Erratum and Addendum
to "Tensor Form of the Breit Equation" by W. Królikowski, *Acta Phys. Pol.* **B14**, 109 (1983)

Eqs. (25) and (26) should be corrected to the form

$$\left[\left(\frac{E-V}{2}\right)^2+\frac{1}{r}\frac{d^2}{dr^2}r-\frac{j(j+1)+2}{r^2}-m^2-\frac{d}{dr}\frac{\frac{dV}{dr}}{E-V}\right]\chi^0_{el}(r)=\frac{2}{r^2}\chi^0_{lon}(r), \tag{E25}$$

and

$$\left[\left(\frac{E-V}{2}\right)^2+\frac{1}{r}\frac{d^2}{dr^2}r-\frac{j(j+1)}{r^2}-m^2+\frac{\frac{dV}{dr}}{E-V}\left(\frac{d}{dr}+\frac{1}{r}\right)\right]\chi^0_{lon}$$
$$=\frac{2j(j+1)}{r^2}\left(1-r\frac{\frac{dV}{dr}}{E-V}\right)\chi^0_{el}, \tag{E26}$$

where $\chi^0_{lon}=\left(\frac{\partial}{\partial\hat{r}}-2\hat{r}\right)\cdot\vec{\chi}^0$ is used instead of the previous $\chi^0_{lon}=\frac{\partial}{\partial\hat{r}}\cdot\vec{\chi}^0$. Now

$$\vec{\chi}^0(\vec{r})=\hat{r}\chi^0_{el}(\vec{r})-\frac{\partial}{\partial\hat{r}}\frac{\chi^0_{lon}(\vec{r})}{j(j+1)}-\left(\hat{r}\times\frac{\partial}{\partial\hat{r}}\right)\frac{\chi^0_{mag}(\vec{r})}{j(j+1)} \tag{28}$$

with

$$\left.\begin{aligned}\chi^0_{el}(\vec{r})&=\hat{r}\cdot\vec{\chi}^0(\vec{r})=\chi^0_{el}(r)Y_{jm}(\hat{r}),\\ \chi^0_{lon}(\vec{r})&=\left(\frac{\partial}{\partial\hat{r}}-2\hat{r}\right)\cdot\vec{\chi}^0(\vec{r})=\chi^0_{lon}(r)Y_{jm}(\hat{r}),\\ \chi^0_{mag}(\vec{r})&=\left(\hat{r}\times\frac{\partial}{\partial\hat{r}}\right)\cdot\vec{\chi}^0(\vec{r})=\chi^0_{mag}(r)Y_{jm}(\hat{r}),\end{aligned}\right\} \tag{29}$$

where $\hat{r}=\frac{\vec{r}}{r}$ and $\frac{\partial}{\partial\hat{r}}=r\frac{\partial}{\partial\vec{r}}-\vec{r}\frac{\partial}{\partial r}$ implying $\hat{r}^2=1$, $\hat{r}\cdot\frac{\partial}{\partial\hat{r}}=0$, $\frac{\partial}{\partial\hat{r}}\cdot\hat{r}=2$, $\left(\frac{\partial}{\partial\hat{r}}\right)^2$

$= -\vec{L}^2 = \left(\frac{\partial}{\partial\hat{r}} - 2\hat{r}\right)^2$, $\hat{r}\times\frac{\partial}{\partial\hat{r}} = \vec{r}\times\frac{\partial}{\partial\vec{r}} = i\vec{L}$ and $\frac{\partial}{\partial\hat{r}}\times\frac{\partial}{\partial\hat{r}} = i\vec{L}$. Note that the operator $\frac{\partial}{\partial\hat{r}} - 2\hat{r}$ is Hermitian conjugate of $-\frac{\partial}{\partial\hat{r}}$. Due to Eqs. (28) or (29) the previous χ^0_{lon} is equal to $\chi^0_{lon}+2\chi^0_{el}$. If $j = 0$ then $\chi^0_{lon} = 0$ and $\chi^0_{mag} = 0$ (where $l = 1$ as $s = 1$).

Eqs. (23) and (27) are correct and remain unchanged. If $j > 0$ then three Eqs. (E25), (E26) and (27) describe in a relativistic way the orbital angular momentum triplet $l = j-1, j+1, j$ (the first two components being mixed) with the total angular momentum j and spin $s = 1$. Eq. (23) describes the singlet $l = j$ with j and $s = 0$. Note that the wave function components with j and $s = 1$ corresponding to the determined orbital angular momentum equal to $l = j-1, j+1, j$ are

$$\chi^0_{l=j-1} = \frac{j\chi^0_{el}-\chi^0_{lon}}{\sqrt{j(2j+1)}}, \quad \chi^0_{l=j+1} = \frac{(j+1)\chi^0_{el}+\chi^0_{lon}}{\sqrt{(j+1)(2j+1)}}, \quad \chi^0_{l=j} = \frac{\chi^0_{mag}}{\sqrt{j(j+1)}}. \tag{30}$$

Eqs. (28) and (30) give

$$\|\vec{\chi}^0\|^2 = \|\chi^0_{el}\|^2 + \frac{\|\chi^0_{lon}\|^2}{j(j+1)} + \frac{\|\chi^0_{mag}\|^2}{j(j+1)} = \|\chi^0_{l=j-1}\|^2 + \|\chi^0_{l=j+1}\|^2 + \|\chi^0_{l=j}\|^2, \tag{31}$$

for norms squared.

Finally, note the useful identities

$$\left.\begin{aligned} \vec{L}^2\hat{r}Y_{jm}(\hat{r}) &= \left\{[j(j+1)+2]\hat{r}-2\frac{\partial}{\partial\hat{r}}\right\}Y_{jm}(\hat{r}),\\ \vec{L}^2\frac{\partial}{\partial\hat{r}}Y_{jm}(\hat{r}) &= j(j+1)\left(\frac{\partial}{\partial\hat{r}}-2\hat{r}\right)Y_{jm}(\hat{r}),\\ \vec{L}^2\left(\hat{r}\times\frac{\partial}{\partial\hat{r}}\right)Y_{jm}(\hat{r}) &= j(j+1)\left(\hat{r}\times\frac{\partial}{\partial\hat{r}}\right)Y_{jm}(\hat{r}), \end{aligned}\right\} \tag{32}$$

which were used jointly with the expansion (28) in deriving from Eq. (24) the corrected Eqs. (E25) and (E26). Here $\vec{e}\vec{L}^2 = \vec{J}^2\vec{e}$ and so

$$\vec{J}^2\vec{e}Y_{jm}(\hat{r}) = j(j+1)\vec{e}Y_{jm}(\hat{r}), \tag{33}$$

where $\vec{e}$ is one of three operators $\hat{r}$, $\frac{\partial}{\partial\hat{r}}$ or $\hat{r}\times\frac{\partial}{\partial\hat{r}}$ and $\vec{J} = \vec{L}+\vec{S}$ with $(S_k\vec{e})_l = -i\varepsilon_{klm}e_m$.