A NOTE ON ELECTROMAGNETISM AND THE NEW COSMOLOGY

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It is shown that in the absence of an electric charge, the cosmology arising from the generalised field theory (nonsymmetric theory of electromagnetism and gravitation, GFT) collapses into a static, Einstein universe. The observed expansion is a direct consequence of the presence of a net charge which is shown to be necessarily negative. The implications for GFT are discussed.

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1. Let us briefly recall the argument which yields a unique cosmological model as a consequence of the Generalised Field Theory (GFT; [1–8]). The static, spherically symmetric solution gives a fundamental $\boldsymbol{g}$ field and a metric tensor $\boldsymbol{a}$ of the (pseudo-Riemannian) space-time which depend on two constant parameters c and r_0. The skew symmetric part

$$\boldsymbol{R}^- \equiv R_{[\mu\nu]}$$

of the Ricci tensor (constructed from the auxiliary connection $\tilde{\Gamma}$ whose torsion vector identically vanishes) is identified as the electromagnetic intensity tensor. In our case, the only nonzero component of $\boldsymbol{R}^-$ is

$$R_{[23]} = c \sin\theta. \tag{1}$$

This means that with respect to the radial coordinate r for which the components $a_{22} = a_{33} \operatorname{cosec}^2\theta$ of $\boldsymbol{a}$ are $-r^2$, the electrostatic field is strictly an inverse square Coulomb field and the dimensionless c is related to charge. On the other hand, the remaining components a_{00} and a_{11} of the metric become imaginary for

$$r > r_0,$$

as well as do the components g_{00}, g_{11} and $g_{23} = -g_{32}$ of $\boldsymbol{g}$. The precise physical role of the $\boldsymbol{g}$ field is as yet an unresolved problem of GFT. We shall make some comments below on the possible interpretation of its skew part g^- remembering that the electromagnetic vector potential is proportional to the torsion vector Γ_μ of the geometrical affine connection Γ. However, the apparent breakdown of geometry at (or rather beyond)

$$r = r_0 \tag{2}$$

can only be understood if this surface represents to a local observer the limits of the universe accessible to observations. This is the genesis of the cosmological interpretation of GFT.

The aim of this article is to seek additional reasons for the above conclusion other than the paradox of a Coulomb field extending to infinity and a cut-off at a finite distance from the observer. We shall find that our interpretation is consistent with the spirit of the theory and with the cruder (and, may we suggest also more certain) facts known about the structure of the world at large. In particular, we shall show that it leads to a natural mechanism for the observed extragalactic expansion as well as an explanation of the somewhat puzzling equations of motion result of Infeld [9].

2. We must note first that whereas Killing equations may inform us about the correct choice of a time coordinate, that of the radial distance becomes arbitrary as soon as an observer begins to contemplate distances much in excess of a "rigid ruler". This is a well known case in astronomy, different definitions of distance being considered according to the method of their determination. It is a major problem in cosmology when cross--checking of various methods is no longer available.

The radial coordinate in the New Cosmology is first chosen so as to recognise explicitly the limiting nature of the surface (2):

$$r \to \frac{r_0 r}{\sqrt{r_0^2 + r^2}} = r\sqrt{w} \text{ say,} \tag{3}$$

so that the metric of space-time becomes

$$ds^2 = \gamma dt^2 - w^2\gamma^{-1}dr^2 - wr^2(d\theta^2 + \sin^2\theta d\psi^2), \tag{4}$$

where

$$\gamma = 1 - \frac{2m}{r},$$

and

$$2m = -cr_0. \tag{5}$$

Introduction of this third constant is of great importance for the interpretation of the model both as far as the suggestive notation and the sign are concerned. We ask what happens when the charge is removed. Clearly c can vanish if either

$$r_0 \to \infty \quad \text{or} \quad m = 0,$$

(or both but then the metric becomes simply that of a Minkowski space-time).

One of us has already pointed out [2] that in the first case ($r_0 \to \infty$) the space-time metric reduces to the classical Schwarzschild case. As required the g field is symmetric and GFT collapses into General Relativity. It is in fact the view of the static universe as it would appear to an observer hypothetically placed outside it since the whole universe would then be just a gravitating, Schwarzschild particle. However, c can be made to vanish if r_0 is kept finite and m put equal to zero. There is then no need to introduce a new "cosmological" distance (transformation (3)) and the spacetime metric

$$ds^2 = dt^2 - \frac{dr^2}{1-r^2/r_0^2} - r^2(d\theta^2 + \sin^2\theta d\psi^2) \tag{6}$$

becomes that of the static, Einstein universe.

Since the charge (c) on the latter is zero, it follows that it is the only neutral model compatible with GFT. However, two more conclusions are implied. We still have a nonsymmetric field g (and an equally nonsymmetric affine connection) but because now

$$R^- = 0,$$

the field equations reduce to the so called strong case of Einstein [10]. It is not surprising therefore that Infeld [9] could not derive from them equations of motion of a charged test particle. For a spherically symmetric field at any rate, his test particle was electrically neutral! It follows also that, as suggested by Russell [8] and extensively studied by Moffat [17], the skew symmetric field now necessarily represents sources of pure gravity. This concurrence with Moffat's theory is particularly pleasing especially since he chooses to derive verifiable conclusions from a solution of precisely the strong field equations (Eq. (4.1) in Ref. [12]).

3. We must next compare the Ricci tensors of the still nonsymmetric theory of gravitation and of General Relativity. Let $R^{\mathrm{E}}_{\mu\nu}$ denote the Ricci tensor of the space-time (6). When $m = 0$, the solution for $\tilde{\Gamma}^{\lambda}_{\mu\nu}$ is (e.g. Ref. [1])

$$\tilde{\Gamma}^1_{11} = \alpha^1/2\alpha, \quad \tilde{\Gamma}^1_{22} = \tilde{\Gamma}^1_{33}\operatorname{cosec}^2\theta = -\frac{r}{\alpha},$$

$$\tilde{\Gamma}^2_{33} = -\sin\theta\cos\theta, \quad \tilde{\Gamma}^3_{(23)} = \cot\theta$$

$$\tilde{\Gamma}^2_{(12)} = \tilde{\Gamma}^3_{(13)} = \frac{1}{r}$$

$$\tilde{\Gamma}^1_{[23]} = \frac{\sqrt{\alpha}}{r_0}\sin\theta, \quad \tilde{\Gamma}^2_{[31]} = \tilde{\Gamma}^3_{[12]}\sin^2\theta = \frac{\sqrt{\alpha}\sin\theta}{r_0}, \tag{7}$$

where

$$\alpha^{-1} = 1 - r^2/r_0^2.$$

Of course, the symmetric components $\tilde{\Gamma}^{\lambda}_{(\mu\nu)}$ are just those of the Einstein world. We readily find that

$$R_{00} = 0$$

and

$$R_{11} = R^{\mathrm{E}}_{11} + \frac{2\alpha}{r_0^2}, \qquad R_{22} = R^{\mathrm{E}}_{22} + \frac{2r^2}{r_0^2}. \tag{8}$$

Hence the field equations of GFT ($\bar{R} = 0$) give

$$R^{\mathrm{E}}_{11} = -\frac{2\alpha}{r_0^2} = -KT^{\mathrm{E}}_{11} - \tfrac{1}{2}\alpha(R^{\mathrm{E}} - 2\Lambda^{\mathrm{E}})$$

$$R^{\mathrm{E}}_{22} = -\frac{2r^2}{r_0^2} = -KT^{\mathrm{E}}_{22} - \tfrac{1}{2}r^2(R^{\mathrm{E}} - 2\Lambda^{\mathrm{E}}) \tag{9}$$

and

$$-\tfrac{1}{2}R^{\mathrm{E}} + \Lambda^{\mathrm{E}} = -KT^{\mathrm{E}}_{00}$$

with the usual definition of the cosmological constant Λ^{E} and energy-momentum tensor $T^{\mathrm{E}}_{\mu\nu}$. Hence, in the pressureless case, we get

$$R^{\mathrm{E}} - 2\Lambda^{\mathrm{E}} = \frac{4}{r_0^2}$$

and

$$T^{\mathrm{E}}_{00} = \frac{2}{kr_0^2}$$

as in the Einstein case with the cosmological constant

$$\Lambda^{\mathrm{E}} = \frac{1}{r_0^2}. \tag{9'}$$

In other words, GFT with $m = 0$ is as anticipated completely compatible with the cosmology of a static Einstein world.

Let us now return to the full model with

$$m \neq 0 \quad \text{and} \quad r_0 < +\infty,$$

and write (as in 3)

$$r = r_0 \tan\frac{2}{r_0}$$

and

$$c = -\frac{2m}{r_0} = -\lambda.$$

The metric becomes

$$ds^2 = \gamma dt^2 - \gamma^{-1} dz^2 - r_0^2 \sin^2 \frac{z}{r_0} (d\theta^2 + \sin^2 \theta d\psi^2), \tag{10}$$

where

$$\gamma = 1 - \lambda \cot \frac{2}{r_0}.$$

It is important now to point out that all information about distant events arrives (on Earth) along radial null geodesics. This of course, is a matter of definition or of conceptualisation: an observer says so because nothing else can be said sensibly. Now, the angular variables θ, ψ can be suppressed because [13] the metric is diagonal and its components are independent of them. In other words, we need only to consider

$$ds^2 = \gamma^2 dt^2 - \gamma^{-1} dz^2, \tag{11}$$

whence,

$$\gamma \frac{dt}{ds} = k, \quad \text{a constant}$$

and

$$\left(\frac{dz}{ds}\right)^2 = k^2 - \gamma. \tag{12}$$

A choice of the units of time measurement enables us to put $k = 1$ and then it is easily shown that a transformation

$$\frac{2}{r_0}\sqrt{\frac{2\lambda}{K}} \ln \omega = \ln \frac{\zeta^2 - \sqrt{2\lambda}\,\zeta + \lambda}{\zeta^2 + \sqrt{2\lambda}\,\zeta + \lambda} - 2 \tan^{-1} \frac{\sqrt{2\lambda}\,\zeta}{\lambda - \zeta^2} \tag{13}$$

where

$$\zeta^2 = \lambda \cot \frac{z}{r_0} = \frac{2m}{r} \tag{14}$$

maps equation (12) into

$$\frac{d\omega}{ds} = \pm \sqrt{K}\,\omega. \tag{15}$$

Hence, as in the de Sitter universe (which however is empty whereas ours is not) a particle, i.e. a galaxy in the usual, smoothed out picture of matter essential to the validity of a spherically symmetric model; remains at rest only if it is at the origin or in other words, if the observer is inside it. Elsewhere, and if the positive sign is chosen in equation (15), it will be seen to recede with exactly the proper Hubble velocity providing ω is chosen as the new radial coordinate (and $\sqrt{K}$ as the Hubble constant). We have already pointed out that this choice is a priori arbitrary. Therefore we can argue that if empirical evidence shows Hubble law to be valid for, say, a number of nearby extragalactic objects, the same observations determine the relation of cosmic distance to local coordinates, i.e. the choice of ω. The GFT model then requires the law to be valid throughout the universe. One of us has argued previously [2] that Hubble expansion holds only approximately for

$$2m \ll r \ll r_0.$$

We now see that this is not the case except at the lower limit since for

$$r < 2m, \tag{16}$$

time and radial distance reverse roles and clearly a new physics is required. It was shown in [3] that the region (16) cannot be transformed to the origin as in the general relativistic case and it was identified with the finitely extended "primeval" atom. We may also notice that in terms of the coordinates which map the metric (4) into a Szekeres-Kruskal form (Ref. [3])

$$\omega = r \exp(-\sqrt{K}\bar{t}). \tag{17}$$

4. The results of the preceding sections throw considerable light on the meaning of GFT. It is necessary to distinguish carefully between local and global situations. In the local case we expect to have the space-time surrounding a stationary electric charge. This solution is quite different from the Reissner-Nordström metric of General Relativity. In principle, it provides us with a laboratory scale test of GFT though it is highly unlikely that such a test can be carried out in practice (for more detailed comments, see [14] and [15]) except in the following sense.

If $r \ll |r_0|$ (the field equations themselves do not imply that $r_0 > 0$), the metric becomes Schwarzschild as when electromagnetic field is absent and the electrostatic field is purely Coulomb. One can then say [2] that the electrostatic and the gravitational fields are locally separated. This however does not seem to allow for the duality of sign of the electric charge. One could of course postulate that the sign of r_0 should be so chosen that irrespective of what the sign of the charge c may be, $cr_0 = -2m$ is always negative (and $m > 0$). Alternatively, it could be asserted (again with m positive), that the space-time about a stationary, positive charge is given by

$$ds^2 = \left(1+\frac{2m}{r}\right)dt^2 - \left(1+\frac{2m}{r}\right)^{-1} dr^2 - r^2(d\theta^2+\sin^2\theta d\varphi^2). \tag{18}$$

The above assertion would then constitute a local test of GFT although its possible failure would still enable us to invoke the first possibility.

On the other hand, the cosmological interpretation requires r_0 to be positive by definition. Thus, if the universe is to appear as a Schwarzschild particle "from outside", as indeed is required by common sense, the excess charge on it is necessarily negative.

We may observe too, that our interpretation is enhanced by the collapse of the GFT model into an Einstein universe in the absence of the excess charge.

It now follows that the expansion of the universe is entirely a consequence of the existence of a net charge on it (and as far as this is concerned, it does not matter whether the net charge is positive or negative). It must be stressed that this is not a hypothesis made arbitrarily to account for a possible mechanism of an observed fact: we do not assume a net charge in order to explain expansion but deduce its existence from interpretations forced by the structure of the theory itself. An electrically neutral universe is the static Einstein world. It follows also from equation (15) and the necessarily excluded region $r < 2m$ that the GFT cosmological model does not require a Big Bang to start off the expansion. On the contrary, a Mere Puff will do!

This brings us to the final point. One of us has argued previously ([5] and [13]) that a more appropriate value than (9′) for the cosmological constant is

$$\Lambda = \frac{2}{r_0^2}.$$

Considerations of matter distribution allow then the assertion that "all of the primeval atom blew up" (or, rather, has taken part in the expansion). Einstein's field equations are still satisfied but the mass density is decreased to $1/kr_0^2$ and there appears a positive pressure

$$T_{11} = \frac{1}{K(r_0^2-r^2)}, \quad T_{22} = T_{33}\operatorname{cosec}^2\theta = \frac{r^2}{Kr_0^2}.$$

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