SOFT CONTINUUM THRESHOLD FACTOR FOR THE SVZ SUM RULES

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The standard step function approximation for the energy dependence of the continuum contribution to the cross-section for heavy quark production in e^+e^- annihilation is replaced by an approximation with a much softer energy dependence. This change is found to have little effect on the predictions from the SVZ sum rules.

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When applying the SVZ sum rules [1] to heavy vector quarkonia, it is necessary to estimate the ratios

$$R_q(s) = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}. \tag{1}$$

Here q is a heavy quark (c or b) and the cross-section in the numerator is summed over all the possible final state interactions of the quarks q and $\bar{q}$. The vector $q\bar{q}$ resonances certainly contribute to $R_q(s)$, the continuum contribution has also to be included, however. Following the pioneering work of Shifman, Vainshtein and Zakharov [1], most analyses use the simple expression

$$R_q^{\text{cont}}(s) = 3Q^2\left(1+\frac{\alpha_s}{\pi}\right)\theta(s-E_T^2), \tag{2}$$

where Q is the electric charge of quark q and θ the step function. The strong coupling constant entering α_s and the continuum threshold energy E_T are either estimated from the

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data, or chosen so as to optimize the agreement of the sum rules with experiment. For large values of s, relation (2) agrees with QCD and seems reliable, therefore, only modifications of the continuum threshold region deserve an analysis. The sharp rise of $R_{\mathrm{q}}^{\mathrm{cont}}(s)$ from zero to its asymptotic value is clearly a crude approximation, but introducing one more parameter to fix the width of the threshold region would greatly reduce the predictive power of the sum rules.

In order to see how sensitive are the results to the Ansatz (2), we introduce an effective quark mass m_{q}^{*} and calculate $R_{\mathrm{q}}^{\mathrm{cont}}(s)$ by substituting m_{q}^{*} for the quark mass m_{q} in the theoretical expressions derived by SVZ for $R_{\mathrm{q}}^{\mathrm{cont}}(s)$. In this approach the parameter E_{T}

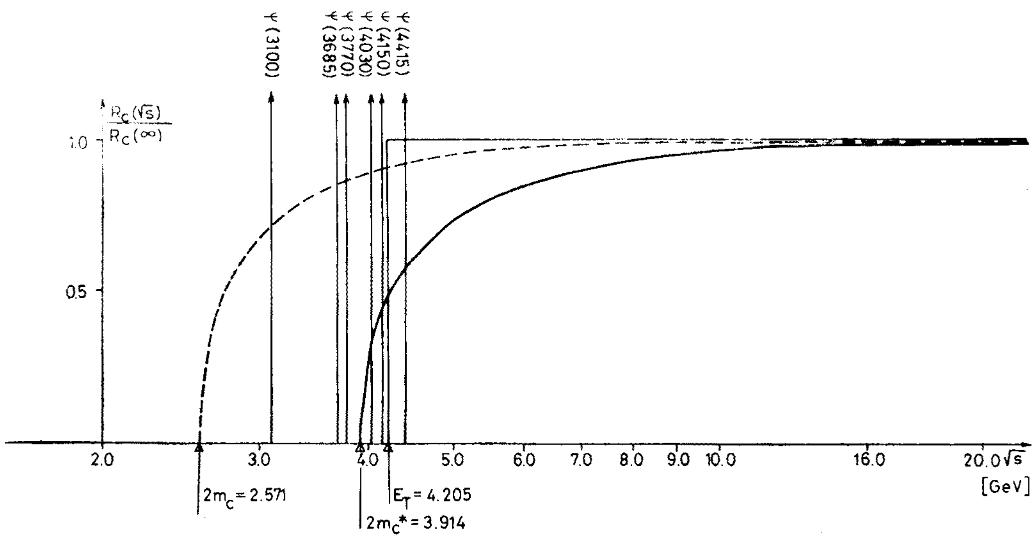

Fig. 1. Energy dependence of $R_{\mathrm{c}}(s)$ used in formulae (3a) and (3b). For comparison the theoretical curve $R_{\mathrm{c}}^{\mathrm{th}}(s)$ is also shown as a dashed line. The curves have been drawn for $\alpha_{\mathrm{s}} = 0.2$ and $G = 0$

is replaced by the parameter m_{q}^{*}, so that the number of parameters does not change. The new continuum contribution has a much softer threshold factor than that given by formula (2) (cf. Fig. 1). Thus, comparing results obtained with the new threshold factor, with those obtained using formula (2), one can get an estimate of the changes due to modifications of $R_{\mathrm{q}}^{\mathrm{cont}}(s)$ in the sum rules.

The usual form of the SVZ sum rules is (cf. e.g. [2])

$$M_n^{\mathrm{th}}(m_{\mathrm{q}}, \alpha_{\mathrm{s}}, G) = \frac{3}{4\pi\alpha^2 Q^2} \sum_{\mathrm{V}} \frac{\Gamma_{\mathrm{V}}}{M_{\mathrm{V}}^{2n+1}} + \frac{1}{4\pi^2}\left(1 + \frac{\alpha_{\mathrm{s}}}{\pi}\right) \frac{1}{nE_{\mathrm{T}}^{2n}}, \tag{3a}$$

where Γ_{V} and M_{V} are the (modified) electronic width and mass of the V-th vector meson from the $\mathrm{q\bar{q}}$ family and

$$M_n^{\mathrm{th}}(m_{\mathrm{q}}, \alpha_{\mathrm{s}}, G) = \frac{M_{0n}}{(m_{\mathrm{q}}^2)^n}\left[1 + A_n\alpha_{\mathrm{s}} + \frac{\pi B_n}{m_{\mathrm{q}}^4}\, G\right]. \tag{4}$$

In this formula M_{0n}, A_n and B_n are known functions of n, while m_q, α_s and the gluon condensate parameter G have to be estimated. M_n^{th} is the theoretical estimate of the right hand side of relation (3a), which is

$$M_n = \frac{1}{12\pi^2 Q^2} \int \frac{R_q(s)ds}{s^{n+1}} . \tag{5}$$

Our proposal is to use instead of (or besides) relation (3a) the relation

$$M_n^{th}(m_q, \alpha_s, G) = \frac{3}{4\pi\alpha^2 Q^2} \sum_V \frac{\Gamma_V}{M_V^{2n+1}} + \left(\frac{m_q}{m_q^*}\right)^{2n} M_n^{th}\left(m_q, \alpha_s, \left(\frac{m_q}{m_q^*}\right)^4 G\right). \tag{3b}$$

For practical calculations this formula is as convenient as formula (3a).

In order to compare the continua in formulae (3a) and (3b), it is necessary to find a correspondence between E_T and m_q^*. For simplicity, we put at first $G = 0$. Let us define

$$\frac{2m_q^*}{E_T} = \mu_n(\alpha_s) = 2\left[\frac{4\pi^2 n M_{0n}(1+A_n\alpha_s)}{1+\frac{\alpha_s}{\pi}}\right]^{\frac{1}{2n}} . \tag{6}$$

When $2m_q^* = E_T\mu_n(\alpha_s)$, the continua for the n-th moment in formulae (3a) and (3b) coincide. For $n = 1, \ldots, 10$ and typical values of α_s the values of $\mu_n(\alpha_s)$ are given in Table I. As seen from the table, for $\alpha_s \approx 0.17$ formulae (3a) and (3b) are almost equivalent. For $\alpha_s > 0.17$ ($\alpha_s < 0.17$) the continuum contribution to formula (3b) decreases faster (more slowly) with increasing n than the corresponding contribution in formula (3a) For the ψ family $\alpha_s > 0.17$. For the Υ family, $\alpha_s \approx 0.17$, but the continuum contribution is much

TABLE I

Parameters $\mu_n(\alpha_s)$

n \ α_s	0.10	0.13	0.16	0.19	0.22	0.25	0.28
1	0.9122	0.9173	0.9223	0.9271	0.9319	0.9365	0.9410
2	0.9184	0.9208	0.9231	0.9254	0.9276	0.9297	0.9318
3	0.9236	0.9244	0.9252	0.9259	0.9267	0.9274	0.9281
4	0.9277	0.9274	0.9270	0.9267	0.9264	0.9260	0.9258
5	0.9309	0.9297	0.9284	0.9272	0.9259	0.9247	0.9235
6	0.9334	0.9313	0.9293	0.9272	0.9251	0.9230	0.9208
7	0.9354	0.9326	0.9297	0.9268	0.9237	0.9206	0.9174
8	0.9370	0.9333	0.9296	0.9257	0.9217	0.9173	0.9128
9	0.9381	0.9337	0.9291	0.9240	0.9187	0.9128	0.9063
10	0.9390	0.9337	0.9280	0.9216	0.9144	0.9061	0.8963

TABLE II

Parameters G_q obtained by comparison of the experimental and theoretical values of r_n

	100 G_c [GeV^4]				100 G_b [GeV^4]	
Formula	3a	3b	3a	3b	3a	3b
E_T or m_q^* [GeV]	4.205+0.098	1.957+0.045	3.881+0.086	1.854+0.038	10.578+0.087	4.877+0.040
m_q [GeV]	1.284+0.016	1.286+0.016	1.289+0.013	1.296+0.015	4.157+0.024	4.154+0.024
n \ α_s	0.2		0.3		0.165	
2	0	0	0	0	0	0
3	1.19+0.28	1.04+0.30	0.88+0.16	0.25+0.04	100+2	88+3
4	1.35+0.31	1.26+0.33	0.76+0.22	0.28+0.30	103+3	97+4
5	1.27+0.27	1.22+0.29	0.63+0.22	0.31+0.29	92+4	91+4
6	1.13+0.22	1.11+0.23	0.54+0.20	0.32+0.25	78+4	80+4
7	1.00+0.17	0.98+0.18	0.47+0.17	0.33+0.20	65+4	68+4
8	0.87+0.14	0.86+0.14	0.42+0.14	0.32+0.17	55+4	58+4
9	0.77+0.11	0.76+0.11	0.38+0.12	0.30+0.14	47+4	50+3
10	0.67+0.09	0.67+0.09	0.34+0.10	0.28+0.11	40+3	43+3

larger than for the ψ family and therefore smaller relative changes may give larger absolute changes than in the ψ case.

In Table II we compare for $n = 2, \dots, 10$ the values of G_q obtained from

$$r_n = \frac{M_n}{M_{n-1}} = \frac{M_{0n}}{M_{0n-1}}\left[1+(A_n-A_{n-1})\alpha_s+\frac{\pi}{m_q^4}(B_n-B_{n-1})G_q\right] \tag{7}$$

when either (3a) or (3b) is subsituted for the experimental moments M_n. The parameters m_q and E_T or m_q^* are calculated assuming that relation (7) for r_2 and a similar relation for M_2/M_1^2 hold exactly, when $G = 0$. For details of the analysis of errors cf. [3]. The results for the continua obtained using the two approximations are similar. In particular, the result $G_b \gg G_c$ [3] is reproduced, also when formula (3b) is used. The variation of G_q with n is slightly reduced, when formula (3b) is used. This could be interpreted as evidence that the parametrization for the continuum discussed in the present paper is a little better than the simple formula (2). We think, however, that the differences are so small that the main merit of the new parametrization is that it provides a check on parametrization (2). The analysis presented here for vector currents (resonances) can be easily extended to other currents.

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