Vol. B14 (1983) ACTA PHYSICA POLONICA No 8

GLUEBALLS IN THE BAG MODELS

By M. Jeżabek

Institute of Nuclear Physics, Cracow*

and J. Szwed

Institute of Computer Science, Jagellonian University, Cracow**

(*Received February 18, 1983*)

The influence of the bag shape on the lowest lying gluonium states is investigated. The MIT and the surface tension version of the bag model are considered.

PACS numbers: 12.90.+b

1. Introduction

The existence of pure-glue states is a clear prediction and test of quantum chromodynamics (QCD) and models inspired by this theory. The bag model [1], which differs from QCD by solving the confinement problem ab initio, is able to give definite predictions concerning the gluonium masses and dimensions, in analogy to the predictions for light hadrons built up of quarks [2]. The parameters of the model can be fixed by fitting the P, Δ, Ω^- and ϱ masses and the problem reduces to the solution of the QCD Maxwell equations

$$D^\mu_{ij}F_{j\mu\nu} = 0, \quad i, j = 1, 2, \dots, 8 \tag{1}$$

where

$$D^\mu_{ij} = \partial^\mu \delta_{ij} - g f_{ijk} A^\mu_k,$$

and

$$F_{j\mu\nu} = \partial_\mu A_{j\nu} - \partial_\nu A_{j\mu} + g f_{jkl} A_{k\mu} A_{l\nu}$$

with the boundary conditions on the bag surface

$$F^{\mu\nu}_i n_\mu = 0 \tag{2}$$

---

* Address: Instytut Fizyki Jądrowej, Kawiory 26a, 30-055 Kraków, Poland.
** Address: Instytut Informatyki UJ, Reymonta 4, 30-059 Kraków, Poland.

and

$$-\tfrac{1}{4}F_i^{\mu\nu}F_{i\mu\nu} = B. \tag{3}$$

B is the bag constant and n_μ — the unit vector which in the surface rest frame takes the form

$$n_\mu = (0, \vec{n})$$

where $\vec{n}$ is the inward normal to the surface. The conditions (2–3) can be rewritten in terms of the gluon electric ($\vec{E}$) and magnetic ($\vec{B}$) fields

$$\vec{n}\cdot\vec{E} = 0$$

$$n_0\vec{E}+\vec{n}\times\vec{B} = 0 \tag{2a}$$

and

$$\tfrac{1}{2}(|\vec{E}|^2-|\vec{B}|^2) = B. \tag{3a}$$

The lowest order results were first given by Jaffe and Johnson [3]. The obtained states range from 1.0 to 1.5 GeV which, together with their Zweig forbidden decays, makes their detection quite probable. These predictions were recently corrected [4, 5] by the inclusion of gluon-gluon interaction inside the cavity. The hyperfine splitting turns out to be large leaving only one [4] or two [5] states below 1.5 GeV.

The detailed studies of gluonia do not take into account possible deviation from the spherical shape of the bag which is assumed throughout these calculations. As a matter of fact the boundary conditions (2a) and (3a) state that the electric field E is tangential to the surface and nonzero on it with the bag constant B positive. This cannot be fulfilled on a sphere or any surface topologically equivalent to it. The simplest topology which is able to account for the boundary conditions (2)–(3) is that of a torus. It was considered by Robson [6] within the MIT bag model assuming a rectangular torus cross-section along the z-axis (Fig. 1). The resulting spectrum turned out to collapse to zero energy and the bag expanded to infinity.

In this paper we study a possible stabilization of the torus-like solutions by introducing the surface tension σ, present in another version of the bag model [7] (the Budapest bag).

The change, as compared to the original MIT version, comes out in the nonlinear boundary conditions (3) which read in this case

$$-\tfrac{1}{4}F_{i\mu\nu}F_i^{\mu\nu} = \sigma\partial_\mu n^\mu. \tag{3b}$$

It means that the pressure exerted by the confined fields from inside is balanced by the surface tension.

Both the Budapest bag model (with $B = 0$) and the original MIT model (with $\sigma = 0$) are equally successful when considering the standard predictions e.g. the masses of the ground state mesons and baryons, magnetic moments etc. [8]. We consider both models separately keeping in mind that any combination of them is also acceptable.

In addition we investigate the influence of edges of the bag surface, which are present in our approximate solutions, by studying bags of cylindrical shape and comparing them

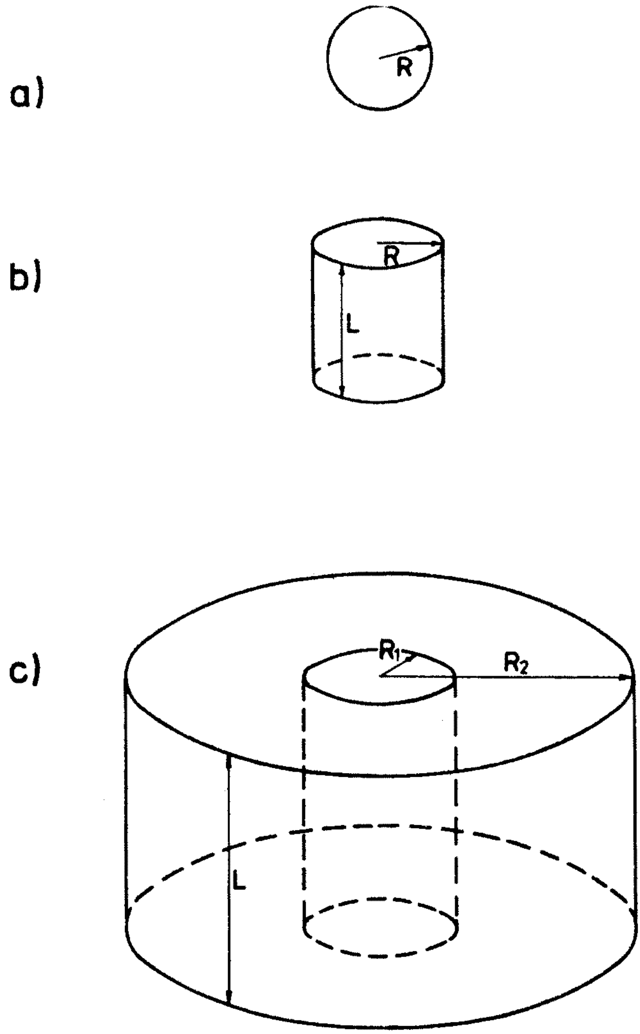

Fig. 1. The parametrizations of the considered bag shapes: a) sphere; b) cylinder; c) torus

with topologically equivalent spherical shapes. All our quantitative results are summarized in Tables I and II.

Our considerations do not take into account the recently proposed vacuum structure [9] which assumes the spherical shape to be built into the theory. In that case the nonlinear boundary conditions (3a) are neglected or fulfilled "on the average".

2. *The toroidal bag with surface tension*

Following Robson [6] we introduce the toroidal coordinates z, ϱ and φ and assume the torus shape as follows (see Fig. 1c)

$$0 \leqslant z \leqslant L, \quad R_1 \leqslant \varrho \leqslant R_2, \quad 0 \leqslant \varphi < 2\pi.$$

Furthermore we introduce new variables f and μ

$$R_1 = f\sinh\mu, \qquad R_2 = f\cosh\mu. \tag{4}$$

Three types of solutions to the Maxwell equations appear and are given in Ref. [6]. We quote only the energies ω of these modes which will be of further use:

— Transverse electric (TE) modes

$$\omega = \sqrt{\frac{(x_m^i)^2}{f^2}+\frac{n^2\pi^2}{L^2}}, \qquad \begin{array}{l} n = 1, 2, \ldots \\ m = \pm 1, \pm 2, \ldots \end{array} \tag{5}$$

where x_m^i is the i-th nonzero root of

$$J_m\left(x\frac{R_2}{f}\right)N_m\left(x\frac{R_1}{f}\right)-J_m\left(x\frac{R_1}{f}\right)N_m\left(x\frac{R_2}{f}\right) = 0$$

(J_m and N_m are the first and second type Bessel functions).

— Transverse magnetic (TM) modes

$$\omega = \sqrt{\frac{(y_m^i)^2}{f^2}+\frac{n^2\pi^2}{L^2}}, \qquad \begin{array}{l} n = 1, 2, \ldots \\ m = \pm 1, \pm 2, \pm 3, \ldots, \end{array} \tag{6}$$

where y_m^i is the i-th nonzero root of

$$J_m\left(y\frac{R_2}{f}\right)N_m'\left(y\frac{R_1}{f}\right)-J_m\left(y\frac{R_2}{f}\right)N_m'\left(y\frac{R_2}{f}\right) = 0.$$

— Transverse electric and magnetic (TEM) modes (for $m = 0$)

$$\omega = \frac{n\pi}{L}, \qquad n = 1, 2, \ldots . \tag{7}$$

The energy of a bag containing g gluons is

$$E = B\pi f^2 L + \sigma 2\pi f(Le^{\mu}+f)+g\omega+E_{\mathrm{rot}}+E_z \tag{8}$$

where B and σ are the volume and surface bag constants, E_{rot} is the rotational energy of the bag and E_z — the zero point energy.

The last two terms in Eq. (8) are neglected in our considerations. The rotational energy of the bag — E_{rot} can be roughly estimated by treating the bag as a rigid rotator and remembering that E_{rot} is inversly proportional to the moment of inertia. The lowest rotational excitations contribute less than 100 MeV to the states of considered angular momentum. This justifies our (Born-Oppenheimer type) approximation which neglects the rotational motion.

The zero point energy E_z was not calculated up to now for a toroidal shape. The usual procedure is to icnlude in it only the finite part, e.g. in the spherical bag — the term proportional to $(\text{radius})^{-1}$. The infinite terms are assumed to renormalize the bag constants B and σ. We are aware of the fact that our analysis of the dependence of the energy spectrum on the bag shape is not complete as both E_{rot} and E_z change with the bag shape.

The case with $\sigma = 0$ and B taken from the fit to the meson and baryon spectrum is given in Ref. [3]. We quote these results in Table I for comparison. We give there also the resulting masses of spherical gluonia (with $E_z = 0$) for the pure volume and pure surface bag models.

In the case when $B = 0$ the bag energy reads

$$E = 2\pi\sigma f(Le^{\mu}+f)+g\omega \tag{9}$$

with ω given by Eqs (5)–(7).

In the case of g TEM modes in the bag the minmialization of the energy with respect to f and L gives

$$\begin{aligned} M_{(\mathrm{TEM})^g} &\to 0\,, \\ L &\to \infty\,, \\ f \to 0 &\quad (R_1 \to 0), \\ \mu \to 0 &\quad (R_1/R_2 \to 0). \end{aligned}$$

It is seen that the introduction of the surface tension term does not stabilize the TEM gluonia.

The TE and TM modes in the bag make the minimalization more complicated. The bag energy reads in such case

$$E = 2\pi\sigma f(Le^{\mu}+f)+g\sqrt{\frac{x^2(\mu)}{f^2}+\frac{n^2\pi^2}{L^2}}\,, \tag{10}$$

where $x(\mu) = x^i_m(\mu)$ for TE and $y^i_m(\mu)$ for TM gluons respectively. In the case of both TE and TM modes present in the bag Eq. (10) contains a sum of square roots corresponding to these two modes.

The minimalization of Eq. (10) with respect to L and f gives an equation for the ratio $\alpha = f/L$

$$\alpha^3+\tfrac{1}{2}e^{\mu}\alpha^2-\frac{x^2e^{\mu}}{2\pi^2n^2} = 0 \tag{11}$$

and

$$L = \sqrt[3]{\frac{gn^2\pi}{2\sigma e^{\mu}}}\;\frac{1}{\sqrt[6]{\alpha^2n^2\pi^2+x^2(\mu)}}\,. \tag{12}$$

One can solve Eq. (11) for any value of μ and insert the solution into the formula for the bag energy

$$E = \frac{g}{\alpha}\sqrt[3]{\frac{2\sigma e^{\mu}}{gn^2\pi^2(\alpha^2n^2\pi^2+x^2(\mu))}}\,[\pi^2n^2\alpha^2(2+\alpha e^{-\mu})+x^2(\mu)]. \tag{13}$$

TABLE I

The obtained masses and dimensions of the cylindrical and toroidal glueballs compared with the spherically shaped glueballs for surface tension and volume pressure bag models. The zero point energy $E_z = 0$

	$E_z = -\frac{z}{R}$		$E_z = 0$					
Shape	Sphere M(GeV); R(GeV^{-1})		Sphere M(GeV); R(GeV^{-1})		Cylinder M(GeV); R(GeV^{-1}); L(GeV^{-1})		Torus M(GeV); R_1(GeV^{-1}); R_2(GeV^{-1}); L(GeV^{-1})	
Mode (J^{PC})	$B^{1/4} = 0.145$ GeV $\sigma = 0$	$B = 0$ $\sigma^{1/3} = 0.101$ GeV	$B^{1/4} = 0.145$ GeV $\sigma = 0$	$B = 0$ $\sigma^{1/3} = 0.101$ GeV	$B^{1/4} = 0.145$ GeV $\sigma = 0$	$B = 0$ $\sigma^{1/3} = 0.101$ GeV	$B^{1/4} = 0.145$ GeV $\sigma = 0$	$B = 0$ $\sigma^{1/3} = 0.101$ GeV
TE^2 (0^{++}, 2^{++})	0.97 5.02	0.97 4.99	1.31 5.57	1.38 5.96	0 0	1.22 7.19 0	0 ∞ ∞ ∞	1.30 6.53 6.53 20.51
TM^2 (0^{++}, 2^{++})	1.60 5.95	1.58 6.38	1.90 6.30	1.91 7.03	1.32 4.58 11.03	1.47 5.45 10.79	1.49 0 5.25 9.72	1.63 0 5.38 10.14
TE^3 (0^{+-}, 1^{++}, 2^{+-}, 3^{++})	1.47 5.78	1.46 6.12	1.78 6.16	1.81 6.82	0 0	1.60 8.23 0	0 ∞ ∞ ∞	1.49 7.47 7.47 23.48
TEM^2 (0^{++})	—	—	—	—	—	—	0 ∞ ∞ ∞	0 0 0 0

To obtain the gluonium mass one still has to find the minimum of Eq. (13) with respect to μ keeping in mind that the crucial dependence is contained in $x(\mu)$. (A plot of y^i_m and x^i_m as a function of μ for lowest values of m and i is given e.g. in Ref. [6]). The resulting mass spectrum is given in Table I.

One sees that the lowest lying $(TE)^2$ state obtains the mass very close to that of the spherical gluonium. The $(TM)^2$ and $(TE)^3$ differ by 200 MeV from their spherical partners. It is interesting that the mass of $(TM)^2$ gluonium is finite despite its infinite diameter.

3. Other shapes and constraints

As we already mentioned our toroidal solution is not an exact one. In addition to the discussed approximations we stress the presence of sharp edges on our torus where even the definition of the normal vector is impossible. To get the idea what is the influence of such edges we calculate the mass of the cylinder-like glueballs and compare them with the known spherical results. The solution to this problem can be found e.g. in Jackson [10] if one interchanges the $\vec{E}$ and $\vec{B}$ fields. In the MIT bag model ($\sigma = 0$) the TE gluon populated bags collapse to flat disks of infinite diameter and zero energy (see Table I). The $(TM)^2$ state is stable and lighter than its spherical partner. The introduction of the surface tension σ ($B = 0$) stabilizes all modes. In this case they are all 150–350 MeV lighter than the corresponding spherical glueballs.

Looking at the bag dimensions given in Table I one notices that the resulting shape is often so degenerate that it cannot be accepted as an approximation to a sphere (in the case of the cylinder) or a torus (in the case of a "rectangular" torus). It may be therefore plausible to put some constraints on the bag dimensions in order to present the bag to collapse. We require as an example that the cross-section along the z-axis of the cylinder bag is a square so that a circle can be inscribed into it. This means that the height of the cylinder is equal to its diameter. The resulting mass spectrum of such glueballs is given in Table II for both volume and surface models separately. All states are now stable and close to their spherical partners, except of the magnetic gluon bags which are considerably lighter. The comparison of these results with the spherical ones gives an idea what is the influence of sharp edges when using the formula (8) for the bag energy with $E_{rot} = E_z = 0$.

Proceedings along the same lines one can calculate the mass spectrum of the toroidal glueballs with square cross sections along the z-axis. In this case the constraint means

$$R_2 - R_1 = L$$

and allows to inscribe a real torus into ours.

The results of the minimalization are given in Table II for the volume and surface tension models separately. All considered states acquire masses even the $(TEM)^2$ glueball! The glueballs built of TE gluons are generally heavier than the spherical ones, the $(TM)^2$ mode is lighter. An interesting result is the resulting shape — in all cases $R_1 = 0$ and R_2 is finite. This means that the zero of the tangential electric field makes minimal job — a single hole in the sphere.

The summary of the paper is given in Tables I and II where all masses and dimensions

TABLE II

As in Table I but with the constraint $2R = L$ for the cylinder and $R_2 - R_1 = L$ for the torus

Shape	Sphere		Cylinder $2R = L$		Torus $R_2 = R_1 = L$	
	M(GeV); R(GeV^{-1})		M(GeV); R(GeV^{-1})		M(GeV); R_1(GeV^{-1}); R_2(GeV^{-1})	
Mode (J^{PC})	$B^{1/4} = 0.145$ GeV $\sigma = 0$	$B = 0$ $\sigma^{1/3} = 0.101$ GeV	$B^{1/4} = 0.145$ GeV $\sigma = 0$	$B = 0$ $\sigma^{1/3} = 0.101$ GeV	$B^{1/4} = 0.145$ GeV $\sigma = 0$	$B = 0$ $\sigma^{1/3} = 0.101$ GeV
TE^2 (0^{++}, 2^{++})	1.31 5.57	1.38 5.96	1.32 4.87	1.45 4.98	1.50 0 6.46	1.67 0 6.55
TM^2 (0^{++}, 2^{++})	1.90 6.30	1.92 7.03	1.32 4.88	1.45 4.99	1.60 0 6.60	1.76 0 6.73
TE^3 (0^{+}I, 1^{++}, 2^{+-}, 3^{++})	1.78 6.16	1.81 6.82	1.79 5.39	1.90 5.70	2.03 0 7.08	2.18 0 7.50
TEM^2 (0^{++})	—	—	—	—	1.34 0 6.23	1.51 0 6.24

are collected. One sees that the glueball masses change within 600 MeV when requiring different shapes. One has to keep this result in mind when looking at the detailed calculations which include higher order corrections. When going to the toroidal shapes which are able to satisfy the nonlinear boundary conditions (3.3b) we encounter one new type of gluon modes — the TEM modes, which form the lightest glueballs. Another general remark is that the surface tension model stabilizes more often the solutions than the volume term.

We would like to thank Marcin Szwed, the son of J.S., for making plastic models of our solutions, which helped us to understand and demonstrate our results.

REFERENCES

[1] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, V. F. Weisskopff, *Phys. Rev.* **D9**, 3471 (1974).
[2] T. De Grand, R. L. Jaffe, K. Johnson, J. Kiskis, *Phys. Rev.* **D12**, 2060 (1975).
[3] R. L. Jaffe, K. Johnson, *Phys. Lett.* **60B**, 201 (1976).
[4] C. E. Carlson, T. H. Hansson, C. Peterson, MIT preprint CTP 1020 (1982).
[5] T. Barnes, F. E. Close, S. Monaghan, *Nucl. Phys.* **B198**, 380 (1982).
[6] D. Robson, *Z. Phys.* **C3**, 199 (1980).
[7] P. Gnädig, P. Hasenfratz, J. Kuti, A. S. Szalay, *Phys. Lett.* **64B**, 62 (1976).
[8] J. Szwed, *Nucl. Phys.* **B138**, 445 (1978).
[9] T. H. Hansson, K. Johnson, C. Peterson, University of California, Santa Barbara preprint NSF-ITP-82-01.
[10] J. D. Jackson, *Classical Electrodynamics*, John Willey and Sons, Inc., 1975.