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INTRANUCLEAR CASCADE IN HIGH ENERGY COLLISIONS

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The experimental analysis of the process of cascading in the target fragmentation region (TFR) is performed on the basis of the available experimental methods and data and the existing phenomenological models. The effect is studied separately for the deuteron and for the heavy nuclei. The following subjects are discussed: 1. the experimental phenomena generated by the existence of the cascading effect in TFR; 2. general features of such process, namely: the effective cascade cross section, the fraction of cascade interactions, multiplicity of particles produced through cascading and their rapidity distributions, the dependence of cascading on energy and on the type of projectile and on the size of the nucleus; 3. the comparison with the phenomenological models and with other proposed mechanisms of particle production in TFR. The possibility of determining the hadronization time (formation time) through the study of the cascading process in TFR is pointed out.

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1. Introduction

It was customary in seventies to start any review of high energy interactions on nuclei with sharp criticism of the cascade model. The cascade model [1] assumed that the projectile interacts with the first nucleon it meets in the nucleus and gives a typical final state of about 10 particles at experimentally available high energies. Each of these first generation particles interacts with the forthcoming nucleons in the nucleus, giving rise to the cascade effect. This simple cascade model describes correctly the most important features of the multiple production on nuclei at relatively low energies ($\leqslant$ 5 GeV). However, at higher energies the model diverges sharply from the experimental data, for instance it predicts a dramatical growth of the multiplicity of particles produced in collision with nuclei. This result had been interpreted as demonstrating the complete absence of any intranuclear interactions of secondary particles. However at the present time this opinion has changed.

At very high energies there are three different regions of particle production on nuclei. Usually this is demonstrated on the plot

$$R(y) = \varrho^{hA}(y)/\varrho^{hp}(y), \tag{1.1}$$

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where $\varrho(y)$ is the repidity density of produced particles in hp and hA interactions. For example in Fig. 1 ($R(y)$ from experiment pAr and pXe at 200 GeV [2]) we can find quite different behaviour in the three regions:

1. a strong maximum in the target fragmentation region (TFR),
2. an approximate "plateau" in the central region,
3. a depletion of particle density in the projectile fragmentation region.

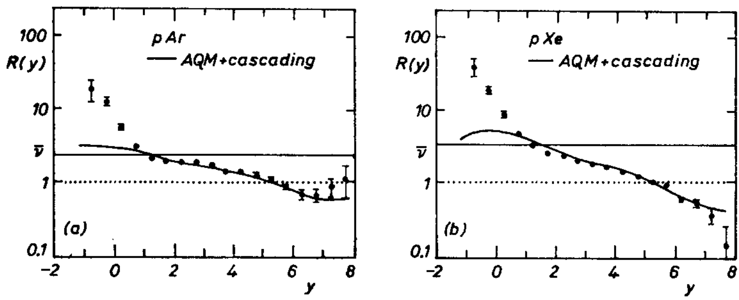


Fig. 1. The ratio $R(y) = \varrho_A(y)/\varrho_p(y)$ for produced particles in pAr (a) and pXe (b) reaction at 200 GeV/c as a function of the rapidity y [2]. The solid lines are the predictions of the additive quark model [57]

Solid line corresponds to the value of the average number of inelastic collisions of the projectile inside nucleus

$$\bar{\nu} = A\sigma_{pp}/\sigma_{pA}, \tag{1.2}$$

where σ_{pp} and σ_{pA} are the inelastic cross section on protons and nuclei and A is the mass number of the nucleus.

If we have $\bar{\nu}$ collisions we expect that the particle production on nuclei should be in average at most $\bar{\nu}$ times bigger than in elementary collision. The ratio $R(y)$ in the TFR exceeds several times the value of $\bar{\nu}$. This fact, observed in many experiments, could be interpreted as evidence for the existence of the cascading effects inside nucleus.

The observation that this intranuclear cascading is restricted to relatively slow particles creates a new very important aspect of the investigation of the process of cascading. Namely, the study of the nature of intranuclear interactions becomes a unique source of information on the space — time evolution of the multiple production process.

The fast particles do not interact inside the nucleus because they are formed outside it. The slow particles do interact inside the nucleus. If we know the size of the nucleus and the range of momenta for which the cascading occurs we can try to estimate the time interval between the collision and the hadronization process, called the formation time. Such information cannot be extracted from any elementary particle-particle interactions.

The interpretation of the absence of the fast particle cascading is different in different approaches. Many models [3–9] propose different ways leading to this effect but almost all of them are based on the uncertainty principle and the Lorentz time dilatation.

The present paper is devoted to the investigation of the intranuclear cascade in this new sense. We would like:

1. to study the target fragmentation region in reactions with nuclei,

2. to complete the arguments for the existence of the process of cascading in this region,

3. to connect possible experimental information with formation time idea,

4. to find possible characteristics of the objects inducing the cascade and of the particles produced in the cascading process.

The experimental conditions lead to two possible ways of these investigations:

1. Rescattering in the deuteron — the simplest one, since in each collision at most only one particle rescatters, the wave function of the deuteron is well known, and the sample of double scattering is very well defined [10, 11].

2. Interactions with heavy nuclei — difficult to numerical interpretation but with more and statistically powerful experimental data giving the dependence on A and on the size of the nucleus.

High energy interactions with nuclei have been the subject of many reviews during the last few years [12–24]. Most of them are prepared from the theoretical point of view using the experimental data only to illustrate the agreement or disagreement with the theory.

The target fragmentation region, difficult to experimental investigations and theoretical description is usually either neglected or reported very briefly, except for Ref. [18]. This paper covers only topics related to the target fragmentation region and stresses the experimental point of view.

The paper is organized as follows. Section 2 contains a review of experimental methods and data in subsections discussing the experiments for hadron-deuteron, hadron-heavy nuclei and lepton-nuclei collisions. In this chapter we also point out the specific character of investigations connected with different techniques of experiments — interactions with nuclear emulsion as target and detector, interactions in bubble and streamer chambers and experiments using the counter techniques for the observation of the target fragmentation region. Section 3 contains a short description of the models of interactions with nuclei which consider effects in the target fragmentation region and models proposed for hadron--deuteron interactions. The formation length idea is described in Section 4. In Section 5 we discuss results of experiments with deuteron on the basis of the theoretical models. Section 6 contains the summary of the information about the process of cascading obtained from experiments with heavy nuclei. Other attempts of interpretation of effects in the TFR without using the cascading idea are reviewed in Section 7. Summary and general conclusions are given in Section 8.

2. *Experimental methods and data*

2.1. Different experimental techniques in the target fragmentation region

To compare the observations obtained by using different experimental techniques one should specify the correspondence between the measured values. The summary of features of the detectors used to study the nuclear interactions especially in TFR is given in Table I.

TABLE I

<table>
<tr><th>Technique</th><th>Nuclear emulsion</th><th>Bubble chamber + magnetic field</th><th>Streamer chamber + magnetic field</th><th>Counter experiment [25]</th></tr>
<tr><td>Target</td><td>CNO or AgBr mixture*</td><td>H, D, Ne or internal nuclear target</td><td>Ne or internal nuclear target</td><td>Any nuclear target</td></tr>
<tr><td>Identification charge momentum</td><td>no</td><td>charge identification
$100 < p_p < 1000$–1200 MeV/c</td><td>charge identification
$150 < p_p < 600$ MeV/c
$p_\pi > 60$ MeV/c</td><td>no</td></tr>
<tr><td>Solid angle of vertex detector</td><td>4π</td><td>4π</td><td>4π</td><td>50% of 4π</td></tr>
<tr><td>Statistics per target, energy projectile</td><td>10^2–$5 \cdot 10^3$</td><td>10^3–10^4</td><td>10^3–10^4</td><td>10^5–10^6</td></tr>
<tr><td rowspan="3">Classification of secondaries</td><td>n_s — shower particles
$\beta = v/c > 0.7$
$p_\pi > 140$ MeV/c
$p_p > 1000$ MeV/c</td><td colspan="2">$n_s \approx n_M = n_{total} - n_p$
n_p — identified protons</td><td rowspan="2">good distinction between shower and grey tracks</td></tr>
<tr><td>n_g — grey tracks
$0.25 < \beta < 0.7$
$35 < p_\pi < 140$ MeV/c
$250 < p_p < 1000$ MeV/c</td><td rowspan="2">$n_p \approx n_g + n_b$
$p_p[100$–$1000]$ MeV/c</td><td rowspan="2">$n_p \approx n_g + n_b$
$p_p[150$–$600]$ MeV/c
evaporation fragments stop in target</td></tr>
<tr><td>n_b — black tracks
$\beta < 0.25$
$p_\pi < 35$ MeV/c
$p_p < 250$ MeV/c</td><td>black tracks absorbed in target and counters</td></tr>
</table>

* In emulsion experiments it is possible to study the events produced in an element. The method consists in incorporating the wires or pellets into nuclear emulsion [26, 27].

Most of the terminology and classification of particles in reactions with nuclei was established in accordance with the tradition in emulsion experiments. Following this tradition we adopt this classification in Table I and compare it with corresponding classes of events observed in bubble chambers, streamer chambers and counter experiments.

The direct observation of target fragmentation region in the standard counter experiments is usually either impossible or incomplete. Therefore column 5 of Table I does not contain general features of electronic experiments but the conditions of a special counter experiment [25]. The aim of this experiment was to continue quickly the emulsion experiments with very high statistics and known nuclear target. Therefore they have the same distinction between shower and grey tracks as in emulsion techniques.

Also in column 4 experimental condition of streamer chamber experiment are mostly based on the experiment performed by Bari, Cracow, Liverpool, Nijmegen, Munich Collaboration [2].

There is one more important point connected with experimental conditions. A convenient parameter for describing the longitudinal motion of particles is the rapidity

$$y = \tfrac{1}{2} \ln [(E+p_{\|})/(E-p_{\|})], \tag{2.1}$$

where E is the total energy of particle and $p_{\|}$ its longitudinal momentum. In many experiments which do not use the magnetic field to determine the momenta of particles only the angles are measured. From these measurements one can obtain an approximate rapidity distribution by using the pseudorapidity variable

$$\eta_{\text{lab}} = -\ln \tan (\theta_{\text{lab}}/2), \tag{2.2}$$

where θ_{lab} is the polar angle of produced particles. The approximation $y_{\text{lab}} \approx \eta_{\text{lab}}$ is good provided

$$\ln [(E+p_{\|})/\sqrt{p_t^2+m^2}] \approx \ln [(p+p_{\|})/p_t] \tag{2.3}$$

i.e. if the mass of produced particle m is small compared to its transverse momentum p_t. It is clear that this approximation is good for pions and very bad for nucleons which mostly populate TFR. We can use both rapidity and pseudorapidity for presenting data but we cannot compare them directly in TFR.

It is convenient to use relative quantity $R(y)$ as defined in formula (1.1) or expressed in terms of pseudorapidity

$$R(\eta) = \varrho(\eta)^{\text{hA}}/\varrho(\eta)^{\text{hp}}, \tag{2.4}$$

where $\varrho(\eta)$ is the density of produced particles in hadron-proton and hadron-A interaction. The reasons for that are following. At first theoretical predictions mostly refer to the increase of particles produced from a nuclear target relative to a single nucleon target. Another reason is that the effects of particle misidentification cancel in the ratio $R(y)$ or $R(\eta)$. It should be stressed that it is necessary to determine both densities entering this ratio in the same experiment to have the same: incoming energy, trigger biases, misidentification effects and so on.

2.2. Experiments with deuteron

As was pointed out in introduction the experiments for hadron-deuteron collisions have particular advantages over experiments with heavier nuclei especially if we investigate TFR. The most important difference is that while the sample of interactions with heavy nucleus contains undistinguishable single, double, triple and more collision events, in deuteron bubble chamber experiment we can in principle separate single and double interactions.

There are two signatures for double scattering events. The first suggested in [11] is the presence in the final state in the forward hemisphere of two protons with momenta exceeding the Fermi momenta in the deuteron. However this very clear cut requires very high statistics experiment. Probability of interaction with both nucleons of the deuteron (double scattering) is about 15% for pion beam and about 20% for proton beam. If we take into account the fact that two protons occur in the final state when interaction with neutron goes through charge exchange and with proton without charge exchange, we obtain probability of registration of events with two protons ~4% of whole sample of interactions. This estimation is valid if we register all protons in full region of momenta. The probability of the identification of recoil proton in bubble chamber experiment equal ~25% (momentum range 300 MeV/c — 1200 MeV/c) reduces this estimation to ~1% obseredv two protons events in whole sample of interactions with deuteron.

The second possibility is based on the analysis of topological cross sections. The description of this procedure for π^- beam [28–31] is following. Interactions on deuterons produce events in which either an odd or an even number of tracks are observed. The odd-prong events are interpreted as π^-n interactions in which a spectator proton is left with too little momentum (less than 80 MeV/c) to produce a visible track in bubble chamber. The even-prong events are of the following three types: (*i*) π^-p interactions with the spectator neutron, (*ii*) π^-n interactions with a spectator proton that is observed in the bubble chamber, (*iii*) interactions involving both the proton and neutron i.e. double scattering. It is possible to obtain a clean sample of π^-n interactions by removing events of type (*ii*) and adding them to the odd-prong events. Even-prong events with recoil protons which are backwards in the laboratory frame are unambiguous π^-n interactions of type (*ii*). After multiplying these events by 1 + flux factor to account for the π^-n interactions with forward spectator protons, these events are subtracted from the even-prong sample and reassigned to the odd-prong sample with multiplicity $N-1$ to form the effective partial cross section σ_N("π^-n"). The remaining even-multiplicity events of type (*i*) and (*iii*), which are a mixture of π^-p interactions and double scattering events, give the effective partial cross section σ_N("π^-p").

Using the σ_N("π^-p") and σ_N("π^-n") obtained by the above procedures we can calculate:

1. the double scattering cross section

$$\sigma^{DS} = \sum_{N\geqslant 3} [\sigma_N(„\pi^-p") - \sigma_N(„\pi^-n")] - [\sigma(\pi^-p) - \sigma(\pi^-n)] \tag{2.5}$$

where $\sigma(\pi^-p)$ and $\sigma(\pi^-n)$ are the free proton and free neutron cross sections,

2. the multiplicity distributions of double scattering sample [30]

$$\sigma_N^{DS} = \sigma_N(„\pi^-p") - \{\sigma(„\pi^-n") - [\sigma(\pi^-p) - \sigma(\pi^-n)]\} \frac{\sigma_N(\pi^-p)}{\sum_N \sigma_N(\pi^-p)} \tag{2.6}$$

3. any double scattering kinematical single particle distribution [31, 35]

$$\frac{d\sigma^{DS}}{di} = \frac{d\sigma}{di}(\text{„}\pi^-\text{p"}) - \frac{d\sigma}{di}(\pi^-\text{p})\frac{\sigma(\text{„}\pi^-\text{p"})-\sigma^{DS}}{\sigma(\pi^-\text{p})}, \tag{2.7}$$

where i could be any kinematical variable e.g. y — rapidity, η — pseudorapidity, x — Feynman variable etc.

To study the subject of internuclear cascade in deuteron we use mainly the results of two experiments performed by Cracow, Davis, Seattle, Warsaw Collaboration in the Fermilab 30 inch deuterium filled bubble chamber. The energy of incident negative pions was 205 and 360 GeV. The experimental details are discussed in [28–31, 34–36]. We use also some results of π^-d experiments at 15, 21, 100 GeV [32–33], pd experiments at 19, 28, 100, 200, 303, 400 GeV and corresponding elementary π^-p and pp experiments collected in Ref. [30]. The single particle rapidity distributions in π^-p elementary interactions at 205 and 360 GeV necessary for the detailed study of rapidity distribution of cascade was obtained from Duke, Toronto, Mc Gill, Notre Dame Collaboration and Iowa State, Maryland, Michigan State, Notre Dame Collaboration [37]. It is very important that these experiments were performed on the same beam and apparatus so they have same experimental conditions and biases.

2.3. Hadron-heavy nuclei experiments

How to observe and to study the process of cascading in the collisions with heavy nuclei? The most direct information comes from the observation of rapidity (pseudorapidity) distributions of produced particles in TFR. The results from emulsion experiments include the pseudorapidity distributions only for shower particles. It means that only pions with momenta bigger than 140 MeV/c are included. Protons with momenta higher than 100 MeV/c are treated as π mesons, are referred to as "produced particles" and artificially shifted to higher rapidities. The rapidity distributions obtained using streamer chamber experiments contain π mesons with momentum higher than 60 MeV/c. They are however distorted by the unidentified protons with momenta bigger than 600 MeV/c. In such an experiment one has a possibility to use the distribution of negative particles to investigate nuclear effects. However, using $R(y)$ for negative particles may be misleading since the nucleus consists of protons and neutrons and the π^- spectra from neutron and proton fragmentation are different. So one needs additionally the h-deuteron interactions or should use the approximate formula

$$\varrho_p(y) = [\varrho_p^-(y)+\varrho_p^+(y)]/2, \tag{2.8}$$

where $\varrho_p^-(y)$ and $\varrho_p^+(y)$ are the rapidity densities of negative and positive produced particles in hadron-proton collisions.

The second possibility of studying intranuclear cascade is the investigation of net charge rapidity distributions [2, 38]. Any collision with proton inside nucleus causes the appearance of the additional charge +1 in the final state. The average net charge can be treated as a measure of average number of target protons participating in the interaction with nuclei

$$\langle Q \rangle^{\text{exp}} = \langle n_{\text{tot}} \rangle - 2\langle n^- \rangle - Q_{\text{h}}, \tag{2.9}$$

where $\langle n_{\text{tot}} \rangle$ is the average total multiplicity, $\langle n^- \rangle$ the multiplicity of negative particles and Q_{h} the charge of projectile. If the projectile interacted only $\bar{v}$ times we would expect

$$\langle Q \rangle = \frac{Z}{A} \bar{v}. \tag{2.10}$$

As we can find, experimental value of $\langle Q \rangle^{\text{exp}}$ exceeds this expected value. This indicates that inside nucleus also the secondaries interact. Using this observation one can try to estimate the number $\bar{v}_{\text{c}}$ of secondary interactions inside the nucleus

$$\bar{v}_{\text{c}} = \frac{A}{Z} \langle Q \rangle^{\text{exp}} - \bar{v}. \tag{2.11}$$

The analysis of the net charge rapidity distribution gives also a possibility of localization in rapidity of products of cascading. Moreover, one can try to estimate the mean number of particles produced in a cascade. Assuming that the cascading effect occurs in the target fragmentation region we can calculate this number using formula

$$\langle n_{\text{c}} \rangle = [\langle n_{\text{M}} \rangle^{\text{hA}}_{\text{TFR}} - \bar{v} \langle n_{\text{M}} \rangle^{\text{hp}}_{\text{TFR}}]/\bar{v}_{\text{c}}, \tag{2.12}$$

where $\langle n_{\text{M}} \rangle^{\text{hA}}_{\text{TFR}}$ and $\langle n_{\text{M}} \rangle^{\text{hp}}_{\text{TFR}}$ corresponds to mean multiplicities of produced particles in target fragmentation region for h-A and h-p interactions.

It has been often suggested that the n_{g} — the number of grey prong particles, or n_{p} — the number of identified protons is a measure of the number of collisions of the primary particle inside nucleus

$$\bar{v} = f(n_{\text{g}}) \quad \text{or} \quad f(n_{\text{p}}). \tag{2.13}$$

The method proposed by B. Andersson et al. [39] gives the prescription how to find a relation between number of collisions $\bar{v}$ and the value n_{g} or n_{p}. Basing on this relation we can also deduce some information about the existence and the features of the cascading process. Using the method proposed by Andersson from data on interaction with one single nucleus of mass A one can obtain information on interactions with very different numbers of projectile collisions $\bar{v}(n_{\text{p}})$. Moreover the range of $v(n_{\text{p}})$ is bigger than $\bar{v}$ (for Uranium $\bar{v} = 4$). Using the subsample of events with different n_{p} we can investigate the following distributions: $R_y(\bar{v}(n_{\text{p}}))$, $\langle \bar{v}_{\text{c}}(n_{\text{p}}) \rangle$, $p_{\text{L}}(\bar{v}(n_{\text{p}}))$ in the target fragmentation region. These variables behave very differently in the various regions of rapidity which can be helpful in determination of the range of the occurrence of cascading effect.

The A dependence of final spectra of produced particles gives also the information about interactions of hadrons or hadronic constituents inside nuclear matter. The following presentation of the A dependence of single particle spectrum appears often in experimental papers

$$\frac{Ed\sigma^{A}}{d^3p}(i) = \frac{Ed\sigma^{P}}{d^3p}(i)A^{\alpha(i)}, \tag{2.14}$$

where $\frac{Ed\sigma}{d^3p}$ is the invariant cross section and i can be rapidity y or tranverse momentum p_t. The values $\alpha = \alpha_0 = 0.69, 0.79, 0.75$ respectively for protons, kaons and pions, correspond to the differential cross sections independent of nuclear size (i.e. independent of the number of collisions of the projectile inside the nucleus). The value of $\alpha = 1$ occurs when the particle multiplicity is proportional to the average number of collisions (in each collision the incident particle produces the same number of particles as in the collision with free nucleon). For $\alpha > 1$ the particle production on nuclei is bigger than $\bar{\nu}$ times the particle production on hydrogen (such effect is expected if the process of cascading exists). In such presentation, the problem for the theoretical description of single particle spectra is to find a physical mechanism which gives the formula

$$\alpha = \alpha(y) \quad \text{or} \quad \alpha = \alpha(p_t). \tag{2.15}$$

To study the cascade we use the results of the following hadron-heavy nuclei experiments: p on Em at 400 GeV [40], p on Em at 67 and 200 GeV [41], π on Em at 200 GeV [41], p on Ar, Xe at 200 GeV [2, 38], π^+, π^-, p, $\bar{\text{p}}$ on Mg, Ag, Au at 100 GeV [42], π^+, π^-, p, $\bar{\text{p}}$ on C, Cu, Pb at 50, 100, 150 GeV [25], p on W, Cr at 300 GeV [27], π^+, π^-, p, $\bar{\text{p}}$, K on C, Al, Cu, Pb, U at 50 to 200 GeV [43].

2.4. Lepton-nucleus interactions

It is possible to study experimentally collisions with nuclei where the leading particle disappears so quickly that it has no chance to scatter more than once. The idea is to look at deep inelastic lepton nucleus scattering [44, 45, 8]. Because of the uncertainty principle the life-time of the virtual photon is simply related to the Bjorken variable ω_B

$$t \approx \frac{1}{E-E'-E_\gamma} \approx \frac{\omega_B}{M} \approx \frac{\omega_B}{5}\,\text{fm} = \frac{1}{5x_B}\,\text{fm}, \tag{2.16}$$

where M is the nucleon mass, $\omega_B = \frac{1}{x_B} = \frac{2M\nu}{Q^2}$, E, E', and E_γ are energies of the initial and final leptons and of the virtual photon, Q^2 is the four-momentum transfer squared and $\nu = E-E'$.

The requirement that only one nucleon participates in the process is equivalent to the following condition

$$t \leqslant 1\ \text{fm}, \tag{2.17}$$

which corresponds to $\omega_B \leqslant 5$. The measurements of momenta and scattering angles of the outgoing lepton allow one to control the virtual photons involved in interactions and to select some classes of production process. The analysis similar as for hadron-nucleus performed on such sample of interactions on nulcei and comparison of the results with production on free nucleon give more accurate information than from multiple scattering. Such an experiment was performed with the Fermilab 150 GeV μ^+ beam [46].

A special possibility of investigation of the cascade gives the experiment ν-deuteron performed also in FNAL [47]. To explain some results of this experiment (ratio $r = \sigma(\nu n \to \mu^- X)/\sigma(\nu p \to \mu^- X)$) the authors use a very simple parametrisation of the dependence of double scattering fraction on the total projectile-nucleon cross section σ_T in particle nucleon interactions. Such a dependence is expected when one considers that the second nucleon in deuteron can be scattered either by the leading particle of the primary interaction which depends on σ_T or by the remaining secondary particles, which are independent of the nature of incident particle [48]. They use a linear dependence

$$d = a + b\sigma_T \tag{2.18}$$

for the double scatter fraction d. The data from pd and πd experiments at different energies was fitted to equation (2.18) and the result extrapolated to the ν-nucleon cross section. The value of d used in the calculation of the ratio of ν-neutron to ν-proton cross section in deuterium gives the result consistent with the quark parton model. It should be stressed that this ratio depends strongly on the frequency of final state interactions. Thus one can treat this as a consistency check of the parametrisation (2.18).

If one accepts this picture and method of estimation of the double scattering fraction d, one should agree that this is simply a fraction of the cascading process in ν-d interactions. Thus this value can be compared with the amount of the cascading process predicted by models for hadron–deuteron interactions.

For comparison with the hadron-neon data we use also the results of bubble chamber experiment ν, $\bar{\nu}$-Ne [49].

3. Phenomenological description of the Target Fragmentation Region (TFR)

3.1. General remarks

None of the standard models of interactions with nuclei is able to describe satisfactorily the effects in TFR. The most basic and experimentally established observation is the behaviour of value $R(y)$ exceeding several times the value of $\bar{\nu}$ in this region. As an example we show in Table II predictions of most discussed models referring to $R(y)$ in TFR and considered range of this region. In most models the problem of existence and range of cascading effects appears essentially as an additional ingredient. Thus the investigation of TFR is rather not suitable for testing the basic postulates of the models. Because of that we limit this survey only to these models which explicitly introduce an additional mechanism of production of particles in TFR.

Again, similarly as in the experimental field, the interactions with deuteron are the simplest for theoretical quantitative description. The advantages of investigation of interactions with deuteron from phenomenological point of view are as follows. The wave-function

TABLE II

Model	Multiplication factor $R(y)$ in TFR	Upper limit of TFR in rapidity
Energy flux cascade model K. Gottfried [6]	$\bar{\nu} = \frac{A\sigma^{hp}}{\sigma^{hA}}$	$y_c = 1/3 y_{inc} + 2/3 \ln \lambda/2\tau_0 c$ λ — mean free path in the nucleus τ_0 — 1 fm/c, characteristic time for strong interactions
Regge type peripheral models J. Koplik, H. Mueller [50]	$\sim A^{1/3} \sim R$	$y_c = \ln mR \approx 1/3 \ln A + \text{const}$ R — radius of nucleus m — mass scale (not known)
L. Caneschi, A. Schwimmer [51]	$\ln A^{1/3}$	$y_c \approx \ln mR$ R — radius of nucleus m — nucleon mass
A. Capella, A. Krzywicki [52]	$\leqslant \bar{\nu}$ cascading neglected	TFR grows with energy of projectile
K. Kinoshita [53]	$\bar{\nu}$ cascading neglected	$y_c = 1$–1.5
Multiple scattering models S. J. Brodsky, J. F. Gunion, J. Kuhn [54]	$\bar{\nu}$	$y_c = 1$–2
A. Capella, J. Tran Thanh Van [55]	no predictions in TFR	$y_c = y_{inc} - \text{const}$
C. B. Chiu, Z. He, D. M. Tow [56]	$\bar{\nu}$	$y_c = \sinh^{-1} d/\tau_0$ $d = 1/m$ internuclear distance $\langle 1/\tau_0 \rangle = 0.5$ GeV
Additive quark models B. B. Levchenko, N. N. Nikolaev [57]	$> \bar{\nu}$ inagreement with experiment for $y_{lab} > 0$	$y_c = \ln Rm \approx 2 + \ln A^{1/3}$ R — radius of nucleus $m = 0.7$–0.8
A. Białas [17]	$\bar{\nu}$	$y_c = \ln 2Rm_T$ m_T — transverse mass of produced meson

of the deuteron is well known. We may also assume that only one object may interact with the second nucleon: the projectile, its constituents or some other object created in the first collision. Therefore we can concentrate on the main problem: what objects scatter off the second nucleon? If one considers cascading effects one can estimate the space--time evolution of this process and additional production of particles in single internuclear collision. Moreover if one subtracts some distribution predicted by the model not considering cascade from experimental distribution one can obtain some characteristics of additional process presumably cascading. If this procedure is applied to deuteron data these characteristics refer to one act of rescattering. We concentrate on the predictions of four models of particle production on the deuteron target

1. The additive quark model proposed by Białas et al. [58], [59].
2. The additive quark model proposed by Nikolaev and Zoller [11].
3. The eikonal type model BRLW proposed by Baker et al. [60].
4. Dual parton model proposed by Capella and Tran Thanh Van [61].

We have a similar possibility to "observe" single secondary interaction under some kinematical condition (see Section 2.4) in interactions of leptons with nuclei.

The theoretical description of TFR for interactions with heavy nuclei require many additional assumptions. We concentrate on the model considering multiple scattering of projectile or its constituents. The additional assumptions should concern the ambiguity coming from two questions:

1. what is the probability of interaction of different products created in collisions of projectile with the remaining nucleons of nuclei,
2. what is the kinematical possibility that products of secondary cascade interactions can interact and produce second or even third generation of cascade.

Nikolaev and Levchenko [57] proposed on the basis of the additive quark model and the quark-hadron duality concept, the correspondence between the quark-nucleus interactions and a specific sequence of particle-nucleon collisions. To obtain quantitative predictions of the model they use the Monte Carlo procedure [57].

The two microscopic models for intranuclear cascade are proposed by Hegab and Hüfner [62] and Suzuki [63]. The idea of these attempts is that the mechanism responsible for production of particles in TFR is the multiple scattering of projectile followed by secondary interactions of knocked out nucleons.

3.2. Models for the deuteron

The additive quark model proposed by Białas et al. [58] was applied to the hadron deuteron collisions [59]. It has been shown that in order to explain:

1. the value of double scattering cross section in πd and pd interactions for different energies,
2. the density of particles produced in double scattering events in central rapidity region

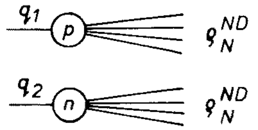
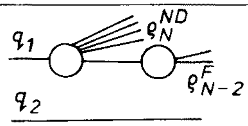
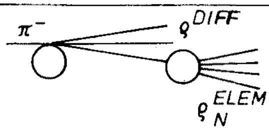
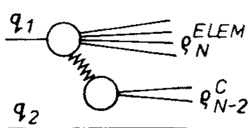
the additive quark model needs an extension to include the cascading effect. The fraction of double scattering coming from cascading was estimated about 50% for πd and pd interactions. The resulting value of effective cascade cross section was obtained

$$\sigma^* = 20.2 \pm 1.7 \text{ mb for } \pi d \text{ experiments,} \tag{3.1}$$

$$\sigma^* = 25.5 \pm 1.9 \text{ mb for pd experiments.} \tag{3.2}$$

This value indicated that particles created in the first collision have hadron-like cross section.

TABLE III

Process (cross section)	Type of process	Multiplicity distribution	m_i -multiplication in TFR
A (σ^A)		$X_N^A = (\rho_N^{ND} \otimes \rho_{N-1}^{ND})\sigma^A$	2
B (σ^B)		$X_N^B = (\rho_N^{ND} \otimes \rho_{N-2}^{F})\sigma^B$	2
DE (σ^{DE})		$X_N^{DE} \approx \rho_N^{ELEM}\sigma^{DE}$	1
$\varkappa$ ($\sigma^\varkappa$)		$X_N^\varkappa = \sigma^\varkappa(\rho_N^{ELEM} \otimes \rho_{N-2}^{C})$	1 (+cascade)

The model [59] specifies processes which contribute to double scattering and gives a prescription how to calculate their partial cross sections. Using the value of effective cross section we can extract from experimental data the mean multiplicity of particles produced in the single act of cascade [64]. The method is based on the construction of double scattering distribution from distributions from hadron nucleon interactions according to the model. Table III shows contributions of different processes to the double scattering effects in deuterium for incident pion. In Table III we use the following notation: ϱ_N is the multiplicity distribution for different processes (ND — non-diffractive, ELEM — elementary, π — nucleon, C — cascade, F — fragmentation of nucleon target, DIFF — diffractive), all normalised to unity; X_N — are the multiplicity distributions for process A, B, DE, κ; σ^A, σ^B, σ^{DE}, σ^κ are partial cross sections for processes A, B, DE, κ and finally

$$\varrho_k = (\varrho^1 \otimes \varrho^2) = \sum_{i=1}^{k} \varrho_i^1 \varrho_{k-i}^2 \tag{3.3}$$

denotes convolution of two multiplicity distributions and m_i denotes the multiplication factor of particles in TFR for particular processes i = A, B, DE, κ. The formulae used for calculation of partial cross sections are explained in Refs. [59, 64].

To discuss the multiplicity distributions and rapidity distributions we split the cross section for double scattering into four terms which we identify as:

Process A — one of the quarks from the incident pion interacts with the proton, the other one with the neutron in the target. In the quark model this situation corresponds to the fragmentation of two coloured strings spanned between two pairs of wounded quarks [65]. The density of particles arising from this fragmentation is doubled in comparison with the elementary process in which only one string fragments. Hence we put in the last column of Table III $m_A = 2$. The multiplicity distribution for this kind of interaction is the convolution of multiplicity distributions for non-diffractive interactions of quark with proton ϱ_N^{ND} and of quark with neutron ϱ_{N-1}^{ND}.

Process B — one of the quarks from the incident pion interacts with both nucleons in the target. The other quark does not interact at all. The rapidity distribution of particles in this process arises from the fragmentation of one string as in elementary interaction. The collision with the second nucleon can add at most the particles produced in the fragmentation process of this second nucleon. The rapidity distribution of products of this fragmentation can be approximated by the rapidity distribution in elementary process in TFR. Hence we put the multiplication factor m_B in TFR equal 2. For the same reason the multiplicity distribution is the convolution of the distribution for non-diffractive quark nucleon interaction with the distribution from the fragmentation of one nucleon in the target. In the fragmentation only produced particles are taken into account.

Process DE — represents the class of processes which cannot be described in terms of the quark model. It contains events in which interaction with one nucleon was diffractive or elastic and the interaction with other nucleon of any type. We assume that the rapidity distribution of particles in TFR for this process and multiplicity distribution are the same as in elementary π nucleon collision. Therefore $m_{DE} = 1$.

Process κ — only one of incident quarks interacts with one of the nucleons in the target and one of products of this collision scatters from the second nucleon. This process represents cascading in the model. The multiplicity distribution is constructed by the convolution of the multiplicity distribution in elementary interaction and of the multiplicity distribution of particles produced by cascade ϱ_{N-2}^{C}. The density of particles in TFR arising from the interaction of projectile is the same as in the elementary process. The interactions of cascade produce in TFR additional amount of particles which can be estimated using experimental data.

To construct the multiplicity distribution described in Table III we make the following assumptions:

1. The multiplicity distribution in the quark nucleon collision is the same as in hadron-nucleon collision (additivity).

2. The interaction of one quark with two nucleons gives multiplicity distribution as in an elementary interaction with one of the nucleons. The collision with the second nucleon can add at most the particles produced in the fragmentation process of the second nucleon ϱ_{N-2}^{F}.

3. Secondary interaction of particles created in the first collision produces in TFR approximately the same multiplicity as in the process of nucleon fragmentation in TFR.

$$\varrho_{N-2}^{F} \approx \varrho_{N-2}^{C}. \tag{3.4}$$

(Also the assumption that the rapidity distribution ϱ_{N-2}^{F} of particles produced in the second collision in process B can be approximated by the rapidity distribution of particles produced in TFR in elementary process was checked. Both approximations give similar results.)

4. The probability of cascading does not depend on the multiplicity in the first collision.

To extract the mean multiplicity of cascade we use the following technique

1. From the experimental multiplicity distribution of double scattering sample we subtract the multiplicity distribution of processes A and DE

$$X_N^{B\kappa} = X_N^{DS} - X_N^{A} - X_N^{DE}, \tag{3.5}$$

where X_N^{DS} is the multiplicity distribution of double scattering events. The distribution which we obtain in this way represents only that part of double scattering sample which contains the cascading process. Therefore according to the model this distribution should be a convolution of some known elementary distribution ϱ_N^{NC} and the cascade distribution ϱ_{N-2}^{C}

$$X_N^{B\kappa} = (\varrho_N^{NC} \otimes \varrho_{N-2}^{C})\sigma^{B\kappa}, \tag{3.6}$$

where

$$\varrho_N^{NC} = \frac{\sigma^{B}}{\sigma^{B\kappa}}\varrho_N^{ND} + \frac{\sigma^{\kappa}}{\sigma^{B\kappa}}\varrho_N^{ELEM} \tag{3.7}$$

and

$$\sigma^{B\kappa} = \sigma^{B} + \sigma^{\kappa}. \tag{3.8}$$

2. We calculate the moments: mean multiplicity $\langle N_{NC} \rangle$ and dispersion squared D^2 for distributions $X_N^{B\kappa}$ and X_N^{NC}.

3. The following relations for convolution of the distribution of cascade, $X_N^{B\kappa}$ and X_N^{NC} distribution are used:

$$\langle N^{B\kappa} \rangle = \langle N^{C} \rangle + \langle N^{NC} \rangle, \tag{3.9}$$

$$D_{B\kappa}^2 = D_C^2 + D_{NC}^2. \tag{3.10}$$

If the model is correct and if we have reliable experimental data these two relations give us the possibility of finding the moments of the cascade multiplicity distribution.

The rapidity distribution of particles produced through cascading can be obtained from the following relation in terms of the model [66]

$$\frac{d\sigma_F^C}{dy} = \frac{d\sigma_F^{DS}}{dy} - r_F \frac{d\sigma_F^{ELEM}}{dy}. \tag{3.11}$$

Here $\frac{d\sigma_F^C}{dy}$, $\frac{d\sigma_F^{DS}}{dy}$ and $\frac{d\sigma_F^{ELEM}}{dy}$ are the rapidity distributions of pions in TFR for cascading (C), double scattering (DS) and elementary nucleon interactions (ELEM). Further ratio r_F is calculated from the model

$$r_F = \frac{\sum_{i=1}^{4} \sigma_i m_i n^F(y)}{(\sigma_{in}^{ELEM} - \sigma_D^{\pi}) n^F(y)} = \frac{\sum_{i=1}^{4} \sigma_i m_i}{(\sigma_{in}^{ELEM} - \sigma_D^{\pi})}, \tag{3.12}$$

where σ_i, m_i for $i = 1 \dots 4$ corresponds to processes A, B, DE and κ discussed above and schematically shown in Table III. The $\sigma_{\rm in}^{\rm ELEM}$ is the inelastic cross section for π-nucleon elementary interaction, $\sigma_{\rm D}^{\pi}$ is the cross section for pion diffraction dissociation process in π-nucleon interactions, $n^{\rm F}(y)$ is the density of particle in rapidity for TFR for elementary interaction. The three last observables are averaged πp and πn interactions.

The density of particles produced in the process of cascading is

$$\frac{dn^{\rm C}}{dy} = \frac{1}{\sigma^{\kappa}} \frac{d\sigma^{\rm C}}{dy}. \tag{3.13}$$

In addition we can obtain the mean multiplicity of particles produced in the cascade

$$\langle n^{\rm C} \rangle = \int \frac{dn^{\rm C}}{dy}\, dy. \tag{3.14}$$

If the model is correct we expect that this value $\langle n^{\rm C} \rangle$ agree with value $\langle N^{\rm C} \rangle$ obtained on the independent way described above.

Another version of the additive quark model proposed by Nikolaev et al. applied to double scattering processes in deuteron [11] uses the following basic assumptions:

1. The projectile rescatters via its own "spectator" quarks i.e. the valence quarks that do not take part in the first collision.

2. The newborn quarks from the first collision cannot rescatter immediately from the other nucleon.

The model introduces the formation length idea. It means that the secondary quarks of momenta $k_{\rm q}$ are present in the final state only at distances Δz exceeding their formation lengths $l_{\rm f}$ from the first nucleon position

$$\Delta z \geqslant l_{\rm f}. \tag{3.15}$$

Formation lengths turn out to be proportional to the momenta $k_{\rm q}$ of the secondary quarks.

$$l_{\rm f} \approx \frac{k_{\rm q}}{m^2}, \tag{3.16}$$

where m is some characteristic mass scale. The spectator quarks already exist and do not need any formation time to be able to rescatter from the second nucleon. The existence of the formation time and the knowledge of internucleon distance in deuteron determine the maximal momentum of quarks produced in the first collision which can interact with the second nucleon. On the other hand if we know from experiment the maximal momentum of quarks produced in first collision which can interact with the second nucleon and the internucleon distance in deuteron, we can estimate the mass scale appearing in formula (3.16).

In this model by cascade we understand the interactions of low momentum quarks produced in the first collision with the second nucleon of the deuteron. The rapidity distribution of particles produced in the cascading process can be obtained from the following equation

$$(1-\eta)\frac{dn^{\mathrm{C}}}{dy} = \frac{dn_{\mathrm{F}}^{\mathrm{DS}}}{dy} - (1+\eta)\frac{dn_{\mathrm{F}}^{\mathrm{ELEM}}}{dy}. \tag{3.17}$$

Here dn^{C}/dy, $dn_{\mathrm{F}}^{\mathrm{DS}}/dy$, $dn_{\mathrm{F}}^{\mathrm{ELEM}}/dy$ are the rapidity distributions of particles in TFR for cascade, double scattering and for elementary interactions. The value η is defined and calculated in [11]. Following assumptions of the model, we consider two kinds of processes in the double scattering sample

1. rescattering of "spectator" quarks of projectile,
2. rescattering of these quarks which are produced in the first collision with momentum low enough to be able to interact with the second nucleon.

The mean multiplicity of such quarks is denoted by $n_{\mathrm{q}}(k \leqslant k_{\mathrm{c}})$, where k_{c} is the upper limit of the momentum of the quark which can interact with the second nucleon, as a consequence of the existence of the formation length. The probability η is the relative contribution of projectile spectator quark to all rescatterings

$$\eta(\pi\mathrm{d}) = \frac{1}{\langle n_{\mathrm{q}}(k \leqslant k_{\mathrm{c}})\rangle + 1}. \tag{3.18}$$

The mean value $\langle n_{\mathrm{q}}(k \leqslant k_{\mathrm{c}})\rangle$ is the multiplicity of prompt quarks which is related to the multiplicity of prompt mesons

$$\langle n_{\mathrm{q}}(k \leqslant k_{\mathrm{c}})\rangle = \langle n_{\text{prompt mesons}}(k \leqslant 2k_{\mathrm{c}})\rangle \frac{\sigma_{\pi\mathrm{N}}}{\sigma_{\mathrm{qN}}}. \tag{3.19}$$

The factor $\frac{\sigma_{\pi\mathrm{N}}}{\sigma_{\mathrm{qN}}} \approx 2$ arises from the basic assumption of the additive quark model. The value $\eta = 0.17$ has been obtained using $\langle n_{\mathrm{q}}(k \leqslant k_{\mathrm{c}})\rangle = 5$ from [11].

The BLRW model [60] describes the double scattering process in deuterium by the conventional space-time development picture of hadronic interactions. The incident hadron interacts with neutron and proton of deuteron via fast virtual constituents. These constituents create two ladders. Each ladder interacts with one of the target nucleons. Using the AGK cutting rules [67] BLRW found a relation between σ^{DS} double scattering cross section and cross section defect in deuterium

$$\delta\sigma = \sigma_{\mathrm{T}}(\mathrm{hp}) + \sigma_{\mathrm{T}}(\mathrm{hn}) - \sigma_{\mathrm{T}}(\mathrm{hd}), \tag{3.20}$$

where $\sigma_{\mathrm{T}}(\mathrm{hp})$, $\sigma_{\mathrm{T}}(\mathrm{hn})$, $\sigma_{\mathrm{T}}(\mathrm{hd})$ are the total cross sections for elementary hp, hn and hd interactions. The relation is following:

$$\sigma^{\mathrm{DS}} = 2\delta\sigma. \tag{3.21}$$

in addition the AGK rules allow one to relate the multiplicity distributions observed in hd Interactions to the multiplicity distributions observed in hadron-nucleon collisions. The model does not take into account the process of cascading at all. Relation (3.21) is in reasonable agreement with the observed amount of double scattering. However, when one considers all available experiments, there is a systematic tendency to underestimate the amount of double scattering. The authors of the BLRW model attribute the excess to the scattering from the slower constituent of projectile. The sources of double scattering in excess of $2\delta\sigma$ are expected to give rapidity distribution similar to single scattering. The simplest possibility of partition of incident momentum between two ladders is that each of them takes $p/2$. Such an assumption gives the predictions consistent with observed multiplicity moments [60, 68]. If we adopt this momentum partition between two ladders, the double scattering charged particle rapidity distribution is equal to

$$\frac{dn^{\mathrm{DS}}}{dy} = [1-2\delta\sigma/\sigma^{\mathrm{DS}}]\frac{dn^{\pi\mathrm{p}}(p)}{dy} + [2\delta\sigma/\sigma^{\mathrm{DS}}]\frac{dn^{\pi\mathrm{p}}(\frac{1}{2}p)}{dy}, \tag{3.22}$$

where $dn^{\pi\mathrm{p}}(\frac{1}{2}p)/dy = 2[1/\sigma_{\mathrm{T}}]d\sigma^{\pi\mathrm{p}}(\frac{1}{2}p)/dy$ and $d\sigma^{\pi\mathrm{p}}(\frac{1}{2}p)/dy$ is the rapidity distribution for $n_{\mathrm{ch}} \geqslant 4$ at half laboratory momentum. The authors of this model do not pretend to describe the rapidity distribution in TFR ($y_{\mathrm{lab}} < 1$) [35].

On similar assumptions are based the predictions of double scattering rapidity distribution in the dual parton model proposed by Capella and Tran Thanh Van [61]. The authors take from the BLRW model the quantitative partition of double scattering cross section into parts giving the rapidity distribution like single and double scattering. The model introduces the dual topological unitarization scheme and parton picture and is one of the multiple scattering type models. The basic assumptions of the model are following:

1. color separation mechanism,
2. universality of the fragmentation,
3. quark momentum distribution functions.

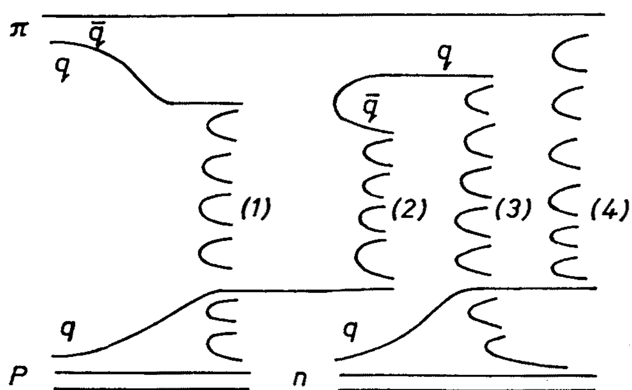


Fig. 2. π-deuteron interaction in the model of Ref. [61]

The four different chains responsible for particle production in the interaction with deuteron are shown in Fig. 2. The chains labelled 1 and 4 involve, respectively, valence antiquark and quark of the projectile, similar as in pion-proton collisions. Chains 2 and 3 involve respectively sea antiquarks and quarks of the projectile.

If we denote by $N_\alpha(y)$ the rapidity density of the chain and $N^{\text{hd}}(y)$ the rapidity density for hd → hX inclusive single particle reaction we have for the scattering involving two nuclons of the target

$$N^{\text{hd}}(y) = 1/\sigma_{\text{in}}^{\text{hd}} \frac{d\sigma^{\text{hd}}}{dy}(y) = N_1(y)+N_2(y)+N_3(y)+N_4(y). \tag{3.23}$$

Using the AGK cutting rules as in the BLRW model, the amount of double scattering resulting from the four chain diagrams is given by $2\delta\sigma$, and the rest of double scattering is expected to give rapidity distributions similar to single scattering. Therefore one has

$$\frac{dN^{\text{DS}}}{dy} = \frac{2\delta\sigma}{\sigma^{\text{DS}}}[N_1+N_2+N_3+N_4]^{n=2}+\left(1-\frac{2\delta\sigma}{\sigma^{\text{DS}}}\right)[N_1+N_4]^{n=1} \tag{3.24}$$

and

$$R(y) = \frac{\varrho^{\text{DS}}(y)}{\varrho^{\text{ELEM}}(y)} = 1-\frac{2\delta\sigma}{\sigma^{\text{DS}}}+\frac{2\delta\sigma}{\sigma^{\text{DS}}}[N_1+N_2+N_3+N_4]^{n=2}/[N_1+N_4]^{n=1}. \tag{3.25}$$

The model gives the predictions valid in the whole region of rapidity. The difference between the experimental distribution and the prediction calculated from the model can be treated as the rapidity distribution of particles produced through cascading process which is not included in the model.

3.3. The description of TFR for heavy nuclei

A model which pretends to describe the full range of rapidity of produced particles from nuclei was proposed by Levchenko and Nikolaev [57]. The basic assumptions of this model are the same as for various versions of constituent quark model [58, 69, 70]. The projectile is made of well spatially separated constituent quarks. These three constituent quarks in the case of the proton beam can interact simultaneously. Generally the interaction of high energy particle with nucleus in terms of this model is described by two equations: the equation for absorption probabilities of ν quarks and the cascade transport equation. The ν quark absorption corresponds to some cuts of the Mandelstam diagrams [57]. The probabilities of ν interactions calculated from the quark model give weights to these cut diagrams. The Mandelstam vertices have simple structure in terms of the constituent quarks. On the other hand using a sort of the quark hadron duality one can translate the quark nucleus interactions into a specific sequence of particle-nucleon collisions: $\nu = 1$ the projectile fragments as on free nucleon target; $\nu = 2$ there are two inelastic collisions. Diquark which is left after first collision is a leading system and it reinteracts. It corresponds to hN collision followed by an inelastic collision of the leading particle from the first collision; $\nu = 3$ baryon and the fastest of mesons from the first collision reinteract.

The cascade transport equation formulated for the constituent quarks can be applied directly to prompt secondary hadrons and there is no difference whether quarks or hadrons are formed (the same formula for the formation lengths). The detailed description of Monte Carlo procedure for intranuclear interactions is given in Ref. [57]. Authors apply a Monte

Carlo program which generates the exclusive final states in hN collisions to hA collisions using the translation coming from the quark hadron duality described above.

A different attempt to describe TFR is presented in Ref. [62]. In this model mechanism responsible for the production of particles in TFR is the multiple scattering of projectile followed by secondary interactions of protons knocked out off the target. The physical picture coming from the model is following. The projectile collides with target nucleons. They are called primary and move essentially forward. On their way out of the nucleus the primary nucleons collide with other target nucleons which are called secondary. The secondary nucleons create tertiary nucleons. One can estimate the degradation of energy for each generation of nucleons. The aim of the model was to describe the development of cascade inside the nucleus, in particular to derive analytical formulae for the relation between the number of collision $\bar{\nu}$ and the number of N_g — grey particles emitted in hadron emulsion experiments. The grey particles ($0.3 < v/c < 0.7$) are mostly protons ~80% and have kinetic energy in range $40 < E < 400$ MeV. The simple kinematical considerations allow one to estimate that only part p_1 of primary nucleons satisfies the requirement of being grey. The secondary nucleons have energies around 100 MeV and are mostly grey, and tertiary nucleons have several tens of MeV and essentially do not belong to the class of grey particles. In simplest approximation the beam hadron is not excited after first collision the dependence of the mean number of grey particles on the number of collisions is following

$$\langle N_g \rangle = p_1 \left(\frac{Z}{A}\right) \langle \nu \rangle + (\sigma_{NN}/\sigma_{hN}) \langle \tfrac{1}{2}\, \nu(\nu-1) \rangle \frac{Z}{A}. \tag{3.26}$$

The first term of equation (3.26) represents the contribution of primary nucleons to the grey particles ($\bar{\nu}$ primary nucleons of which the fraction Z/A is charged and which are grey with probability p_1). The secondary grey nucleons are produced with the effective cross section σ_{NN}. If a primary nucleon is produced in the μ-th hN collision it will scatter $(\nu-\mu)\sigma_{NN}/\sigma_{hN}$ times and produce secondaries before it escapes. Summing contributions from all μ's and multiplying them by the charge ratio Z/A leads to the second term. Glauber theory [71] relates the average $\langle \nu(\nu-1) \rangle$ to $\langle \nu \rangle$. Formula (3.26) is more complicated if we take into account that the hadron after the first collision is excited and has the cross section σ_{h^*N}[1].

$$\langle N_g \rangle = \frac{Z}{A}\left(p_1 + \frac{\sigma_{NN}}{\sigma_{hN}} \langle \nu-1 \rangle\right) + \frac{Z}{A}\,\frac{\sigma_{h^*N}}{\sigma_{hN}}\left(p_1 \langle \nu-1 \rangle + \tfrac{1}{2}\,\frac{\sigma_{NN}}{\sigma_{hN}} \langle (\nu-1)(\nu-2) \rangle\right), \tag{3.27}$$

where averages involving ν are calculated with σ_{hN} from the Glauber relation [71]. This formula fitted to the data allows one to estimate cross section σ_{h^*N} which can be treated as an effective cross section of projectile on a target nucleons.

Such an attitude was criticised for example in review [13]. The argument was that there is nothing that could distinguish the recoil nucleons from all the other secondary

[1] The simple argument for introducing this complication is following. The first hN collision in a nucleus is similar to hadron-free nucleon interaction. In both cases an excited hadron h is formed. In free space h* decays immediately and gives final state particles. The h* formed in a nucleus collides with other nucleons of the target with the cross section probably different from the hN cross section.

particles which may also interact inside nucleus. On the other hand the universal description of cascading process is very difficult and needs very complicated many parameter input. The simplification of the physical picture proposed by Hegab and Hüfner in Ref. [62] leads to a very good one parameter fit and one may hope that it represents correctly the mechanism of the process. The interpretation of the σ_{h^*N} cross section by including also secondary interactions of mesons could be a next step in the understanding of this process.

4. *Hadronization time*

One of the characteristic features of the process of particle production is that the collision and hadronization processes are separated by some time interval. The interactions with nuclei give a possibility of investigation of the time ordering of multiparticle production. The experimental discovery of the absence of intranuclear cascading of fast secondaries implies that fast secondary hadrons are created outside the nucleus. The observation of interactions of slow secondaries inside nucleus would allow one to delimit the time or spatial distance after which the created hadron is able to interact. We call this interval the hadronization time or sometimes the formation time (the length of formation zone with reference to spatial distance).

The phenomenon of long hadronization time was observed experimentally for the first time by the Cracow High Energy Group (see e.g. [4]). It was discussed in several reviews in terms of the general principles of quantum mechanics and field theory, [6, 7, 72–74], and, generally speaking, can be interpreted as a consequence of the uncertainty principle and of the Lorentz time dilatation [4, 5, 8].

The formation time concept comes from the Landau Pomeranchuk phenomenon [72]. The intensity of bremsstrahlung radiation from high energy electron undergoing multiple scattering as it passes through a material is much smaller than the value predicted as the number of rescatterings multiplied by the intensity of the radiation per scattering in one atom. Landau and Pomeranchuk noticed that the emission of photons in two successive scatterings is uncorrelated only when the distance between the two scattering points is large in comparison with the formation zone. We can understand the formation zone in this phenomenon as follows [3]. After the first collision the new shape of the stable field of electron appropriate to its changed momentum is not established instantaneously. Until this field is reestablished (formation time) the scattering in the external field occurs without emission.

In other words the bremsstrahlung process for soft, small angle radiation from relativistic electron is affected by a coherent action of many nuclei situated within the effective length. If the distance between scattering points is large in comparison with the formation zone then the coherence is destroyed, and emission in successive scatterings is uncorrelated. The condition for coherent or incoherent action (big or small time intervals between collisions) is formulated (measured) in terms of the formation time τ

$$\tau = \frac{1}{E_\gamma - \vec{k} \cdot \vec{v}}, \tag{4.1}$$

where E_γ and $\vec{k}$ are the energy and momentum of the emitted photon and $\vec{v}$ is the velocity of incident electron. If τ is big in comparison with the time distance between scattering points, the amplitudes from successive scatterings add coherently.

The formation zone can be also interpreted as the formation time of the photon. We rewrite formula (4.1) into

$$\tau = \frac{E_e}{k \cdot p} = \frac{E_e}{m} \frac{1}{\omega}, \tag{4.2}$$

where E_e and m are energy and mass of the electron, $(k \cdot p)$ is the four-vector product of the photon and electron momentum and is the frequency of the photon in the rest frame of the electron. We can treat τ as the minimum time necessary to define the photon (in the wave mechanics the emission of the photon described by the wave of some frequency ω takes the time of order $T = 1/\omega$).

If one accepts the formation zone concept as a general principle, one can translate the picture from the bremsstrahlung process into the interactions of high energy hadrons with nuclei [7]. According to this idea, the distance in the laboratory necessary for the emission of a field quantum is

$$\tau \approx \frac{2E_h}{(Mx)^2 + k_T^2 + m_h^2} \approx \frac{2E_h}{m_T^2}, \tag{4.3}$$

where E_h is the energy of emitted particle in the laboratory system, M is the mass of projectile, $x = E_h/E_{proj}$ is the energy fraction of radiated hadron, $m_T^2 = k_T^2 + m_h^2$ is the transverse mass of hadron.

It should be noticed that from this analogy the formation time is proportional to the energy of emitted particle. This means that the low energy hadrons have shorter formation zones than the fast ones. This is an opposite situation to the photon production in bremsstrahlung process where $\tau \sim 1/E$ and slow photons have long formation time.

Similar formula for formation time could be obtained of we interpret long time of hadronization as a consequence of the uncertainty principle. The minimal time necessary to emit a slow energy hadron is of the order

$$\tau_0 = 1/\Delta E, \tag{4.4}$$

where ΔE is an energy typical for strong interactions. This energy scale is different in different approaches to the problem of formation time. Some authors do not precise the value of ΔE. In other models ΔE is simply equal to the mass m_h [13, 14], or to the transversal mass m_T [17] of the produced particle.

When the hadron produced is moving, the relativistic time dilatation should be taken into account. Then

$$\tau_f = \gamma \tau_0 = \frac{E_h}{m_h} \frac{1}{\Delta E} \tag{4.5}$$

or

$$l_f = \beta\tau_f = \frac{k_h}{m_h}\frac{1}{\Delta E}, \tag{4.6}$$

where E_h, k_h, m_h, γ, β are the energy, momentum, mass, Lorentz factor and relative velocity of the moving hadron.

The other approach is that the process of fragmentation into the hard hadron comes through decay of an unstable particle i.e. quark. In this case the characteristic time is

$$\tau_f = \frac{E_q}{m_q}\tau_q^{(0)}, \tag{4.7}$$

where $\tau_q^{(0)}$ is the life-time of the quarks and m_q and E_q are its mass and energy. This hypothesis was proposed by Białas and Chmaj [75] for leptoproduction of high energy hadrons on heavy nuclei.

The formation time phenomenon in nuclei manifests itself by the lack of cascading of hadrons whose formation length in laboratory is longer than the radius of nucleus ($l_f > R$).

Since the details of the time dependence of hadron production are not known and the value of ΔE is still controversial we can use the parametrization:

$$\tau_f = \frac{2E_h}{m_T^2} \quad \text{(Stodolsky [7])}, \tag{4.8}$$

$$\tau_f = \frac{E_h}{m_T^2} \quad \text{(Białas [17])}, \tag{4.9}$$

$$\tau_f = \frac{E_h}{m_h^2} \quad \text{(Nikolaev [11, 13, 14])}, \tag{4.10}$$

or simply

$$\tau_f = \frac{E_h}{m^2} \tag{4.11}$$

treating the mass scale m as a parameter. It allows us to compare the results of theoretical predictions and of measurements of the formation zone obtained using the different methods.

Nikolaev [14] predicts on the ground of the multipheripheral model that the scale dimension is

$$m^2 \approx m_\varrho^2 \approx 0.7\ \text{GeV}^2. \tag{4.12}$$

If we assume that the effect of cascading occurs for

$$l_f < R_A \tag{4.13}$$

we obtain the maximal value of momentum of the hadron which can interact inside nucleus

$$k_c = R_A m^2, \tag{4.14}$$

which corresponds to rapidity

$$y_c \approx \ln R_A \frac{m^2}{m_T}. \tag{4.15}$$

Here m_T is the transverse mass of the produced hadron. (The relation between y_c and k_c is valid for particles with longitudinal momentum bigger than the transverse mass.) If the mass scale, as some authors propose, is simply the transverse mass of hadron then

$$y_c \approx \ln R_A m_T \approx \ln A^{1/3} m_T. \tag{4.16}$$

The experimental determination of the mass scale m is complicated. The transition point $y_c(\eta_c)$ in rapidity (pseudorapidity) between plateau and cascading region can be observed on the plot $R(y)$ ($R(\eta)$) see Fig. 1. This observation was used for the first time in 1978 to compare the experimental data for p-nucleus collisions at 50, 100 and 200 GeV published by MIT and FNAL group [82] with the condition (4.3) [76]. It was shown that the multiplication $R(\eta)$ really increases significantly in the region of pseudorapidities predicted by condition of localization of emission of the secondary particle inside nucleus. In Ref. [76] an observation has also been made that this condition seems to be independent of energy and the nature of the primary hadron.

The rough estimate based on the optical model and on the experimental observation of absorption effects on heavy nuclei gives

$$m^2 = 0.15-0.20 \text{ GeV}^2 \tag{4.17}$$

[77]. The similar value of m^2 could be obtained by using the energy dependence of multiplicity of grey particles [22]. The very low value of

$$m^2 < 0.03-0.05 \text{ GeV}^2 \tag{4.18}$$

comes from the investigation of π mesons and antiprotons of energy ~ 1 GeV in p-nuclei interactions [77]. A method of determining the formation zone by measurement of the absorption of the medium energy hadrons created in nuclear matter was proposed by Białas [78]. The method was applied to the data on the process $\pi A \to \bar{p}X$. For antiprotons of momentum ~ 16 GeV/c the formation zone was estimated as $\tau = 15 \pm 5$ fm. This result gives in parametrization $l_f = k/m^2$

$$m^2 = 0.21 \pm 0.07 \text{ GeV}^2. \tag{4.19}$$

The value

$$m^2 = 0.16^{+0.07}_{-0.04} \text{ GeV}^2 \tag{4.20}$$

was obtained in a neutrino-freon experiment in the energy range $3 \leqslant E \leqslant 30$ GeV [79].

The estimate was done from the slow proton multiplicity in the framework of an intranuclear cascade model combined with the formation zone idea.

High statistics experiments on different nuclei should help us in understanding the significance of the formation zone phenomenon. E.g. as was proposed by Białas [78], the universality of the formation time for different hadrons independently of their mass would suggest the interpretation of as the life-time of quasi-free quarks. The experiments giving the high statistics in deep inelastic interactions of leptons with nuclei are very promising in this field.

5. *Discussion of results of experiments with the deuteron*

The problem of the existence and of the characteristics of intranuclear cascade in the deuteron was investigated in the models described in Section 3.2. We have used methods presented in Section 2.2 to obtain relevant experimental distributions. The papers [59], [64], [66] contain a complete discussion of experimental details. The results concern the following aspects of the cascading process:

1. the multiplicity distribution of particles produced through cascading,
2. their rapidity distribution,
3. fraction of cascading in the double scattering process,
4. the rapidity range of the effect.

The analysis of the multiplicity distribution of double scattering events performed in terms of the additive quark model including the cascading effect [64] gives the values of mean multiplicity of particles produced in the process of cascading:
$\langle N^c \rangle = 1.47 \pm 0.57$ for πd experiment at 100 GeV/c [80],
$\langle N^c \rangle = 0.95 \pm 0.47$ for πd experiment at 205 GeV/c [28],
$\langle N^c \rangle = 1.25 \pm 0.63$ for πd experiment at 360 GeV/c [29].
These values seem to be independent of momentum of the incident hadron. Such an observation confirm indirectly the hypothesis that in the process of cascading only slow particles produced in the first collision may interact. It has been also noticed that the dispersion of experimental multiplicity distribution of double scattering sample is narrower than this obtained from the model [59]. This result can be explained by the dependence of the probability of the cascading process on multiplicity in the collision with the first nucleon. The detailed argumentation is brought forward in Ref. [59]. On the other hand this conclusion confirms indirectly the hypothesis of limited momenta of cascading objects. The simple reasoning leading to this result is following. For elementary high energy interactions the momentum distribution of produced particles depends on the multiplicity. In particular for low multiplicities we observe fewer particles with low momenta than for higher multiplicities. If we assume that only slow particles created in the first collision can interact with the second nucleon in deuteron, we can also argue that cascading is smaller for low multiplicities in the first collision. The value $\langle N^c \rangle$ of the mean multiplicity of particles produced in cascading process averaged over experiments is

$$\langle N^c \rangle^{\text{Prod}} = 1.15 \pm 0.31 \tag{5.1}$$

and corresponds to

$$\langle N\rangle^{\pi p} = \langle N^c\rangle^{\text{Prod}}+2 = 3.15\pm 0.31 \tag{5.2}$$

for π-nucleon interactions. Such a number suggests that if the interaction of cascade does not differ from the elementary nucleon interactions the average momentum of particles which initiate the cascade is not bigger than 3 GeV/c. This result also confirms the observation that only slow particles produced in the first collision may interact with the second nucleon. The mean multiplicity of particles produced through cascading was obtained in an independent way but using the same model. According to formula (3.14) the mean multiplicity is simply the integral of rapidity distribution of particles produced in cascading process. This value obtained only for πd experiment at 205 GeV is

$$\langle N^c\rangle = 1.08\pm 0.29. \tag{5.3}$$

The calculations based on the Nikolaev model give similar value for this experiment

$$\langle N^c\rangle = 1.11\pm 0.24. \tag{5.4}$$

The BLRW model has not included the cascading process. The dependence of mean multiplicity of negative particles produced in double scattering events on the incident momentum is shown in Fig. 3 for eleven πd and pd experiments [30]. One can notice that the values

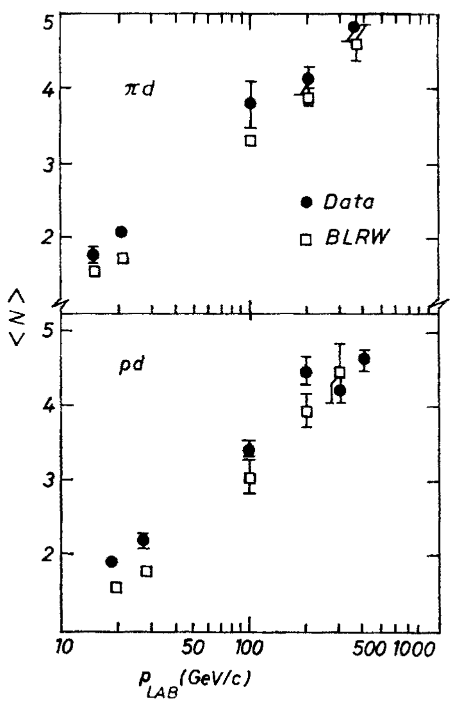


Fig. 3. Average multiplicity ⟨N⟩ of produced particles as a function of incident momentum for πd and pd double scattering [30] together with BLRW model predictions [60]

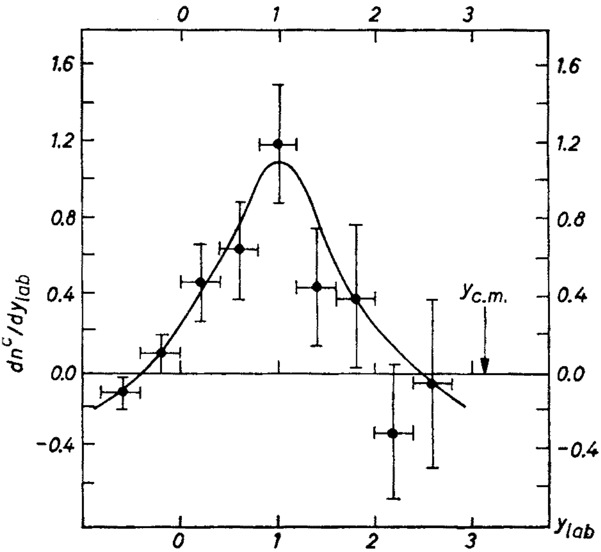


Fig. 4. The rapidity distribution of charged particles produced in cascade obtained using the model of Ref. [59]. The curve has been drawn to guide the eye

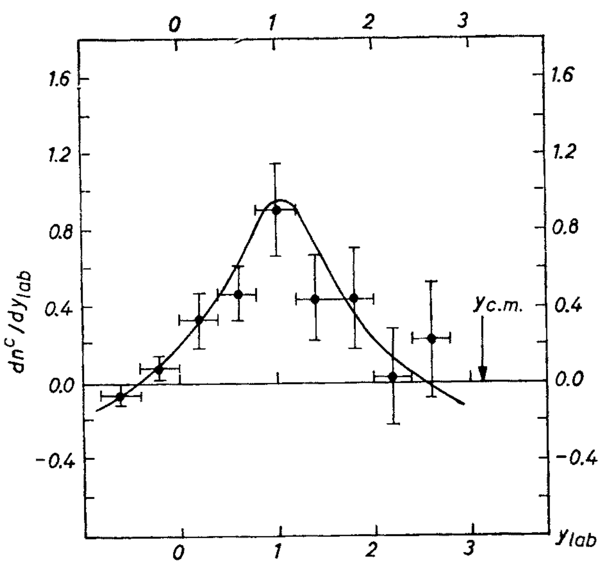


Fig. 5. The rapidity distribution of charged particles produced in cascade obtained using the model of Ref. [11]. The curve has been drawn to guide the eye

predicted by the BLRW model are slightly but systematically lower than experimental mean multiplicities. We can interpret this observation as the existence of some additional mechanism presumably cascading which is responsible for the increase of multiplicity.

The rapidity distribution of particles produced through cascading was calculated by using the methods described in Sections 2.2 and 3.2. The detailed description of formulae can be also found in Refs. [59, 64, 66]. The two versions of the additive quark model [59] and [11] was used. The results are shown in Fig. 4 and 5. The experimental data were taken from the πd experiment at 205 GeV [28] and from the πp experiment at the same energy [37]. The distributions look very similar and within experimental errors do not depend on the model which was used to obtain them. It should be stressed that the assumptions of the considered models are substantially different. The model of Białas et al. considers inclusion of cascade as an extension to the additive quark model necessary to describe correctly the πd and pd double scattering cross section at various energies and also the density of particles produced in central rapidity region. Nikolaev et al. introduce the formation length concept and specify that the cascade is initiated by quarks produced in the interaction with the first nucleon. The assumed upper limit of the momentum of quarks which are able to interact with the second nucleon of the target is also checked by using the available values of double scattering cross section in πd and pd collisions.

The rapidity distributions in Fig. 4 and 5 are very similar as in elementary low energy hadron nucleon interactions. From the upper limit of rapidity of particles produced in cascade $y_{max} \approx 2.5$ ($k_{max} \approx 2.2$ GeV/c) we can expect that objects initiating the cascade have the momentum not higher than 3 GeV/c.

As was mentioned in Section 3.2 the dual parton model proposed by Capella et al. gives quantitative predictions for full range of rapidity of produced particles. The process of cascading is not included in the model. The difference dn^{d}/dy_{lab} between the experimental rapidity distribution for double scattering events and the distribution calculated from the model [61] is shown in Fig. 6. If we assume that this distribution corresponds to the process of cascading we can only compare the shape of rapidity distribution and the rapidity range of particle production in this effect with the corresponding features of distributions in Fig. 5 and 6. The distribution in Fig. 6 is normalised to the total double scattering cross section. This is equivalent to the assumption of the presence of cascading in each act of double scattering. The absolute values dn^{c}/dy which can be compared with the distributions in Fig. 4 and 5 can be obtained by introducing an additional parameter — the fraction of double scattering in which cascading effect occurs.

The fraction of double scattering going through cascading predicted by the models is different — 83% in the Nikolaev model and 52% in the Białas model. In other words Nikolaev predicts that cascading takes part in about $d = 13.3 \pm 1\%$ of π-deuteron interactions at 205 GeV. The same value in the model proposed by Białas et al. is $d = 8.3 \pm 1\%$.

In Section 2.4 we have described the method of estimating the fraction of double scattering for neutrino-deuteron reactions [47]. This value is energy independent and equals $d = 9.4 \pm 3.5\%$. We can treat it simply as the fraction of cascading in πd interactions. Unfortunately the errors are too big to decide conclusively which model gives the proper value. The amount of the cascading process in the Nikolaev model is directly connected to the formation length, in particular to the mass scale value m in formula (4.2). The m^2 is taken equal to 0.7 GeV^2 and seems to be too high in comparison with estimates made using different methods quoted in Section 4.

We can also roughly estimate this value using our results from the deuteron experiment.

From formula (4.8) $m_0^2 = k_c R_A$, where k_c is the maximal momentum of the hadron which can interact inside nucleus and R_A for deuteron is the internucleon distance. We assume the mean value of internucleon distance equal to $\langle R_d^2 \rangle^{1/2} = 2$ fm. We can estimate the mean value of k_c as a mean value of the momentum of the products of cascading equal

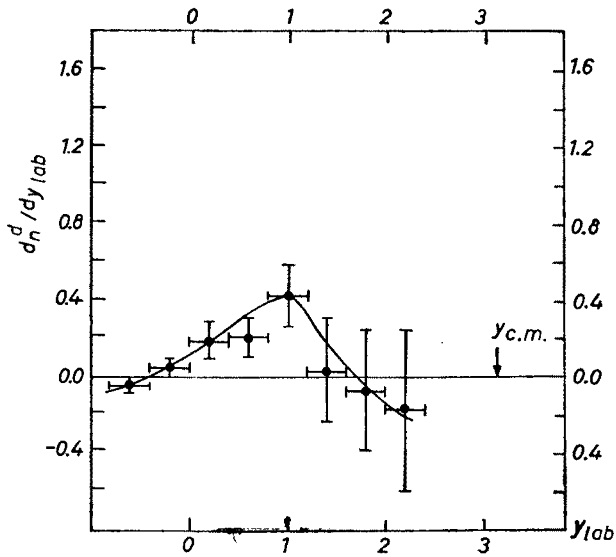


Fig. 6. The difference $dn^d/dy = dn^{\text{DSexp}}/dy - dn^{\text{DSmod}}/dy$ between the experimental rapidity distribution for double scattering events and the distribution calculated from the model [61]. The curve has been drawn to guide the eye. The normalization is to the total double scattering cross section

to about 0.5 GeV/c multiplied by their multiplicity $\langle n^c \rangle + 2 = 3$. Finally we obtain

$$m_0^2 = 1.5\ \text{GeV}/2\ \text{fm} = 0.15\ \text{GeV}^2. \tag{5.5}$$

This value is in a good agreement with the estimate coming from other experiments (see Section 4).

6. *Intranuclear cascade in heavy nuclei*

The application of the methods described in Section 2.3 allows us to classify the information about intranuclear cascade coming from different experiments on heavy nuclei. Following the theoretical considerations discussed in Section 3.3 and the conclusions from the experiments with the deuteron we concentrate on the phenomena occurring in the target fragmentation region.

To examine the question of cascade in heavy nuclei in all its possible bearings we collect the results of experiments in the following order:

1. the experimental phenomena generated by cascading process,
2. the kinematical localization and range of the effect,
3. the dependence on: energy, projectile and size of nucleus,

4. the possibilities of estimation of the mean number of interactions of secondaries inside nucleus,

5. the comparison with the predictions of the intranuclear cascade models.

All hadron emulsion experiments show that the total number of protons among the grey tracks is much larger than the number of recoil protons $\bar{\nu}$ which could be struck in the interactions of projectile with nucleons inside nucleus. For example it was found in

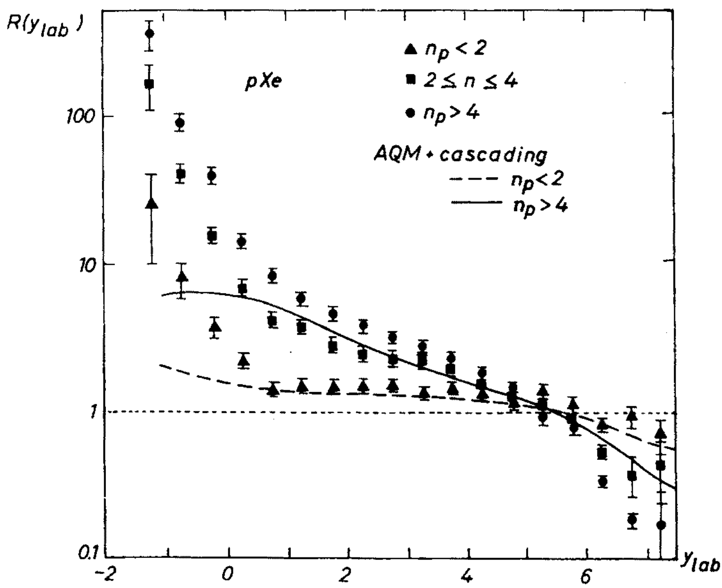


Fig. 7. The ratio $R(y)$ for different numbers of identified protons n_p in pXe interactions at 200 GeV/c [2]

p-Em experiments at 67 and 200 GeV that 50–60% of grey tracks belong to protons produced in the secondary processes in the nucleus [41]. Similar estimate is given by [81]. They calculate that $\langle n_g \rangle \sim A^{0.7}$. This dependence is stronger than $\bar{\nu} \sim A^{1/3}$ expected from the models without any cascading of slow particles. The method proposed by Andersson et al. [39] gives the relation between $\bar{\nu}(n_g)$ and n_g. This correspondence is independent of the energy of projectile and weakly dependent on the type of incident particle. It indicates that only slow particles produced in collisions of the projectile with nucleons inside nucleus are responsible for cascading.

The excess of the ratio $R_y = \varrho^{hA}/\varrho^{hp}$ for low rapidities can also be interpreted as an evidence for the cascading effect. Very large values of R_y for $y < 1$ exceed significantly the average number of collisions. Such behaviour is shown in Fig. 1 for experiment p on Ar and Xe target at 200 GeV/c [2] and was observed in many experiments: $\pi^{\pm}$, $K^{\pm}$, p, $\bar{p}$ on C, Cu, Pb, U at 50, 100, 200 GeV/c [43], p on Cr, W at 300 GeV/c [27], $\pi^{\pm}$ p, $\bar{p}$ on Mg, Ag, Au at 100 GeV/c, $\pi^{\pm}$, $K^{\pm}$, p on C, Cu, Pb at 50, 100, and 150 GeV/c [25], $\pi^{\pm}$, ν, $\bar{\nu}$ on Ne [49]. It was also noticed that the large value of R_y in TFR comes mainly from events with large value of n_p the observed protons. For small values of n_p in Fig. 7 [2] the R_y

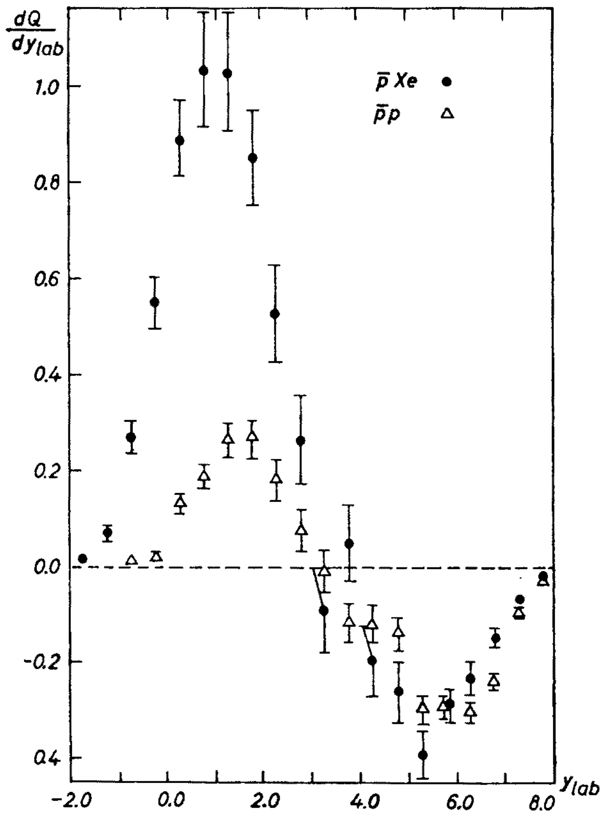


Fig. 8. The average net charge $dQ/dy = \varrho^{+}(y) - \varrho^{-}(y)$ versus y at 200 GeV/c $\bar{\text{p}}$ beam [2]

approaches 1 as would be expected for the reaction with the single nucleon. This is a further evidence for the importance of the intranuclear cascading. The result of the analysis of the average net charge distribution in experiment [2] is the value

$$dQ/dy = \varrho^{+}(y) - \varrho^{-}(y), \tag{6.1}$$

which is shown in Fig. 8 for $\bar{\text{p}}$ Xe interaction. The comparison of $\bar{\text{p}}$p and $\bar{\text{p}}$ Xe net charge distributions shows large positive net charge in the backward CMS hemisphere for the Xe target. It also could be interpreted as an evidence of interactions of secondaries inside nucleus.

The other way of presentation of the rapidity and A dependence of final spectra of produced particles is used in Ref. [43]. Pions, kaons, protons, and antiprotons with momenta ranging from 50–200 GeV/c were scattered on the targets ranging from beryllium to uranium. The differential multiplicities $N(\Delta\eta)/\Delta\eta$ from the 12 angular regions were seperately fitted to the form $A^{\alpha'(\eta)}$. This method of looking for possible intranuclear cascade is described in Section 2.3. The authors of Ref. [43] show in Fig. 9 the value of $\alpha'(\eta) = \alpha(\eta) - \alpha_0$ in order to get rid of the dependence of total cross section. The values $\alpha_0 = 0.69, 0.79, 0.75$ describe the A dependence of the total cross section of protons, kaons,

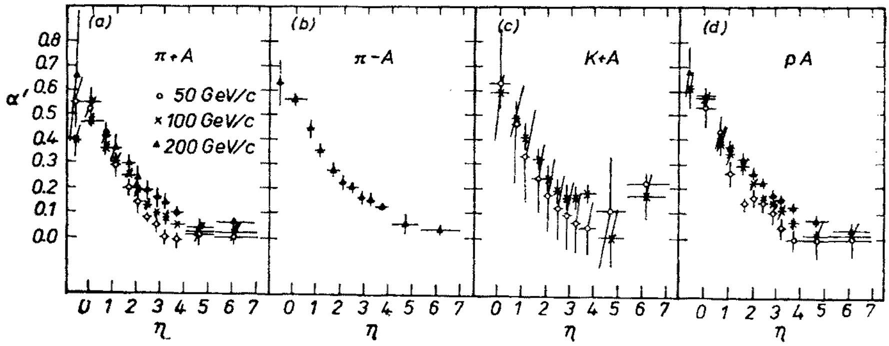


Fig. 9. Exponent α', where the normalized inclusive cross section $1/\sigma_{tot}\sigma(\Delta\eta)/\Delta\eta$ is assumed to follow the form $A^{\alpha'(\eta)}$. π^+, π^-, K^+ and p induced data are shown in (a), (b), (c) and (d) respectively [43]

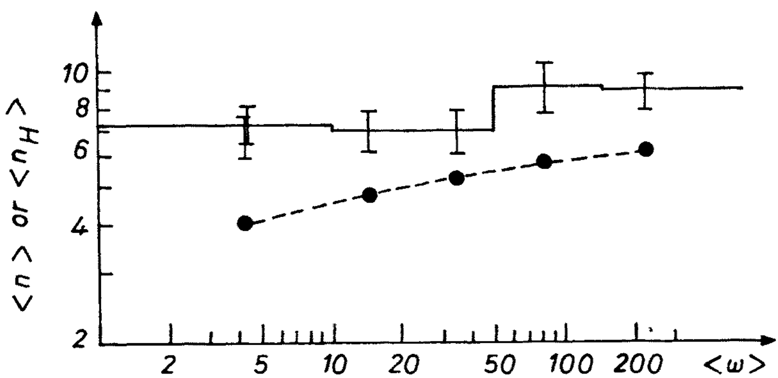


Fig. 10. Mean multiplicities of produced hadrons $\langle n\rangle$ in μ^+-emulsion interactions (solid line) plotted versus $\langle\omega_B\rangle$, together with mean multiplicities $\langle n_{nucl}\rangle$ of produced hadrons in μ^+-nucleon interactions (dots connected by a dashed line) calculated from (6.2) for the value of $\langle W\rangle$ in a given ω_B bin. The two values for $\langle n\rangle$ at the lowest ω_B bin (solid and dotted lines) show the estimates of the upper and the lower bounds of $\langle n\rangle$ [46]

and pions respectively. The results in Fig. 9 show that $\alpha(\eta)$ becomes greater than unity in the region $\eta \leqslant 1.5$ indicating that cascading within the nucleus is taking place.

As was mentioned in Section 2.4, the study of deep in elastic lepton nucleus interactions gives a possibility to select the processes in which particle multiplication must occur through intranuclear cascading. This idea was tested experimentally in the μ^+ Em experiment at 150 GeV/c [46]. Fig. 10 shows the mean multiplicities of produced hadrons $\langle n\rangle$ in μ^+ Em interactions (solid line) plotted versus $\langle\omega_B\rangle$ together with mean multiplicities $\langle n_{nucl}\rangle$ of produced hadrons in muon nucleon interactions (dots connected by dashed line). The $\langle n_{nucl}\rangle$ was calculated from the formula

$$\langle n_{nucl}\rangle = 2.3+0.28\ln(W-m)+0.53\ln^2(W-m) \tag{6.2}$$

for the value of $\langle W\rangle$ in a given ω_B bin. The two values for $\langle n\rangle$ at the lowest ω_B bin (solid and dotted lines) show the estimates of the upper and lower bounds of $\langle n\rangle$. For small,

$\omega_B(\langle\omega_B\rangle = 4.3$ for the first bin) where the virtual photon interacts with only one nucleon, one can observe that $\langle n\rangle > \langle n_{nucl}\rangle$. It means that the products of the first collision on their way out of the nucleus interact with the other nucleons and produce the excess of particles. Another way of looking at nuclear effects presented in Ref. [46] is the comparison of the pseudorapidity distributions of produced particles for two classes of events: with $N_h = 3$–8 and $N_h > 8$($N_h = n_g + n_b$ in the emulsion technique terminology). This comparison is based on the well known observation in hadron-emulsion experiments. The average multi-

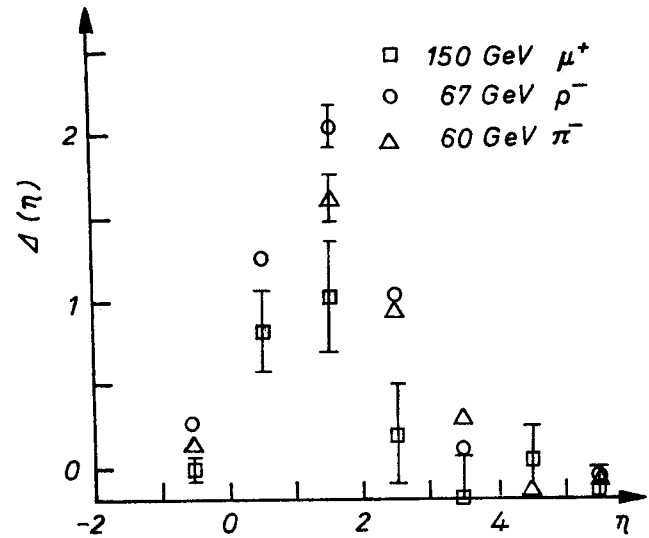


Fig. 11. The value $\Delta\eta = 1/N\, dn/d\eta|_{N_h>8} - 1/N dn/d\eta|_{N_h=3-8}$ plotted for μ^+-emulsion interactions at 150 GeV, p-emulsion interactions at 67 GeV and π-emulsion interactions at 60 GeV [46]

plicities and angular distributions of interactions having small N_h are close to those in hadron hydrogen reactions, whereas the interactions with high N_h behave differently from collisions with hydrogen target. The value

$$\Delta\eta = \frac{1}{N}\frac{dn}{d\eta}\bigg|_{N_h>8} - \frac{1}{N}\frac{dn}{d\eta}\bigg|_{N_h=3-8} \tag{6.3}$$

is plotted in Fig. 11 for μ^+ Em interactions at 150 GeV together with p-Em data at 67 GeV and π^--Em data at 60 GeV. It is clearly seen that the increase of multiplicity due to the nuclear effects exists and is localized in the range of small values of η.

The precise determination of the rapidity range of the cascading process is difficult on the basis of existing experimental data. The transition area between TFR and the central plateau in rapidity R_y fluctuates between 1 and 2 for various experiments. The estimate depends on the accuracy of the experimental data and of the current theoretical suggestions concerning the height of the central plateau.

The exponent $\alpha(\eta)$ Eq. (2.14) in Fig. 9 becomes greater than unity in the region $\eta \leqslant 1.5$. The average net charge distributions for pXe and pp interaction shown in Fig. 8 indicate that the rapidity limit is about $y \approx 2$. The similar rapidity limit is suggested by $\mu^+\pi^+$, p-Em data presented in Fig. 11. However in these experiments it is difficult to estimate the uncertainties coming from the nuclear composition of the emulsion target. The assumption that the events with $N = 3$–8 behave as a sample of interactions with the hydrogen target leads also to additional uncertainties. The relative multiplication R_y of produced particles in TFR in the $\nu/\bar{\nu}$ Ne experiments ($3 < W < 6$ GeV) presented in Ref. [49] is greater than 1 up to the rapidity +1.

The range and the A dependence of the cascading effect seem to be independent of the incident particle type and of the incident momentum. This hypothesis was put for the first time by Mięsowicz [76] in 1978. He has shown that the rapidity range of multiplication of secondaries in the collision with nuclei is determined by the length of the formation zone and that it is independent of nature and energy of projectile. Such a conclusion can be also deduced from Fig. 9. The values of α' for the π^+, π^-, K^+ and p beams for three energies 50, 100 and 200 GeV show the same A dependence in TFR within experimental errors. The comparison of results of π^+/π^--Ne with $\nu/\bar{\nu}$-Ne experiments [49] and of μ^+-Em at 150 GeV with p-Em at 67 GeV and π-Em at 60 GeV leads to the same conclusion. The changes of value of η_c, the critical rapidity between TFR and central region, was measured in experiment [23, 25]. The result indicates the independence of η_c of the projectile momentum i.e. that $\Delta\eta_c$ is less than 0.2 for incident momentum of 50–150 GeV. The authors of this experiment observe also small projectile dependence for small η.

The attempt of the estimation of the mean number of interactions of secondaries inside nucleus was undertaken in Ref. [38]. If one assumes that the mean number of collisions of the projectile is given by formula (1.2) then from simple charge conservation considerations described in Section 2.3 one can estimate the average value of secondary interactions $\bar{\nu}_c$ inside nucleus. This value obtained for Ar and Xe target is 4.8 ± 0.4 and 10.7 ± 0.8 respectively and shows the strong A dependence.

The comparison of experimental results with the predictions of the model including the cascading effect [57] was performed in [2]. General conclusion is that the model describes the data quite well for $y > 0$. The model cannot describe the data for $y < 0$ (see Fig. 1). The subsample of events characterised by the number of observed protons n_p bigger than 2 is also not satisfactorily described (Fig. 7).

A simple phenomenological model [62] applied to most of available data reproduces very well the distribution of grey tracks. The comparison of the data with the model prediction is shown in Ref. [62], [25]. The model assumes a simplified physical picture of interaction (Section 3.3) and describes the data using one free parameter σ_c – the effective cross section of cascading objects. This cross section is between $0.7\sigma_{hN}$ and $1.4\sigma_{hN}$. It is a typical hadronic cross section. It indicates that cascading is caused by hadrons formed in the target nucleus. One can imagine that this effective cross section describes the effective cascading of some products of primary interactions.

7. *Production of fast backward particles. Cumulative effects*

To complete the discussion of the cascade mechanism of particle production in TFR we give a short description of other proposed production mechanism. The cumulative effects causing the production of cumulative particles give a similar experimental consequences as the process of cascading [83]. By cumulative particles we mean secondary particles produced on nuclei in the region forbidden for scattering off free nucleon. Such particles, mostly fast backward protons (but also mesons) populate TFR. The classical mechanism of the Fermi motion of nucleons inside nucleus cannot explain the momenta of backward protons greater than 300 MeV/c. The effect can be quantitatively described as a manifestation of short-range few nucleon correlations in the nucleus wave function (mostly two-nucleon correlations). The oversimplified picture of such effect is following. For short range phenomena one can expect the high momenta correlated nucleons. The incident hadron going through the nucleus knocks out a nucleon moving forward (in the nucleus rest frame) releasing the backward moving nucleon of the correlated pair. A natural mechanism of production of fast backward mesons is the scattering of incident hadron from fast backward nucleons. Similar picture can be adopted for lepton-nuclei interactions. Here γ or W strikes one of the nucleons of the correlated system.

The possible sources of short range few nucleon correlation inside nucleus are discussed in [84]. On the other hand the effect of cumulative particle production in high energy processes can be used to find the answer to the basic questions about the short-range nuclear structure i.e. amount and shape of high momentum components in the wave function.

The inclusive cross section for the production of cumulative protons can be parametrized as follows [83]:

$$Ed^3\sigma^{\mathrm{a+A\to p+X}} = \sigma_{\mathrm{in}}(\mathrm{aN})C_0 \exp\left(-B(\theta)p^2\right), \tag{7.1}$$

where σ_{in} (aN) is the elementary a-N cross section, E and p are the energy and momentum produced proton, θ is the angle of production. The following experimental features characterize the production of cumulative protons. The slope B is independent of the type of projectile and the nucleus mass A, and also of the energy of incident particle above few GeV. The value B strongly depends on the angle of production of the cumulative proton. The value C_0 for high energy (<10 GeV) is also constant. These features (except of the θ dependence) are similar to the characteristics of the cascading process so that it is rather difficult to separate these two mechanism.

Another approach which relates fast backward particle production to the presence of high momentum nucleon component in nuclear wave function are the average field models [85–87]. In these models it is assumed that the fast nucleon momentum is balanced by the rest of the nucleus, but not by one or two nucleons. The hadron projectile interacts with several nucleons producing $\nu \sim A^{1/3}$ forward nucleons in association with fast backward particle. It was also suggested [88–89] that fast backward particles are produced in the projectile interaction with a nucleon-cluster called the fluctuon.

The problem of fast backward particle production was also considered on the basis of the coherent tube model or the effective target model [90]. In this approach it is assumed that the incident hadron interacts coherently with ν nucleons at its impact parameter as if it were a single hadron of mass νm_N.

It should be noticed that the mechanisms described in this section introducing the dependence of fast backward particle production on shape of nuclear wave function can also affect the other kinematical regions of particle production in TFR.

8. *Conclusions*

The experimental analysis of the available data performed in this paper shows that the process of intranuclear cascading exists and can be observed in interactions with the deuteron as well as with the heavy nuclei. Due to the different experimental and phenomenological possibilities the effect was studied separately in deuteron and in heavy nuclei. The results obtained in these two ways are in most cases complementary but general features can be deduced from both the deuteron and the heavy nuclei data.

Summary of results contains: the list of experimental phenomena generated by the existence of the cascading effect in TFR, general features of such process, the comparison of the experimental results with the phenomenological models, and with other proposed mechanisms of particle production in TFR.

The following experimental observations point to the existence of cascading phenomenon in TFR:

1. The mean number of protons observed in the interactions with nuclei is much larger than the expected value — the average number of collisions of projectile with target protons;

2. The multiplication of particles in TFR in comparison with elementary process is greater than predicted by models of multiple collisions of projectile or its constituents;

3. The positive net charge in TFR cannot be explained without assumption of the secondary interactions inside nucleus;

4. The mean multiplicity of particles produced in μ^+-nucleus interactions is bigger than for elementary interactions. The result was obtained for the subsample of interactions with $\omega_B < 5$, where virtual photon is expected to interact with only one nucleon of the target;

5. The investigation of deuteron interactions in terms of two versions of additive quark model shows that besides the double scattering of projectile one should introduce interactions of objects produced in the first collision. It should be stressed that the assumptions in the considered models referring to the cascading effect are substantially different;

6. In the neutrino deuteron interactions the assumption of the amount of double scattering is necessary to obtain the ratio of ν-neutron to ν-proton cross section consistent with the quark parton model prediction. The projectile in the πd collisions interact only with one nucleon. Then we can assign the ocurrence of double scattering to the existence of cascading.

The features of single act of cascading process can be deduced only from interactions with deuteron. The results are model dependent. However, two versions of additive quark model applied in analysis give very similar results:

1. The value of the effective cascade cross section is the typical hadronic cross section equal to:

$\sigma^* = 20.2 \pm 0.7$ mb obtained from πd interactions,
$\sigma^* = 25.5 \pm 1.9$ mb obtained from pd interactions;

2. The fraction of cascade interactions in deuteron scattering at 205 GeV is:

$d = 13.3 \pm 1\%$ from the model proposed by Nikolaev et al.;
$d = 8.3 \pm 1\%$ from the model proposed by Białas et al.

The same quantity estimated from neutrino-deuteron interactions using a linear fit $d = a + b\sigma_T$ and confirmed by the measurement of the ratio of ν-neutron to ν-proton cross section is $d = 9.4 \pm 3.5\%$;

3. The mean multiplicity of particles produced through cascading obtained using two different methods and two models is:

$\langle n^c \rangle = 1.11 \pm 0.24$ from the Nikolaev model,
$\langle n^c \rangle = 0.95 \pm 0.47$ from Białas model using formula (3.9),
$\langle n^c \rangle = 1.08 \pm 0.29$ from Białas model using formula (3.14);

4. The rapidity distribution of particles produced in cascade is very similar to that observed in low energy elementary hadron-nucleon interactions and does not depend on the model used;

5. From the upper limit of rapidity of particles produced through cascading $y_{max} \approx 2.5$ we can expect that the objects initiating the cascade have the momenta not higher than 3 GeV/c.

This last value can be also deduced from experiments with heavy nuclei. The upper limit of the rapidity range of cascading fluctuates between 1.5 and 2.5 for different experiments and different applied methods.

The questions of the dependence of cascading on: energy, projectile and size of nucleus, and also amount of cascades inside nucleus can be studied in the interactions with heavy nuclei. On the basis of the analysed data we can summarize these problems as follows:

1. The rapidity range and the A dependence of the cascading seems to be independent of the incident momentum and incident particle type. It seems to be a simple consequence of the fact that secondary interactions inside nucleus can be induced by low momentum objects;

2. The mean number of interactions of secondaries obtained in pAr and pXe experiment is 4.8 ± 0.4 and 10.7 ± 0.8 for Ar and Xe respectively and thus shows strong A dependence.

We can conclude that the models including the mechanism of cascade proposed for hadron-deuteron interactions describe quite well the phenomena in TFR. The model proposed by Levchenko and Nikolaev for hadron-heavy nucleus scattering reproduce the data well for $y_{lab} > 0$ but cannot describe the region $y_{lab} < 0$. Also for the sample of events with a big number of observed protons which corresponds probably to interactions characterized by greater amount of secondary collisions inside nucleus, the agreement of the predictions of the model and the experiment is not satisfactory.

The other mechanism of production of particles in TFR based on the assumption of short range few nucleon correlations in the nucleus wave function was only applied to the description of fast backward produced particles. The effect of dependence of the multiplication of particles in TFR on the number of visible protons is difficult even to qualitative understanding in terms of this model. Also a large value of the net charge in interactions with heavy nuclei cannot be explained by this mechanism. Nevertheless the influence of such effects should be taken into account in experimental analyses of TFR.

To use the TFR phenomena to study such basic concepts as space time development of hadronic interactions, we need more data of good accuracy especially for interactions with the deuteron. The lepton-heavy nucleus experiments are also very promising. The most important experimental problem is to have precise evidence and description of all particles produced in TFR.

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