THE CONCEPT OF SUPERLUMINAL REFERENCE FRAME AND THE THEORY OF RELATIVITY. A SHORT CRITICAL ANALYSIS

By J. K. Kowalczyński

Institute of Physics, Polish Academy of Sciences, Warsaw*

(Received May 19, 1983)

The relation between the theory of relativity and the concept of superluminal inertial reference frame is analysed in terms of the operational definitions of spacelike, timelike, and null directions. It is concluded that such a concept does not exist inside the theory of relativity and that every consistent addition of that concept to the theory gives consequences being strange from the point of view of contemporary physics. It is also concluded that the interpretation of that concept in the literature is contradictory to the theory of relativity. The existence of that contradiction is explained here as an effect of the confusion between mappings and transformations.

PACS numbers: 03.30.+p, 14.80.Kx

The concept of a superluminal inertial reference frame has been appearing in the tachyonic literature for almost twenty years and this concept was related to the theory of relativity in an unfortunate way. We present here an abbreviated[1] analysis of the problem hoping to stop that unfortunate trend. (Though probably I am too optimistic about that.)

In order to make our considerations simpler we assume a four-dimensional flat spacetime M having only one timelike dimension, i.e., the usual physical spacetime of special relativity[2], that is endowed with a standard system of the Lorentz coordinates x, y, z, t.

The inertial reference frame in M is denoted by f.

We shall start by discussing some auxiliary concepts, i.e., those of mapping, transformation, reference frame, and spacelike, timelike, and null directions.

* Address: Instytut Fizyki PAN, Al. Lotników 32/46, 02-668 Warszawa, Poland.

[1] The reader interested in the problem can find a more detailed analysis in my recent paper [1] where the most representative literature of the subject is also given.

[2] Our considerations can easily be generalized to a curved spacetime and to a spacetime having an arbitrary number (but more than one) spacelike dimensions, what is shown in Ref. [1], where the unphysical and topologically different case of two-dimensional spacetime as well as the attempts of using spacetimes having a few timelike dimensions are also discussed.

Let us consider a diffeomorphism given by the system of four equations:

$$x^{\mu'} = Y^{\mu}(x, y, z, t), \tag{1}$$

where $\mu = 1, 2, 3, 4$, and $x^{\mu'}$ are other coordinates. In our case Y^{μ} can represent linear functions. It is commonly known that Eqs. (1) have in the theory of relativity two different meanings.

In the first meaning Eqs. (1) determine the one-to-one mapping $\phi : M \to M'$ where spacetimes M and M' can *differ*.

In the second meaning Eqs. (1) determine the transformation of coordinates x, y, z, t to coordinates $x^{\mu'}$ in *one and only one* spacetime M.

In other words, we can say that in the first meaning (mapping) Eqs. (1) determine a transition from a point of M to a point of M', while in the second meaning (transformation) Eqs. (1) do not change any point of M but merely alter the coordinates of the point [2].

In mathematical terms of the theory of relativity a reference frame (a set of independent vectors that is the coordinate basis) is defined in a spacetime by means of a coordinate language. Thus, changes of the reference frames are equivalent to changes of the coordinate systems in *one and only one* spacetime [1], and hence those changes are realized via *transformations* (*not* via mappings).

In physics, the essence of the concept of f is related to the concept of *rest* (immobility), what is commonly emphasized by the figurative expression "the observer being at rest in f". In other words, if rest cannot be defined, then there is no sense to speak of f from the point of view of physics. The following definition is well known:

Definition 1. A point of space is at rest in f if and only if the world line (in M) of that point is parallel to the time axis of f.

Thus the rest in f is *invariantly* defined in terms of the theory of relativity (for details see: Section 4.2 of Ref. [1] where the definition of rest in f is precisely formulated; or Ref. [3]).

In order to make understanding of physical experiments precise in terms of physical theory, today everyone proceeds in the spirit of operationism. Thus, it is only essential that the determination be clear of the kind of instrument in the following relation: measuring instrument — result of measurement — mathematical notation; but not the choice of the mathematical notation[3]. In that sense the use of the concepts of rod and clock is common in kinematics. In operational terms we can say that space (distance) and time are such entities that are *directly* measured with a rod and a clock, respectively.

Now we shall introduce the operational definitions of spacelike, timelike, and null directions.

Definition 2. A direction in M is a spacelike (timelike) one if and only if there exists f such that the direct measurement in M in that direction is realizable in f by means of a rod (clock) alone.

---

[3] That is why the discussion conducted between the supporters of the superluminal frame idea on whether the use of imaginary coordinates is proper or not is completely futile.

Definition 3. A direction in M is a null one if and only if for every f the direct measurement in M in that direction is not realizable in f by means of a rod alone and it is not realizable in f by means of a clock alone.

If the standard physical interpretation that has been used in the theory of relativity (in its geometrical language) is assumed, then definitions 2 and 3 are equivalent to the standard definitions of spacelike, timelike, and null directions.

Note that in definition 3 the null direction is defined without using the term "velocity of light". Note also that by virtue of definitions 2 and 3 every direction in M is a spacelike or timelike or null one in the operational sense of those definitions (for details see Section 4.3 of Ref. [1]); and it is in that sense that the terms "spacelike, timelike, and null directions" will be used in the following.

Let us determine regions $N, S, T, P, Q \subset M$ as follows: the three-dimensional region N is a two-sheet light cone whose vertex is fixed at the origin of the system of coordinates x, y, z, t; the three-dimensional region P is a two-sheet cone with axis x and tangent to N; and the four-dimensional regions S, T, and Q are the exterior of N, the interior of N, and the interior of P, respectively (see Fig. 1). Of course $Q \subset S$, $P \cup Q \subset N \cup S$, $N \cup S \cup T = M$, and $N \cap S = S \cap T = T \cap N = 0$ (empty set).

Attention. Henceforth the term "direction" will mean a direction in M that passes through the origin of the x, y, z, t coordinate system.

Let $\boldsymbol{s}$, $\boldsymbol{t}$, and $\boldsymbol{n}$ be directions that lie inside the S, T, and N regions, respectively. Thus N separates all $\boldsymbol{s}$'s from all $\boldsymbol{t}$'s. Note that this is a purely geometrical fact and that we do not yet assign the directions $\boldsymbol{s}$, $\boldsymbol{t}$, and $\boldsymbol{n}$ any features of being spacelike, timelike, or null.

Let us denote a superluminal f by F.

Now we can proceed ad rem.

If the standard physical (kinematic) interpretation that has been used in the theory of relativity is assumed, then $\boldsymbol{s}$, $\boldsymbol{t}$, and $\boldsymbol{n}$ have the features of being spacelike, timelike, and null directions, respectively, and only such features. This means by definition 1 that we cannot define a state of rest in terms of the theory of relativity for the tachyon (and luxon). This means that the concept of F does not exist inside that theory.

If the concept of F is contradictory to that theory, then the problem is terminated.

Let us assume that the concept of F is independent of the theory of relativity. Then there exists a consistent extension of that theory including the concept of F. Every F is f by definition. Thus by virtue of definition 3 the directions $\boldsymbol{n}$ and only directions $\boldsymbol{n}$ are null in the F sense. Since F is f, then by definition 1 one of the directions $\boldsymbol{s}$ must be parallel to the time axis of F and hence that $\boldsymbol{s}$ is a timelike direction in the F sense. Note that this is not pure verbalism, but that the problem concerns a kind of measuring instrument used by the observer being at rest in F (definition 2).

If one would postulate that some other direction $\boldsymbol{s}$ is spacelike in the F sense, then one would have to assume a discontinuous physical (kinematic) interpretation over the continuous distribution of $\boldsymbol{s}$'s in S. Then he would have to accept among others such a phenomenon in F that a homogeneous piece of metal (rod) held in the hand transforms during a slight motion into a pocket watch with a dial and machinery (clock). This would be due to the fact that there are no null directions in S also in the F sense.

The use of a discontinuous interpretation in any domain of physics would certainly be a new and so far unknown quality in the methodology of physics.

However, if somebody renounces paying such a price and wants to preserve the normal continuous physical interpretation, then he has to assume that all **s**'s are timelike directions in the F sense and that all **t**'s are either timelike (first case) or spacelike (second case) directions in the F sense, since N separates all **s**'s from all **t**'s.

The observer being at rest in F sees the universe, i.e. M, without any spacelike dimensions but only with four independent kinds of time in the first case (but what are then clocks if there are no spacelike dimensions?), or with only one spacelike dimension and with three independent kinds of time in the second case. In both cases the null directions exist for that observer. Many embarrassing questions arise now, e.g., what is rest for that observer? (maybe there are various kinds of rest), what about speed (or speeds), etc., etc. Note that this applies to all kinds of phenomena seen by that observer, i.e., subluminal, luminal, and superluminal in relation to F. Note also that both the subluminal and superluminal observers are inside the same common M in accordance with what was said above.

We now see that if the theory of relativity is assumed, then we have to abandon, among other things, the hope of our journey by a faster-than-light spaceship in a well--furnished cabin. This does not mean of course that we have to reject (from the point of view of relativity) the possibility of faster-than-light phenomena existing, since firstly F's are not necessary for the observations of such phenomena and usual subluminal f's suffice, and secondly the tachyons may be microscopic objects [1] or may be systems of suitably shaped fields expanding with the velocity of light, as for instance in the tachyon model presented in Ref. [4].

We have presented above all possible versions of adding the F concept to the theory of relativity. Although they seem to be strange from the point of view of our contemporary intuitions in physics, they are methodologically correct.

The supporters of the F idea assume, however, the theory of relativity and at the same time want the observer being at rest in F to see the universe as a usual spacetime. We know now that such a conjunction is impossible as it is self-contradictory. Thus the embarrassing question arises: how could it happen that an extensive literature exists about an empty concept?

Two explanations can be given.

The first is that the supporters have confused mappings and transformations, and that in reality they have made a one-to-one mapping ϕ_1 of M onto another usual flat spacetime M' ($\phi_1 : M \to M'$) that is endowed with a standard system of the Lorentz co-ordinates x', y', z', t'. More precisely this mapping looks as follows (see Fig. 1):

$$\phi_1 : N \to P', \quad \phi_1 : P \to N', \quad \phi_1 : T \to Q',$$

$$\phi_1 : Q \to T', \quad \phi_1 : S-P-Q \to S'-P'-Q'. \tag{2}$$

Of course such a mapping can also be understood as the mapping ϕ_2 of M onto itself ($\phi_2 : M \to M$), if we assume that $M = M'$. A more precise presentation of ϕ_2 is also given

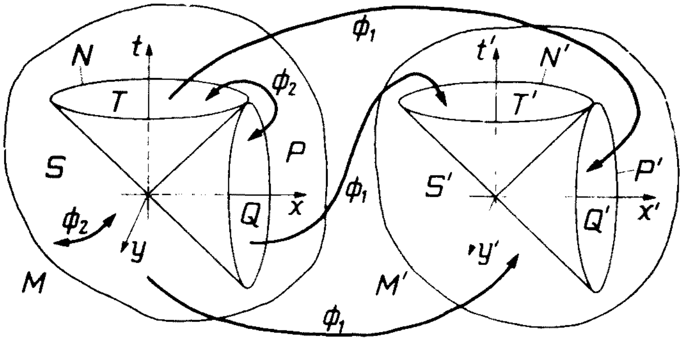

Fig. 1. The mappings ϕ_1 and ϕ_2, erroneously called superluminal Lorentz transformations. The third spatial dimensions (and thus the axes z and z') as well as the second sheets of N, P, N', and P' cones are not shown

by relations (2) if we abandon there the primes (see Fig. 1). The mappings ϕ_1 or ϕ_2 are given in many papers as explicit formulae (i.e., as an explicit form of Eqs. (1) where coordinates $x^{\mu'}$ then mean x', y', z', t') under an unfortunate name of "superluminal Lorentz transformations". With the use of those formulae one can easily verify relations (2).

The second explanation is that the supporters have really made transformations from frame to frame, but those have been usual subluminal Lorentz transformations and f's [5]. Simply, if we assume, as they did, that in their transformation formulae the symbols x and t denote the spacelike and timelike coordinate, respectively, then really (in the operational meaning) x' is a timelike and t' is a spacelike coordinate in those formulae, i.e., quite opposite to what they have assumed. Thus the symbol v (or u) appearing in those formulae does not represent velocity but its inverse (for $c = 1$).

REFERENCES

[1] J. K. Kowalczyński, *Int. J. Theor. Phys.* **23** (1984).
[2] S. Gołąb, *Tensor Calculus*, Elsevier Scientific, Amsterdam, PWN, Warszawa 1974, Sec. 3.
[3] J. L. Synge, *Relativity: the Special Theory*, North-Holland, Amsterdam 1965, Chap. I, §§ 16, 17, 20 and Chap. II, § 4.
[4] J. K. Kowalczyński, *Phys. Lett.* **74A**, 157 (1979).
[5] A. Yaccarini, *Lett. Nuovo Cimento* **9**, 354 (1974).