REMARKS ON THE ROLE OF CHIRAL SYMMETRY BREAKING IN FIRST-ORDER DECONFINING PHASE TRANSITIONS*

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We discuss in some simple cases — where mean-field or strong coupling methods for lattice gauge theories are applicable — the stabilizing role on first-order deconfining phase transitions played by the spontaneous breaking of chiral invariance for euclidean Kogut-Susskind fermions.

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One of the most interesting theoretical problems concerning the behaviour of hadronic matter under extreme conditions is the fate of the first-order deconfining phase transition of lattice QCD at finite temperatures, when light dynamical quarks are taken into account. This is a delicate dynamical problem that demands a good control of fermions effects beyond the quenched approximation in Monte Carlo simulations of the theory. The purpose of this talk is to review the results of some simple analytic calculations — basically the results contained in Ref. [1] and some further work of my own — which tend to suggest that light (or even massless) Kogut-Susskind fermions will not necessarily wash out the latent heath of the first order transition of the pure gauge theory. These simple calculations also indicate that the stabilizing mechanism is the spontaneous breaking of chiral symmetry: the potentially dangerous screening effects of light fermions are depressed because they acquire a dynamical mass through chiral symmetry breaking.

Since first order deconfining phase transitions occur even at zero temperature if the number of space-time dimensions d is big enough ($d \geqslant 5$), I shall first briefly review the role of dynamical Kogut-Susskind fermions in this case, where a systematic mean-field expansion can be carried out. Then, I will discuss the case of a SU(N) lattice gauge theory at finite temperature in the strong-coupling regime.

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1. Mean-field theory in presence of dynamical fermions

Let us consider SU(N) lattice gauge fields coupled to massless euclidean Kogut-Susskind fermions [2, 3]. Sites are labelled by a d-dimensional vector x whose components $(x_1, \ldots, x_d)$ are integers, and links are labelled by the site to which they are attached and a unit vector in the positive μ direction to be denoted $\hat{\mu}$. The partition function we consider is

$$Z = \int \prod_{x,\hat{\mu}} d\mu(U_{x,\hat{\mu}}) \prod_x d\chi_x d\bar{\chi}_x \exp [S_G + S_F], \tag{1.1}$$

where $d\mu(U)$ is the Haar measure over the gauge group, S_G the Wilson action

$$S_G = \beta \sum_{x,\hat{\mu}\neq\hat{\nu}} \mathrm{Tr}\, [U_{x,\hat{\mu}} U_{x+\hat{\mu},\hat{\nu}} U^+_{x+\hat{\nu},\hat{\mu}} U^+_{x,\hat{\nu}}] \tag{1.2}$$

and S_F the fermionic action corresponding to massless Kogut-Susskind fermions.

$$S_F = \tfrac{1}{2} \sum_{x,\hat{\mu}} \eta_\mu(x) \{\bar{\chi}(x+\hat{\mu}) U^+_{x,\hat{\mu}} \chi(x) - \bar{\chi}(x) U_{x,\hat{\mu}} \chi(x+\hat{\mu})\}, \tag{1.3}$$

where $\eta_\mu(x) = (-1)^{x_1+x_2+\cdots+x_{\mu-1}}$. The Grassmann variables $\chi(x)$ and $\bar{\chi}(x)$ carry color (and possibly flavor) but no spin indices. We shall deal with the gauge degrees of freedom in Eq. (1.1) by using mean-field methods, which are accurate at large d. The reason for choosing Kogut-Susskind fermions is that their effects can also be systematically studied in a $1/d$ expansion: a mean field theory can also be developed to deal with the fermionic degrees of freedom [4]. To leading order, this will be the standard Hartree-Fock approximation, but there is a systematic way of computing corrections to it. This program has been vigourously pursued in the strong coupling regime [4, 5]. Similar methods do not work for ordinary Dirac fermions because in this case the number or spin components grow like $2^{\left[\frac{d}{2}\right]}$, and one needs the additional assumption that $N \to \infty$ in order to justify the loop expansion.

Let us for the moment consider the mean-field approximation for the gauge theory in the absence of fermions. The starting point of the mean-field approach is to decouple the group variables $U_{x,\hat{\mu}}$ in the Wilson action by replacing the plaquette interaction by the interaction of $U_{x,\hat{\mu}}$ with a random external field $A_{x,\hat{\mu}}$ at each link [6]. This leads to the following representation of the partition function (1.1) (in the absence of fermions)

$$Z = \int [dVdA] \exp \left\{ S_G[V] - \frac{i}{2} \sum_{x,\hat{\mu}} \mathrm{Tr}\, (A_{x,\hat{\mu}} V^+_{x,\hat{\mu}} + \text{h.c.}) + \sum_{x,\hat{\mu}} \omega(A_{x,\hat{\mu}} A^+_{x,\hat{\mu}}) \right\}, \tag{1.4}$$

where $V_{x,\hat{\mu}}$ and $A_{x,\hat{\mu}}$ are now ordinary $N \times N$ matrices at each link (a fluctuating mean-field and external field, respectively) and $\omega(AA^+)$ is the one-link integral which contains the information on the nature of the gauge group [6]

$$\omega(AA^+) = \ln \int d\mu(U) \exp \left\{ \frac{i}{2} \mathrm{Tr}\, (AU^+ + A^+U) \right\} \tag{1.5}$$

and which is explicitly known for the case of U(1), SU(2) and SU(N) as $N \to \infty$ [7]. The mean-field expansion is just the saddle-point evaluation of Eq (1.4). Looking for a translationally invariant saddle point of the form

$$V_{x,\hat{\mu}} = \varrho 1_N; \quad A_{x,\hat{\mu}} = i\alpha 1_N \tag{1.6}$$

one finds that:
a) For any value of the coupling constant $\bar{\beta} = \beta(d-1)$, there is a trivial saddle point $\varrho = \alpha = 0$;
b) For $\bar{\beta} > \bar{\beta}_c$, a new non-trivial saddle point $\varrho \neq 0$ takes over. At $\bar{\beta}_c$, ϱ jumps to a value slightly below 1 and α jumps to a large value (0.9 and 5.3, respectively) for the case of U(1). As $\bar{\beta} \to \infty$ (weak coupling limit), ϱ approaches 1 and α tends to infinity. This behaviour reflects the compactness of the underlying gauge group: one needs a huge external field to polarize the system to its maximum allowed value at weak coupling. We refer the reader to the original paper of Brezin and Drouffe for a discussion of how this approach evades Elitzur's theorem [6].

In the mean-field approximation, the vacuum expectation value of a Wilson loop $\langle W(c) \rangle$ behaves like ϱ^L, L being the length of the path c. If $\varrho \neq 0$ (ordered phase) this is the perimeter law. If $\varrho = 0$ (disordered phase), the result can be interpreted as an area law with infinite string tension, which is the infinite coupling ($\bar{\beta} = 0$) result. The mean-field ϱ is then order parameter related to the first-order deconfining phase transition. Moreover, $1/d$ corrections to the mean-field approximation can be computed by using by now well established methods [1, 7, 8].

Let us now switch on the Kogut-Susskind fermions and consider the full partition function (1.1). One would be tempted to replace the gauge field $U_{x,\hat{\mu}}$ in the fermionic action (1.3) by the fluctuating mean-field $V_{x,\hat{\mu}}$, and to carry out right away the Grassmann integration. However, it has been emphasized in Ref. [1] that this procedure does not match the known strong-coupling results [4], and that it is important to keep the fermions interacting with the compact gauge variable $U_{x,\hat{\mu}}$. Therefore, the correct procedure is to consider the fermion bilinears $\bar{\chi}\chi$ coupled to $U_{x,\hat{\mu}}$ as part of the fluctuating external field $A_{x,\hat{\mu}}$. This yields the same representation (1.4) for the partition function, up to the following modifications [1]:

a) Include the fermion integration $\int \prod_x d\chi_x d\bar{\chi}_x$

b) Replace, in the argument of the one-link integral

$$\omega(A_{x,\hat{\mu}} A^+_{x,\hat{\mu}}): \begin{cases} A_{x,\hat{\mu}} \to A_{x,\hat{\mu}} + \eta_\mu(x)\bar{\chi}(x+\hat{\mu})\chi(x) \\ A^+_{x,\hat{\mu}} \to A^+_{x,\hat{\mu}} - \eta_\mu(x)\bar{\chi}(x)\chi(x+\hat{\mu}), \end{cases}$$

where color indices are understood: $\bar{\chi}_\alpha \chi_\beta$ is to be interpreted as a matrix in color space. An effective Lagrangian for the Fermi fields thus emerges by expanding the one link integral ω in the Grassmann variables $\chi(x)$, $\chi(x+\hat{\mu})$, $\bar{\chi}(x)$ and $\bar{\chi}(x+\hat{\mu})$. This expansion of course satures at some given order which depends on the number of color indices. This effective Lagrangian contains multifermion near-neighbour interactions with coefficients which depend on $A_{x,\hat{\mu}}$.

To leading order in a $1/d$ expansion, Kogut-Susskind fermions decouple from the gauge field and the results of the mean-field approximation remain valid. The reason is trivial to understand: since there is only one spin degree of freedom per site, the number of gauge degrees of freedom grows with d faster than the number of matter field degrees of freedom. The fermion contribution to the free energy per link starts at order $1/d$ with respect to the gauge mean-field result, and they are of the same order as the corrections to the mean-field approximation in the pure gauge sector [1]. In order to compute the leading $1/d$ contributions to the free energy arising from the fermion contribution, one can then safely neglect the fluctuations of $A_{x,\hat{\mu}}$ and replace it by its saddle-point value. One then has to deal with an effective Lagrangian which reads, in the case of an U(1) lattice gauge theory

$$\mathscr{L}_{\text{eff}} = \sum_{x,\hat{\mu}} \tfrac{1}{2} A(\alpha)\eta_\mu(x) \{\bar{\chi}(x+\hat{\mu})\chi(x) - \bar{\chi}(x)\chi(x+\hat{\mu})\} + \sum_{x,\hat{\mu}} \tfrac{1}{2} B(\alpha) [\bar{\chi}(x)\chi(x)] [\bar{\chi}(x+\hat{\mu})\chi(x+\hat{\mu})], \tag{1.7}$$

where

$$A(\alpha) = \frac{I_0'(\alpha)}{I_0(\alpha)} = \varrho, \tag{1.8}$$

$$B(\alpha) = \tfrac{1}{2}\left[1 - \left(\frac{I_0'(\alpha)}{I_0(\alpha)}\right)^2\right] = \tfrac{1}{2}(1-\varrho^2). \tag{1.9}$$

In Eqs (1.8) and (1.9) the zeroth order saddle-point equations have been used to write $A(\alpha)$ and $B(\alpha)$ in terms of the mean-field ϱ, which is in turn a function of the coupling constant $\bar{\beta}$ having the behaviour described above. For the SU(N) gauge group, the effective Lagrangian contains interactions higher than the four-fermion interaction in Eq. (1.7). However, only the four-fermion interaction is important in the calculation of the leading $1/d$ contributions to the free energy [1, 4].

The physics contained in the effective Lagrangian (1.7) is easy to understand. In the strong coupling disordered phase ($\bar{\beta}$ small) the mean-field ϱ vanishes. Therefore, from Eq. (1.8) it follows that there is no free quark propagation and we are only left with the four-fermion interaction which is known to induce spontaneous chiral symmetry breaking [4]. As we move towards the weak-coupling ordered phase ($\bar{\beta} \to \infty$) ϱ tends to 1. Then the four-fermion interaction fades away and we are left with just a free fermion Lagrangian. Somewhere in the way chiral symmetry has been restored. The effective Lagrangian (1.7) can be studied by mean-field (Hartree-Fock) methods, which allows one to set up a systematic $1/d$ expansion for the fermion contributions to the free energy. We shall not review here the technical details of these calculations, which are similar in spirit to the one we shall describe in more detail in the next section, and refer the reader to the original papers for more details [1, 4]. To leading order in a $1/d$ expansion, the dynamical quark mass λ_0 satisfies a Nambu-Jona-Lasinio equation:

$$\frac{\lambda_0}{Bd} = \int_{-\pi/2}^{\pi/2} \frac{d^d p}{\pi^d} \frac{\lambda_0}{\lambda_0^2 + \varrho^2 \sum_\mu \sin^2 p_\mu}. \tag{1.10}$$

This equation admits at any ϱ the trivial solution $\lambda_0 = 0$. For $\varrho^2 < 1/2$ there is also a non--trivial solution:

$$\lambda_0 = [(\tfrac{1}{2}-\varrho^2)d]^{1/2}\,[1+O(1/d)], \tag{1.11}$$

which breaks chiral smmyetry and dominates the free-energy whenever it exists. As a function of ϱ, chiral symmetry is then restored at $\varrho^2 = 1/2$. However, the mean-field is not a free parameter but rather a function of the coupling constant $\bar{\beta}$ through the mean-field equations for the gauge field. We know that, as the gauge system deconfines, ϱ jumps from zero to a value close to 1 and in any case bigger than $1/\sqrt{2}$. Then, we conclude that chiral symmetry is restored as soon as the gauge system deconfines. The leading $1/d$ calculation gives the fermion free energy as a sum of uncorrelated fermion loops of fermions of mass λ_0. This is equivalent to the quenched approximation made in Monte-Carlo simulations of lattice gauge theories, with the fluctuating gauge degrees of freedom replaced by a mean-field. We then see that in this quenched approximation the sudden transition of the mean-field ϱ from the strong coupling to the weak coupling region drives the restoration of chiral invariance. Had ϱ jumped at $\bar{\beta}_c$ to a value smaller than $1/\sqrt{2}$, we would have seen a phase where the gauge system is not confined but chiral symmetry is still broken. But this phase does not exist (in the quenched approximation) for fermions in the fundamental representation of the gauge group.

Strictly speaking, once fermions have been included in the theory, there is no order parameter for deconfinement. One would expect a non-vanishing mean field — related to a perimeter law for Wilson loops — even at strong coupling, arising from screening effects. A careful analysis of this problem — by using arguments close in spirit to the ones developed in the next section — reveals that, in the presence of dynamical quarks, the expectation value of $W(c)$ behaves at strong coupling as,

$$\langle W(c)\rangle \sim \left(\frac{1}{\lambda_0}\right)^L. \tag{1.12}$$

We see here the stabilizing role played by chiral symmetry breaking; because there is a dynamical quark mass λ_0 the effective mean-field at strong coupling is very tiny, namely $\varrho \propto d^{-1/2}$. The latent heath of the transition is not washed out at large d.

All the results we have described here follow essentially from the simple and rather trivial remark that Kogut-Susskind fermions are harmless at large d, provided chiral symmetry breaking is properly taken into account. The question of whether $d = 4$ is big enough is of course beyond the reach of this kind of mean-field arguments.

2. *SU(N) lattice gauge theory at finite temperature in the strong coupling regime*

We now turn to the discussion of the deconfining phase transition that occur in SU(N) lattice gauge theories at finite temperature T, when light Kogut-Susskind fermions are turned on. In the absence of fermions, this deconfining phase transition is known to be related to the spontaneous breaking of a global $Z(N)$ symmetry [9]. This symmetry is induced by the periodic boundary conditions in euclidean time for the gauge fields, needed in order to describe finite temperature effects in the path integral formulation of the theory. It arises in the following way: let us consider a lattice of finite length La in the euclidean time direction. In the rest of this section, d will be number of *space* directions. Sites in the lattice will be labelled by $(\bar{x}, \tau)$, $\bar{x}$ being a d-dimensional vector and τ the discrete time coordinate. If one imposes periodic boundary conditions in the Euclidean time direction for the gauge field variables pointing in space directions

$$U_i(\bar{x}, 0) = U_i(\bar{x}, La)\,(i = 1, \ldots, d) \tag{2.1}$$

both the action and the boundary conditions (2.1) are invariant under gauge transformations which are periodic up to a *constant* element z of the center of the gauge group, $Z(N)$

$$\Omega(\bar{x}, 0) = \Omega(\bar{x}, La)z. \tag{2.2}$$

The order parameter related to the deconfining phase transition is the thermal Wilson loop $W(\bar{x})$, a Wilson loop that winds along the time direction, which is not invariant under the class of gauge transformations (2.2)

$$W(\bar{x}) \to W(\bar{x})z \tag{2.3}$$

and which carries information about the free energy excess F_q of a single quark at point $\bar{x}$

$$\langle W(\bar{x})\rangle = e^{-F_q}. \tag{2.4}$$

Therefore, if $\langle W(\bar{x})\rangle = 0$, the center symmetry $Z(N)$ is unbroken, $F_q = \infty$ and the gauge system is in a confining phase. If $\langle W(\bar{x})\rangle \neq 0$, the $Z(N)$ symmetry is spontaneously broken, F_q is finite and the gauge system is in an unconfined phase.

As pointed out by Svetitsky and Yaffe [10], if an effective action for the thermal Wilson loops $W(\bar{x})$ is computed in the pure gauge sector by integrating away the gauge degrees of freedom $U_i(\bar{x}, \tau)$ $(i = 1, \ldots, d)$, the result is $Z(N)$ invariant. Explicit strong-coupling calculations [11–12] as well as Monte-Carlo studies [13] have verified their conjecture, namely, that the deconfining phase transition is of second order for SU(2) and of first order for SU(3). When quark matter fields in the fundamental representation are coupled to the gauge field, their contribution to the effective action for $W(\bar{x})$ breaks explicitly the global $Z(N)$ symmetry, the reason being that the fermionic action in a periodic lattice is not invariant under the class of gauge transformations (2.2). By analogy with a $Z(N)$ spin system in an external magnetic field, one can immediately see that the fermion-induced term in the effective action for $W(\bar{x})$ will reduce the latent heath of the first-order transition of the SU(3) theory or, if strong enough, will wash the transition away.

This problem has been examined in the strong-coupling limit, for Wilson and Kogut-Susskind fermions [14]. Even at strong coupling, the fermion dynamics is not exactly soluble, and further approximations are needed such as, for example, the hopping parameter expansion $K = (m_0 + d + 1)^{-1}$ for Wilson fermions with $r = 1$, m_0 being the bare quarks mass. One obtains in this case that the strength of the $Z(N)$ symmetry breaking term in the effective action for $W(\bar{x})$ induced by the quark fields is proportional to K^L [14].

The same analysis for Kogut-Susskind fermions gives a strength proportional to m_0^{-L} [14], so it looks as if in this case the first-order phase transition is doomed to disappear as soon as the quarks are light enough.

We reexamine in the rest of this section the strong coupling problem for Kogut--Susskind fermions and exhibit once again the stabilizing role played by spontaneous chiral symmetry breaking. Indeed, we shall find that, when this effect is taken into account, the Z(N) symmetry breaking effects are no longer controlled by the bare quark mass but rather by the dynamical mass acquired by the fermions through the spontaneous breaking of chiral symmetry. We shall use a $1/d$ expansion to compute the fermion path integral. The leading term in this expansion contains terms to all orders in the hopping parameter $K = m_0^{-1}$.

We consider again the SU(N) lattice gauge theory defined by Eqs (1.1)–(1.3). Our lattice has a finite size La in the euclidean time direction, and the physical temperature is given by $T = (La)^{-1}$. Fermi fields satisfy antiperiodic boundary conditions at $\tau = 0$ and $\tau = La$. We shall take the integer L to be even. The basic reason is that a consistent reinterpretation of Kogut-Susskind fermions in terms of quark fields with flavor quantum numbers requires the introduction of a lattice of lattice spacing $2a$ [2]. Likewise, a transfer matrix for flavor-carrying quark fields can only be defined between time slices separated by 2 units of a.

In the presence of the periodic boundary conditions (2.1), a gauge can always be found in which the gauge variables $U_0(\bar{x}, \tau)$ sitting on time-like links are time-independent and diagonal [12]

$$U_0(\bar{x}, \tau) = \mathrm{diag}\,(e^{i\phi_1(\bar{x})}, \ldots, e^{i\phi_N(\bar{x})}), \tag{2.5}$$

where the angles $\phi_\alpha(\bar{x})$ obey the SU(N) constraint $\sum_{\alpha=1}^{N} \phi_\alpha = 0 \pmod{2\pi}$ and their range is restricted to $\phi_\alpha \in \left[-\frac{\pi}{L}, \frac{\pi}{L}\right]$ to avoid Gribov copies. In this gauge, the partition function (1.1) takes the form:

$$Z = \int_{-\pi/L}^{\pi/L} \prod_{\vec{x}} \prod_{\alpha=1}^{N} d\phi_\alpha(\bar{x}) \int \prod_{\tau} \prod_{\bar{x},i} d\mu(U_i(\bar{x}, \tau))$$
$$\times \Delta[\phi_\alpha] \exp\{S[U_0, U_i; \chi, \chi]\}, \tag{2.6}$$

where $\Delta[\phi_\alpha]$ is a term arising from a partial integration of the Haar measure of the $U_0(\bar{x})$ variables.

Next, we perform the $U_i(\bar{x}, \tau)$ integration in the strong-coupling region. As far as the Z(N)-symmetric piece of the effective action is concerned, everything goes as described in Ref. [12], so we shall only discuss here the fermion contribution. The fermionic action (1.3) being linear in the U_i variables, the integration can be performed explicitly at strong coupling (where the pure gauge piece of the action is neglected). This integration yields an effective fermion Lagrangian involving interactions between Fermi fields at equal times τ and sitting on nearest neighbour sites in space.

$$\mathscr{L} = \tfrac{1}{2}\sum_{\bar{x},\tau} \eta_0(\bar{x})\,\{\bar{\chi}(\bar{x},\tau+1)U_0^{+}(\bar{x})\chi(\bar{x},\tau) - \bar{\chi}(\bar{x},\tau)U_0(\bar{x})\chi(\bar{x},\tau+1)\}$$

$$+\frac{1}{4N}\sum_{\tau}\sum_{\bar{x},\hat{i}} [\bar{\chi}(\bar{x}+\hat{i},\tau)\chi(\bar{x}+\hat{i},\tau)]\,[\bar{\chi}(\bar{x},\tau)\bar{\chi}(\bar{x},\tau)] + \ldots \tag{2.7}$$

where contractions over color indices are understood inside the brackets. The dots in Eq. (2.7) represent higher-order between composite $\bar{\chi}\chi$ fields (as well as baryon fields) which can be neglected to leading order in a $1/d$ expansion [4]. The next step is to integrate the Grassmann variables χ, $\bar{\chi}$. Following the procedure of Ref. [4], a scalar field $\lambda(\bar{x},\tau)$ can be introduced to linearize the four-fermion interaction:

$$\exp\{\tfrac{1}{2}\sum_{\bar{x},\bar{x}'} \bar{\chi}\chi(\bar{x}',\tau)\,(V^{-1})_{\bar{x}',\bar{x}}\bar{\chi}\chi(\bar{x},\tau)\}$$

$$= (\det V)^{1/2}\int\prod_{\bar{x}} d\lambda(\bar{x},\tau)\,\exp\{-\tfrac{1}{2}\sum_{\bar{x},\bar{x}'}\lambda(\bar{x}',\tau)V(\bar{x}',\bar{x})\chi(\bar{x},\tau)$$

$$+\sum_{\bar{x}}\lambda(\bar{x},\tau)\bar{\chi}(\bar{x},\tau)\chi(\bar{x},\tau)\}, \tag{2.8}$$

where we have defined:

$$(V^{-1})_{\bar{x}',\bar{x}} = \frac{1}{4N}\{\delta_{\bar{x}',\bar{x}+\hat{\imath}} + \delta_{\bar{x}',\bar{x}-\hat{\imath}}\}. \tag{2.9}$$

The Lagrangian for the Fermi fields is now local in space, but quarks can propagate in the time direction and the mass term is time dependent. However, the $\lambda(\bar{x},\tau)$ integration will be performed next by a saddle point method, and, because of the choice of gauge (2.5), there is a time-independent saddle point $\lambda_0(\bar{x})$. For a time-independent $\lambda = \lambda(\bar{x})$ the Grassmann integration, with antiperiodic boundary conditions at $\tau = 0$ and $\tau = La$, gives the following effective action

$$S_{\text{eff}} = -\tfrac{1}{2}\sum_{\bar{x},\bar{x}'}\lambda(\bar{x})V(\bar{x}-\bar{x}')\lambda(\bar{x}')+\tfrac{1}{2}\operatorname{Tr}\ln V$$

$$+\sum_{\alpha=1}^{N}\sum_{\bar{x}}\sum_{k=0}^{L/2}\ln\left\{\lambda^2(\bar{x})+\sin^2\left[\phi_\alpha(\bar{x})+\frac{(2k+1)\pi}{L}\right]\right\}. \tag{2.10}$$

This expression still has to be minimized with respect to $\lambda(\bar{x})$. The corresponding saddle-point equation is

$$\frac{L}{2}\sum_{\bar{x}'} V(\bar{x}-\bar{x}')\lambda(\bar{x}') = \sum_{\alpha=1}^{N}\sum_{k=0}^{L/2}\frac{\lambda(\bar{x})}{\lambda^2(\bar{x})+\sin^2\left[\phi_\alpha(\bar{x})+\dfrac{(2k+1)\pi}{L}\right]} \tag{2.11}$$

which admits always the trivial solution $\lambda_0(\bar{x}) = 0$. Any non-trivial solution will necessarily be $\bar{x}$ dependent due to the presence of the phases $\phi_\alpha(\bar{x})$. But it can easily be verified that this lack of translational invariance is a $1/d$ effect. Indeed, from the relation $\sum_{\bar{x}'} V(\bar{x}-\bar{x}') = 2Nd^{-1}$, [4] it follows that Eq. (2.11) admits a non-trivial solution of the form:

$$\lambda_0(\bar{x}) = \sqrt{\frac{d}{2}}\,[1+O(1/d)]. \tag{2.12}$$

It can also be verified that this saddle point dominates over the trivial one. To leading order in the $1/d$ expansion there is therefore a dynamical mass $\lambda_0 = (d/2)^{1/2}$, as expected. Finally, introducing the thermal Wilson loop phases $\theta_\alpha(\bar{x}) = L\phi_\alpha(\bar{x})$, the part of S_{eff} which explicitly depends on this variable reads

$$S_{\text{eff}} = \sum_{\alpha=1}^{N} \sum_{\bar{x}} \sum_{k=0}^{L/2} \ln\left\{\lambda_0^2 + \sin^2\left(\frac{\theta_\alpha(\bar{x})+(2k+1)\pi}{L}\right)\right\}$$
$$= \sum_{\alpha=1}^{N} \sum_{n=-\infty}^{\infty} e^{in}(\theta_\alpha+\pi) f_n, \tag{2.13}$$

where

$$f_n = \sum_{K=1}^{\infty} \frac{1}{K}\frac{(-1)^K}{(2\lambda_0)^K} \sum_{s=0}^{K} \binom{K}{s} \delta_{K-2s,nL}. \tag{2.14}$$

At large d, the leading term in the expansion (9) is then given by

$$S_{\text{eff}} = -\frac{1}{L(2\lambda_0)^L} \sum_{\alpha=1}^{N} \cos\theta_\alpha, \tag{2.15}$$

which explicitly breaks the $Z(N)$ symmetry, because of Eq. (2.2). Eq. (11) shows the desired result: in spite of the fact that we started from massless Kogut-Susskind fermions, the dynamical mass λ_0 controls the strength of the symmetry breaking effective interaction.

The calculation we have outlined above has a simple interpretation in terms of fermionic paths in the lattice. Eq. (2.15) represents thermal Wilson loops with a renormalized hopping parameter, $K = (m_0+\lambda_0)^{-1}$. The renormalization of the hopping parameter in the time direction arises from the sum over all fermion paths which generate planar zero area loops in the space directions. These are the diagrams responsible for spontaneous breaking of chiral symmetry in the strong-coupling d-dimensional theory. In the case of Wilson fermions these diagrams do not contribute because, a it is well known, fermion paths are self-avoiding. Nevertheless, the term proportional to $(d+1)$ in the expression for the hopping parameter $K = (m_0+(d+1))^{-1}$ for Wilson fermions can evidently be interpreted as a "dynamical" mass generated by the *explicit* breaking of chiral symmetry in this formulation. Therefore, the strong coupling physics for Wilson and Kogut-Susskind fermions is not so different after all: the effects of light quarks on the deconfining phase transition at finite temperature are stabilized by the explicit (or spontaneous) breaking of chiral symmetry.

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