AN EFFECTIVE ENERGY-DEPENDENT ONE-BODY POTENTIAL FOR THE LOW-ENERGY PARTICLE-NUCLEUS SCATTERING PROBLEM. THE ONE OPEN CHANNEL CASE

By J. Tōke* and T. Matulewicz

Institute of Experimental Physics, Warsaw University**

(Received February 10, 1983)

A simple method of generating the relative motion continuum wave functions in the region of isolated resonances by using an energy-dependent effective one-body potential is proposed. The method accounts for the effects of the configuration mixing and predicts splitting of the single-particle resonances in quantitative agreement with the shell model diagonalization approach.

PACS numbers: 24.10.-i

1. Introduction

It is common practice to generate the wave functions of relative motion of a projectile plus target nucleus system using a suitably chosen optical potential [1]. In many applications, where the corresponding compound system excitation energy is high, a smooth energy dependence of the optical potential parameters is sufficient to account properly for the behaviour of the phase shifts (see e.g. [2]). In the region of isolated resonances, however, such a commonly used potential fails to reproduce the phase shifts and in specific applications recourse to other approaches, such as, e.g., the R-matrix approach [3], has to be made. At the same time, from the point of wiew of some particular applications, such as, e.g., the DWBA calculations or other direct reaction amplitude evaluation, it is desirable to have a possibly simple method for generating the wave function of the relative motion with proper phase shift, i.e. describing properly the elastic scattering amplitudes.

In the present paper we propose a simple phenomenological extension of the common optical potential approach consisting in the introduction of a special energy dependence of the one-body optical potential. The proposed approach is much more simple than the

* Present address: GSI, 6100 Darmstadt, W. Germany.

** Address: Instytut Fizyki Doświadczalnej, Uniwersytet Warszawski, Hoża 69, 00-681 Warszawa, Poland.

more complete theories of continuum plus bound states problem worked out in the past (see e.g. [3] and [4]) and at the same time it accounts for major (from point of view of the particular applications mentioned above) effects of the mixing of the pure single particle (cluster) states with the bound states embedded into the continuum (BSEC). Therefore we expect it to be a convenient tool for analysing direct reaction data in the region of isolated resonances and especially of those having considerable single-particle strength.

In Section 2 the structure of the effective one-body potential for the considered scattering problem is derived starting from the Ritz-Reyleigh variational principle. At the cost of a few approximations extreme simplicity of the final formula has been gained. Although the assumptions made are not controversial (they do not go beyond those lying behind some commonly accepted and applied procedures) their effects cannot be assessed beforehand and therefore the question of the usefulness of the proposed approach is to be solved by applying tests.

In Section 3 the proposed approach is subjected to tests which reveal its equivalence with the shell model diagonalization problem as far as the predicted positions of the resonances (each of which now carry only a fraction of the original single-particle strength) and their single-particle (cluster) spectroscopic factors are concerned. This we view as an important property of the proposed energy-dependent potential which proves its usefulness no matter how well founded were the assumptions made when deriving its general structure.

Section 4 discusses some possible applications, analogies and further developments of the proposed approach.

2. *Derivation of the general structure of the energy-dependent effective one-body potential*

We consider a target plus projectile system with one open reaction channel. For the sake of simplicity we consider one partial wave only and disregard the angular momenta coupling game.

We start with the Ritz-Reyleigh variational principle:

$$\langle \delta\Psi_E(\xi, r) \,|H-\tilde{E}|\Psi_E(\xi, r)\rangle = 0, \tag{1}$$

where H is the full Hamiltonian of the system, $\tilde{E}$ is the total energy and $\Psi_E(\xi, r)$ is a tria wave function as determined by the physical problem.

For the trial function we assume the following ansatz:

$$\Psi_E(\xi, r) = \Phi_c(\xi)\chi_E(r) + \sum_{i=1}^{N} b_i \Phi^{(i)}_{\mathrm{BSEC}}(\xi, r), \tag{2}$$

where $\Phi_c(\xi)$ is the product of intrinsic wave functions of the target and projectile nuclei, assumed to be given, $\chi_E(r)$ is the wave function of relative motion of the projectile and target nuclei, $\Phi^{(i)}_{\mathrm{BSEC}}(\xi, r)$ are the wave functions of the bound states embedded into the continuum which are also assumed to be given and b_i are the unknown spectroscopic amplitudes of the BSEC.

At this stage it is assumed additionally that:

$$\langle \Phi_c(\xi) | H | \Phi_c(\xi) \rangle_\xi = h_0(r) + E_c, \tag{3a}$$

where $h_0(r)$ is the optical model one-body Hamiltonian for the relative motion of the projectile and target nuclei, E_c is the intrinsic energy of the projectile and target nuclei and $\langle\ \rangle_\xi$ denotes integration over the intrinsic coordinates only,

$$\langle \Phi_c(\xi) | \Phi^{(i)}_{\mathrm{BSEC}}(\xi, r) \rangle_\xi = 0, \tag{3b}$$

$$\langle \Phi^{(i)}_{\mathrm{BSEC}} | \Phi^{(k)}_{\mathrm{BSEC}} \rangle = \delta_{ik} \quad \text{and} \tag{3c}$$

$$\langle \Phi^{(i)}_{\mathrm{BSEC}} | H | \Phi^{(k)}_{\mathrm{BSEC}} \rangle = \mathscr{E}_i \delta_{ik}. \tag{3d}$$

Next, we decompose the total Hamiltonian:

$$H = H_0(\xi, r) + V_{\mathrm{res}}(\xi, r), \tag{4}$$

so that:

$$\langle \Phi_c | H_0 | \Phi^{(i)}_{\mathrm{BSEC}} \rangle_\xi = 0 \tag{5}$$

and introduce the following quantities:

(*i*) norms $$N_E = \langle \chi_E(r) | \chi_E(r) \rangle_R^{1/2}, \tag{6a}$$

where $\langle\ \rangle_R$ denotes integration over the relative distance r from zero up to a "reasonably" chosen channel radius R (the choice of the channel radius value is not crucial),

(*ii*) formactors $$w_i(r) = \langle \Phi_c(\xi) | V_{\mathrm{res}} | \Phi^{(i)}_{\mathrm{BSEC}}(\xi, r) \rangle_\xi, \tag{6b}$$

(*iii*) matrix elements $$V_{ci} = \langle \chi_{\mathscr{E}_0}(r) | w_i(r) \rangle_R N^{-1}_{\mathscr{E}_0}, \tag{6c}$$

where $\mathscr{E}_0$ denotes the energy of the single-particle (cluster) resonance,
(*iv*/*v*) formfactors $f_i(r)$ and $g_E(r)$ defined by:

$$w_i(r) = f_i(r) \chi_{\mathscr{E}_0}(r) V_{ci} N_{\mathscr{E}_0} F_i^{-1}, \tag{6d}$$

where $$F_i = \langle \chi_{\mathscr{E}_0} | f_i(r) | \chi_{\mathscr{E}_0} \rangle_R \tag{6e}$$

and $$\chi_{\mathscr{E}_0}(r) = g_E(r) \chi_E(r). \tag{6f}$$

Now, equation (1) can be replaced by a set of $N+1$ coupled integro-differential equations:

$$\langle \Phi_c(\xi) | H - \tilde{E} | \Phi_c(\xi) \chi_L(r) + \sum_{i=1}^{N} b_i \Phi^{(i)}_{\mathrm{BSEC}}(\xi, r) \rangle_\xi = 0 \tag{7a}$$

and

$$\langle \Phi^{(k)}_{\mathrm{BSEC}}(\xi, r) | H - \tilde{E} | \Phi_c(\xi) \chi_E(r) + \sum_{i=1}^{N} b_i \Phi^{(i)}_{\mathrm{BSEC}}(\xi, r) \rangle = 0 \tag{7b}$$

(k goes from 1 to N) which under the above listed assumptions and definitions can be rewritten as:

$$(h_0 - E)\chi_E(r) + \sum_{i=1}^{N} b_i f_i(r) g_E(r) \chi_E(r) V_{ci} N_{\mathscr{E}_0} F_i^{-1} = 0 \tag{8a}$$

and

$$V_{ck} N_{\mathscr{E}_0} F_k^{-1} \langle \chi_{\mathscr{E}_0}(r) | f_k(r) g_E^{-1}(r) | \chi_{\mathscr{E}_0}(r) \rangle_R + b_k(\mathscr{E}_k - E) = 0, \tag{8b}$$

where E and $\mathscr{E}_k$ are now energies counted with respect to the sum of intrinsic energies of the nonexcited projectile and target nuclei. The approximation which allows for a major simplification of set (8) and which at the same time is decisive for the range of applicability of our approach consists in putting:

$$f_i(r) g_E^{-1}(r) = f_i(r) N_E. \tag{9}$$

This means that within the range of nuclear interaction (set by the formfactor $f_i(r)$), the wave function of relative motion of the projectile and target nuclei is, except for normalization (N_E), independent of energy (see Eq. 6f). This approximation is valid as long as the range of energy variation is small compared to the interaction potential depth i.e. for the energy range of the order of a few MeV.

Assumption (9) allows to rewrite (8b) as:

$$V_{ci} N_{\mathscr{E}_0} N_E - b_i(E - \mathscr{E}_i) = 0 \tag{10}$$

and consequently rewrite (8a) as:

$$\left[h_0 + \sum_{i=1}^{N} V_{ci}^2 N_{\mathscr{E}_0}^2 F_i^{-1} \frac{1}{E - \mathscr{E}_i} f_i(r) \right] \chi_E(r) = E \chi_E(r), \tag{11}$$

which is a Schrodinger equation for the relative wave function $\chi_E(r)$. The effective one-body potential has thus the form:

$$V_{\text{eff}}(r) = V_0(r) + \sum_{i=1}^{N} \frac{a V_{ci}^2}{E - \mathscr{E}_i} f_i(r), \tag{12}$$

where

$$a = N_{\mathscr{E}_0}^2 F_i^{-1}. \tag{13}$$

As approximation (9) breaks down also then, when the correction to $V_0(r)$ (i.e. the sum over the BSEC in Eq. (12)) becomes comparable with $V_0(r)$ itself, formula (12) is invalid in the close vicinities of $\mathscr{E}_i$. Fortunately, however, the regions where our approach based on formula (12) fails seem to be of negligible widths in many practical applications.

Parameters V_{ci}, a, $\mathscr{E}_i$ and the formfactors $f_i(r)$ are unknown. Therefore they are to be considered as adjustable quantities when applying our energy-dependent effective potential approach in the course of analysis of experimental data. It is very much in the spirit of a phenomenological approach to adopt a simple function for the formfactors $f_i(r)$ based

on general considerations. In the tests described in the next section, like in the first practical applications ([8] and [9]) common Woods-Saxon formfactors were assumed for $V_0(r)$ and $f_i(r)$, so the energy dependence was limited to the potential depth parameter only. This simplification is exactly in line with the commonly accepted practice of generating the bound state wave functions for DWBA purposes. (See the last paragraphs of the Discussion.)

3. *Analytical and numerical tests*

The action of potential (12) can be understood from Fig. 1. The almost straigth line in Fig. 1b labelled $U_{res}(E)$ describes the dependence of the single-particle (cluster) resonance energy for a particular partial wave on the Woods-Saxon well depth. Variation of the well depth with particle energy in accordance with (12) (Fig. 1b) causes that the pure single--particle (cluster) resonance (Fig. 1a) splits into $N+1$ resonances, where N is the number of relevant BSEC (here $N = 1$). Those new resonances (Fig. 1c) occur at the energies where the effective potential and the resonance-adjusted depths are equal, i.e. their positions can be found by equating $U_{eff}(E)$ to $U_{res}(E)$:

$$U_{eff}(E_{res}) = a(E_{res} - \mathscr{E}_0) + U_0. \tag{14}$$

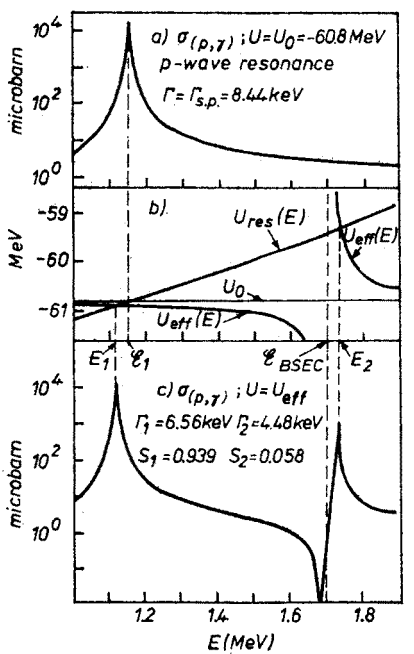

Fig. 1. Excitation function for the (p, γ) reaction for a hypothetical $A = 28$, $Z = 14$ nucleus as calculated using in the entrance channel a constant depth potential (upper section) and the energy dependent effective potential with $U_{eff}(E)$ shown in the middle section (lower section). The $U_{res}(E)$ line in the middle section represents the dependence of the single-particle resonance energy on the well depth for the particular well geometry

The right hand side of Eq. (14) is an approximate parametrization of the $U_{\text{res}}(E)$ dependence. It is worth pointing out that the parameter a in formula (14) has the same meaning as that in (13). This is quite encouraging (though not crucial) as it leads to a detailed correspondence with the shell model approach. Namely, equation (14) can be cast into the familiar determinant form:

$$\begin{vmatrix} (\mathscr{E}_0 - E_{\text{res}}) & V_{c1} & V_{cN} \\ V_{c1} & (\mathscr{E}_1 - E_{\text{res}}) & 0 \\ \vdots & \vdots & \vdots \\ V_{cN} & 0 & (\mathscr{E}_N - E_{\text{res}}) \end{vmatrix} = 0 \tag{15}$$

exhibiting the equivalence of our approach with the shell model diagonalization problem as far as the predicted positions and therefore also the spectroscopic factors of the resonances (resulting from admixing fractions of single-particle strength to the BSEC) are concerned. It may be verified that the structure of the determinant in (15) corresponds exactly to the assumptions and definitions adopted in Section 2.

Numerical testing of our approach was carried out by calculating the "direct" proton capture (p, γ) reaction excitation functions for a hypotherical nucleus with $A = 28$ and $Z = 14$. In those calculations the continuum state wave function of the entrance channel, calculated using the energy-dependent effective one-body potential approach, was inserted into the standard direct capture code [6] to yield the cross sections shown in Fig. 1. It is interesting to note the "interference" pattern, which was obtained even though the reaction amplitude was not calculated in parts but rather as a single El transition amplitude from the entrance channel scattering state to the final bound state.

In Table I, displaying some other results of the calculations it is interesting to note the rather impressive compliance with the sum rule for the spectroscopic factors which were calculated as quotients $\Gamma_k/\Gamma_{\text{sp}}^{(k)}$, where Γ_k is the width of the k-th resonance and $\Gamma_{\text{sp}}^{(k)}$ is the width of the single-particle resonance generated (by an appropriate choice of the constant well depth) at the energy where the k-th resonance occurs.

In the calculations the El transition from the p-wave to the $2s_{1/2}$ bound orbital was assumed. A standard Woods-Saxon geometry with $r_0 = 1.3$ fm and $a_R = 0.5$ fm was adopted. The values of other parameters involved in generating the entrance channel wave function were: $U_0 = -60.8$ MeV, $\mathscr{E}_1 = 1.7$ MeV, $aV_{ci}^2 = 0.0492$ MeV (corresponding to the interaction matrix element of 0.169 MeV) and the Coulomb radius parameter $r_C = 1.3$ fm. It is perhaps worth mentioning that both the direct and resonant captures contribute here to the calculated cross sections, although in more realistic calculations one has to account also for the decay of the BSEC itself, e.g. by renormalizing appropriately the effective charge [7].

4. *Discussion*

The proposed energy-dependent effective one-body potential approach has been designed for the subbarrier energy ranges where only a few resonances for a given partial wave occur. It has been applied by us successfully for analyzing the direct capture (p, γ)

TABLE I

Analysis of the resonances computer-generated by using the energy-dependent effective one-body potential approach

Case	$\mathscr{E}_0$[a] $\mathscr{E}_i$[b]	U_0 $a^{1/2}V_{ci}$	E_k a	Γ_k	$\Gamma_{sp}^{(k)}$	S_k[c] ΣS_k
	MeV	MeV	MeV	keV	keV	
I	1.470	−60.0	1.508	32.6	38.5	0.847
	1.300	0.453	1.261	2.20	14.4	0.153
						1.000
II	1.152	−60.8	0.939	1.09	2.24	0.488
	1.100	0.222	1.183	1.83	9.91	0.185
	1.300	0.222	1.387	6.24	24.7	0.253
	1.700	0.222	1.738	5.71	77.8	0.073
						0.999

[a] i varies from 1 to N and k varies from 1 to $N+1$ where N is the number of BSEC coupled to the entrance channel.

[b] The energies are counted with respect to the energy of the system of noninteracting target and projectile nuclei.

[c] Errors of the S values are of the order of 0.001. No special effort was made to minimize them.

reaction data on ^{28}Si target nuclei ([8] and [9]), where it allowed us to avoid the use of the R-matrix formalism for accounting for the observed broad resonances [10]. Generally, we do believe that our approach may be considered as alternative to the R-matrix one as far as the description of the subbarrier resonances is concerned. It has, however, the advantage of being able to produce directly the wave functions which are required by the direct or semidirect reaction analysis codes.

One possible way of extending the range of applicability of the presented approach consists in introducing the energy dependence of the one-body potential formfactor itself. So, e.g., a self-consistent formfactor $g_E(r)$, which can be derived by iteration from Eq. (8), would most probably be a remedy to the $E \sim \mathscr{E}_i$ problem (where our approach fails), however, derivation of such a self-consistent formfactor destroys the simplicity of the proposed approach.

A natural generalization to the many open channel cases seems to be possible via setting the BSEC energies complex, with the imaginary parts related to the decay widths of those BSEC into other channels. This would affect the real potential well, though the main effect is the introduction of the energy-dependent effective imaginary well. Qualitatively, effects such as broadening of resonances and the loss of flux in the only explicitly treated channel are correctly reproduced. Further investigation in this direction is in progress.

It is worth pointing out the analogy of the proposed approach with the one which is widely used when generating bound state one-body wave-functions for DWBA purposes.

In that case too, the residual interactions cause that the strength of a shell model one-body (bound) state of interest spreads among a number of discrete physical states. It is then assumed (and has been proved to be a reasonable assumption by the long history of DWBA analyses) that some important effects of mixing the particular shell model bound state of interest with more complicated states (the BSEC) can be mocked up by an appropriate choice of the one-body potential depth. This depth is different for each physical state (i.e. the effective one-body potential is energy-dependent) which contains an admixture of the shell model state of interest and is being in practice found by requiring that the binding energy be equal to the experimentally observed separation energy.

Our proposition (12), simplified additionally by assuming an energy independent Woods-Saxon formfactor, can be considered as a generalization of such a bound state treatment. Except that now, when the energy changes in a continuous manner rather than in a discrete, a formula for the depth of the potential is being given. (A similar formula is valid also for the bound state case, however, there its usefulness seems doubtful.) This formula contains a (small) number of parameters which are to be determined by fitting the "theoretical" resonances (their positions and widths) to the experimentally observed ones. This formula tells, how the potential depth should be chosen now (i.e. in the continuum wave function case) in order to account for the same effects that in the case of bound state wave function are accounted for by applying the fameous separation energy recipe. The assumptions lying behind that recipe are the same as those lying behind our approach (which are exhibited in Section 2).

The authors wish to acknowledge discussions with H. A. Weidenmüller, P. Decowski and B. Sikora.

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