WOUNDED NUCLEONS IN α-α COLLISIONS AT HIGH ENERGIES

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Distribution of the number w of wounded nucleons and of number ν of inelastic collisions in α-α scattering at high energies is calculated using nuclear probability calculus. Correlations between w and ν are also studied. The results are compared with generalized optical approximation.

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1. Introduction

Collisions of α particles at high energy have recently been studied experimentally at the CERN ISR [1, 2]. The general characteristics of particle production at low $p_\perp$ were measured and interpreted in terms of some simple models. In particular a model of "wounded nucleons" [3] (which is also the first approximation to the dual-parton model [4]) proved rather successful [5]. This observation prompted us to consider again the model of wounded nucleons in some detail. In the present paper the results of the calculations of various distributions of number (w) of wounded nucleons in α-α collisions are presented. We also show distribution of the number of collisions (ν) and several parameters describing correlations between w and ν and also between left- and right-hand hemisphere. All these results were obtained using nuclear probability calculus. They are also compared with quasi-optical approximation [6] which is known to work reasonably well for total cross-sections [7].

In Section 2 and 3 we define probabilities of different scattering configurations and develop formulae by which they can be calculated. The results of these calculations are described in Section 4. They are compared with quasi-optical approximation in Section 5. Our conclusions are listed in the last Section. The Appendices describe various algorithms used in numerical estimates.

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2. *Probabilities in a collision between two nuclei*

In this paper we consider only inelastic nondiffractive collisions. The probability for such a collision of two nuclei A and B, colliding at impact parameter b is

$$\sigma(b) = \int \prod_{i=1}^{A} d^2 s_i \prod_{j=1}^{B} d^2 \bar{s}_j D_A(\vec{s}_1, \ldots \vec{s}_A) D_B(\vec{\bar{s}}_1, \ldots \vec{\bar{s}}_B) \sigma(\vec{b}; \vec{s}_1, \ldots \vec{s}_A; \vec{\bar{s}}_1, \ldots \vec{\bar{s}}_B) \tag{2.1}$$

where $\sigma(\vec{b}; \vec{s}_1 \ldots \vec{s}_A; \vec{\bar{s}}_1 \ldots \vec{\bar{s}}_B)$ is the probability of the collision for a fixed transverse configuration of the nucleons inside the nuclei and

$$D_A(\vec{s}_1, \ldots \vec{s}_A) = \int dz_1 \ldots dz_A \varrho_A(\vec{r}_1, \ldots \vec{r}_A) \tag{2.2}$$

where ϱ_A is the nuclear density and $z_1 \ldots z_A$ are the coordinates along the beam direction.

Assuming that the individual nucleon-nucleon collisions are independent of each other we have

$$\sigma(\vec{b}; \vec{s}_1, \ldots \vec{s}_A; \vec{\bar{s}}_1, \ldots \vec{\bar{s}}_B) = 1 - \prod_{i=1}^{A} \prod_{j=1}^{B} \{1 - \sigma_{ij}\} \tag{2.3}$$

where

$$\sigma_{ij} \equiv \sigma(\vec{b} - \vec{s}_i + \vec{\bar{s}}_j) \tag{2.4}$$

is the probability of nucleon-nucleon inelastic collision at impact parameter $\vec{b} - \vec{s}_i + \vec{\bar{s}}_j$. All these geometrical relations are illustrated in Fig. 1.

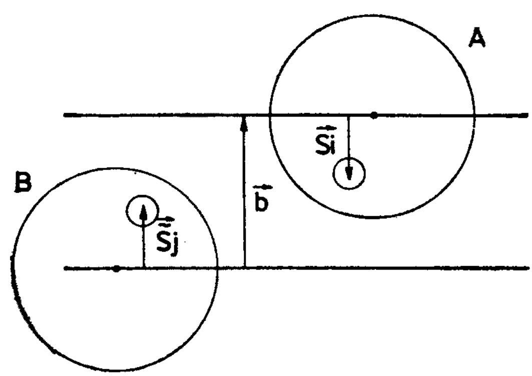


Fig. 1. Geometrical picture of nucleus-nucleus scattering

The physical interpretation of the expression (2.3) is well-known: σ_{ij} is the (unconditional) probability of a collision of the nucleon i from nucleus A with the nucleon j from nucleus B (irrespectively if other nucleons did or did not interact). Thus $1 - \sigma_{ij}$ is the probability that these two nucleons do not interact with each other. Consequently, the product $\prod_{i=1}^{A} \prod_{j=1}^{B} (1 - \sigma_{ij})$ is the probability that no interaction between any of the nucleons took place.

We are interested in this paper in finding probabilities for collisions of given groups of nucleons from nucleus A with given groups of nucleons from nucleus B under the condition that no other collision between the nucleons took place. Let us denote the number of collisions in such a particular case by ν. The required probability is then given by a product of ν factors σ_{ij} (indicating which pairs of nucleons interacted with each other) and $AB-\nu$ factors $(1-\sigma_{ij})$ (indicating which pairs of nucleons did not interact). For illustration, let us consider two simple examples: probability that there is just one collision, say nucleon 1 in A with nucleon 1 in B is given by

$$\sigma(\vec{b};\vec{s}_i;\vec{s}_j)P = \frac{\sigma_{11}}{1-\sigma_{11}} \prod_{i=1}^{A} \prod_{j=1}^{B} (1-\sigma_{ij}). \tag{2.5}$$

Similarly, probability that the nucleon 1 from A collided with two nucleons, say 1 and 2 from B and no other collision took place is

$$\sigma(\vec{b};\vec{s}_i;\vec{s}_j)P = \frac{\sigma_{12}\sigma_{11}}{(1-\sigma_{11})\,(1-\sigma_{12})} \prod_{i=1}^{A} \prod_{j=1}^{B} (1-\sigma_{ij}). \tag{2.6}$$

It is not very difficult to verify that the sum of probabilities for all configurations is correctly normalized to unity. This is seen by considering the identity

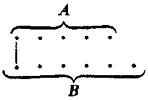
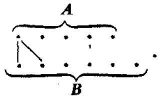
$$1 \equiv \prod_{i=1}^{A} \prod_{j=1}^{B} (1-\sigma_{ij}+\sigma_{ij}) \tag{2.7}$$

and observing that the right-hand side can be written as $\prod_{i=1}^{A} \prod_{j=1}^{B} (1-\sigma_{ij})+$sum of all products of σ_{ij} multiplied by corresponding $(1-\sigma_{ij})$. We thus obtain

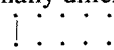
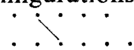
$$\sigma(\vec{b};\vec{s}_i;\vec{s}_j) \equiv 1-\prod_{i=1}^{A} \prod_{j=1}^{B} (1-\sigma_{ij}) = \sum_{\substack{\text{all confi-}\\ \text{gurations}}} \frac{\sigma_{i_1 j_1} \cdots \sigma_{i_\nu j_\nu}}{(1-\sigma_{i_1 j_1}) \cdots (1-\sigma_{i_\nu j_\nu})} \times \prod_{i=1}^{A} \prod_{j=1}^{B} (1-\sigma_{ij}). \tag{2.8}$$

This expansion of the expression for the total cross-section may be contrasted with the standard "multiple scattering expansion" [8] which is an expansion in terms of different products σ_{ij}. The expansion (2.8) has two clear advantages: each term is positive and each has the definite physical (probabilistic) meaning, as explained above.

It is convenient to represent graphically each term in the expansion (2.8). We used the representation in which nucleons are represented by two series of points (one representing nucleus A and another nucleus B) and the factors σ_{ij} are represented by lines connecting the points from one series with the points of the other series (missing lines represent the factors $(1-\sigma_{ij})$). For example, the two contributions (2.5) and (2.6) are represented as

 and .

Another way of denoting different terms is by $A \times B$ matrices with elements corresponding to σ_{ij} equal to 1 and elements corresponding to $(1-\sigma_{ij})$ equal to 0.

The number of different configurations, or terms in expansion (2.8) is $2^{AB}-1$, a very large number. Fortunately, the symmetry between different nucleons inside each of the colliding nuclei implies that many different configurations give the same contribution to the formula (2.1). For example  and  give obviously the same result. Therefore, in actual calculation it is essential to identify all *non-equivalent* configurations (we shall call them *graphs* and denote them by G_n) and to find how many times a given graph enters in the expression (2.1) or (2.8) (this number will be called *multiplicity* K_n of the graph). Using this notation, the Eq. (2.8) can be rewritten as

$$\sigma(\vec{b}) = \sum K_n G_n \tag{2.9}$$

where

$$G_n = \int \prod_i d^2 s_i \prod_j d^2 \tilde{s}_j D_A(\{\vec{s}_i\}) D_B(\{\vec{\tilde{s}}_j\}) \prod \sigma_{ij} \prod (1-\sigma_{ij}). \tag{2.10}$$

We do not know solution of this problem for general A−B collision. However, we found a solution for a particular case of α-α scattering considered in this paper. The list of the graphs and of their multiplicities are given in the Appendix 1.

3. *Graphs in α-α collisions*

Calculation of contributions of the different graphs from Eq. (2.8) to the total cross-section is in general a formidable task, requiring multidimensional $(2A+2B)$ integration of complicated expressions. For heavy nuclei, this does not seem to be feasible without some drastic simplifications and approximations. However, the case of α-α scattering seems to be still tractable (although rather complicated). This is due to the particularly simple form of the nuclear density of He^4, which can be very reasonably described by a Gaussian. We have taken

$$D(\{\vec{s}_i\}) = \frac{1}{(\pi R^2)^3} \exp\left\{-\frac{s_1^2+s_2^2+s_3^2+s_4^2}{R^2}\right\} \delta^{(2)}\left(\frac{\vec{s}_1+\vec{s}_2+\vec{s}_3+\vec{s}_4}{4}\right) \tag{3.1}$$

where the last factor takes care of recoil corrections (assuring the straight line motion of the transverse position s of the center-of-mass of the α particle).

Also the nucleon-nucleon interaction probability can be approximated by a Gaussian form. We thus take

$$\sigma(\vec{b}) = \frac{\sigma}{\pi r^2} \exp\left\{-\frac{b^2}{r^2}\right\} \tag{3.2}$$

where $\sigma = \int \sigma(\vec{b}) d^2 b$ is the total nucleon-nucleon (nondiffractive) cross-section. With these choices, all functions we consider are Gaussians and we can use the well-known formulae for integration of two-dimensional Gaussian expressions.

The algorithm for calculating the integral

$$g_n = \int \sigma_{i_1 j_1}, \ldots \sigma_{i_\nu j_\nu} D(\{\vec{s}_i\}) D(\{\vec{\tilde{s}}_j\}) \prod_{i=1}^{4} d^2 s_i \prod_{j=1}^{4} d^2 \tilde{s}_j \tag{3.3}$$

with σ_{ij} and $D(\{\vec{s}_i\})$ given by Eqs. (3.2) and (3.3) is known and is summarized in Appendix 2.

The expansion of the graphs (2.8) into the integrals of the type (3.3) is straightforward, but requires lengthy calculations. The description of the method of calculation and of the results are presented in Appendix 3.

Using the formulae from Appendices 1–3 we calculated probabilities of all configurations (graphs) entering the Eq. (2.8). These results were used in turn to calculate the distributions of the number of collisions, of wounded nucleons and of wounded quarks. Also some parameters describing correlation between these quantities were calculated. These results are described in the next two sections.

4. Number of collisions and number of wounded nucleons

Using the parameters listed in Table I we obtained total α-α non-diffractive cross section equal to 220.8 mb.

Distribution of the number ν of non-diffractive collisions in α-α scattering is shown in Fig. 2. One sees that the distribution falls steadily with increasing ν. The average value and dispersion of the distribution are also given in Fig. 2.

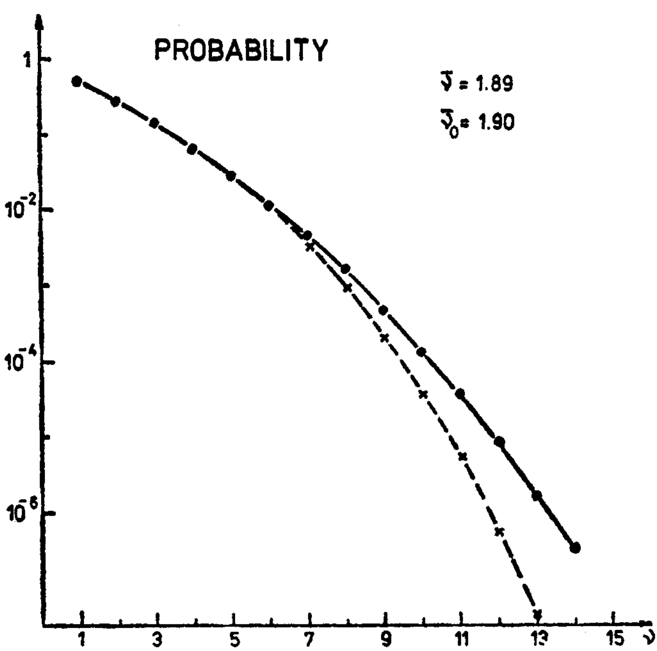


Fig. 2. Probability distribution of number of collisions. Dashed line: quasi-optical approximation

Distribution of the number w of wounded nucleons is shown in Fig. 3 together with the distribution of wounded nucleons in one of the α-particles. We see again that the minimal number of wounded nucleons is most probable. It is interesting to note that the pro-

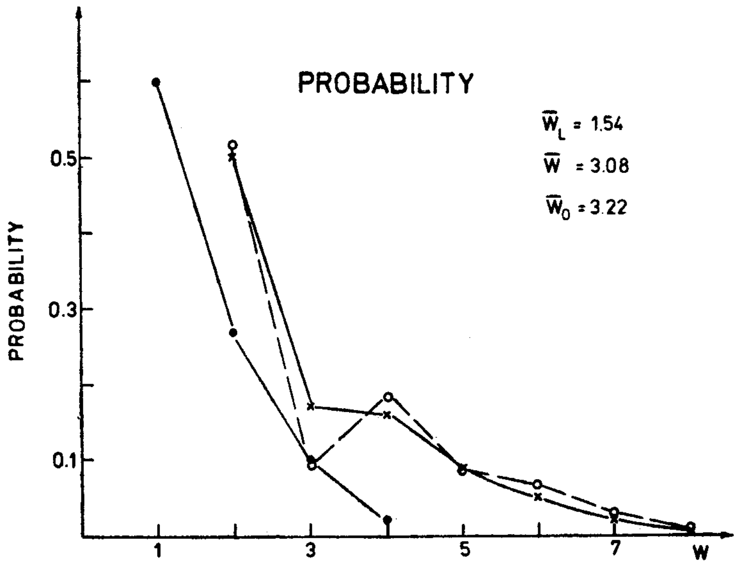


Fig. 3. Probability distribution of total number of wounded nucleons (crosses) and of number of wounded nucleons in one α-particle (solid line). Dashed line: quasi-optical approximation

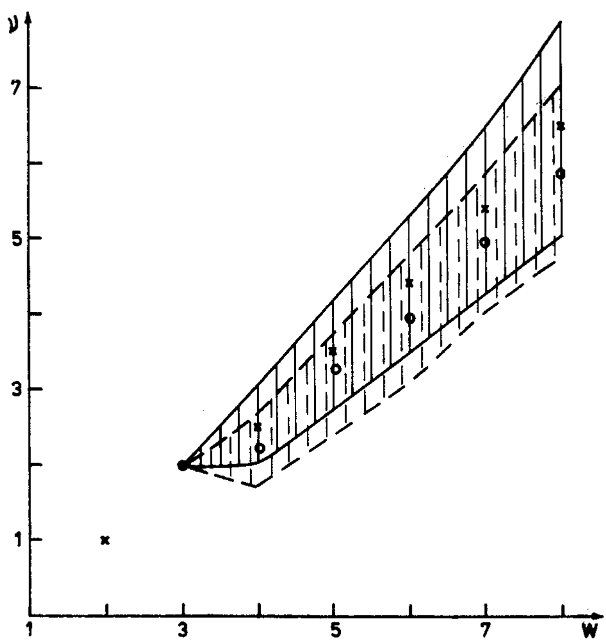


Fig. 4. Correlation between number of wounded nucleons and average number of collisions. Dashed lines: quasi-optical approximation

bability of $w = 8$, i.e. of all nucleons participating in the collision is not negligible (being $\sim .44\%$) corresponding to the cross-section of about 1 mb. It seems thus quite possible to obtain a large statistics of such events even with not very high intensity beams.

As the next problem we studied correlations between ν and w. In Fig. 4 the average number of collisions is plotted versus the number of wounded nucleons. One sees an almost linear relation, with about 6.5 collisions on the average for $w = 8$. The dispersion of the multiplicity distribution is also indicated in the Fig. 4. The relatively large values of the dispersion show that the number of wounded nucleons is not very precise measure of the number of collisions. This observation is further illustrated in Fig. 5 where the distribution

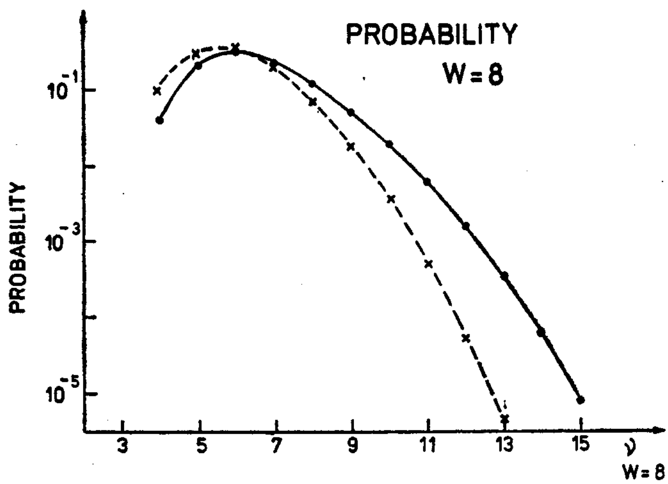


Fig. 5. Probability distribution of number of collisions for events with 8 wounded nucleons

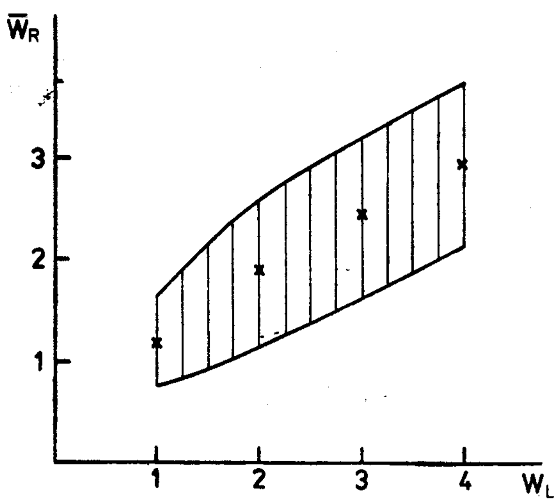


Fig. 6. Correlation between number of wounded nucleons in one hemisphere and average number of wounded nucleons in the other hemisphere

of the number of collisions for $w = 8$ is plotted. The distribution is asymmetric, with maximum at $\nu = 6$ and a substantial tail extending up to $\nu \sim 10$ and higher. This last feature is interesting because it indicates that one may perhaps study experimentally events with many collisions by appropriate cuts, i.e. on very high multiplicity.

Finally, we studied correlations between interactions in left-moving and right-moving α particles. In Fig. 6 the average number w_R of right-moving wounded nucleons is plotted versus number w_L of left-moving wounded nucleons. The observed relation is again close to a linear one with small deviation at $w_L = 4$. The dispersion of the distribution of w_R is also indicated in Fig. 6.

5. *Comparison with quasi-optical approximation*

As we have seen in the previous sections, explicit calculation of the probabilities in α-α scattering is very complicated and a successful extension of this method to larger nuclei seems unlikely. It is therefore interesting to compare our results with the so-called quasi-optical approximation which is often used in estimates of the total cross-section of heavy nuclei.

In quasi-optical approximation, the total non-diffractive A–B cross-section is given by the formula

$$\sigma_{AB}^{ND} = \int d^2b\{1-[1-\sigma_{AB}^{ND}(\vec{b})]^{AB}\} \tag{5.1}$$

with

$$\sigma_{AB}^{ND} = \int d^2x_\perp d^2y_\perp \tilde{D}_A(x_\perp)\tilde{D}_B(y_\perp)\sigma_{pp}^{ND}(b-x_\perp+y_\perp) \tag{5.2}$$

where

$$D_A(x_\perp) = \frac{A}{(A-1)\pi R^2}\exp\left\{-\frac{A}{A-1}\frac{x_\perp^2}{R^2}\right\}. \tag{5.3}$$

The substitution $R^2 \to \dfrac{A-1}{A}R^2$ in Eq. (5.3) takes into account recoil corrections.

The values of total cross-section for different set of parameters, obtained from Eq. (5.1) is given in Table I. One sees that they indeed are very close to those obtained by "exact" calculation. Formula (5.1) gives the following probability distribution of number of collisions

$$\sigma_{AB}P(\nu) = \int d^2b \binom{AB}{\nu}\{\sigma_{AB}^{ND}(b)\}^\nu\{1-\sigma_{AB}^{ND}(b)\}^{AB-\nu} \tag{5.4}$$

i.e. all graphs $G_i(\nu)$ for a given ν have the same value equal to

$$G(\nu) = \int d^2b\sigma_{AB}^{ND}(b)\{1-\sigma_{AB}^{ND}(b)\}^{AB-\nu}. \tag{5.5}$$

TABLE I

Parameters of the calculations presented in this paper, and results for cross-section, average values and dispersion s of quantities indicated. The bottom line corresponds to quasi-optical approximation (see text)

σ_{pp}^{ND}	R	r
26.5 mb	1.37 f	0.94 f

$\sigma_{\alpha\alpha}^{ND}$	$\bar{\nu}$	$\bar{w}$	$\bar{w}_L$	D_ν	D_w	D_{w_L}
220.8 mb	1.89	3.08	1.54	1.57	1.92	0.59
224.9 mb	1.90	3.22	1.61	1.49	2.38	0.67

Thus we see that the approximation (5.1) does not distinguish between different configurations for fixed ν.

The probability distribution (5.4) is plotted in Fig. 2. One sees that this distribution is narrower than the exact one and differs particularly from the exact one for $\nu \gtrsim 10$.

Distribution of the number of wounded nucleons is shown in Fig. 3. It is seen that the general trend is reproduced by the approximation. However, the significant structure at $w = 2$ and 3 is not present in exact distribution. The distribution of number of wounded nucleons in one of the α particles does not differ significantly from the exact one shown in Fig. 3 and is not plotted.

The correlation between ν and w is also affected as is seen from Fig. 4. This is illustrated particularly by a quite dramatic difference in the distribution of number of collisions at $w = 8$ shown in Fig. 5.

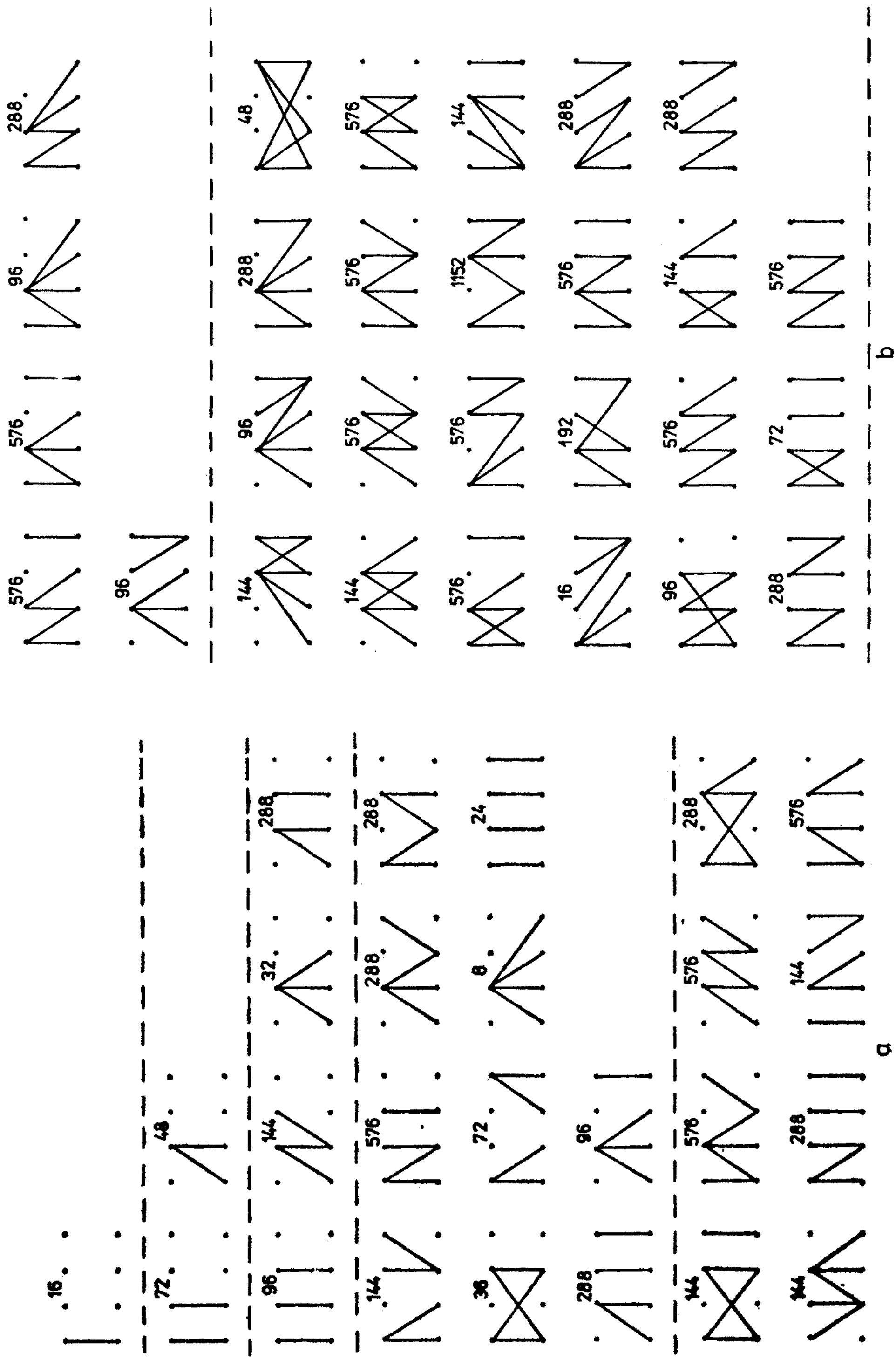
Finally, we observed that there is no significant difference in estimates of right-left correlations.

We thus conclude that quasi-optical approximation gives reasonable results for average quantities and the probabilities close to the maximum of the distribution. However, it cannot be used for estimates of the tails of the distribution i.e. for estimates of the fluctuations very far off the average.

APPENDIX 1

Graphs in α-α scattering

The Figures 7a–7d show 111 graphs representing different configurations of collisions in α-α scattering including collisions from 1 to 8. The multiplicities of the graphs are also indicated in the Figures. The graphs with more than 8 collisions are obtained from those with less than 8 collisions by adding all the missing lines and removing these which were present. The corresponding multiplicities are identical. The multiplicity of the last graph with 16 lines is equal to one.


a
b

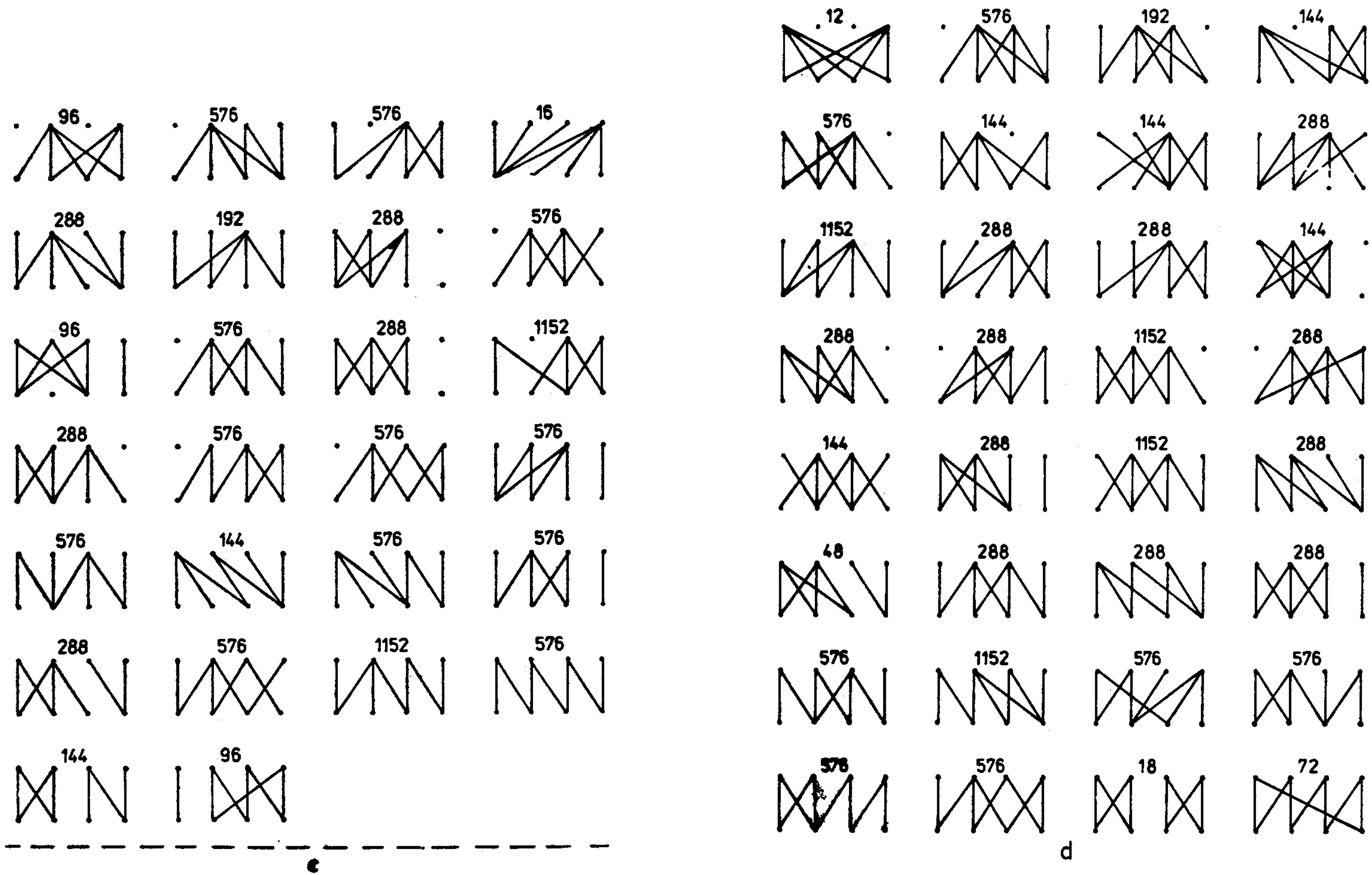


Fig. 7. Graphs describing probabilities in α-α collisions. The numbers indicate multiplicity of each graph

APPENDIX 2

Algorithm for evaluation of Gaussian integrals

The general form of integrals considered is

$$F(b) = \int \prod_{i=1}^{A} d^2 s_i \prod_{j=1}^{B} d^2 \tilde{s}_j D_{\mathrm{A}}(\{\vec{s}_i\}) D_{\mathrm{B}}(\{\vec{\tilde{s}}_j\}) G(\{\vec{b} - \vec{s}_i + \vec{\tilde{s}}_j\}), \tag{A2.1}$$

where

$$D_A(\{\vec{s}_i\}) = A R_{\mathrm{A}}^2 \delta^{(2)}\left(\sum_{i=1}^{A} \vec{s}_i\right) \tilde{D}_{\mathrm{A}}(\{\vec{s}_i\}) \tag{A2.2}$$

with

$$\tilde{D}_{\mathrm{A}}(\{\vec{s}_i\}) = \frac{1}{(\pi R_{\mathrm{A}}^2)^A} \exp\left\{- \sum_{i=1}^{A} s_i^2 / R_{\mathrm{A}}^2\right\} \tag{A2.3}$$

and corresponding formulae for $D_{\mathrm{B}}(\{\vec{\tilde{s}}_i\})$.

We first show how to treat the factors $\pi A R_{\mathrm{A}}^2 \delta^{(2)}(\sum \vec{s}_i)$ and $\pi B R_{\mathrm{B}}^2 \delta^{(2)}(\sum \vec{\tilde{s}}_i)$ which take into account the recoil corrections [8].

Introducing Fourier transform of $F(b)$ by the formula

$$f(\vec{\Delta}) = \int d^2 b e^{i\vec{\Delta}\vec{b}} F(\vec{b}), \tag{A2.4}$$

using the expansions for δ functions

$$\delta^{(2)}\left(\sum \vec{s}_i\right) = \frac{1}{(2\pi)^2} \int d^2 k e^{i\vec{k}\,\Sigma \vec{s}_i}; \; \delta^2\left(\sum \vec{\tilde{s}}_j\right) = \frac{1}{(2\pi)^2} \int d^2 \tilde{k} e^{i\vec{\tilde{k}}\,\Sigma \tilde{s}_j} \tag{A2.5}$$

and changing variables

$$\vec{s}_i \to \vec{s}_i + i\,\frac{\vec{k} R_{\mathrm{A}}^2}{2}, \qquad \vec{\tilde{s}}_j \to \vec{\tilde{s}}_j + i\,\frac{\vec{\tilde{k}} R_{\mathrm{B}}^2}{2}, \tag{A2.6}$$

$$\vec{b} \to \vec{b} + i(\vec{k} R_{\mathrm{A}}^2 - \vec{\tilde{k}} R_{\mathrm{B}}^2)/2 \tag{A2.6}$$

we obtain from (A2.1) and (A2.4)

$$f(\vec{\Delta}) = \frac{A R_{\mathrm{A}}^2}{4\pi} \frac{B R_{\mathrm{B}}^2}{4\pi} \int d^2 k \exp\left\{- \frac{R_{\mathrm{A}}^2}{4} (A k^2 - 2\vec{k}\vec{\Delta})\right\}$$
$$\times \int d^2 \tilde{k} \exp\left\{- \frac{R_{\mathrm{B}}^2}{4} (B \tilde{k}^2 + 2\vec{\tilde{k}}\vec{\Delta})\right\} \int d^2 b e^{i\vec{\Delta}\vec{b}} \tilde{F}(\vec{b}), \tag{A2.7}$$

where

$$\tilde{F}(\hat{b}) = \int \prod_{i=1}^{A} d^2 s_i \prod_{j=1}^{B} d^2 \tilde{s}_j \tilde{D}_{\mathrm{A}}(\{\vec{s}_i\}) \tilde{D}_{\mathrm{B}}(\{\vec{\tilde{s}}_j\}) G(\{\vec{b} - \vec{s}_i + \vec{\tilde{s}}_j\}) \tag{A2.3}$$

is the integral (A2.1) calculated *without* recoil corrections. This integral is much easier to perform, because the densities $\tilde{D}_A$ and $\tilde{D}_B$ factorize.

The integrals over d^2k and $d^2\bar{k}$ can be performed and the final answer is

$$f(\vec{\Delta}) = \exp\left\{\frac{\Delta^2}{4}\left(\frac{R_A^2}{A} + \frac{R_B^2}{B}\right)\right\} \int d^2b\tilde{F}(\vec{b})e^{i\vec{\Delta}\vec{b}}. \tag{A2.9}$$

One special case is of importance. If we are interested only in *total rates* and not in b-dependence we need only integral

$$\int d^2bF(\vec{b}) \equiv f(\vec{\Delta} = 0). \tag{A2.10}$$

As is easily seen from formula (A2.9), in this case the recoil corrections disappear. Thus, for calculation of the *total rates* one can use the nuclear densities (A2.3) uncorrected for the recoil effects.

The b-dependence can be easily determined if $\tilde{F}(b)$ is a Gaussian

$$\tilde{F}(b) = \tilde{F}(0)\exp(-b^2/\lambda^2). \tag{A2.11}$$

It follows from Eqs. (A2.9) and (A2.4) that also $f(\Delta)$ and $F(b)$ are Gaussians in this ca-e, and they are readily calculated as

$$f(\Delta) = \pi\lambda^2\tilde{F}(0)\exp\left\{-\frac{\Delta^2}{4}\left[\lambda^2 - \frac{R_A^2}{A} - \frac{R_B^2}{B}\right]\right\} \tag{A2.12}$$

and

$$F(b) = \frac{1}{(2\pi)^2}\int e^{-ib\Delta}f(\Delta)d^2\Delta = \frac{\lambda^2}{\lambda^2 - R_A^2/A - R_B^2/B}\tilde{F}(0)\exp\left\{-\frac{b^2}{\lambda^2 - R_A^2/A - R_B^2/B}\right\}. \tag{A2.13}$$

From this last formula $F(b)$ can be calculated, provided $\tilde{F}(b)$ is a Gaussian. If $\tilde{F}(b)$ is a superposition of Gaussians (as is the case in our problem) formula (A2.13) must be applied to each term separately.

Let us now turn to evaluation of $\tilde{F}(b)$. In the case considered in this paper the function $G(\{\vec{b} - \vec{s}_i + \vec{\bar{s}}_j\})$ in the formula (A.28) can be written as a superposition of the products of Gaussians. This expansion is discussed in the Appendix 3. We are thus led to considering the integrals of the type

$$I(b) = \int \prod_{i=1}^{4} d^2s_i \prod_{j=1}^{4} d^2\bar{s}_j \exp\{-w(\vec{s}_1, \ldots \vec{\bar{s}}_4; \vec{b})\}, \tag{A2.13}$$

where

$$w(\{\vec{s}_i, \vec{\bar{s}}_j; \vec{b}\}) = \sum_{i=1}^{4}\frac{s_i^2}{R^2}\sum_{j=1}^{4}\frac{\bar{s}_j^2}{R^2} + \sum_{i=1}^{4}\sum_{j=1}^{4} m_{ij}(\vec{b} - \vec{s}_i + \vec{\bar{s}}_j)^2 \tag{A2.14}$$

with $m_{ij} = 0$ or $m_{ij} = 1/r^2$. The matrix m_{ij} defines the integral which is to be performed. The result of the integration is [9]:

$$I(b) = \frac{\pi^8}{\det H} \exp\{-b^2[z - V^T H^{-1} V]\}, \tag{A2.15}$$

where

$$Z = \sum_{i=1}^{4} \sum_{j=1}^{4} m_{ij}, \quad V_i = \sum_{j=1}^{4} m_{ij} \quad \text{for} \quad i = 1, \ldots, 4,$$

$$V_i = -\sum_{j=1}^{4} m_{j,i-4} \quad \text{for} \quad i = 5, \ldots, 8,$$

$$H_{ii} = V_i + \frac{1}{R^2} \quad \text{for} \quad i = 1, \ldots, 4, \quad H_{ii} = -V_i + \frac{1}{R^2} \quad \text{for} \quad i = 1, \ldots, 8,$$

$$H_{ij} = G_{ji} = -m_{i,j-4} \quad \text{for} \quad i = 1, \ldots, 4, \quad j = 5, \ldots, 8, \quad = 0 \quad \text{for other} \quad i < j.$$

Thus the matrix H has a form

$$H = \left[\begin{array}{cc|cc} H_{11} & 0 & & \\ & \ddots & -\hat{m} & \\ 0 & H_{44} & & \\ \hline & & H_{55} & 0 \\ -\hat{m}^{\mathrm{T}} & & & \ddots \\ & & 0 & H_{88} \end{array}\right].$$

APPENDIX 3

Expansion of graphs into products of Gaussians

As we saw in Appendix 2, for evaluation of the graphs G_m from Eqs (2.8) and (2.9), it is essential to expand integrands into products of Gaussians. The calculation is then reduced to evaluation of the integrals of the general form given by Eq. (A2.13). Such expansion is quite straightforward and amounts to expressing the products of the type $\prod (1-\sigma_{ij})$ into sum of the products of the type $\prod \sigma_{ij}$.

We now observe that all different integrals

$$g_n = \int \prod d^2 s_i \prod d^2 \tilde{s}_j D_{\mathrm{A}}(\{\vec{s}_i\}) D_{\mathrm{B}}(\{\vec{\tilde{s}}_j\}) \prod_{i,j} \sigma_{ij} \tag{A3.1}$$

can be *labelled* by the same diagrams which were used for labelling the graphs G_n. Indeed, g_n differs from G_n defined in Eq. (2.10) only by absence of factors $\prod (1-\sigma_{ij})$ and thus the labelling is unaffected.

Using this notation, we can write generally

$$G_m = \sum_n (-1)^{\nu_m - \nu_n - 1} K_{mn} g_n, \tag{A3.2}$$

where the coefficients K_{mn} indicate how many times a given graph g_n enters into G_m.

Since we have the algorithm for calculation of the graphs g_n, the remaining problem is to determine the coefficients K_{mn}. To do this we first observed that it is convenient to introduce another set of parameters L_{mn} which give the number of ways in which a given graph G_m can be inserted into g_n. It is then straightforward to see that

$$K_m G_m = \sum (-1)^{\nu_m - \nu_n - 1} L_{mn} K_n g_n,$$

i.e.

$$K_m K_{mn} = L_{mn} K_n.$$

The parameters L_{mn} can be calculated using the following algorithm
(a) we observe that for $\nu_m > \nu_n$ $L_{mn} = 0$ and for $\nu_m = \nu_n$ $L_{mn} = \delta_{mn}$.
(b) For $\nu_m = \nu_n - 1$ L_{mn} were calculated directly from the definition.
(c) For $\nu_m < \nu_n - 1$ L_{mn} were calculated by iteration.
There are two sum rules which are very useful in testing the result in order to eliminate the mistakes during the calculation. They involve coefficients which connect graphs with given numbers of lines:
(a) For every n

$$\sum_m L_{mn} = \binom{\nu_n}{\nu_m},$$

where the sum runs over all graphs with ν_m lines
(b) for every m

$$\sum_n K_{mn} = \binom{AB - \nu_m}{AB - \nu_n},$$

where the sum runs over all graphs with ν_n lines.

Note added in proof

We decided to add the full list of probabilities $w_{LR} = w_{RL}$ of wounding L left-moving and R right-moving nucleons in α-α collisions.

$w_{11} = .504$	$w_{12} = .086$	$w_{13} = .012$
$w_{14} = .001$	$w_{22} = .135$	$w_{23} = .044$
$w_{24} = .006$	$w_{33} = .039$	$w_{34} = .010$
$w_{44} = .004.$		

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