e_{NM}(\Sigma N) \cong 2\sigma^e(\Sigma N)$ in (2.20):

$$2\sigma^x(\Sigma N) = \sigma^x(\Sigma p) + \sigma^x(\Sigma n), \qquad x = c, e, \tag{2.22}$$

where for Σ^- we have:

$$\sigma^c(\Sigma^- p) = \sigma(\Sigma^- p \to \Lambda n), \qquad \sigma^c(\Sigma^- n) = 0,$$

$$\sigma^e(\Sigma^- p) = \sigma(\Sigma^- p \to \Sigma^- p) + \sigma(\Sigma^- p \to \Sigma^0 n),$$

$$\sigma^e(\Sigma^- n) = \sigma(\Sigma^- n \to \Sigma^- n), \tag{2.23}$$

where the cross sections on the right hand side are total cross sections for unpolarized particles. Notice that $\sigma^e(\Sigma^- p)$ contains cross sections for both elastic and charge exchange scattering.

For Σ^0, we have:

$$\sigma^c(\Sigma^0 p) = \sigma(\Sigma^0 p \to \Lambda p), \qquad \sigma^c(\Sigma^0 n) = \sigma(\Sigma^0 n \to \Sigma^0 \Lambda),$$

$$\sigma^e(\Sigma^0 p) = \sigma(\Sigma^0 p \to \Sigma^0 p) + \sigma(\Sigma^0 p \to \Sigma^+ n),$$

$$\sigma^e(\Sigma^0 n) = \sigma(\Sigma^0 n \to \Sigma^0 n) + \sigma(\Sigma^0 n \to \Sigma^- p). \tag{2.24}$$

Expressions for Σ^+ may be obtained from Eqs (2.22) by subsituting $\Sigma^+ \to \Sigma^-$, p → n, and n → p. Notice that if isospin is conserved, we have

$$\sigma^x(\Sigma^+N) = \sigma^x(\Sigma^-N) = \sigma^x(\Sigma^0N), \quad x = c, e. \tag{2.25}$$

With the substitutions (2.22), expressions (2.19) and (2.20) (with approximation (2.21)) take the form:

$$\Gamma^c = \hbar^2\tilde{v}2/(2\pi)^3 \int d\boldsymbol{k}_N(k_{\Sigma N}/\mu_{\Sigma N})Q_\Lambda(K, k'_{\Lambda N})\,[\sigma^c(\Sigma p)+\sigma^c(\Sigma n)]$$
$$= \varrho\hbar^2\tilde{v}/(2\mu_{\Sigma N})\,\langle Q_\Lambda(K, k'_{\Lambda N})k_{\Sigma N}[\sigma^c(\Sigma p)+\sigma^c(\Sigma n)]\rangle_{AV}, \tag{2.26}$$

$$\Gamma^e = \hbar^2\tilde{v}2/(2\pi)^3 \int d\boldsymbol{k}_N(k_{\Sigma N}/\mu_{\Sigma N})Q_\Sigma(K, k'_{\Sigma N})\,[\sigma^e(\Sigma p)+\sigma^e(\Sigma n)]$$
$$= \varrho\hbar^2\tilde{v}/(2\mu_{\Sigma N})\,\langle Q_\Sigma(K, k'_{\Sigma N})k_{\Sigma N}[\sigma^e(\Sigma p)+\sigma^e(\Sigma n)]\rangle_{AV}, \tag{2.27}$$

where $\langle\rangle_{AV}$ denotes the average value in the Fermi sea.

Eqs. (2.26) and (2.27) are the basic expressions for our calculations. They are of the form of the semiclassical expressions, except for the appearance of the exclusion principle operators Q_Y, and the effective mass factor $\tilde{v}$ which together with the energy conservation equations $\alpha_Y = 0$ takes care of the dispersive effects.

2.2. The single particle energies

For the s.p. energy of nucleons in the Fermi sea, we use the approximation:

$$e_N(k_N \leqslant k_F) \cong \varepsilon_N(k_N)+\langle V_N\rangle_{AV}, \tag{2.28}$$

where ε_N denotes the nucleon kinetic energy,

$$\varepsilon_N(k_N) = \hbar^2k_N^2/2M_N, \tag{2.29}$$

and V_N is the nucleon s.p. potential, approximated in (2.28) by its average value in the Fermi sea, $\langle V_N\rangle_{AV}$. We assume that $\langle V_N\rangle_{AV}$ leads to the empirical energy per nucleon in the ground state of NM, E/A,

$$\tfrac{3}{5}\,\varepsilon_N(k_F)+\tfrac{1}{2}\,\langle V_N\rangle_{AV} = E/A. \tag{2.30}$$

To determine E/A as a function of the density ϱ of NM, or of the Fermi momentum k_F ($\varrho = 2k_F^3/3\pi^2$), we write E/A in the simple form:

$$E/A \equiv f(k_F) = \tfrac{3}{5}\,\varepsilon_N(k_F)+bk_F^3+ck_F^4, \tag{2.31}$$

and require that NM saturates at $k_F = k_{F0} = 1.35\ \text{fm}^{-1}$ with $f(k_{F0}) = -15.8$ MeV. This requirement fixes the values of $b = -44.1$ MeV fm^3 and $c = 21.1$ MeV fm^4. The NM compressibility obtained with expression (2.31), $k_{F0}^2[d^2f/dk_F^2]_0 = 240$ MeV, agrees nicely with empirical estimates (see, e.g., the review by Blaizot [11]).

From Eqs. (2.31) and (2.30), we get

$$\langle V_N\rangle_{AV} = 2bk_F^3+2ck_F^4. \tag{2.32}$$

For $k_N > k_F$, we use the effective mass approximation,

$$e_N(k_N \geq k_F) = \hbar k_N^2/2M_N^* - \hbar^2 k_F^2/2M_N^* + e_N(k_F), \tag{2.33}$$

where $e_N(k_F)$ is assumed to be equal to the nucleon separation energy $\partial E/\partial A$:

$$e_N(k_F) = \partial E/\partial A = E/A + \tfrac{1}{3} k_F d(E/A)/dk_F. \tag{2.34a}$$

With the help of Eq. (2.31), we get

$$e_N(k_F) = \varepsilon_N(k_F) + 2bk_F^3 + (7/3)ck_F^4. \tag{2.34b}$$

To determine $M_N^*/M_N = \tilde{\nu}_N$, we write Eq. (2.34a) in the form $e_N = \varepsilon_N + V_N$, and get

$$\hbar^2/M_N^* = \hbar^2/M_N + k_N^{-1}\partial V_N(k_N)/\partial k_N. \tag{2.35}$$

If we make a reasonable assumption that $V_N(k_N)$ depends linearly on the density ϱ, we get

$$\tilde{\nu}_N = 1/[1 + (\nu_N^{-1} - 1)\varrho/\varrho_0], \tag{2.36}$$

where ν_N is the value of $\tilde{\nu}_N$ at the equilibrium density ϱ_0. In our calculations, we use Eq. (2.36) with the value $\nu_N = 0.7$ which is compatible with theoretical estimates and with the empirical energy dependence of the real part of the nuclear optical potential [12].

For the hyperon s.p. energies, we apply the effective mass approximation,

$$e_Y(k_Y) = (1/\tilde{\nu}_Y)\varepsilon_Y(k_Y) + D, \tag{2.37}$$

where ε_Y is the hyperon kinetic energy, D the hyperon well depth in NM, and $\tilde{\nu}_Y = M_Y^*/M_Y$. In (2.37), we use a common value of D for both hyperons Λ and Σ which approximately corresponds to the experimental indications. For the density dependence of the effective mass parameter $\tilde{\nu}_Y$, we use the form analogous to Eq. (2.36). Furthermore, we make the simplifying assumption:

$$\tilde{\nu}_\Lambda = \tilde{\nu}_\Sigma = \tilde{\nu}_N \equiv \tilde{\nu}, \tag{2.38}$$

which appears reasonable. According to Bando and Nagata [13] $\tilde{\nu}_\Lambda = 0.8$, and according to Chong, Nogami and Satoh [14] $\tilde{\nu}_\Lambda = 0.7$. And for ν_Σ, we expect a similar value.

2.3. Energy conservation in ΣN scattering in NM

We start with the $\Sigma N \to \Lambda N$ conversion process, for which we write the energy conservation equation, $\alpha_\Lambda = 0$, in the form:

$$e_N(k'_N) + e_\Lambda(k'_\Lambda) = e_N(k_N) + e_\Sigma(k_\Sigma) + \Delta. \tag{2.39}$$

We substitute for $e_N(k_N)$ expression (2.28), for $e_N(k'_N)$ expression (2.33), and for $e_Y(k_Y)$ expression (2.37), and express the momenta k_N, k_Σ, k'_N, and k'_Λ through the relative momenta $k_{\Sigma N}$, Eq. (2.3), and $k'_{\Lambda N}$, Eq. (2.9), and through the conserved total momentum K, Eq. (2.8). In this way Eq. (2.39) takes the form:

$$\hbar^2 k_{\Lambda N}'^2/2\mu_{\Lambda N}\tilde{\nu} = \hbar^2 k_{N\Sigma}^2/2\mu_{\Sigma N}$$
$$-\hbar^2 K^2[M_\Sigma/\tilde{\nu}+(1/\tilde{\nu}-1)M_N-M_\Lambda]/[2(M_N+M_\Sigma)(M_N+M_\Lambda)]+\Delta^*, \tag{2.40}$$

where

$$\Delta^* = \Delta+\langle V_N\rangle_{AV}-e_N(k_F)+\varepsilon_N(k_F)/\tilde{\nu}+(1/\tilde{\nu}-1)\varepsilon_\Sigma(k_F), \tag{2.41}$$

where $e_N(k_F)$ is given by Eq. (2.33).

The energy conservation equation for the $\Sigma N \to \Sigma N$ scattering, $\alpha_\Sigma = 0$, may be written in the form:

$$e_N(k_N')+e_\Sigma(k_\Sigma') = e_N(k_N)+e_\Sigma(k_\Sigma). \tag{2.42}$$

Proceeding in the same way, as with Eq. (2.39), we obtain

$$\hbar^2 k_{\Sigma N}'^2/2\mu_{\Sigma N}\tilde{\nu} = \hbar^2 k_{\Sigma N}^2/2\mu_{\Sigma N}-\hbar^2 K^2(1/\tilde{\nu}-1)/2(M_N+M_\Sigma)$$
$$+\langle V_N\rangle_{AV}-e_N(k_F)+\varepsilon_N(k_F)/\tilde{\nu}+(1/\tilde{\nu}-1)\varepsilon_\Sigma(k_F), \tag{2.43}$$

where $e_N(k_F)$ is given by Eq. (2.34b).

Notice that with the help of Eqs. (2.3) and (2.8), we may express K in terms of k_N, $k_{N\Sigma}$, and k_Σ:

$$K = [(k_N^2/M_N+k_\Sigma^2/M_\Sigma-k_{\Sigma N}^2(\mu_{\Sigma N})(M_N M_\Sigma/\mu_{\Sigma N})]^{1/2}. \tag{2.44}$$

Consequently, $k_{\Lambda N}'$ and $k_{\Sigma N}'$ are determined by energy equations (2.40) and (2.43) as functions of k_N and $k_{\Sigma N}$ (for a fixed value of k_Σ).

2.4. Results for Γ^c and Γ^e in NM

If isospin is conserved, the widths of Σ^-, Σ^+, and Σ^0 in symmetrical NM are the same (see Eq. (2.25)). Here, we specialize to Σ^0 for which the experimental cross sections are best known.

To calculate Γ^c, Eq. (2.26), we need the experimental total cross section for $\Sigma^- p \to \Lambda n$. We use the parametrization

$$(v/c)\sigma(\Sigma^- p \to \Lambda n) = (1+13v/c)^{-1}\, 5.1\ \mathrm{fm}^2, \tag{2.45}$$

where v is the relative $\Sigma^- p$ velocity, i.e.,

$$v/c = (\hbar/\mu_{\Sigma N}c)k_{\Sigma N}. \tag{2.46}$$

This parametrization was adjusted by Gal, Toker, and Alexander [5] to the whole Σ^- low energy regime up to 300 MeV/c in the laboratory system.

To calculate Γ^e, Eq. (2.27), we need the sum of the total elastic and charge exchange cross sections:

$$\sigma^e(\Sigma^-) \equiv \sigma^e(\Sigma^- p)+\sigma^e(\Sigma^- n) = \sigma(\Sigma^- p \to \Sigma^- p)+\sigma(\Sigma^- p \to \Sigma^0 n)$$
$$+\sigma(\Sigma^- n \to \Sigma^- n) = \sigma(\Sigma^- p \to \Sigma^- p)+\sigma(\Sigma^+ p \to \Sigma^+ p)+\sigma(\Sigma^- p \to \Sigma^0 n), \tag{2.47}$$

where in the last step we assumed isospin invariance, and replaced $\sigma(\Sigma^- n \to \Sigma^- n)$ by $\sigma(\Sigma^+ p \to \Sigma^+ p)$.

As the experimental cross sections appearing in the last part of Eq. (2.46), we consider the cross sections tabulated by Rijken [15] for Σ laboratory momenta up to 1000 MeV/c. They represent an extrapolation (by means of the Nijmegen baryon-baryon interaction) of the measured cross sections, known for Σ laboratory momenta below 200 MeV/c. For the sum of the three cross sections in Eq. (2.46), we use the parametrization:

$$k^2\sigma^e(\Sigma^-) = 24+41k-[24+(k/0.14)^2]\exp\{-(k/1.2)^2\}$$
$$+18\exp\{-[(k-1.8)/0.4]^2\}, \tag{2.48}$$

where k is the Σ laboratory momentum (in fm^{-1}),

$$k = (M_N/\mu_{\Sigma N})k_{\Sigma N}. \tag{2.49}$$

The form of expression (2.48) is slightly complicated due to the behaviour of the "experimental" cross section $\sigma(\Sigma^+ p \to \Sigma^+ p)$ which shows two maxima (see Fig. 3 of Ref. [7]).

To obtain Γ^c and Γ^e, Eqs. (2.19) and (2.20), we have to perform k_N-integrations. The cross sections, Eqs. (2.45) and (2.48) are functions of $k_{\Sigma N}$. The momenta $k'_{\Lambda N}$ and $k'_{\Sigma N}$ (the arguments of the Q_Y operators) are determined by k_N and $k_{\Sigma N}$ through energy conservation Eqs (2.40) and (2.43). Also the total momentum K is expressed by k_N and $k_{\Sigma N}$, Eq. (2.44). Thus the k_N-integrations involve functions depending on k_N and $k_{\Sigma N}$, and we perform the integration numerically, by applying the formula:

$$\int dk_N \mathscr{F}(k_N, k_{\Sigma N}) = (M_N M_\Sigma/\mu_{\Sigma N}^2)(2\pi/k_\Sigma)\int_0^{k_F} dk_N k_N \int_{\beta(-)}^{\beta(+)} dk_{\Sigma N} k_{\Sigma N} \mathscr{F}(k_N, k_{\Sigma N}). \tag{2.50}$$

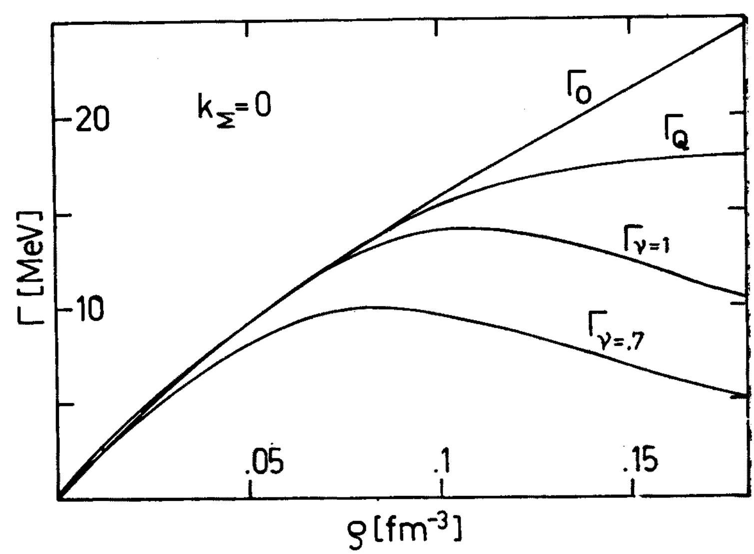

Fig. 1. The width Γ of Σ ground state in NM as function of NM density ρ

where

$$\beta(\pm) = \mu_{\Sigma N}|k_N/M_N \pm k_\Sigma/M_\Sigma|. \tag{2.51}$$

First, let us present our results for Γ^c. For the sake of comparison, we calculate Γ^c not only for $\nu = 0.7$ but also for $\nu = 1$. Furthermore, we also consider the case in which we take for all the s.p. energies pure kinetic energies, but retain the exclusion principle operator Q_Λ. In this case, we denote the width by Γ^c_Q. If in addition, we disregard the exclusion principle, we use the notation Γ^c_0.

Our results for Γ^c as a function of ϱ, for $k_\Sigma = 0$, are shown in Fig. 1. Notice that in the ground state of the Σ+NM system, $k_\Sigma = 0$, we have $\Gamma^e = 0$, and $\Gamma = \Gamma^c$, i.e., the total width is entirely due to the conversion process. Whereas $\Gamma_0 = \Gamma^c_0$ increases

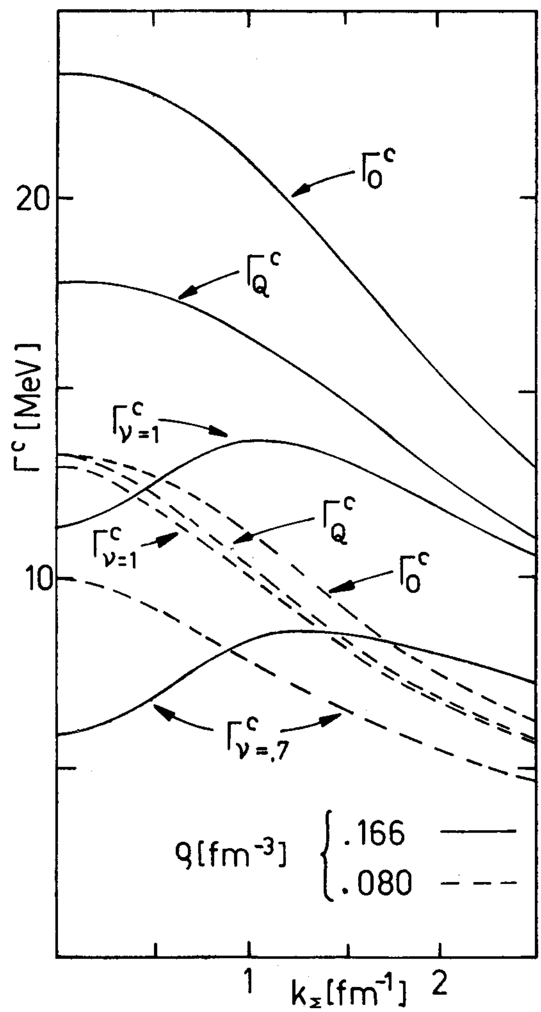

Fig. 2. The width Γ^c of Σ in NM as function of Σ momentum at two NM densities

approximately linearly with ϱ, $\Gamma_Q = \Gamma_Q^c$ increases much slower at high density where the exclusion principle becomes important. The role of the exclusion principle is further enhanced by dispersive effects present in Γ_ν. This, combined with the decrease in $\sigma(\Sigma^- p \to \Lambda n)$ with increasing Σ momentum, leads to the decrease in Γ_ν with increasing ϱ at high densities. At equilibrium density $\varrho_0 = 0.166\ \mathrm{fm}^{-3}$ ($k_F = 1.35\ \mathrm{fm}^{-1}$) we have: $\Gamma_0 = 23.5$ MeV, $\Gamma_Q = 17.8$ MeV, $\Gamma_{\nu=1} = 11.4$ MeV, and $\Gamma_{\nu=0.7} = 5.9$ MeV. The results are obviously sensitive to the value of ν. E.g., an acceptable value of $\nu = 0.6$ would lead to $\Gamma_{\nu=0.6} = 4.4$ MeV at $\varrho = \varrho_0$. Consequently, our value of $\nu = 0.7$, which we shall keep in the following calculations, should be considered as a conservative estimate of the dispersive effects.

The calculated dependence of Γ^c on k_Σ for $\varrho = \varrho_0$ and $\varrho = 0.08\ \mathrm{fm}^{-3}$ is shown in Fig. 2. With increasing momentum $\sigma(\Sigma^- p \to \Lambda n)$ decreases, and consequently Γ_0^c is a decreasing function of k_Σ. On the other hand the exclusion principle, enhanced by dispersive effects, is most effective in reducing Γ^c at $k_\Sigma = 0$, and this reduction is stronger at higher densities. This explains the difference in the behaviour of Γ_Q^c and Γ_ν^c compared to that of Γ_0^c at the two densities considered.

Results for Γ^e as a function of k_Σ at $\varrho = \varrho_0$ and $\varrho = 0.08\ \mathrm{fm}^{-3}$ are shown in Fig. 3 which contains also a plot of Γ^c and $\Gamma = \Gamma^e + \Gamma^c$. All the curves were obtained with $\nu = 0.7$. At small Σ momenta $\Gamma^e \ll \Gamma^c$ because here, especially at high densities, Γ^e is almost completely suppressed by the exclusion principle enhanced by dispersive effects. (Actually the suppression at very small momenta is exaggerated by our crude approximation of the s.p.

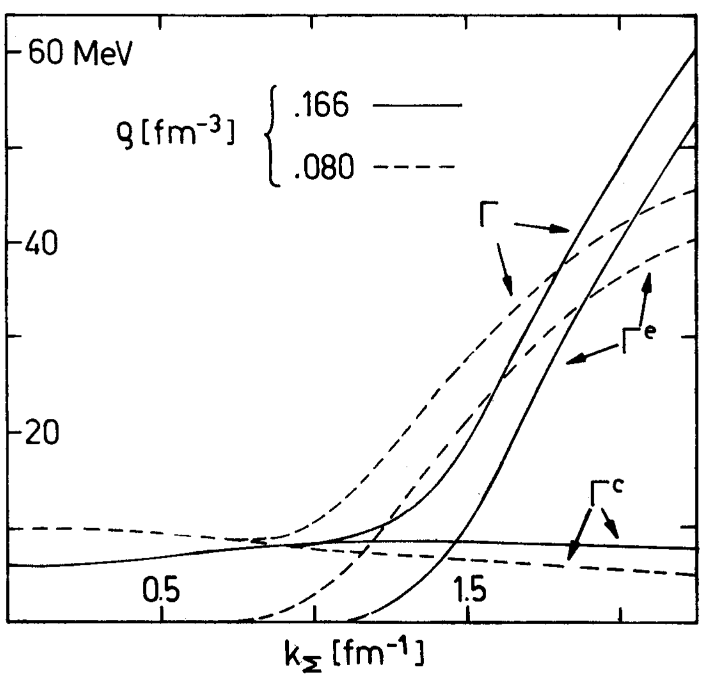

Fig. 3. The widths Γ^c, Γ^e, and Γ of Σ in NM as function of Σ momentum at two NN densities

nucleon potential $V_N(k_N \leqslant k_F) \cong \langle V_N \rangle_{AV}$, Eq. (2.26).) At higher momenta, where the Pauli blocking is not important, $\Gamma^e \gg \Gamma^c$ because here the total elastic cross section is much bigger than the conversion cross section (of course Γ^e reflects the absorption of Σ due to elastic scattering and has nothing to do with the life time of Σ in NM).

Notice that $k_{\Sigma N}$ in expression (2.27) for Γ^e is determined from energy conservation Eq. (2.43), and consequently $k'_{\Sigma N} < k_{\Sigma N}$. This decrease in the relative ΣN momentum in ΣN elastic scattering in NM was disregarded in Ref. [8]. For this reason our present results for Γ^e (and the results for the absorptive Σ-nucleus potential presented in Sect. 4) differ from the results presented in [8], especially at small Σ momenta.

3. *The width of Σ bound states in finite systems*

As the result of Sect. 2, we have at our disposal the width of Σ states in NM as a function of ϱ and k_Σ,

$$\Gamma_{NM}(\varrho, k_\Sigma) = \Gamma^c_{NM}(\varrho, k_\Sigma) + \Gamma^e_{NM}(\varrho, k_\Sigma), \tag{3.1}$$

where by $\Gamma^{c(e)}_{NM}$ we denote the value of $\Gamma^{c(e)}$ obtained in Sect. 2 with $\nu = 0.7$. To estimate the width Γ of a state of Σ bound in a finite system in which the nuclear density is given by a function $\varrho(r)$, and the Σ state is described by a wave function $\Psi_\Sigma(\boldsymbol{r})$, we approximate Γ by

$$\Gamma \cong \Gamma^c_{NM}(\bar{\varrho}, \bar{k}_\Sigma), \tag{3.2}$$

where the average density and Σ momentum, $\bar{\varrho}$ and $\bar{k}_\Sigma$, are defined by

$$\bar{\varrho} = \int d\boldsymbol{r} \varrho(r) |\Psi_\Sigma(\boldsymbol{r})|^2, \tag{3.3}$$

$$\bar{k}^2_\Sigma / 2M_\Sigma = \langle \Psi_\Sigma | T_\Sigma | \Psi_\Sigma \rangle, \tag{3.4}$$

where T_Σ is the operator of Σ kinetic energy. (Obviously, in applying Eqs. (3.2)–(3.4), the CM motion has to be taken into account.) This approximate way of calculating Γ was used successfully by Köhler [16] in explaining the width of deep hole states in nuclei.

Notice that in Eq. (3.2) we use Γ^c_{NM} and disregard Γ^e_{NM}. Practically disregarding Γ^c_{NM} is of no importance as $\Gamma^e_{NM} \ll \Gamma^c_{NM}$ for the relavant values of k_Σ. Also in principle Γ^e_{NM} should be disregarded. Imagine that we calculate Γ for the ground state of a Σ hypernucleus. In the absence of the conversion process ($\Gamma^c_{NM} = 0$) the width Γ of this state should be zero. However, by keeping Γ^e_{NM} in Eq. (3.2) we would get a nonvanishing Γ. Obviously, elastic processes in a state, which is not coupled to the continuum, cannot cause the state to acquire a finite width.

Now, we will apply approximation (3.2) to Σ hypernuclei and Σ atoms.

3.1. Σ hypernuclei

We describe a Σ hypernucleus with a nuclear core with A nucleons by a s.p. model in which $\Psi_\Sigma(\boldsymbol{r})$ is determined by the Schroedinger equation

$$\left\{ -\frac{\hbar^2}{2\mu_{\Sigma A}} \Delta + V(r) \right\} \Psi_\Sigma(\boldsymbol{r}) = E \Psi_\Sigma(\boldsymbol{r}), \tag{3.5}$$

where $\boldsymbol{r}$ is the vector from the nuclear core to the Σ hyperon, $\mu_{\Sigma A} = M_\Sigma M_A/(M_\Sigma + M_A)$, and E is the Σ energy ($-E$ = Σ binding energy). For the mass of the nuclear core, M_A, we use the value $M_A = 931\ A\ \mathrm{MeV}/c^2$.

For the s.p. potential $V(r)$, we use

$$V(r) = -V_0 \varrho(r)/\varrho_0, \tag{3.6}$$

with $V_0 = 30$ MeV, and for the density $\varrho(r)$ we use the Saxon-Woods form

$$\varrho(r) = \varrho_0/\{1+\exp[(r-c)/z]\}. \tag{3.7}$$

From the normalization,

$$4\pi \int dr r^2 \varrho(r) = A, \tag{3.8}$$

we get (see Elton [17]) for the half-way radius c (for $z \ll c$)

$$c = r_0 A^{1/3} - (\pi^2 z^2/3r_0) A^{-1/3}, \tag{3.9}$$

where $r_0 = (4\pi\varrho_0/3)^{-1/3} = 1.128$ fm. With the value of the surface thickness $s = 2.49$ fm, we get $z = s/\ln(81) = 0.567$ fm.

Eq. (3.5) was solved numerically for the 1s ground state, and for the 1p excited state, for several values of A. In each case the expectation value of V was calculated, and used to determine $\bar{\varrho}$ and $\bar{k}_{\Sigma A}$, according to the relations:

$$\bar{\varrho} = -(\varrho_0/V_0)\langle \Psi_\Sigma | V | \Psi_\Sigma \rangle, \quad \hbar^2 \bar{k}_{\Sigma A}^2 / 2\mu_{\Sigma A} = E - \langle \Psi_\Sigma | V | \Psi_\Sigma \rangle. \tag{3.10}$$

From the average relative Σ-nuclear core momentum, $\bar{k}_{\Sigma A}$, we get the average Σ momentum in the rest frame of nuclear medium, $\bar{k}_\Sigma = M_\Sigma \bar{k}_{\Sigma A}/\mu_{\Sigma A}$. Values of $\bar{\varrho}/\varrho_0$, $\bar{k}_\Sigma$, and E as functions of A are shown in Fig. 4. With $\bar{\varrho}$ and $\bar{k}_\Sigma$ inserted into expression (3.2), we obtain the results for Γ shown in Fig. 5.

For light hypernuclei, with decreasing A, E approaches zero (for our potential $V(r)$ the 1p state is hardly bound for $A = 16$, and is not bound at all for $A = 12$), and the Σ wave function spreads out beyond the size of the nuclear core. Consequently, $\bar{\varrho}$ drops sharply. This is the essential reason for the drop in Γ for small values of A. As A decreases this happens first with the 1p state. Hence for the lightest nuclei we have $\Gamma(1p) < \Gamma(1s)$. Obviously, as E approaches zero, the kinetic energy, and consequently $\bar{k}_\Sigma$ decreases, which also effects Γ (see Fig. 2) but in a less drastic way.

With increasing A, $-E$ approaches its limiting, and $\bar{\varrho}$ is increasing. For heavy hypernuclei $\bar{\varrho} \simeq \varrho_0$ for both the 1s and 1p states. Here, the kinetic energy of the 1p state is bigger than that of the 1s state. Consequently, $\boldsymbol{k}_\Sigma(1p) > \boldsymbol{k}_\Sigma(1s)$, and $\Gamma(1p) > \Gamma(1s)$ (see Fig. 2).

For heavy hypernuclei, whose nuclear cores contain more neutrons than protons, the width Γ of Σ^- hypernuclei is smaller than Γ obtained from Γ^c_{NM} for symmetric NM. To estimate this effect, we replace ϱ in Eq. (2.26) by $[1-(N-Z)/A]\varrho$ and use for the neutron excess the value for β stable nuclei, $N-Z \cong 6\times 10^{-3} A^{5/3}$. The resulting values of Γ are shown as broken lines in Fig. 5. (The corresponding correction factor for Σ^+ hypernuclei is $[1+(N-Z)/A]$, and the width of Σ^0 hypernuclei is not affected by neutron excess.)

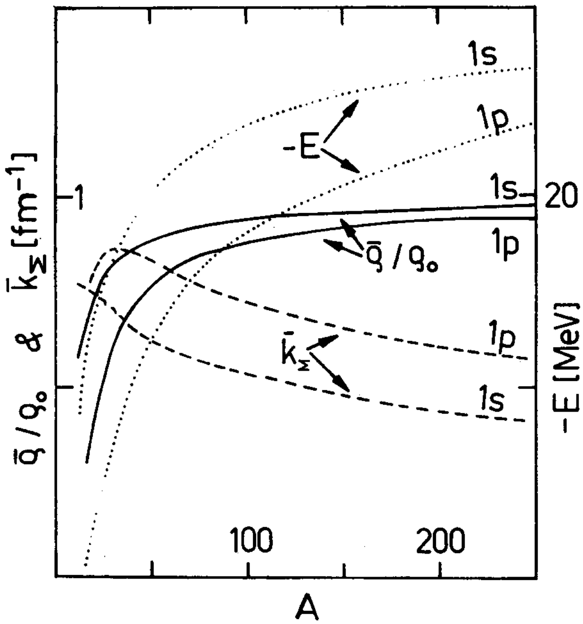

Fig. 4. The average density $\bar{\varrho}$ and momentum k_Σ, and the energy E for the 1s and 1p states in Σ hypernuclei

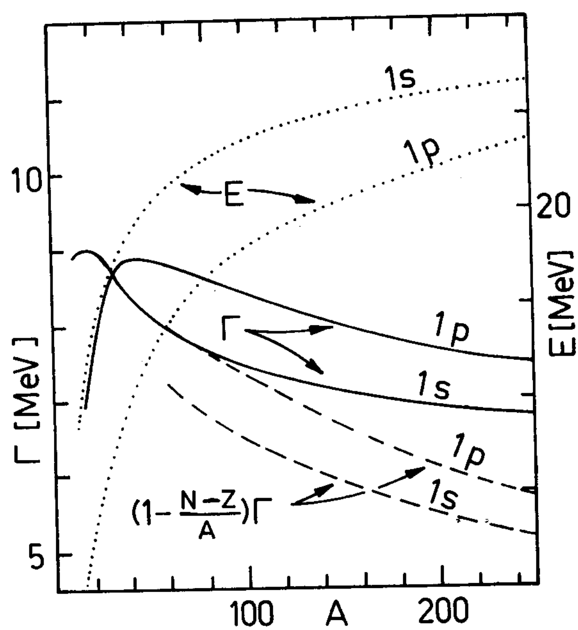

Fig. 5. The width Γ and the energy E of the 1s and 1p states in Σ hypernuclei

In Sect. 2.2 we have considered a momentum dependent s.p. Σ potential in NM with a momentum dependent part (see Eq. (2.37)):

$$\varepsilon_\Sigma(k_\Sigma)(\tilde{v}^{-1}-1) = (\hbar^2 k_{\Sigma A}^2/2\mu_{\Sigma A})(M_\Sigma/\mu_{\Sigma A})(\tilde{v}^{-1}-1). \tag{3.11}$$

To be consistent with Sect. 2.2, we should add a corresponding momentum dependent part to $V(r)$. Doing it in the case of $\tilde{v} = \tilde{v}(\varrho(r))$ complicates the resulting Schroedinger equation. If $\tilde{v} = v_0 = \text{const}$, we obtain the momentum dependent part of V by substituting Δ for $k_{\Sigma A}$ in (3.11), and in place of Eq. (3.5) we get

$$\left\{-\frac{\hbar^2}{2\mu_{\Sigma A}v_0}\left[1+\frac{M_\Sigma}{M_A}(1-v_0)\right]\Delta+V(r)\right\}\Psi_\Sigma(\boldsymbol{r}) = E\Psi_\Sigma(\boldsymbol{r}). \tag{3.12}$$

To see the effect of the momentum dependence of V on Γ we have calculated Γ as described before but with Ψ_Σ determined from Eq. (3.12) (the right hand side of the second of Eqs. (3.10) has to be multiplied by $v_0/[1+(M_\Sigma/M_A)(1-v)]$). For v_0 we used the value $v_0 = \tilde{v}(\varrho_0) = 0.7$ (a properly averaged value of $\tilde{v}(\varrho(r))$ would lie between 0.7 and 1). The results for Γ for heavy hypernuclei practically do not differ from the results shown in Fig. 5. The point is that the average Σ momentum in heavy hypernuclei is small because of the big size of these hypernuclei. Also for light hypernuclei the effect of using $v_C = 0.7$ is not big. For instance, for $A = 40$ we get a 2% increase in Γ for the 1s state. For the lightest hypernuclei $\bar{\varrho} \ll \varrho_0$, and a proper value of v_0 should be close to 1, and consequently the effect of the momentum dependence of V is also very small.

3.2. Σ atoms

We describe the motion of Σ^- on a circular orbit ($n = l-1$, where l and n are the orbital and principal quantum numbers) around a nucleus (A nucleons, Z protons) by the hydrogen wave function:

$$\Psi_\Sigma(\boldsymbol{r}) = \frac{R(r)}{r}Y^l_{em}(\hat{\boldsymbol{r}}), \tag{3.13}$$

$$R(r) = (2nr/a_n)^n \exp\{-nr/a_n\}/[(2n-1)!a_n]^{1/2}, \tag{3.14}$$

where a_n is the radius of the orbit,

$$a_n = (n^2/z)\hbar/\mu_{\Sigma A}. \tag{3.15}$$

Since Σ hyperons are absorbed from orbits for which is about an order of magnitude bigger than nuclear radius, we need $\varrho(r)$ which would be possibly accurate at the tail of the nucleon distribution in order to obtain a reliable value for $\bar{\varrho}$, Eq. (3.3). Here, we assume $\varrho(r)$ to be of a 3 parameter Fermi form,

$$\varrho(r) = \varrho(0)[1+w(r/c)^2]/[1+\exp\{(r-c)/z\}], \tag{3.16}$$

with parameters c, z, w determined by electron scattering [19]. In cases of negative values of w, we put $\varrho(r) = 0$ for $r > c/\sqrt{-w}$. The value of $\varrho(0)$ is determined by normalization condition (3.8), which for $c \gg z$, and $w > 0$ takes the form:

$$\tfrac{4}{3}\pi c^3 \varrho(0)\left\{1+\left(\frac{\pi z}{c}\right)^2+\tfrac{3}{5}w\left[1+\tfrac{10}{3}\left(\frac{\pi z}{c}\right)^2+\tfrac{7}{3}\left(\frac{\pi z}{c}\right)^4\right]\right\} = A. \tag{3.17}$$

The average density $\bar{\varrho}$, Eq. (3.3), was obtained by calculating numerically the integral

$$\bar{\varrho} = \int_0^\infty dr R(r)^2 \varrho(r). \tag{3.18}$$

The average Σ momentum in nuclear medium $\bar{k}_\Sigma = M_\Sigma \bar{k}_{\Sigma A}/\mu_{\Sigma A}$ and the relative ΣA momentum is determined by:

$$\hbar^2 \bar{k}_{\Sigma A}^2/2\mu_{\Sigma A} = ze^2/2a_n. \tag{3.19}$$

To obtain Γ, these values of $\bar{\varrho}$ and $\bar{k}_\Sigma$ were inserted into expression (3.2). Obviously, at the small densities $\bar{\varrho}$, exclusion principle and dispersive effects are irrelevant, and Γ_{NM}^c may be obtained from expression (1.1).

Our results for $\bar{\varrho}$, $\bar{k}_\Sigma$, and Γ are shown in Table I which also contains the experimental results for Γ of Batty et al. [2], the parameters of the density distribution [19], and the radii of Σ orbits.

4. *Absorption in Σ-nucleus scattering*

We want to calculate the absorptive potential, i.e., the imaginary part of the optical model potential $\mathscr{V}_\Sigma$, for Σ-nucleus scattering. Our starting point is a Σ hyperon moving with momentum $\bar{k}_\Sigma$ in NM of density ϱ. If we write its energy, Eq. (2.2), in the form

$$E_\Sigma = \hbar^2 k_\Sigma^2/2M_\Sigma + \mathscr{V}_\Sigma(\varrho, k_\Sigma), \tag{4.1}$$

then for the imaginary part of the Σ optical potential in NM, we have

$$\mathrm{Im}\,\mathscr{V}_\Sigma(\varrho, k_\Sigma) \equiv W_{NM}(\varrho, k_\Sigma) = -\tfrac{1}{2}\Gamma_{NM}(\varrho, k_\Sigma). \tag{4.2}$$

Here Γ_{NM} is the total width which consists of the conversion part Γ_{NM}^c, and of the elastic part Γ_{NM}^e (see Eq. (3.1)). We shall denote the corresponding parts of W_{NM} by W_{NM}^c and W_{NM}^e.

To calculate the absorptive potential $W(r)$ for Σ-nucleus scattering, we apply the local density approximation:

$$W(r) \cong W_{NM}(\varrho(r), k_\Sigma(r)), \tag{4.3}$$

where $\varrho(r)$ is the nuclear density distribution in the target nucleus, and $k_\Sigma(r)$ is the local Σ momentum in that nucleus. We determine $k_\Sigma(r)$ from the energy conservation equation (here, the recoil of the nucleus is neglected),

$$E_0 = (1/\tilde{v}_\Sigma)\varepsilon_\Sigma(k_\Sigma) + D, \tag{4.4}$$

TABLE I

Results for Σ atoms

	c^a	z^a	w^a	$\varrho(0)$ [fm^{-3}]	n	a_n [fm]	$\bar{\varrho}$ [fm^{-3}]	$\bar{k}_\Sigma$ [fm^{-1}]	Γ [eV]	
									Calc.	Exp.[b]
^{16}O	2.608	0.513	−0.051	0.165	3	27.5	1.6 E-6	0.118	644	—
					4	48.8	1.9 E-9	0.089	0.84	$1.0^{+1.7}_{-0.4}$
^{24}Mg	3.108	0.607	−0.163	0.165	4	31.8	1.5 E-7	0.133	61	< 70
					5	49.6	2.6 E-10	0.106	0.11	0.11 ± 0.09
^{27}Al	3.070	0.519	0	0.174	4	29.1	3.4 E-7	0.144	127	43 ± 75
					5	45.5	7.4 E-10	0.115	0.29	0.24 ± 0.6
^{28}Si	2.93	0.569	0	0.194	4	27.0	6.1 E-7	0.155	232	220 ± 110
					5	42.2	1.3 E-9	0.124	0.54	0.41 ± 0.10
^{32}S	3.503	0.633	−0.250	0.182	4	23.5	2.5 E-6	0.177	927	870 ± 700
					5	36.4	8.0 E-9	0.142	3.1	1.5 ± 0.8

[a] from Ref. [19]; [b] from Ref. [2]

where E_0 is the (kinetic) energy of the incoming Σ outside of the nucleus, and the right hand side is the s.p. Σ energy, Eq. (2.37). For the (density dependent) Σ well depth D, we substitute $D = -V_0\varrho(r)/\varrho_0$ (see Eq. (3.6)). For $\tilde{\nu}_\Sigma = \tilde{\nu}_N = \tilde{\nu}(\varrho)$ we use expression (2.36) with ϱ equal to the local density $\varrho(r)$. In this way, we get from Eq. (4.4)

$$k_\Sigma(r) = \left\{\frac{2M_\Sigma}{\hbar^2}\tilde{\nu}(\varrho(r))\left[E_0 + V_0\varrho(r)/\varrho_0\right]\right\}^{1/2}. \tag{4.5}$$

For $\varrho(r)$ we assume the Saxon-Woods form of Sect. 3.1. We use the values of $V_0 = 30$ MeV, and $\tilde{\nu} = \tilde{\nu}(\varrho_0) = 0.7$.

Results obtained for $W(r)$ for $\Sigma-{}^{16}O$ scattering at $E_0 = 0$, 30, and 60 MeV are shown as solid curves in Fig. 6. The conversion part of $W(r)$, $W^c(r)$ shown in Fig. 6 as a broken curve for $E_0 = 30$ MeV, is practically the same for the three energies E_0 considered, and

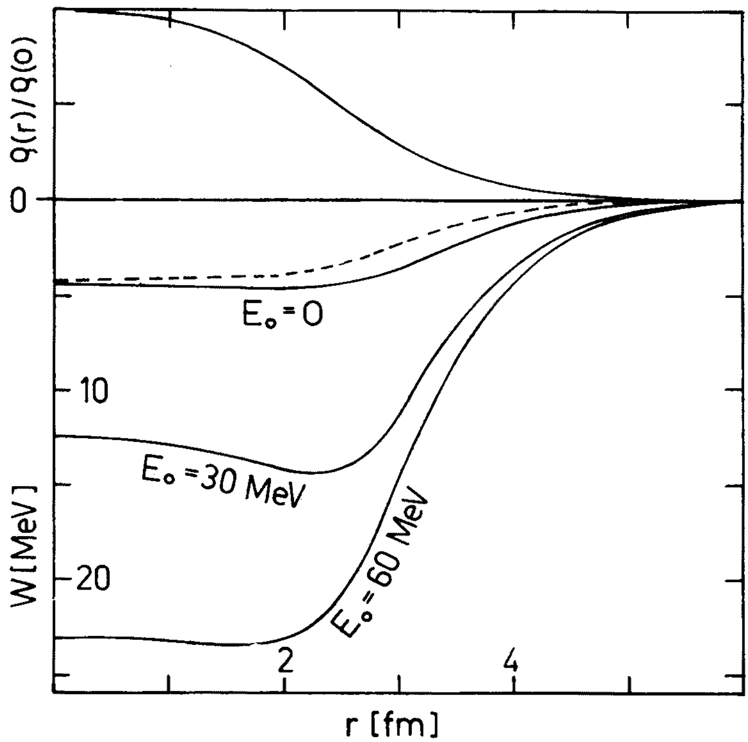

Fig. 6. The absorptive potential $W(r)$ for $\Sigma-{}^{16}O$ scattering for three Σ incoming energies E_0. The broken curve represents $W^c(r)$ for $E_0 = 30$ MeV

becomes unimportant at higher energies where $W(r)$ is determined predominantly by elastic processes. Notice that our $W(r)$ is surface peaked, especially for $E_0 = 30$ MeV. This is the result of the suppression of ΣN scattering by exclusion principle which is stronger at higher density inside of the nucleus than at the lower density at the surface.

5. *Discussion*

The simple theory presented in this paper does not contain any essentially free adjustable parameters. The only input are the experimental cross sections for ΣN scattering. The simplicity of the theory is achieved by approximating the effective ΣN cross sections in NM by cross sections for the scattering of an isolated ΣN pair, Eq. (2.21).

In NM, we predict the existence of a Σ ground state with a width $\Gamma \simeq 6$ MeV, much narrower than the semiclassical estimate, $\Gamma \simeq 24$ MeV. This substantial reduction of the Σ width in NM is due to the strong suppression of the $\Sigma N \to \Lambda N$ process in NM. Namely, the $\Sigma N \to \Lambda N$ process in NM is accompanied by the excitation of NM, which uses part of the released energy (i.e., nucleons and hyperons in NM are quasiparticles). This diminishes

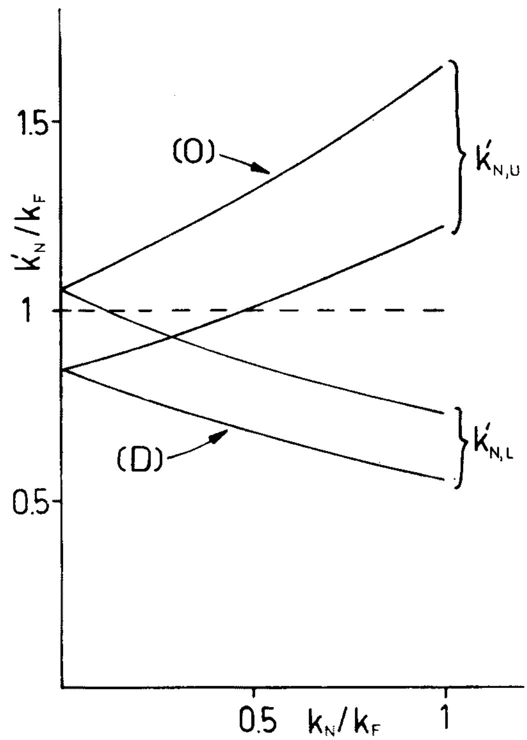

Fig. 7. Ranges of final nucleon momenta k'_N in ΛΣ conversion in NM (for $k_\Sigma = 0$ and $\varrho = \varrho_0$) as functions of initial nucleon momenta k_N without (*O*) and with dispersive effects (*D*)

the final nucleon momenta to such a degree that an essential part of them are smaller than the Fermi momentum, and are excluded by the Pauli principle. This point is visualized in Fig. 7 which shows the upper and lower limits of the final nucleon momenta, $k'_{N,U}$ and $k'_{N,L}$ as functions of the initial nucleon momentum k_N, for $k_\Sigma = 0$, and $\varrho = \varrho_0$. We see that due to the dispersive effects (with $v = 0.7$) only a small fraction of final nucleon momenta exceed the Fermi momentum.

To estimate the width Γ of bound states of Σ hypernuclei, we use our NM results and apply approximation (3.2). For very heavy hypernuclei ($A \simeq 200$), we predict that $\Gamma_{1s} \simeq 7$ MeV for the ground state (Σ in 1s state + nuclear core in the ground state), and $\Gamma_{1p} = 8$ MeV for Σ in the 1p state (+ nuclear core in the ground state). With decreasing A the width of both states increases slightly, but for $A \gtrsim 60$, the situation does not change very much (for $A \simeq 60$, we have $\Gamma_{1s} \simeq 8$ MeV, and $\Gamma_{1p} \simeq 9$ MeV).

With further decreasing A, the width eventually starts decreasing, because here, in light hypernuclei, Σ is less tightly bound, and the overlap between Σ wave function and nuclear density is small. This effect is more pronounced for the 1p state than for the 1s state for which it shows only in the lightest hypernuclei. Obviously this effect is sensitive to the Σ well depth V_0. With our choice of $V_0 = 30$ MeV (approximate equality of the Σ and Λ well depth suggested in [1]), we have for the $\Sigma + {}^{16}O$ system: $\Gamma_{1s} = 9.0$ MeV ($E_{1s} = -11.3$ MeV), $\Gamma_{1p} = 6.9$ MeV ($E_{1p} = -0.5$ MeV), and for the $\Sigma + {}^{12}C$ system: $\Gamma_{1s} = 8.9$ MeV ($E_{1s} = -8.6$ MeV) with 1p state unbound. With a smaller value of $V_0 \simeq 20$ MeV (suggested in [4]), we would obtain smaller values of Γ_{1s}, and the 1p state would be unbound in both systems: $\Sigma + {}^{16}O$ and $\Sigma + {}^{12}C$. Bearing in mind the uncertainty of V_0, we restrict ourselves to the following conclusions concerning light hypernuclei $A \lesssim 20$: $\Gamma_{1s} \lesssim 9$ MeV and Γ_{1s} is decreasing with decreasing A, and as long as the 1p state is bound we have $\Gamma_{1p} < \Gamma_{1s}$ and Γ_{1p} is decreasing with decreasing A much faster than Γ_{1s}.

Instead of applying approximation (3.2) we could proceed in a different way. We could calculate a complex Σ-nucleus potential $\mathscr{V}$ (see Sect. 4), insert $\mathscr{V}$ (in place of V) into Schroedinger equation (3.5), and solve this equation for Ψ_Σ and $E = E_R - \frac{i}{2}\Gamma$. The difficulty of this procedure is caused by the fact that $\mathscr{V}$ depends on E. This procedure was applied by Stępień-Rudzka and Wycech [20], and recently by Johnston and Thomas [21], who overcome this difficulty by introducing a number of simplifying assumptions. To solve the Schroedinger equation with the complex potential $\mathscr{V}$ one looks for exponentially damped Ψ_Σ characterized by complex momentum eigenvalues $k = \sqrt{2\mu_{\Sigma A}E/\hbar^2} = k_R + ik_I$, $k_R < 0$, $k_I > 0$ ($E_R = \hbar^2(k_R^2 - k_I^2)/2\mu_\Sigma$, $\Gamma = -2\hbar^2 k_R k_I/\mu_{\Sigma A}$). As noticed by Stępień-Rudzka and Wycech, if $|k_R| > k_I$ one has an exponentially decaying, normalizable state with $E_R > 0$. These states were discussed in detail by Gal, Toker, and Alexander [5] who called them bound states embedded in the continuum (BSEC) (see also [21]).

The suggestion that the narrow resonances in Σ hypernuclei should be identified with these exotic BSEC is very interesting. One possibility of reaching these resonances is to consider a Σ state that without absorption is weakly bound. By switching in the absorption (which diminishes the Σ wave functions and acts similarly as repulsion) we may shift the Σ energy to positive values. In our approach in which the absorption is treated as a perturbation, we would predict a narrow width of the weakly bound state without predicting the shift in energy.

We agree with everybody else in the field with the reduction of Γ for light hypernuclei caused by the decreasing Σ binding (possibly through allowance of BSEC which are beyond the reach of our simple approach). This mechanism of reducing Γ, supplemented by the selectivity mechanism suggested by Gal [22], is effective only in a limited range of Σ hypernuclei. If this was the only mechanism, narrow Σ states would be the exception rather than the rule. However, our present results, as well as our earlier results [6], demonstrate that the mechanism of Pauli blocking and dispersive (i.e., binding) effects reduce Γ on all Σ bound states (see Fig. 5).

Experimentally, no ground states of heavy Σ hypernuclei have been observed so far. The point is that the recoilless or almost recoilless Σ production in heavy nuclear targets leads predominantly to excited hypernuclear states. Similarly, no heavy Λ hypernuclear ground states have been observed so far in the (K, π) reaction (see, e.g., Bertini et al. [23]). In the heaviest Σ hypernucleus reported, ${}^{16}_{\Sigma}C$, the level structure could not be identified [4]. It appears that at the moment only in the ${}^{12}_{\Sigma}C$ hypernucleus the ground state with Σ-binding of ~4 MeV could be identified [24]. This is also the only hypernucleus in which a 1p Σ level, al ~5 MeV excitation, is probably observed. No estimate of the width of those levels was reported in [24], but both of them are narrow, with $\Gamma \sim$ 5–10 MeV (according to our visual estimate of the spectra given in [24]), in agreement with our results in Fig. 5. It would be very important for us if more levels in heavier Σ hypernuclei could be identified, in particular for those states in which the nuclear core is in its ground state (a hole in the nuclear core introduces additional width which for deep hole states would be much bigger than the conversion width, estimated here).

In Σ atoms our calculated widths Γ agree very nicely with the measured widths. Our procedure of calculating Γ was essentially the same as in the case of Σ hypernuclei, except that we used for Ψ_Σ the hydrogen functions, and thus neglected the Σ-nucleus potential in determining Ψ_Σ. Let us consider then $n = 4$ orbit in the Si atom. The radius of this orbit (the distance r at which $R(r)^2$ attains maximum) is $a_N = 27$ fm, and $R^2(r)\varrho(r)$ in the overlap-integral $\bar{\varrho}$, Eq. (3.18) attains maximum at $r \simeq 4$ fm. Now, at $r \simeq 4$ fm the Σ-nucleus potential $V(r)$, Eq. (3.6), is approximately as strong as the Coulomb potential, and thus its effect on $\Psi_\Sigma(r)$ might have a noticeable effect on $\bar{\varrho}$. In discussing Σ hypernuclear states we mentioned a different procedure of calculating first the whole complex Σ-nucleus potential $\mathscr{V}$, and inserting it into the Schroedinger equation for Ψ_Σ. In this procedure, applied for Σ atoms in [20], also the absorptive potential has an effect on Ψ_Σ, which is disregarded in our approach. These effects, as well as the effect of the finite size of the charge distribution, should certainly be investigated.

Our results for the strong absorptive Σ-nucleus optical potential, Im $\mathscr{V}_\Sigma$, were obtained from our NM results with the help of the local density approximation, Eq. (4.2). The main uncertainty in our results stems from the poor knowledge of the experimental ΣN elastic cross sections. We relied on the extrapolation to higher energies based on the Nijmegen interaction. Experimentally, a strong Σ absorption is deduced from the measurement of the emission of charged Σπ pairs resulting from the capture at rest of K^- mesons on complex nuclei (see [3]). In the interpretation of these measurements, Wycech [25] used Re $\mathscr{V}_\Sigma$ and Im $\mathscr{V}_\Sigma$ similar to our V and our Im $\mathscr{V}_\Sigma$. A more direct comparison of our results with experiment is not possible since there are no data on Σ-nucleus scattering.

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