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THE ELECTROMAGNETIC INTERFERENT ANTENNAE FOR GRAVITATIONAL WAVES DETECTION

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An electromagnetic wave propagating in the toroidal waveguide is considered as an electromagnetic gravitational antenna. An interferometric method is applied to measure the disturbances of phase of the electromagnetic field caused by the incident gravitational wave. The calculations presented take into account the dispersive and dissipative phenomena occurring during the interaction between electromagnetic and gravitational fields. The active cross-section of the antenna interacting with coherent and pulsed gravitational radiation is estimated. Experimental possibilities presently available are discussed. Limiting fluxes in the astrophysical range of frequencies measured by the interferometric electromagnetic antenna are a factor of ten or so smaller than in the case of a classic mechanical antenna. Moreover the antenna could be used for carrying out a gravitational Hertz experiment.

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1. *Introduction*

Among different gravitational antennae the mechanical antennae are still intensively investigated and used in experiments. It is due to the historical development of the pioneer ideas of Weber.

The mechanical antennae are very promising in their further development, because there are many experimental possibilities of their refinement through Quantum Nondemolition Measurements, the progress in material technology, and cryogenics.

From the beginning of sixties the idea of gravitational antennae with constant or varying electromagnetic fields (EM) have been introduced and developed. The EM field, assumed as the main component of the antenna, interact directly with a gravitational field, whereas in the mechanical antennae the EM field plays only an auxiliary role.

The idea of application of EM field to the detection of gravitational waves requires an estimation of such a field configuration which yields strong interaction of the field with the gravitational wave, and a development of the electromagnetic detection technique of the disturbances of EM field caused by the interaction.

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There are many solutions of these two problems. For example it is possible to construct electromagnetic systems in which a strong resonant interaction of both fields takes place. The principal EM field parameters, the phase shifts and the energy, might be experimentally measured with an accuracy comparable to the quantum uncertainty limit.

In the case of mechanical antennae the technique of vibration measurements has been developed in the last twenty years, and their basic principles of construction have remained unchanged. Contrarily, there are many possible constructions of the electromagnetic antennae, but till now no optimal construction was developed. Thus, there were no attempts to construct such antennae.

In this work a theoretically optimal solution of a EM field antenna construction is proposed, that seems to be experimentally optimal as well.

2. *The graviton-photon resonance*

The most interesting suggestion of the EM field gravitational antenna in the form of a toroidal waveguide with induced propagating coherent electromagnetic wave was given in 1971 [1, 2].

The resonance exchange of the momentum between the gravitational and EM fields occurs, when the torus lies in a plane perpendicular to the direction of propagation of the gravitational wave and EM field circulation period approaches twice the value of the gravitational period. Four spatially distributed areas of EM field wave packets are formed in the torus, due to EM and gravitational wave interaction, with momenta alternatively larger and smaller than the primary undisturbed momentum.

A measurement of the amplitude of an EM field disturbance gives the possibility of determination of the disturbing gravitational radiation intensity. Changes of frequency, amplitude and phase shift follow the momentum disturbances, that allow us to measure one of those quantities only.

In this work the toroidal electromagnetic antenna, in which the phase shift changes are measured by use of an interferometer is further considered. Its simple model has been discussed earlier [3, 4]. In the present model the influence of the EM field dispersion on the graviton-photon resonance is taken into account, because the antennas with a dispersional waveguide may have better properties than the antennae with a nondispersional waveguide.

2.1. Equation of frequency deviation

Let us consider two wave packets of photons which propagate inside the toroidal waveguide along two geodesics lying very closely to each other. To describe a photon propagation we ought to introduce two parameters: an affinical parameter λ which varies along the photon trajectory and a parameter n corresponding to separate geodesics. The vector η^μ, defined by

$$\eta^\mu \equiv \frac{\partial x^\mu}{\partial n} dn = n^\mu dn \tag{2.1}$$

is an infinitesimal vector joining points with the same value of λ on neighboring geodesics with values of n differing by dn. The η^μ vector is invariable in the flat space because of the equidistance of parallel geodesics. In the curved space the η^μ vector fulfills the equation of geodesics deviation

$$\frac{\delta^2\eta^\mu}{\delta\lambda^2} = -R^\mu{}_{\alpha\beta\gamma}\frac{\partial x^\alpha}{\partial\lambda}\eta^\beta\frac{\partial x^\gamma}{\partial\lambda}, \tag{2.2}$$

where symbol $\frac{\delta}{\delta\lambda}$ means the covariant derivation with respect to λ. That equation is easily adapted to description of photon propagation. The wave vector of photon may be defined as

$$k^\mu \equiv \frac{\partial x^\mu}{\partial\lambda}. \tag{2.3}$$

The covariant derivative of $\frac{\partial x^\mu}{\partial n}$ with respect to λ can be written in the form

$$\frac{\delta}{\delta\lambda}\left(\frac{\partial x^\mu}{\partial n}\right) = \frac{\delta}{\delta n}\left(\frac{\partial x^\mu}{\partial\lambda}\right) = \frac{\delta k^\mu}{\delta n}$$

hence, using (2.1) we have

$$\frac{\delta\eta^\mu}{\delta\lambda} = \frac{\delta k^\mu}{\delta n}dn.$$

The above expression introduced into (2.2), with (2.1) taken into account, gives

$$\frac{\delta^2 k^\mu}{\delta\lambda\delta n} = -R^\mu{}_{\alpha\beta\gamma}k^\alpha n^\beta k^\gamma. \tag{2.4}$$

The equation (2.4) describes the space-time deviation of the wave vector of photons propagating in the gravitational field. In the waveguide the photons undergo the four-dimensional "acceleration" caused by the interaction of the EM field with the waveguide walls. The waveguide influences the changes of the direction of the wave vector in space-time not affecting the photon energy $\omega_e = k^0 c$. Thus the $\frac{\delta^2 k^0}{\delta\lambda\delta n}$ component of the deviating "acceleration" of photons vanishes. The mutual change of k^0 in space-time, which will be of interest for us further, may be connected with an interaction of the EM field with the gravitational field only.

Let the locally plane, weak gravitational wave be expressed as

$$h_{ik}(x^j, t) = \mathrm{Re}\,\{h_1 e_{ik}^{(1)} + h_2 e_{ik}^{(2)}\}, \tag{2.5}$$

where $e_{ik}^{(1,2)}$ are the unit tensors corresponding to two states of wave polarisation. That wave fulfills the orthogonality $h_{ij,j} = h_{ij}k_j = 0$ and no trace $h_{kk} = 0$ conditions.

Let us assume, that the local frames connected with any points lying on the waveguide circumference are Lorentzian, and that the EM wave field frequencies, which are registered by in the same frame, equals $\omega_e = k^0 c$. Then, according to (2.3) $\frac{\delta}{\delta\lambda} = \frac{\omega_e}{c^2}\frac{\partial}{\partial t}$, where t is the local time of these observers. Similarly $\frac{\delta}{\delta n} = \frac{\partial}{\partial x}$, where x denotes the space coordinate measured along the waveguide circumference.

Taking into account that $n^\mu = (0, n^i)$, the wave vector can be written in the form $k^\mu = (k^0, k^i)$, where $k^i = \frac{\omega_e}{v_p} n^i$. The phase velocity of the EM field in the waveguide and group velocity denoted as v_p and v_g respectively, fulfil the condition $v_p v_g = c^2$.

The presented above considerations allow us to express equation (2.4) for the k^0 component in the form

$$\frac{\partial^2 k^0}{\partial x \partial t} = -v_g k^0 R_{i0k0} n^i n^k. \tag{2.6}$$

Equation (2.6) gives the relative changes of the photon frequency in the gravitational field as well.

The only non-vanishing components of the Riemann tensor for the plane gravitational wave are R_{i0k0} components. The location of the detector waveguide at the null point of frame does not affect the universality of the considerations. Then, for the wave described by equation (2.5) we have

$$R_{i0k0} = -\frac{1}{2c^2}\{e_{ik}^{(1)}\dot{H}_1 + e_{ik}^{(2)}\dot{H}_2\}, \tag{2.7}$$

where

$$H_1(t) \equiv \mathrm{Re}\,\{\dot{h}_1(t, 0)\} \text{ and } H_2(t) \equiv \mathrm{Re}\,\{\dot{h}_2(t, 0)\}.$$

$H_1(t)$ and $H_2(t)$ are the time dependent amplitudes of the gravitational wave field in the first and the second polarisation state, respectively. These quantities have spectral properties similar to the amplitudes of the electric and magnetic fields of the electromagnetic waves. If the gravitational wave propagates along the axis x^1, and the EM wave field moves in the anti-clockwise direction along the circle with a radius r, lying in the plane x^2, x^3, we have: $R_{2020} = -R_{3030}$ and $R_{2030} = R_{3020}$ and all these components are the non zero ones, corresponding to the two states of polarisation. After performing appropriate calculations, to within of an unimportant phase factor, we obtain

$$R_{i0k0} n^i n^k = -\frac{1}{2c^2}\{\dot{H}_1 \cos k_r x + \dot{H}_2 \sin k_r x\}, \tag{2.8}$$

where $k_r = \frac{2}{r}$. Substituting (2.8) into (2.6) we have the deviation frequency equation in the final form

$$\frac{\partial^2 \omega_e}{\partial x \partial t} = \frac{v_g \omega_e}{c^2}\{\dot{H}_1 \cos k_r x + \dot{H}_2 \sin k_r x\}. \tag{2.9}$$

Its solutions are $\omega_e(t, x) = \omega_{e0}+\omega_\delta(t, x)$, with boundary conditions for $t = 0$; $\omega_e = \omega_{e0}$. Because the wave amplitudes always are $|\dot{H}| \ll 1$, thus $|\omega_\delta| \ll \omega_{e0}$. In the above analysis the principles of geometrical optics were used. It demands that the space regions of disturbances of the EM field extended over rougly to 1/4 of the waveguide circumference are much larger than the EM field wavelength, which means that $r \gg \lambda_e \sim \frac{c}{\omega_e}$. Such systems are considered in this work.

2.2. The phase deviation equation

The wave packets propagating in the dispersive detector waveguide will be considered. Their frequency is continuously perturbed by the gravitational field in the way described by equation (2.9). A solution of a linear dispersive problem might be reduced to calculation of the Fourier Integral

$$\varphi(t,x) = \varphi_0 \frac{1}{2\pi} \int_{-\infty}^{\infty} \gamma(i\omega)e^{i\psi(t,x)}d\omega, \tag{2.10a}$$

where

$$\psi(t, x) = \omega(t, x)t-K[\omega(t, x)]x \tag{2.10b}$$

is the phase of the wave. The $K[\omega]$ function is a dispersive relation of the waveguide medium. The spectral function $\gamma(i\omega)$ corresponds to an initial profile of a wave packet. To obtain a phase deviation equation means in fact to deduce a relation between equation (2.9) and (2.10b). From the experimental point of view, considering small relative values of frequency deviations, we will be interested in an amplitude and a wave phase after the perturbation evolution which satisfies the condition $t \gg \frac{1}{\omega}$. That allows us to use asymptotic methods to obtain a solution of (2.10). The solution is sought with the following boundary conditions:

$$\text{for} \quad t = 0 \quad \begin{array}{ll} 1.\ x = \eta & 4.\ \dfrac{\partial\gamma}{\partial t}(0, \eta) = \gamma_1 \\ 2.\ \gamma(0, \eta) = 1 & 5.\ \left.\dfrac{\partial\psi}{\partial t}\right|_{t=0} = \omega_{e0} \\ 3.\ \gamma(i\omega) = \mathscr{F}[\gamma(t, \eta)] & 6.\ \left.\dfrac{\partial\psi}{\partial x}\right|_{t=0} = -k_{e0} = -K[\omega_{e0}]. \end{array} \tag{2.11}$$

Using a stationary phase approximation or the method of steepest descent [8, 9] we obtain non-zero values of (2.10) for $\omega = \omega_e$, for which the equation

$$x = \frac{1}{K'}t+\eta \tag{2.12}$$

is fulfilled, with $K' = \left.\frac{\partial K}{\partial \omega}\right|_{\omega=\omega_e}$. Somewhere in space-time volume described by (2.12) the "energy center" of the wave-packet is located, and hence the maximal density of the energy dispersed in the medium.

It is very convenient to analyse the phase deviations in the antenna within the frame moving with a mean velocity of the EM field, in which the coordinate is

$$\xi_{\tau=t} = x - v_{g0} \cdot t, \tag{2.13}$$

where $v_{g0} = \left.\frac{1}{K'}\right|_{\omega=\omega_{e0}}$ is the group velocity of the undisturbed field. In the volume corresponding to the stationary points (2.12) the wave phase equation (2.10b) may be expressed in new coordinates as

$$\psi(\tau, \xi) = \omega(\tau, \xi) \cdot \tau - \frac{K[\omega(\tau, \xi)]}{K'[\omega(\tau, \xi)]}\, \tau - K[\omega(\tau, \xi)]\eta. \tag{2.14}$$

Differentiation of (2.14) with respect to τ and ξ leads to equation

$$\frac{\partial^3 \psi}{\partial \xi \partial \tau^2} = \frac{K''K}{(K')^2}\left\{2\frac{\partial^2 \omega}{\partial \xi \partial \tau} + \tau \frac{\partial}{\partial \tau}\left(\frac{\partial^2 \omega}{\partial \xi \partial \tau}\right)\right\} - K'\eta \frac{\partial}{\partial \tau}\left(\frac{\partial^2 \omega}{\partial \xi \partial \tau}\right) + O(h^2). \tag{2.15a}$$

In the nondispersive medium $K'' = \left.\frac{\partial^2 K}{\partial \omega^2}\right|_{\omega=\omega_e} = 0$ and the phase deviation is described by the second term of (2.15a). Its physical meaning might be more clear when noticed, that in the nondispersive medium the condition (2.12) is identical with the transformation (2.13) and the phase equation (2.14) is then reduced to $\psi(\tau, \xi) = -K\xi$. After differentiation with respect to ξ and τ the phase equation for nondispersive medium is obtained

$$\frac{\partial^3 \psi}{\partial \xi^2 \partial \tau} = -K'\left\{2\frac{\partial^2 \omega}{\partial \xi \partial \tau} + \xi \frac{\partial}{\partial \xi}\left(\frac{\partial^2 \omega}{\partial \xi \partial \tau}\right)\right\} + O(h^2). \tag{2.15b}$$

The partial derivatives $\frac{\partial^2 \omega}{\partial \xi \partial \tau}$ from equations (2.15a) and (2.15b) are directly related to the frequency deviation equation (2.9). Equation (2.9) is identified as a standard type equation $\frac{\partial^2 \omega}{\partial x \partial t} = C\frac{\partial H}{\partial t} e^{ikx}$ with condition $\frac{\partial H}{\partial x} = 0$.

Assuming $\frac{\partial}{\partial \tau} = \frac{\partial}{\partial t} - v_{g0}\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial \xi} = \frac{\partial}{\partial x}$, this equation might be expressed in the coordinates in the form

$$\frac{\partial^2 \omega}{\partial \xi \partial \tau} = C\frac{\partial H}{\partial \tau} e^{i\omega_r \tau} e^{ik_r \xi}. \tag{2.16}$$

The frequency ω_r described as

$$\omega_r = k_r v_{g0} = \frac{2}{r} v_{g0} = 2\omega_0, \tag{2.17}$$

where ω_0 is the angular frequency of photon circulation in the waveguide, represents the resonant frequency of the antenna. The resonance of both fields takes place, if the condition $\omega_g = \omega_r$ is satisfied. Finally, after taking into account (2.16), the equation (2.9) expressed in ξ and τ coordinates becomes

$$\frac{\partial^2 \omega}{\partial \xi \partial \tau} = \frac{v_{g0}\omega_{e0}}{2c^2} \{\dot{H}_1 \cos(k_r \xi + \omega_r \tau) + \dot{H}_2 \sin(k_r \xi + \omega_r \tau)\}. \tag{2.18}$$

After substitution of (2.18) into equations (2.15a) and (2.15b) they represent the phase deviation equations which were sought. The structure of the above equations exhibits the resonant character of the EM field interaction with the gravitational field. For ξ = const and ω_g approaching to ω_r we have $\dot{H} \sin \omega_r \tau \sim \sin \omega_g \tau \cdot \sin \omega_r \tau$, which is a slow oscillating function of time. The equations (2.15) become of the type $\frac{\partial^n \psi}{\partial \tau^n}$ = const. Their solutions increase monotonically in time. The substantial difference of the graviton-photon resonance in dispersive and nondispersive media is obvious on the basis of equations (2.15). Equation (2.15a) contains the second derivative with respect to time and (2.15b) has the first derivative of time. This allows us to obtain the solution $\psi \sim \tau^2$ in the first case and $\psi \sim \tau$ in the second case.

During the derivation of formulae (2.15) the dispersive and nondispersive effects were separated. In the dispersive waveguide both dispersive and nondispersive effects of phase deviation take place and the general solution is a superposition of solutions of (2.15a) and (2.15b). In the case of sufficiently large $\left.\frac{K''K}{(K')^2}\right|_{\omega=\omega_e}$ value and $t \gg \frac{1}{\omega_r}$ the dispersive effect is dominant, which allows us to neglect the contribution of nondispersive effects.

Integrating the equation over ξ it is necessary to take into account the periodical boundary conditions of the EM field in the toroidal waveguide. If $\lambda_e = \frac{2\pi v_p}{\omega_e}$ represents the EM wave length in the waveguide, the boundary condition is

$$N\lambda_e = 2\pi r, \tag{2.19}$$

where N is a natural number. As it follows from (2.17) and (2.19) the ratio of the EM field frequency ω_e and the detector resonant frequency ω_r is

$$\mathcal{N} = \frac{\omega_e}{\omega_r} = \frac{N}{2} \frac{v_p}{v_g}. \tag{2.20}$$

The number $\mathcal{N}$ appears to be one of the most important parameters of the antenna.

The simplifications made before derivation of the formulae (2.15) ought to be remembered. Precise calculations need an initial profile of the wave packet $\gamma(i\omega)$ and the exact solution of equations (2.10), with a given dispersion relation, consistent with the frequeny deviation equations (2.6) and the dissipation taken into account. These calculations lead to additional terms of order $\frac{1}{\mathcal{N}}$ and $\frac{1}{\omega_r\tau}$ in the solutions of (2.15), which describe unsteady state in the antenna. Further on these terms are neglected, and equilibrium processes in the antenna, based on the solutions of (2.15), will be considered.

2.3. Solutions of the phase deviation equation

A general solution of equations (2.15), with boundary conditions (2.11), has a form $\psi(\tau, \xi) = -k_{e0}\cdot\xi + \psi_\delta(\tau, \xi)$. When neglecting nondispersive term, we have for (2.15a)

$$\psi_\delta(\tau, \xi) = \frac{KK''}{(K')^2}\,\psi_\delta(\tau)e^{ik_r\xi} \tag{2.21a}$$

and for equation (2.15b)

$$\psi_\delta(\tau, \xi) = -K'\psi_\delta(\tau)\xi e^{ik_r\xi}. \tag{2.21b}$$

The variable ξ may have values from the $0 \leqslant \xi \leqslant 2\pi r$ interval, where r is the radius of antenna waveguide. The interferent process of the phase deviations measurement in which a spatial phase difference plays a fundamental role $\psi_\Delta(\Delta\xi, \tau) = \psi_\delta(\tau, \xi_2) - \psi_\delta(\tau, \xi_1)$ for $\Delta\xi = \Delta x = \xi_2 - \xi_1 = \frac{\lambda r}{2} = \frac{1}{4}$ waveguide circumference will be analysed in Section 3. After integration of (2.15) over ξ between $\xi_1 = \xi_i$ and $\xi_2 = \xi_i + \Delta\xi$ with the above condition, the phase deviation equation depending on time only is obtained

$$\frac{d^2\psi_\Delta}{d\tau^2} = \beta^2\mathcal{N}D\{\dot{H}_1 \sin(\omega_r\tau - \psi_i) - \dot{H}_2 \cos(\omega_r\tau - \psi_i)\} \tag{2.22a}$$

$$\frac{d\psi_\Delta}{d\tau} = \beta^2\mathcal{N}\frac{1}{\omega_r}\{\dot{H}_1 \cos(\omega_r\tau + \psi_i) + \dot{H}_2 \sin(\omega_r\tau + \psi_i)\}, \tag{2.22b}$$

where $\psi_i = k_r\xi_i$ is the initial phase, $D = \left.\frac{K''K}{(K')^2}\right|_{\omega=\omega_{e0}}$ and $\beta = \frac{v_g}{c}$.

The electromagnetic resonant gravitational antenna described by equation (2.22) represents the dynamic linear system with two inputs $H_1(t)$ and $H_2(t)$ and an output $\psi_\Delta(t)$. The $\psi_\Delta(t)$ function is a slowly increasing function of time, describing a phase shift between two regions of field at the distance of 1/4 of the waveguide circumference. A general solution (2.22) may be found with the help of the following conditions

1. $\hat{H}_1(t)$ and $\hat{H}_2(t)$ are the real stationary processes in a broader sense (5),
2. the following conditions are satisfied $\langle\hat{H}_1(t)\rangle = \langle\hat{H}_2(t)\rangle = 0$,
3. time of interaction $t \gg \frac{1}{\omega_r}$. (2.23)

Equations (2.22) might be understood as stochastic differential equations then, an the conditions 2 and 3 make the integration easier. Conditions 1, 2 and 3 are fulfilled for the antenna interacting with the radiation having a thermal spectrum and a high time coherency as well. Equations (2.22) might be solved using the substitution method, for example

$$\psi_\Delta(t) = \tilde{\psi}(t)\,\{\cos\psi_i(\sin\omega_r t - \cos\omega_r t) - \sin\psi_i(\sin\omega_r t + \cos\omega_r t)\}. \tag{2.24}$$

For the boundary conditions $\dot{H}_1(t) = \delta(t)$, $\dot{H}_2(t) = 0$ and $\dot{H}_1(t) = 0$, $\dot{H}_2(t) = \delta(t)$ we have two solutions of equation (2.22a)

$$\tilde{\psi}_1(t) = \tfrac{1}{2}\,\beta^2 \mathscr{N} D U(t)\cdot t\{\cos\psi_i(\cos\omega_r t - \sin\omega_r t) - \sin\psi_i(\cos\omega_r t + \sin\omega_r t)\}$$

$$\tilde{\psi}_2(t) = \tfrac{1}{2}\,\beta^2 \mathscr{N} D U(t)\cdot t\{-\sin\psi_i(\cos\omega_r t - \sin\omega_r t) - \cos\psi_i(\cos\omega_r t + \sin\omega_r t)\}, \tag{2.25}$$

where $U(t) = \begin{cases} 1, & t \geqslant 0 \\ 0, & t < 0 \end{cases}$ is a unitary jump representing the pulse responses of the antenna for both states of gravitational wave polarisation. The solution $\tilde{\psi}(t)$ for any function $H(t)$ fulfilling conditions (2.23) is a convolution of $H(t)$ function with the pulse responses (2.25). After the substitution to (2.24) we obtain the expected solution.

$$\psi_\Delta(t) = \beta^2 \mathscr{N} D\{[\dot{H}_1(t)U(t)\sin(\omega_r t - \psi_i)$$
$$-\dot{H}_2(t)U(t)\cos(\omega_r t - \psi_i)] * tU(t)\}. \tag{2.26}$$

The multiplying functions $U(t)$ in equation (2.26) describe switching of the interaction of a group of photons, which at the moment $t = 0$ is introduced to the waveguide and begins to interact with the gravitational field of the wave, while the element $* \, tU(t)$ describes the time development of interaction of a photon group. Calculating the integral of the convolution, solution (2.26) may be expressed in an equivalent form

$$\psi_\Delta(t) = \beta^2 \mathscr{N} D \int_0^t \{\dot{H}_1(\tau)\sin(\omega_t \tau - \psi_i)$$
$$-\dot{H}_2(\tau)\cos(\omega_r \tau - \psi_i)\}\,(\tau - t)d\tau. \tag{2.27}$$

If $\hat{\dot{H}}_1(t)$ and $\hat{\dot{H}}_2(t)$ belong to the family of normal processes, then the $\hat{\psi}_\Delta$ is a nonstationary Wiener-Levy process [5]. For $t \geqslant 0$ the mean value of that process $\langle\hat{\psi}_\Delta\rangle = 0$, the variance $\langle\hat{\psi}_\Delta^2\rangle \sim t$, that means, that the antenna phase deviation accumulates in time. That equation has a very important meaning in a case of gravitational radiation with thermal spectrum. For the coherent radiation, when $\dot{H}_1(t)$ or $\dot{H}_2(t)$ are harmonic functions of the $\dot{H}(t) = \omega_g^2 h_c \cos\omega_g t$ type and $\omega_g \approx \omega_r$, the solution $\psi_\Delta \sim t^2$ are obtained. Continuing an analogous procedure for the nondispersive antenna we obtain the solutions of the phase deviation equation (2.22b) in the form

$$\psi_\Delta(t) = \beta^2 \mathscr{N} \frac{1}{\omega_r}\{[\dot{H}_1(t)U(t)\cos(\omega_r t + \psi_i)$$
$$+\dot{H}_2(t)U(t)\sin(\omega_r t + \psi_i)] * U(t)\}. \tag{2.28}$$

The process $\hat{\psi}_\Delta$ gives $\langle\hat{\psi}_\Delta\rangle = 0$ and the variance $\langle\hat{\psi}_\Delta^2\rangle = 0$, if $\dot{\hat{H}}_1(t)$ and $\dot{\hat{H}}_2(t)$ are assumed to be normal processes. When $\dot{H}_1(t)$ or $\dot{H}_2(t)$ are harmonic functions with frequency ω_r, we obtain $\psi_\Delta \sim t$. This type of antenna was considered earlier [4].

3. *The electromagnetic gravitational antenna with the interference phase detection*

The solution (2.21b) implies, that the maximal difference of phase deviation appears for two points of the antenna at the distance of 1/4 of the waveguide circumference length. The method of the gravitational wave detection analysed here is based on an interference measurement of the phase deviations in the electromagnetic antenna.

Let us analyse at first a simple model of the antenna which consists of a lossless toroidal waveguide and four ideal lossless switches of the EM field placed every 1/4-th of the antenna circumference. Also, let us assume, that the switches introduce no disturbances of the EM field till the moment when they are switched on to transmit the EM field from the waveguide to the external waveguide connected with an interferometer. In the interferometer an interference of the fields will take place in the time interval equal to 1/4-th of the circulating period of the photon in the antenna waveguide.

According to (2.13) and (2.21) the equations of the EM field for the two points of the waveguide with coordinates $x_1 = x_i$ and $x_2 = x_i+\frac{1}{4}2\pi r$ may be expressed as

$$\varphi_1(t, x_i) = \varphi_0 \exp\left[-i(\omega_{e0}t-k_{e0}x_i)\right] \exp\left\{i\psi_\Delta(t)\exp\left[-i(\omega_r t-k_r x_i)\right]\right\}$$

$$\varphi_2(t, x_i) = \varphi_0 \exp\left[-i(\omega_{e0}t-k_{e0}x_i)\right]$$

$$\times\exp\left[-ik_{e0}\frac{\lambda_r}{2}\right]\exp\left[i\frac{\lambda_r k_r}{2}\right]\exp\left[-i(\omega_r t-k_r x_i)\right]\}, \tag{3.1}$$

where φ_0 is a field amplitude. If $|\psi_\Delta| \ll 1$ and N is an even number, at the output of the interferometer summing fields amplitudes we obtain the amplitude

$$\varphi_\Delta(t, x_i) = 2i\varphi_0\psi_\Delta(t)\exp\left[-i(\omega_{e0}t-k_{e0}x_i)\right]\exp\left[-i(\omega_r t-k_r x_i)\right]. \tag{3.2}$$

Because an output amplitude of the antenna given by equation (3.2) is directly proportional to ψ_Δ, i.e. to the gravitational wave amplitude H, according to (2.27) and (2.28), so we have

$$\varphi_\Delta \sim H_0\exp\left[-i\omega_r t\right]\varphi_0\exp\left[-i\omega_e t\right] \tag{3.3}$$

which means that the detection signal of the interference antenna appears at the output of the interferometer in the vicinity of two frequencies

$$\omega_R = \omega_e \pm \omega_r. \tag{3.4}$$

The above peculiarity is characteristic for the heterodyne systems. The heterodyne characteristics of the antenna make the system significally intensive to external disturbances during the detection process.

A measurement of the detected signal energy allows us to estimate the gravitational radiation energy flux, which penetrates the antenna. If the energy of the EM field existing in the antenna waveguide $\varepsilon_0 \sim |\varphi_0|^2$, then at an output of the interferometer after switching on the field switches, we have the energy

$$\varepsilon_\Delta = \varepsilon_0 |\psi_\Delta|^2. \tag{3.5}$$

This model of antenna does not take into account the influence of the EM field energy losses in the antenna, which limit the interaction time of both fields. In fact, the construction of the waveguide with sufficiently long lifetime of photons is the most serious technical problem. A construction of the EM field switches seems to be even a more difficult problem.

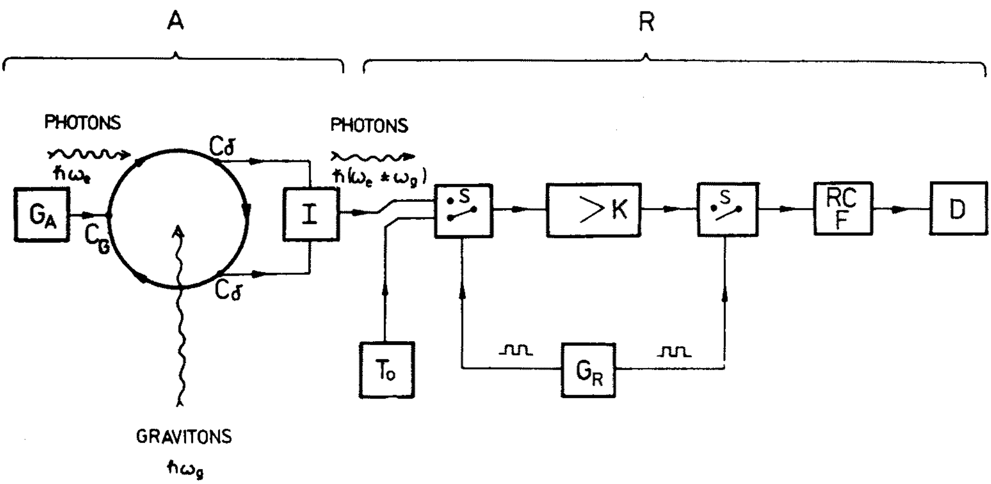

Fig. 1. The diagram of interference electromagnetic detector of the gravitational waves, realizing continuous interference process. A — interference detector antenna; C_d — directional coupler with a small coupling coefficient joining antenna waveguide and external photon source; C_A — directional couplers with small coupling coefficients joining antenna waveguide and interferometer; I — antenna interferometer; G_A — generator source of photons; R — Dicke system radiometer; T_0 — standard noise source; S — switch of the receiver radiometer; K — high frequency amplifier; F — RC integrating filter; G_R — switching generator

To eliminate the above mentioned difficulties let us analyse a model of an antenna, in which a measurement of the phase shifts is continuous. Such a measurement may be performed by continuous extraction of small field fluxes from two points at the distance of 1/4 the waveguide circumference and performing the continuous interference detection. Extraction of the field is possible, with minimal field disturbance in the waveguide of the antenna, by use of directional couplers, which are well known in the waveguide technics both for microwave and optical bands. A diagram of the antenna is presented in Fig. 1. Detection power occurring at the output of the antenna interferometer is accordingly smaller than that in the pulse antenna, but might be measured continuously. To preserve in the antenna the stable EM field energy, which decreases due to dissipation introduced by the waveguide and the interference process, the external souıce of field energy may be used. In the case of the antenna with continuous detection, losses and space dispersion of energy, together with energy completing, lead to its dynamical equilibrium. That equilibrium may be described by use of the distribution function of photons in the antenna.

3.1. The photon distribution function

The waveguide of the antenna filled with the monochromatic EM field may be treated as a harmonical oscillator in terms of quantum-mechanical description. Feeding of the EM field into the waveguide and its flow out to interferometer is controlled by the couplers with a very small coupling coefficient. Absorption of the EM field energy is a quantum process, the probability of photon absorption in the waveguide walls may be regarded as very small. In such conditions the quantum state of the field in the antenna and at the coupler output is given by the Poisson distribution. The mean square deviations of the photon number and the phase in the antenna are $\Delta n = \sqrt{\bar{n}}$ and $\Delta\psi = \frac{1}{2\sqrt{\bar{n}}}$, where $\bar{n}$ is the mean number of the EM field energy quanta in the antenna. It follows from the quantum-mechanical considerations that the probability of passage of the waveguide segment of length x by a photon is $\mathscr{P}(x) = \exp[-\alpha_{ex}x]$, where α_{ex} is a quantity corresponding to the classical unitary damping factor of the waveguide. The damping factor corresponding to a single circulation of the photon in the antenna equals $\alpha_{e0} = 2\pi r\alpha_{ex}$. A total damping factor corresponding to a single circulation equals

$$\alpha_e = \alpha_{e0}+\alpha_\Delta, \tag{3.6}$$

where α_Δ is a factor describing decrease of photons lead out to the interferometer. If $\alpha_e \ll 1$, then α_{e0} and α_Δ are the probabilities of photon absorption by the waveguide and of a hit of the directional couplers by photon respectively, i.e. $\mathscr{P} \approx \alpha_{e0}$ and $\mathscr{P} \approx \alpha_\Delta$. That means, that $\bar{n}[1-\exp(-\alpha_e)] \simeq \bar{n}\alpha_e$ photons get out of the antenna during one turn, and source in order to maintain the equilibrium. The photons fed into the waveguide bear no information about the phase deviations caused by interaction with gravitational field, because there was no such interaction in the source. These photons may contribute to the interference only after some time of their circulation in the antenna waveguide.

The dynamical equilibrium of the EM field interacting with gravitational field must take into account not only the number of photons, but also their weighted contribution to the resulting interference product. The probability of a photon occurrence at the antenna interferometer output for small phase deviations is directly proportional to phase shift deviations $\mathscr{P} \sim |\psi_\Delta|^2$. For $\psi_\Delta = 0$, which means a photon not interacting with the gravitational field, that photon may occur at the interferometer output with the probability $\mathscr{P} \sim |\Delta\psi|^2 = \frac{1}{\Delta\bar{n}}$, producing a quantum noise.

Absorption and creation of photons in the antenna has a random character, thus a number of photons in the antenna may be described by use of stochastic process $\hat{n}(t)$, with $\bar{n} = \langle \hat{n}(t) \rangle$. To describe a statistical weight of photon groups in an interference product, le us introduce an additional parameter r, describing the number of turns executed by a photon group in the antenna waveguide. In such a way a random vector is constructed $\{\hat{n}(1, t), \hat{n}(2, t), \ldots \hat{n}(r, t), \ldots\}$. The mean value of each component $\bar{n}(r) = \langle \hat{n}(r, t) \rangle$ describes the mean number of photons in the group after r turns. The $\bar{n}(r)$ function will be called the photon distribution function in the antenna. It fulfills an obvious condition

$$\sum_{r=0}^{\infty} \bar{n}(r) = \bar{n}. \tag{3.7}$$

In the distribution function $\bar{n}(r)$ an influence of energy dispersion on the graviton-photon resonance in the dispersive antenna may also be taken into account. This function permits us to determine an important parameter of the detector in the dynamical equilibrium state.

3.2. An influence of energy dispersion on graviton-photon resonance

In the resonance of the gravitational field with the EM field in the antenna waveguide, the four space regions of the field may be distinguished, in which the red-shift and blue-shift of photons alternate. These regions move with the group velocity of the EM field, which determines the detector resonance frequency (Eq. (2.17)). Any small perturbation of the exact synchronism of the interacting fields, for example through small perturbation of the group velocity, leads to a displacement of blue-shift photons to red-shift region and vice versa, and as a result the amplitude of the graviton-photon resonance decreases. The fulfilling of the condition of an exact resonance is not possible in the linear dispersive medium. The EM field, existing in each of four analysed regions, evolves as a spatially finite wave packet. The wave packet spectrum spreads over the frequency range around the carrier frequency of the field. Each of the spectral components of the wave packet propagates in the dispersive waveguide with a different group velocity, which means, that the energy will be leaking from the discussed regions, gradually filling up the whole antenna waveguide volume.

Intensity of dispersion process depends on the ratio of the packet spectrum width $\Delta\omega_e$ to the carrier frequency ω_e. Because the field regions situated in quadrants of a waveguide circumference are discussed, and $\Delta\omega_e \sim \frac{2c}{\lambda_r} \sim 2\omega_r$, the intensity of energy dispersion is characterized by the ratio

$$\frac{\Delta\omega_e}{\omega_e} \sim \frac{\omega_r}{\omega_e} \sim \frac{1}{\mathcal{N}}, \tag{3.8}$$

where $\mathcal{N}$ is described by Eq. (2.20). The spatial instability of the EM field is caused by the gravitational frequency deviation, this is negligibly small in relation to the above discussed process. However the gravitational frequency deviation is obviously important for the formation of the phase deviations in the dispersive antenna.

The equation describing a wave packet with the spectrum $\gamma(i\omega)$ propagating in a dispersive medium with a dispersion relation $K[\omega]$ is given with use of the integral (2.10).

An exact solution of integral (2.10) is difficult to obtain for both the spectrum and the dispersion ıelation arbitrary chosen. To estimate quantitatively an influence of the dispersion on the graviton-photon resonance, a gaussian envelope of the wave packet is considered: $\gamma(i\omega) = \frac{2\pi}{\Delta\omega_e} \exp\left[-\pi\left(\frac{\omega-\omega_e}{\Delta\omega_e}\right)^2\right]$. Assuming $\Delta\omega_e \ll \omega_e$, i.e. $\mathcal{N} \gg 1$, we may apply the envelope of $K[\omega]$ around ω_e,

$$K[\omega] = k_e + K'|_{\omega=\omega_e}(\omega-\omega_e) + \tfrac{1}{2} K''|_{\omega=\omega_e}(\omega-\omega_e)^2 + O\{(\omega-\omega_e)^3\}.$$

Taking into account three first terms we have an approximate solution

$$\varphi(t, x) = \frac{1}{\sqrt{\frac{K''|_{\omega=\omega_e}\Delta\omega_e^2 x}{4\pi^2}}} \exp\left[i(\omega_e t + k_e x)\right] \exp\left\{\frac{-\pi\left(t-\frac{x}{v_g}\right)^2}{\frac{4\pi^2}{\Delta\omega_e^2} + K''\Big|_{\omega=\omega_e} x}\right\}. \tag{3.9}$$

The wave packet center propagates with the group velocity $v_g = \frac{1}{K'}\Big|_{\omega=\omega_e}$ in the dispersive medium, and the packet broadens during propagation. The space density of the packet energy may be defined as $\varepsilon(t, x) = |\varphi(t, x)|^2$. Using standard dispersive relations it may be proved, that $\varepsilon(t, x)$ fulfills the equation of energy propagation

$$\frac{\partial\varepsilon(t, x)}{\partial t} + v_g(\omega)\frac{\partial\varepsilon(t, x)}{\partial x} = 0,$$

where $v_g(\omega)$ is a group velocity of each of discussed spectral components of the packet. In the case of one dimensional propagation the energy dispersion is symmetrical in relation to the propagation direction. The energy contained in the element of length $\Delta\xi$, which propagates with the resonant group velocity v_{g0} with its center exactly coinciding with the center of the packet, is expressed by

$$\varepsilon(\tau, \Delta\xi) = \int_{-\frac{\Delta\xi}{2}}^{\frac{\Delta\xi}{2}} \varepsilon(\tau, \xi)d\xi. \tag{3.10}$$

The relative energy changes in the element $\Delta\xi$ as depending on time are described by the characteristic function

$$\delta_d(\tau) = \frac{\varepsilon(\tau, \Delta\xi)}{\varepsilon(0, \Delta\xi)}, \tag{3.11}$$

which for $t = 0$ is $\delta_d(0) = 1$ and for $\tau \geqslant 0$ is $\delta_d \leqslant 1$. Substituting (3.9) into (3.10) and (3.11) and integrating within the limits $\Delta\xi = \frac{1}{4}2\pi r$ for the gaussian packet with a spectral width $\Delta\omega_e$ we obtain

$$\delta_d(\tau) \simeq \frac{1}{1 + \frac{1}{\pi^2} K''\Big|_{\omega=\omega_e} v_g\Delta\omega_e^2\tau}. \tag{3.12}$$

The energy contained in the element $\Delta\xi$ decreases to half of its value after the time

$$\tau_d \simeq \frac{\pi^2}{K''|_{\omega=\omega_e} v_g \omega_r^2}. \tag{3.13}$$

That quantity may be defined as a characteristic time constant of the packet energy dispersion. The quantity τ_d, similarly as the finite life-time of photons τ_e, limits the time of a resonant interaction of fields in a dispersive antenna. The distribution function $n(r)$ depends on those two quantities. A single circulation energy dispersion coefficient may be introduced similarly as for a dissipation process

$$\alpha_d = \frac{4\pi}{\tau_d \omega_r} = \frac{4}{\pi} K''\bigg|_{\omega=\omega_e} v_g \omega_r. \tag{3.14}$$

3.3. The photon distribution function $\bar{n}(r)$

The dissipation of the EM field energy appears to be the only factor limiting the interaction time in a nondispersive antenna. The characteristic function describing energy losses after r circulations is

$$\delta_e(r) = [\exp(-\alpha_e)]^r = \exp(-r\alpha_e) = \delta_e^r. \tag{3.15}$$

The function describing the remaining energy is

$$\gamma_e(r) = 1 - \exp(-r\alpha_e) = 1 - \delta_e(r) = \gamma_e^r, \tag{3.16}$$

and $\delta_e^r + \gamma_e^r = 1$. Let us assume, that there is a dynamical energy equilibrium in the antenna at the moment when the interaction with the gravitational EM field begins. The initial number of photons $\bar{n}$ decreases after one circulation to $\bar{n}\delta_e^1$. These photons are not absorbed. The remaining number of photons $\bar{n}\gamma_e^0$ is absorbed, therefore this number of photons must be introduced from an external source. The newly introduced photons begin their first circulation now.

After two circulations we have four groups of photons: *i*) $\bar{n}\delta_e^2$ group interacting with the gravitational field during two circulations, *ii*) $\bar{n}\delta_e\gamma_e$ group interacting during one circulation (no absorbed part of $\bar{n}\gamma_e^0$ photons introduced after first circulation), *iii*) $\bar{n}\gamma_e^2$ and *iv*) $\bar{n}\delta_e\gamma_e$ not interacting yet, which were introduced to compensate appropriate losses of the above mentioned groups. Generally, after k circulations we have 2^k groups of photons after $k, k-1, \ldots, k-i, \ldots, 1$ circulations, during which they interacted with the gravitational field, and also 2^k groups of photons introduced after k-th circulation to compensate total losses.

The group after $k-i$ circulations interacted $k-i$ times with the gravitational field and contains

$$\bar{n}(k-i) = \bar{n}\left\{\binom{i-1}{0}\delta_e^{k-1}\gamma_e^{k-1} + \binom{i-1}{1}\delta_e^{k-2}\gamma_e^2 + \ldots + \binom{i-1}{i-1}\delta_e^{k-i}\gamma_e^i\right\}(k-i)$$

photons. After performing the summation we have

$$\bar{n}(k-i) = \bar{n}\gamma_e\{\delta_e^{\overline{\frac{k-1}{i-1}}} + \gamma_e^{\overline{\frac{k-i}{i-1}}}\}^{(i-1)}.$$

For $r = k-i$ representing the number of interactions with the gravitational field, the number of photons $\bar{n}(r)$ for $k \to \infty$ is

$$\bar{n}(r) = \lim_{k\to\infty} \bar{n}\gamma_e\{\overline{\delta_e^{k-r-1}}^{k-1} + \gamma_e\overline{\delta_e^{k-r-1}}^{r}\}^{(k-r-1)} = \gamma_e \exp(-r\alpha_e). \tag{3.17}$$

A decay of the interaction of the EM field and the gravitational field in the dispersive antenna is described by two characteristic functions, one of them described by Eq. (3.15) and the characteristic energy dispersion function (3.12) which we express now as

$$\delta_d(r) = \frac{1}{1+r\alpha_d}. \tag{3.18}$$

The distribution function for the dispersive antenna may be found above. Assuming $\alpha_e \ll 1$ and $\alpha_d \ll 1$ we have

$$\bar{n}(r) = \bar{n}\frac{1}{(1+r\alpha_d)^2}\{(\alpha_e+\alpha_d)\exp(-r\alpha_e)+\alpha_e\alpha_d r\exp(-r\alpha_e)\}. \tag{3.19}$$

The above function also fulfills the condition (3.7).

The broadening of wave packets causes no energy losses but the information concerning gravitational interactions is lost in that case. In the state of dynamical equilibrium we have $\bar{n}\alpha_e$ photons absorbed, $\bar{n}\alpha_\Delta$ photons are lead out to interferometer and $\bar{n}\alpha_d$ are spatially delocalised due to dispersion effects, and $\bar{n}\{1-(\alpha_e+\alpha_\Delta)\}$ photons are introduced by the external source in every circulation.

4. *The basic parameters of the antenna in dynamical equilibrium*

Description of the antenna operating at the continuous detection regime must be based on the equation of output power of the antenna. The power of the interference product yielded by a group of photons after the r-th circulation may be described, using (3.5), as follows

$$P_\Delta(r) = P_0|\psi_\Delta(r)|^2\bar{n}(r)\alpha_\Delta. \tag{4.1}$$

$P_0 = \frac{1}{4\pi}\hbar\omega_e\omega_r$ is the power transmitted by one photon during one circulation, $\psi_\Delta(r)$ is a solution of the phase deviation equation in which $t = \frac{2\pi r}{\omega_0}$. The total power yielded by all the photon groups is

$$P_\Delta = \alpha_\Delta P_0 \sum_{r=0}^{\infty} |\psi_\Delta(r)|^2\bar{n}(r). \tag{4.2}$$

Because $|\psi_\Delta|^2 \sim \dot{H}^2 \sim S$, this equation determines the antenna active cross-section when the spectral characteristics of the gravitational radiation which interacts with the antenna are known.

The mean weighted number of circulations when both fields are interacting serves as the next important antenna parameter

$$\langle r\rangle = M_2\{\bar{n}(r)\} = \frac{1}{\bar{n}}\sum_{r=0}^{\infty} r\bar{n}(r). \tag{4.3}$$

The frequency bandwidth B_Δ around the resonant frequency ω_r of the fields interacting and the width of the antenna resonance curve is directly connected with the above quantity through a simple relation

$$B_\Delta \simeq \frac{1}{\langle t\rangle} = \frac{\omega_r}{4\pi}\,\frac{1}{\langle r\rangle}. \tag{4.4}$$

For a nondispersive antenna: $\langle r\rangle \simeq \dfrac{1}{\alpha_e}$ and $B_\Delta = \dfrac{\omega_r\alpha_e}{4\pi}$. In the case of a dispersive antenna: for $\alpha_d \gg \alpha_e$ we have $\langle r\rangle = \dfrac{1}{\alpha_d} E_1\left(\dfrac{\alpha_e}{\alpha_d}\right)\exp\left(\dfrac{\alpha_d}{\alpha_e}\right)$ with $E_1(x) = \displaystyle\int_1^{\infty}\frac{\exp(x\cdot u)}{u}$; whereas for $\alpha_e \gg \alpha_d$ it is $\langle r\rangle \simeq \dfrac{1}{\alpha_d}$.

4.1. The active cross-section for coherent radiation

The solution of the phase deviation equation together with the assumption about coherent gravitational radiation having polarization described by $\dot{H}_1(t) = \ddot{h}_1(t) = -h_c\omega_g^2 \sin\omega_g t$ can be given as follows

$$\psi_\Delta(t) = \tfrac{1}{2}\beta^2\mathcal{N}Dh_c\omega_r^2 t^2 I(\Delta\omega), \tag{4.5}$$

where

$$I(\Delta\omega) = \frac{\sin^2\dfrac{\Delta\omega\cdot t}{2}}{\left(\dfrac{\Delta\omega t}{2}\right)^2}, \qquad \Delta\omega = \omega_r - \omega_g.$$

In a dynamic equilibrium state i.e. when $t \gg \tau_e$ and $t \gg \tau_d$, the output power of antenna (4.2) is

$$P_\Delta = \alpha_\Delta P_0\psi_0^2 M_4\bar{n}, \tag{4.6}$$

where $\psi_0 = 8\pi^2\beta^2\mathcal{N}Dh_c$.

$$M_4 = \frac{1}{n}\sum_{r=0}^{\infty} r^4\bar{n}(r) \tag{4.7}$$

is the fourth moment of the distribution function (3.19). Assuming, that $\alpha_d \ll \alpha_e$ we obtain

$$M_4 \simeq \frac{24}{\alpha_e^4}. \tag{4.8a}$$

If $\alpha_e \lesssim \alpha_d$, the fourth moment of the distribution function may be well approximated by the expression

$$M_4 \simeq \frac{8}{\alpha_d \alpha_e^3} - \frac{4}{\alpha_d^2 \alpha_e^2} + \frac{\alpha_e - 3\alpha_d}{\alpha_d^5} \exp\left(\frac{\alpha_e}{\alpha_d}\right) E_1\left(\frac{\alpha_e}{\alpha_d}\right). \tag{4.8b}$$

The equation of the antenna output may be expressed in the form

$$P_\Delta = P_0 \psi_0^2 \bar{n} Q_\Delta. \tag{4.9}$$

The quantity $Q_\Delta = M_4 \alpha_\Delta$, which depends only on α_Δ, α_e and α_d, plays a role of a resonant coefficient of the antenna interacting with the coherent gravitational radiation.

The ratio of the antenna output power to the coherent radiation power density flux, that is to say the active cross section of the dispersive antenna for the coherent radiation, we find after substituting (3.5) and (4.5) to (4.9)

$$\sigma_c = \frac{512\pi^4 G\hbar}{c^3} \beta^4 \mathcal{N}^3 D^2 \bar{n} Q_\Delta. \tag{4.10}$$

The coefficient Q_Δ reaches the maximum for an optimally chosen value of the coupling coefficient α_Δ, of the antenna waveguide with the interferometer. Using the condition $\frac{\partial}{\partial \alpha_\Delta} Q_\Delta = 0$ for $\alpha_d \ll \alpha_e$ we find $\alpha_\Delta = \frac{1}{3}\alpha_{e0}$ and $Q_\Delta \simeq 1.9 \frac{1}{\alpha_{e0}^3}$, for $\alpha_e \ll \alpha_d$, $\alpha_\Delta = \frac{1}{2}\alpha_{e0}$ and $Q_\Delta \simeq 1.2 \frac{1}{\alpha_d \alpha_{e0}^2}$.

An expression describing the output power of the nondispersive antenna interacting with the coherent radiation is identical as (4.9), where $\psi_0 = 2\pi\beta^2 \mathcal{N} h_c$, $Q_\Delta = M_2 \alpha_\Delta$, and M_2 is the second moment of the distribution function (3.17), which for $\alpha_e \ll 1$ is

$$M_2 \simeq \frac{1}{\alpha_e^3}. \tag{4.11}$$

Analogously, optimizing the coefficient Q_Δ we obtain $\alpha_\Delta = \alpha_{e0}$ and $Q_\Delta = \frac{1}{4\alpha_{e0}}$. The active cross-section of nondispersive antenna interacting with the coherent radiation equals

$$\sigma_c = \frac{32\pi^2 G\hbar}{c^3} \beta^4 \mathcal{N}^3 \bar{n} Q_\Delta. \tag{4.12}$$

4.2. The active cross-section for the pulse radiation

Most of the astrophysical sources of the gravitational radiation emits gravitational waves in a form of short pulses with a broad spectrum. From the point of view of the analysed above antenna the pulse of the gravitational radiation may be treated as short, if $\dot{H}_p(t) = \omega_g h_p \delta(t)$. Then we may analyse a stimulation of the antenna, for example, in the form: The energy flux of such a field

$$S_p = \frac{c^3}{32\pi G} h_p^2. \tag{4.13}$$

For the nondispersive antenna we obtain the solution of the phase deviation equation $\psi_\Delta(t) = \beta\mathcal{N}\frac{1}{\omega_r} h_p U(t)$. The phase difference resulting from crossing it the gravitational field pulse through the nondispersive antenna is shifted with jump having certain value and later on remains unchangeable. The solution of the phase deviation equation for the dispersive antenna leads to an expression

$$\psi_p(t) = \beta^2\mathcal{N}Dh_p\omega_r tU(t) = \psi_0 r, \tag{4.14}$$

where $\psi_0 = 4\pi\beta^2\mathcal{N}D$. The above phase deviation is increasing also after the crossing of the pulse of gravitational field through the antenna. It results from the space focussing of the wave packets propagating in the dispersive waveguide with different group velocities. In the antenna interacting with the field pulse the dynamical equilibrium is not achieved, and the processes in the antenna have an unsteady character.

According to discussion in Section 4, the output energy of the antenna interferometer after transition of the gravitational field pulse may be expressed as

$$\varepsilon_{\Delta p} = \alpha_\Delta \varepsilon_0 \psi_0^2 M_{2p}\bar{n}, \tag{4.15}$$

where $\varepsilon_0 = \hbar\omega_e$. M_{2p} is the second moment of the distribution function $\bar{n}_p(r)$, representing the number of photons, which interacted with the gravitational field at $t = r = 0$. $\bar{n}_p(r)$ continuously decreases in time due to dissipation effects. Taking into account the characteristic functions (3.15) and (3.18) we have

$$\bar{n}_p(r) = \bar{n}\delta_e(r)\delta_d(r) = \bar{n}\frac{\exp(-\alpha_e r)}{1+r\alpha_d}. \tag{4.16}$$

Assuming $\alpha_e \gg \alpha_d$ we obtain

$$M_{2p} \simeq \frac{2}{\alpha} \tag{4.17a}$$

and, if $\alpha_d \gg \alpha_e$

$$M_{2p} \simeq \frac{1}{\alpha_e^2\alpha_d} - \frac{1}{\alpha_e\alpha_d^2} + \frac{1}{\alpha_d^3}\exp\left(\frac{\alpha_e}{\alpha_d}\right)E_1\left(\frac{\alpha_e}{\alpha_d}\right) \simeq \frac{1}{\alpha_e^2\alpha_d}. \tag{4.17b}$$

The output energy $\varepsilon_{\Delta p}$ is proportional to the coefficient $P_\Delta = \alpha_\Delta M_{2p}$, which depends on α_e, α_d and α_Δ. Similarly as for coherent radiation, the optimalization of the coupling coefficient of the antenna waveguide with the interferometer α_Δ may be carried out. From the condition $\frac{\partial}{\partial \alpha_\Delta} P_\Delta = 0$ we have $\alpha_\Delta = \frac{1}{2}\alpha_{e0}$ for $\alpha_e \gg \alpha_d$ and $\alpha_\Delta = \alpha_{e0}$ for $\alpha_d \gg \alpha_e$. The values of P_Δ are then $P_\Delta = \frac{8}{27\alpha_{e0}^3}$ and $P_\Delta = \frac{1}{4\alpha_{e0}\alpha_d}$ respectively. Substitution (4.14) and (4.15) to (4.13) allows us to determine the active cross-section short pulse of the gravitational field

$$\sigma_p = \frac{512\pi^3 G\hbar}{c^3} \beta^4 \mathcal{N}^3 D^2 \omega_r \bar{n} P_\Delta. \tag{4.18}$$

4.3. The detection of the gravitational radiation with nonspecified spectral properties

It is difficult to formulate a universal expresssion describing an antenna cross section interacting with radiation having nonspecified spectral properties. Such an expression would be a superposition of solutions with stationary and nonstationary character. The averaging processes in the antenna make impossible, in principle, a firm conclusion about the spectral character of the input signal of an amplitude of the gravitational field wave on the basis of the features of the output signal. Anyhow this problem, to some extent, is characteristic for all types of antennae.

With respect to the polarisational properties, the discussed antenna has the characteristics typical for the axial symmetric antennae. The cross-section might be given then in the form

$$\sigma = \sigma_0[a_i], \tag{4.19}$$

where $[a_i] = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$ is a column matrix of the reduced Stokes parameters characterizing the antenna [10, 11]. The output power of the antenna interacting with the gravitational radiation flux $S[s_i]$, where $[s_i]$ is a matrix of reduced Stokes parameters of the incident radiation, is

$$P_\Delta = \sigma_0[a_i]^{\mathrm{T}} S[s_i]. \tag{4.20}$$

The toroidal electromagnetic antenna detects one of the circular polarisation states. A direction of the EM field circulation in the waveguide determines the antenna torsion, and the antenna with the right hand EM field circulation detects radiation with the same kind of circular polarisation.

From both practical and theoretical point of view it is interesting to detect the microwave radiation with a thermal spectrum using electromagnetic antennae. Then, the amplitudes of the gravitational field wave $\hat{H}_1(t)$ and $\hat{H}_2(t)$ are the gaussian stationary processes and a solution of the phase deviation equation (2.26) in the dispersive antenna takes a form of nonstationary process with the mean value increasing in time. In the antenna with a continuous detection the dynamical equilibrium exists, which causes that the output power is a stationary gaussian process. The discussion of detection possibilities of the gravitational microwave cosmological background radiation, being a good example of radiation with a thermal spectrum, is presented in [6].

5. *Technical problems of detection*

From the technical point of view the dispersion equation

$$K[\omega] = \frac{\omega_c}{c}\frac{1}{\xi_c}(1-\xi_c^2)^{1/2}, \qquad \xi_c = \frac{\omega_c}{\omega}, \tag{5.1}$$

has an essential meaning. Here ω_c is the critical frequency which is characteristic both for the microwave metallic waveguides and the dielectric waveguides applied in optics. The critical frequency ω_c is connected with choosing of the EM field wave mode and the transverse dimensions of the waveguide. For example, for the metallic waveguide with the rectangular cross section, operating in the simplest mode TE_{01}, we have $\omega_c = \frac{\pi c}{b}$, where b is the longest side in the transverse cross section.

Le us discuss the possibilities of construction of the antenna with such a waveguide. All the basic elements of the antenna may be constructed from the same type of the waveguide. The directional couplers between the EM field source, the interferometer arms and the antenna waveguide, might be made conventionally, boring appropriate holes with small diameters. The output power measurement may be accomplished using a standard microwave radioastronomical radiometer operating on one of the frequencies described by the relation $\omega_R = \omega_e \pm \omega_r$. The gravitational antenna constructed from the circular waveguide and an interferometer is equivalent to the classical radioastronomical antenna, from the experimental point of view. The threshold power quantity detectable by the radiometer, expressed in Kelvins, equals

$$\Delta T = \frac{\alpha_R}{2} T_R \frac{1}{(B_R \cdot \tau_R)^{1/2}}, \tag{5.2}$$

where T_R is the equivalent temperature of the radiometer receiver noises, τ_R is a time constant of the integrating filter at the detector output and B_R is a bandwidth of the receiver. The coefficient α_R is of order of unity and depends on the work-diagram of the radiometer. In the case of the gravitational antenna, when choosing the receiver bandwidth B_R, it is convenient to use the "informative" matching criterion $B_\Delta = B_R$, where a typical value of $B_\Delta = 10^{-2}$ Hz is much smaller than those occurring in the radioastronomical systems. That gives, assuming a Dicke type system of work, where $\alpha_R = \pi\sqrt{2}$, the minimal detected power

$$\Delta P = kT_R\eta_R, \tag{5.3}$$

where $\eta_{\mathrm{R}} = \frac{\pi}{2}\left(\frac{B_{\mathrm{R}}}{\tau_{\mathrm{R}}}\right)^{1/2}$. In the following calculations we assume $\tau_{\mathrm{R}} = \frac{1}{B_{\Delta}}$ which gives $\eta_{\mathrm{R}} = \frac{\pi}{\sqrt{2}} B_{\Delta}$.

In the case of the detection of gravitational radiation pulses, the radiometer serves to measure the power produced in time period $t \approx \tau_e$ at the output of the interferometer. Thus we may assume, that the minimal detectable energy $\Delta\varepsilon = \Delta P \cdot \tau_e$ which gives from (5.3)

$$\Delta\varepsilon = kT_{\mathrm{R}}\eta_{\mathrm{PR}}, \tag{5.4}$$

where $\eta_{\mathrm{PR}} = \frac{\pi}{2}\left(\frac{\tau_e}{\tau_{\mathrm{R}}}\right)^{1/2}$ is a dimensionless constant of order of unit.

5.1. The loss coefficient of the waveguide

The active cross-sections of the antenna are strongly dependent on α_{e0} parameter, which should be relatively small. It is possible to reduce α_{e0} values very effectively in the waveguide with superconductive elements. For the dispersion equation (5.1), the group and phase velocities in the waveguide v_g and v_p, respectively, are

$$v_g = c(1-\xi_c^2)^{1/2} \quad \text{and} \quad v_p = \frac{c}{(1-\xi_c^2)^{1/2}} \tag{5.5}$$

which give, on the basis of (2.20), $\mathcal{N} = \frac{1}{2}\frac{N}{1-\xi_c^2}$ and the waveguide length at the resonance $2\pi r = \frac{4\pi c}{\omega_r}(1-\xi_c^2)^{1/2}$. The one-way circulation loss coefficient is $\alpha_{e0} = \alpha_{ex} \cdot 2\pi r$ and from that

$$\alpha_{e0} = \frac{4\pi c}{\omega_r}(1-\xi_c^2)^{1/2}\alpha_{ex}. \tag{5.6}$$

The coefficient α_{ex} of the microwave waveguide with the rectangular cross section

$$\alpha_{ex} \sim \sqrt{\varrho_n}\, \omega_e^{3/2} \tag{5.7}$$

depends on the specific resistivity ϱ_n of the material used for the waveguide walls. The wave propagation inside the waveguide made of a superconducting material is connected with energy losses typical for the mechanism of the alternating current superconductivity

$$\varrho_n = A\omega_e^2 \exp\left(-\frac{\Delta}{kT}\right) + \varrho_r, \tag{5.8}$$

where Δ is the energy gap in the superconductor and A is a constant depending on material used. This relation is experimentally confirmed for the $T < 0.5\, T_c$ temperature range [12]. ϱ_r is the specific resistivity of the residual surface. From (5.6) and (5.8) it comes out, that $\alpha_{e0} \sim \frac{\alpha_{ex}}{\omega_r} \sim \frac{\omega_e^{5/2}}{\omega_r}$ and because $\mathcal{N} \sim \frac{\omega_e}{\omega_r}$ it may be finally written that in the antenna the one-way circulation field energy losses coefficient equals

$$\alpha_{e0} = C\mathcal{N}^{5/2}\omega_r^{3/2}. \tag{5.9}$$

Due to the actual state of technology it may be assumed that $C = 6\times 10^{-27}$ [$s^{3/2}$] [6, 13, 14].

5.2. The energy dispersion coefficient

For the waveguide with the dispersion equation (5.1)

$$K^{\parallel}|_{\omega=\omega_e} = \frac{1}{c\omega_c}\frac{\xi_c^2}{(1-\xi_c^2)^{3/2}}. \tag{5.10}$$

Hence, using (3.18) we have

$$\alpha_d = \frac{2}{\pi}\frac{\xi_c^2}{1-\xi_c^2}\frac{1}{\mathcal{N}}. \tag{5.11}$$

5.3. An estimation of $\bar{n}$

For the microwave rectangular waveguide, with the EM field amplitude E_0 the mean value of photons equals

$$\bar{n} = \frac{2\pi^4 c^2}{\eta_0 \hbar}\frac{E_0^2}{\mathcal{N}^3\omega_r^4}\frac{(1-\xi_c^2)^{1/2}}{\xi_c^2}. \tag{5.12}$$

The parameter η_0 is a wave resistivity of the medium filling the waveguide, and for the vacuum $\eta_0 = 377[\Omega]$. The maximal $\bar{n}$ value in the antenna depends on the electric field amplitude E_0 which can be applied in the experiment. The transmission of the EM field inside the waveguide is accompanied by the magnetic field $H_0 = \dfrac{E_0}{\eta_0}$, which must fulfill the condition $H_0 < H_{cAC} < H_c$, H_{cAC} is a critical value of the field for the AC superconductor. Nowadays technology enables us to reach $E_{0max} \leqslant 10^{10}$ V/m, and for the waveguides made of nobium $E_0 \simeq 10^7$ V/m ought be taken.

5.4. An influence of other factors on the detection process

The most important factors which may disturb the detection process are
1) sources which cause appearance of noncoherent component of EM field inside the waveguide,
2) reflected waves in the antenna waveguide, produced by the waveguide curvature and technological irregularities of the waveguide cross section,
3) a giroscopic effect caused by a rotation of the antenna around its avis of symmetry.

In fact, the EM field source, always having some frequency instability and phase noise, appears to be a source of noncoherent component also. The superconducting waveguide of an antenna becomes a stable frequency standard because of a very long time of the EM energy is maintained in it. An external source of the field supplying energy to the antenna ought to have such a small phase noises that, after averaging by the antenna waveguide, there would not be a measurable signal at the antenna interferometer output. From a technical point of view it may be easily fulfilled for any antenna type [15]. In the microwave frequency range a source may be constructed, for example, by an activity stabilised and, necessarily, a passively stabilised clystron (by use of an additional superconducting cavity in the latter case).

The gravitational wave is one of the sources of mechanical influences on the antenna system. However these influences do not lead to measurable phase fluctuations of the EM field in the electromagnetic resonators near the frequency ω_R. The problem of perturbations of boundary conditions of the EM field in the electromagnetic resonators, caused by mechanical vibrations, has been analysed in [16]. Owing to the large number of $\bar{n}$, the influence on the detection process of the thermal photons, emitted by the antenna, may be neglected.

The summary influence of the factors leading to appearance of noncoherent component of the EM field in the antenna waveguide may be expressed in the form of an equivalent temperature of the antenna. This quantity makes simple an estimate of the degree of the interference caused by random phase disturbances in the vicinity of one of ω_R frequencies, also make possible a comparison of different antennae. In the case of an ideal antenna of an ideal EM field source and a total lack of other disturbing factors, it equals $T_A = \frac{\hbar}{k}\omega_e$, which is the consequence of quantum field noncoherency. An experimental T_A value will be always larger than the above given value. An influence of the most important phase disturbing factor, i.e. an instability of frequency of the EM field source, on the value T_A is discussed as an example in the Appendix.

The local effects of the loss of superconductivity on technological roughness of the waveguide walls and the free charges generated, for example, by the high-energy cosmic rays, may be important factors significantly increasing the equivalent temperature of the antenna in the range of used high intensities of the E_0 field amplitudes. Both above mentioned factors lead to thermalisation of the EM field in the antenna. Lack of experimental data does not allow for estimation of T_A in the range of fields larger than $E_0 \simeq 10^7$ V/m.

The construction of the electromagnetic toroidal antenna resembles the electromagnetic giroscope in many aspects. The giroscopic effects appear at the antenna interferometer output, whereas inside the antenna there exists a standing wave. The spectral maximum of those effects is at the frequency ω_e, whereas the antenna radiometer is equipped with the narrow-band receiver operating at frequency $\omega_{e'}$. After appropriate radiometer tunning, the gravitational antenna may be a sensitive giroscopic system.

5.5. Tunning of the antenna

The operating frequency of the antenna is calculated by the resonance (2.17), and may be changed either by variation of the antenna waveguide circumference length or by changes of the EM field group velocity $\omega_r = \omega_{r0}\beta$, where $\omega_{r0} = \frac{2c}{r}$. The EM field group velocity in the dispersive waveguide may be easily changed by variation of the EM field frequency ω_e. The waveguide systems, which were mechanically precise enough are able to work correctly in the range $0.01 < \beta < 0.7$ and in such a range the antenna may be tunned. Due to the EM field boundary conditions (2.19), it is impossible to tune the antenna in the continuous manner, because the number N must be an integer. For $N \gg 1$ the successive tunning frequencies are separated by $\Delta\omega_r \sim \frac{\omega_r}{N}$, which is so small a value, that the problem of discontinuities is loosing its importance.

6. *An analysis of the possible detection system*

Numerical calculations were performed for the antennae made of three types of the microwave rectangular waveguide having critical frequencies $\lambda_c = \frac{\omega_c}{2\pi} = 6.5 \times 10^8$ Hz, 6.5×10^9 Hz and 6.5×10^{10} Hz, which corresponds to the standards WR 770, WR 90 and WR 8. It was assumed that the waveguides operate in a superconducting state, the technological coefficient has the value $C = 6 \cdot 10^{-27}$ [$s^{3/2}$], and the amplitude of the EM field in the waveguide is $E_0 = 10^7$ V/m. Three groups of antennae with the lengths $l = 2\pi r = 10$, 100 and 1000 m were considered, with the corresponding maximal resonant frequencies $\frac{\omega_{r0}}{2\pi} = \frac{2c}{l} = 6 \times 10^7$, 6×10^6 and 6×10^5 Hz respectively. For all above considered antenna systems the criterion $\alpha_d > \alpha_e$ was satisfied. The temperature of the

TABLE I

l [m]	10	10^2	10^3	Remarks
ν_r [Hz]	$6 \cdot 10^6$	$6 \cdot 10^5$	$6 \cdot 10^4$	$\beta = 0.1$
α_{e0} [—]	$5 \cdot 10^{-9}$	$5 \cdot 10^{-8}$	$5 \cdot 10^{-7}$	
$\bar{n}$ [—]	$2 \cdot 10^{24}$	$2 \cdot 10^{25}$	$2 \cdot 10^{26}$	
P_G [W]	$5 \cdot 10^{-4}$	$5 \cdot 10^{-3}$	$5 \cdot 10^{-2}$	The power of source of photons
$\dot{Q}_{LHe}$ [W]	0.25	2.5	25	The power of parasity flow of heat
$\dot{Q}_{LN_2}$ [W]	2	20	200	The power of parasity flow of heat
P [kW]	2.5	5	20	The total power of refrigerating system
$S_c \left[\frac{W}{m^2}\right]$	$2 \cdot 10^{-15}$	$2 \cdot 10^{-20}$	$2 \cdot 10^{-25}$	$\beta = 0.1$
$S_p \left[\frac{J}{m^2 Hz}\right]$	$1.5 \cdot 10^{-12}$	$1.5 \cdot 10^{-15}$	$1.5 \cdot 10^{-18}$	$\beta = 0.1$
$\gamma \left[\frac{Hz}{Hz}\right]$	10^{-6}	$3 \cdot 10^{-8}$	10^{-9}	$\tau_w = 1$ [s]

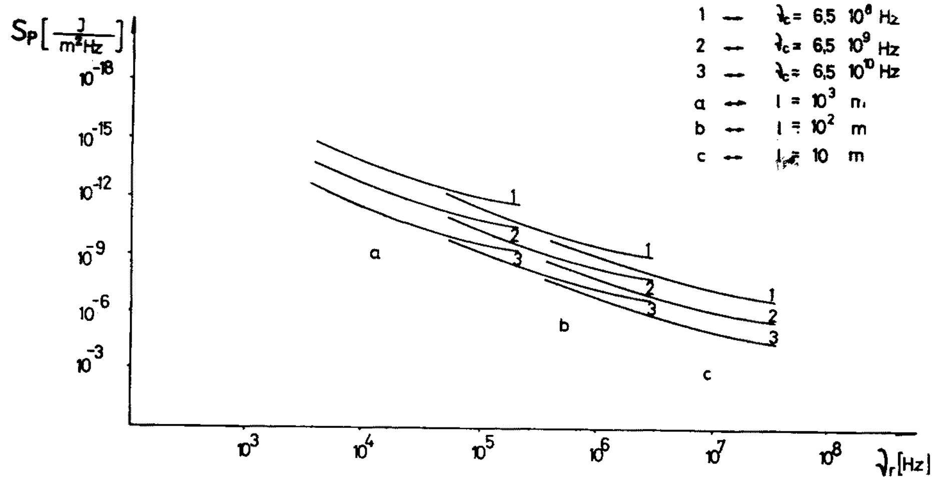

Fig. 2. The expected possibilities of the registration of the coherent gravitational radiation limiting fluxes using interference detectors based on superconductivity microwave waveguides. The curves are parametrized by the length l and critical frequency ν_c of the waveguides

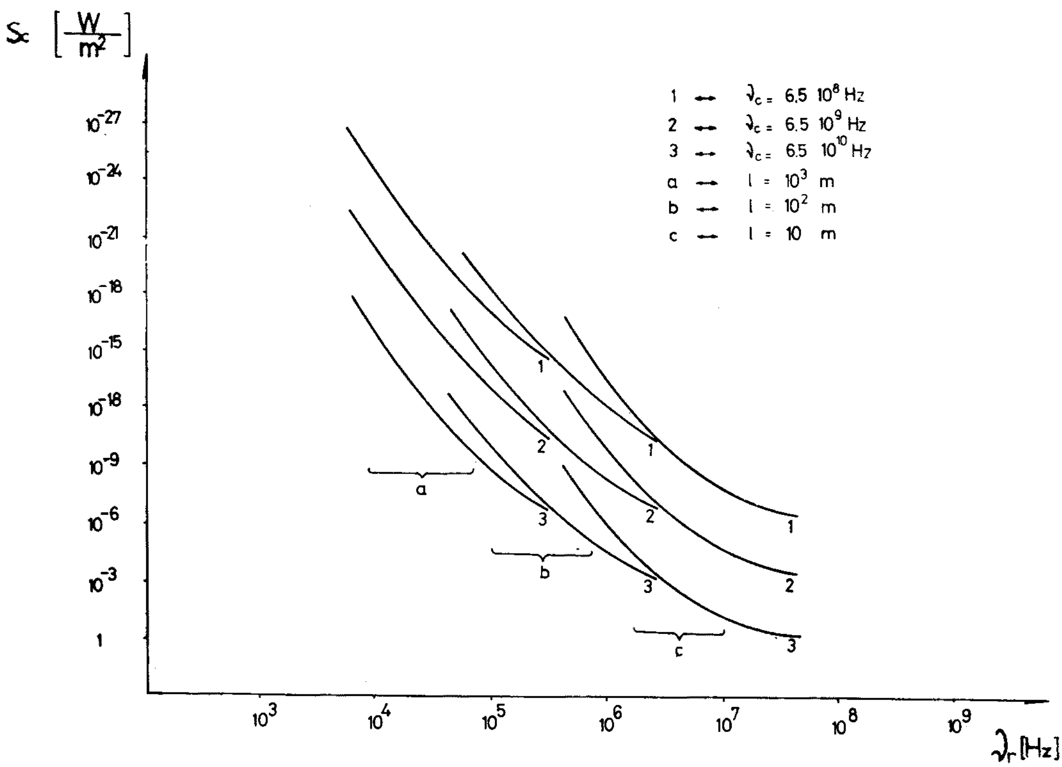

Fig. 3. The expected possibilities of the registration of the gravitational radiation having form of short impulses with a broad spectrum. The parametrization of curves is the same as in Fig. 2

internal noises of the radiometer receiver was taken as $T_R = 10$ K, which can be done easily. An exemplary set of parameters for antennas equipped with the waveguide having the critical frequency $\nu_c = 6.5 \cdot 10^9$ Hz for $E_0 = 10^7$ V/m is given in Table I. A construction of cryogenic systems with lengths of order of hundreds of meters, and with small cross section of order 5 cm^2 represents no serious problem, because it has been solved in connection with construction of superconducting power transmission lines. Some parameters of the cryogenic instalation calculated on the basis of application of known technical solutions [17] are given in Table I. The values of limiting fluxes for several antennas are shown in Fig. 2 and Fig. 3. These dependences show the possibility of the tunning of the antenna by the change of the EM field group velocity β within the limit from 0.7 to 0.01.

It comes out from Fig. 3, that for antennae with lengths $l = 1000$ m and small group velocities $\beta = 0.01$, an operation in the astrophysical frequency range is possible. Moreover, the fluxes of ten orders of magnitude weaker than those detected by classical mechanical antennae are detectable.

7. Conclusions

The idea of the electromagnetic gravitational antenna presented above leads to possibilities of gravitational radiation flux detection of intensities several orders of magnitude lower than in the other classical methods.

The proposed antenna exhibits strong resonant properties and, due to its heterodyne properties, is not sensitive to the, foreseen or not, disturbing factors. Such detection method seems to be fully experimentally realizable, and moreover, shows some simplicity in construction.

A circular beam of photons playing a role of gravitational radiation detector is in many aspects similar to a circular beam of monoenergetical electrons making an electromagnetic radiation detector [3]. As a result of the interaction of an electron beam with an alternating EM field the modulation of the velocity takes place and, due to that, after some time the space grouping of the charge will appear in the beam and makes detection of small fields possible. In the electrodynamics it is possible to obtain a coherent charged beam in the superconducting state, however there is not known a mechanism making the modulation of the beam momentum possible in that state. It appeared to be possible, in the framework of the gravitational field theory, to create a coherent beam of monoenergetic particles interacting with the gravitational field, and to propose a method of precise interference measurement of flux perturbations. The photons play a role of these particles, with success.

The quantum uncertainty of the phase $\Delta\psi = \frac{1}{2\sqrt{\bar{n}}}$, which results from the interference phase measurement, seems not to be the definite limit of the beam phase perturbation measurement possibilities. And, is it possible to construct such a quantum state of the EM field inside the waveguide, and such a method of that field detection, which would allow us to realize a measurement of a phase shift with an uncertainty $\Delta\psi = \frac{1}{2\bar{n}}$ or even better $\Delta\psi = 0$? Such a question was raised by R. Serber and C. H. Townes [8] for the first time about twenty years ago, and it found no satisfactory answer up today. An ideal phase detector, which realizes quantum nondemolition phase measurement $\Delta\psi = 0$, if applied in the electromagnetic gravitational antenna, could provide a possibility of a measurement of infinitely small fluxes of the gravitational radiation, at least theoretically. The quantum nature of the gravitational wave would play a dominant role in an operation of such a detector. That detector would allow to register single quanta of gravitational radiation. Theoretically it would mean equal possibilities of the gravitational and the electromagnetic radiations detection, by use of antennae with similar dimensions. Further, more that would mean a possibility of construction of gravitational antennae with relatively small linear dimensions and large active cross-section.

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APPENDIX

The equivalent temperature of the interferent antennae

A broad class of random fluctuations of EM field phase of nongravitational origin in the antenna may be described using a stationary process $\hat{\psi}(t)$. Generally, appearance of a maximum near the frequency ω_e is a characteristic feature of the power spectrum of such a process. A radiometer cooperating with antenna registers phase fluctuations with frequencies about $\omega_R = \omega_e \pm \omega_r$. When the spectral density of the power spectrum of the process is known for one of these frequencies, a variance of phase fluctuations σ_ψ^2 of the EM field may be estimated. With it a noise interference power product registered by radiometer is connected

$$P_\Delta = \alpha_\Delta P_0 \bar{n} \sigma_\psi^2. \tag{1}$$

In the ideal antenna $\sigma_\psi^2 = \sigma_{\psi\,\text{quantum}}^2 = |\Delta\psi|^2 = \frac{1}{4\bar{n}}$ and the radiometer registers only quantum phase noise. In the real antenna $\delta_\psi^2 > \frac{1}{4n_{\text{quantum}}}$. Let us define the equivalent temperature T_A of the antenna

$$kT_A B_\Delta = P_\Delta, \tag{2}$$

where B_Δ is given by (4.4). So defined quantity may be expressed in the form

$$T_A = \frac{\hbar}{k}\,\omega_e\{1+F\}, \tag{3}$$

where $F = \bar{n}\sigma^2_{\psi\,\text{technical}}$. A term $S\dfrac{\sigma}{\hbar\omega_e}$ describing increase of antenna temperature, caused by its interaction with gravitational field, may be also added to expression (3). For the antenna with an ideally coherent EM field $F = 0$ and $T_A = T_{\text{quantum}} = \dfrac{\hbar}{k}\omega_e$. In practice, according to (5.3) most useful is a construction of such antenna, which fulfills a condition $T_A \leqslant T_R\eta_R$, which with the help of (3) gives

$$1+F \leqslant \frac{kT_R}{\hbar\omega_e}\eta_R. \tag{4}$$

In the opposite case, the boundary flux will be yielded by the equivalent temperature T_A instead of the radiometer temperature T_R.

Later on the equivalent temperature of the antenna will be evaluated taking into account one of the main reasons of phase fluctuations of a technical origin, i.e. instability of the frequency of the EM field source. The frequency of a field source may be treated as a stationary process having the mean value

$$\langle\hat{\omega}_e(t)\rangle = \omega_{e0}. \tag{5}$$

The frequency and phase fluctuations are described by the process $\hat{\Delta}(t) = \hat{\omega}_e(t)-\omega_{e0}$ for which $\langle\hat{\Delta}(t)\rangle = 0$. Let us consider a process with a lorentzian spectrum of power $W_\Delta(\omega)$ with the autocorrelation function

$$W_\Delta(\omega) = \frac{w^2}{1+(\omega\tau_w)^2} \rightleftarrows R_\Delta(\tau) = \frac{w^2}{\tau_w}e^{-\tau/\tau_w}. \tag{6}$$

w^2 is a spectral density and τ_w is a time constant of fluctuations. For a real source, for example a laser or an atomic clock, the fluctuation process may be described by a sum

$$W_\Delta(\omega) = \sum_i \frac{w_i^2}{1+(\omega\tau_{wi})^2}, \tag{7}$$

with w_i and τ_{wi} known from experiment and tabulated. To describe the spectral purity of a source a following parameter is used in practice

$$\gamma = 2\pi\frac{\sigma_\Delta}{\omega_e}, \tag{8}$$

where $\sigma^2_\Delta = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} W_\Delta(\omega)d\omega = \dfrac{w^2}{\tau_w}R_\Delta(0)$. In the interference process a summation of processes with frequencies $\hat{\Delta}(t)$ and $\hat{\Delta}\left(t+\dfrac{T_0}{4}\right)$ takes place, where T_0 is a EM field circulation period in a waveguide. A phase difference is described by

$$\hat{\psi}_\delta(t) = \int_0^{\tau^*} \hat{\delta}(t)dt, \tag{9}$$

where $\hat{\delta}(t) = \hat{\Delta}\left(t + \frac{T_0}{4}\right) - \hat{\Delta}(t)$, and τ^* denotes a time constant connected with taking a mean of the field in the waveguide. In the case of slow fluctuations $\tau_w \gg \frac{T_0}{4} \sim \frac{1}{\omega_r}$ and $\hat{\delta}(t) = \frac{d}{dt}[\hat{\Delta}(t)] \cdot \frac{T_0}{4}$, which gives

$$W_\delta(\omega) = \frac{\pi^2\sigma_\Delta^2}{\omega_r^2\tau_w} \frac{1}{1+(\omega\tau_w)^2} \rightleftarrows R_\delta(\tau) = \frac{\pi^2\sigma_\Delta^2}{\omega_r^2\tau_w^2} e^{-\tau/\tau_w}. \tag{10}$$

The maximum of spectral density of power spectrum $\hat{\delta}(t)$ is at the frequency $\omega = \omega_e$. Estimation of spectral density of $\hat{\delta}(t)$ process around the frequency $\omega_R = \omega_e \pm \omega_r$ will be carried out taking into account conditions $\omega_R \gg \frac{1}{\tau_w}$ and $B_R \approx B_\Delta \ll \omega_r$. A power spectrum curve of the radiometer receiver may be approximated using the expression

$$L(\omega) = \left\{\frac{1}{1+(\omega-\omega_R)^2 B_R^2}\right\}^n, \tag{11}$$

where n — number of receiver resonance circuits. Further $n = 1$, as a most disadvantageous case is assumed. So, the power spectrum of frequency fluctuations within the wideband of the radiometer is expressed by

$$W_L(\omega) = W_\delta(\omega)L(\omega). \tag{12}$$

Using (10), (11) and (12) we have

$$W_L(\omega) = \frac{\pi^2\sigma_\Delta^2}{\omega_r^2\tau_w^2} \frac{1}{1+(\omega_R\tau_w)^2} \frac{1}{1+[(\omega-\omega_R)B_R]^2}$$

$$\downarrow\uparrow$$

$$R_L(\tau) = \frac{\pi^2\sigma_\Delta^2}{\omega_r^2\tau_w^2} \frac{1}{1+(\omega_R B_R)^2} \cos\omega_r\tau \cdot e^{-\tau B_R}. \tag{13}$$

The $W_L(\omega)$ function describes finally the $\hat{r}(t)$ process, with which the phase fluctuations $\hat{\psi}_r(t)$ at the input of the antenna interferometer, having spectral components lying near ω_R, is connected

$$\hat{\psi}_r(t) = \int_0^{\tau^*} \hat{r}(t)dt. \tag{14}$$

Using (13) the variance of the process (14) may be approximated. Using appropriate integration theorems for stochastic processes we have

$$\sigma_{\psi r}^2 = \frac{\pi}{2} \frac{\sigma_\Delta^2}{\omega_r\tau^*} \frac{V(x)}{V(y)}, \tag{15}$$

where $x = \frac{\tau^*}{\tau_R}$, $y = \tau_w \frac{\omega_r}{2\pi}$ and $V(x) = \left\{1 - \frac{1}{x} + \frac{1}{x} e^{-x}\right\}$, $V(y) = y[1+(2\pi y)^2]$. Assuming $\tau^* \approx \tau_w \approx \tau_R$ and using relations (3) and (8) we obtain

$$T_A = \frac{\hbar}{k}\omega_e\left\{1+\gamma^2\frac{\mathcal{N}^2\bar{n}}{4\pi e\alpha_e y^3}\right\}. \tag{16}$$

In the above presented analysis dispersive phenomena were not taken into account. Conditions of source spectral purity which follow from inequality (4) for model antennae are presented in Table I.

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