FERMION MASSES IN POTENTIAL MODELS OF CHIRAL SYMMETRY BREAKING*

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A class of models of spontaneous chiral symmetry breaking is considered, based on the Hamiltonian with an instantaneous potential interaction of fermions. An explicit mass term $m\bar{\psi}\psi$ is included and the physical meaning of the mass parameter discussed. It is shown that if the Hamiltonian is normal-ordered (i.e. self-energy omitted), then the mass m introduced in the Hamiltonian is *not* the current mass appearing in the current algebra relations.

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1. *Introduction*

The spontaneous chiral symmetry breaking, which seems to occur in nature, is one of the most fascinating and yet not solved problems in the theory of strong interactions (for a recent review see e.g. [1]). Various dynamical mechanisms have been proposed to explain it, in particular in the framework of QCD, but essentially all have one common feature: the chirally non-symmetric ground state of the theory (the vacuum) is a fermion-antifermion (quark-antiquark) condensate, which develops as a consequence of a sufficiently strong fermion-antifermion attraction.

In this article I discuss a certain simple (perhaps oversimplified), but physically appealing approach to the problem [2–5]: one takes the existence of those attractive fermion-antifermion interactions for granted, assumes that they can be approximated by an instantaneous vector potential interaction, and tries to deduce physical consequences (the structure of the vacuum, effective fermion masses, etc.). To be more specific, let us write the Hamiltonian assumed in these models:

$$H(m) = \int d^3x : \bar{\psi}(\vec{x})\,(-i\vec{\gamma}\cdot\vec{\partial}+m)\psi(\vec{x}) : +\tfrac{1}{2}\int d^3x d^3y V(\vec{x}-\vec{y})\varrho(\vec{x})\varrho(\vec{y}), \tag{1}$$

where

$$\varrho(\vec{x}) = :\bar{\psi}(\vec{x})\gamma_0\psi(\vec{x}):$$

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is the "charge" density (we suppress possible flavour and colour indices; or we can think of (1) as a model for QED). The normal ordering in (1) is understood with respect to the creation/destruction operators in the field expansion

$$\psi(\vec{x}) = \sum_{\vec{p},h} e^{i\vec{p}\cdot\vec{x}}(u_{\vec{p}h}b_{\vec{p}h}+v_{-\vec{p}h}d^{+}_{-\vec{p}h}) \tag{2}$$

in terms of free spinors of mass m, three-momentum $\vec{p}$, and helicity h (we quantize in a finite volume, omitting the trivial factors like volume$^{-1/2}$, etc.). The fields ψ are assumed to satisfy the usual canonical anticommutation relations.

The Hamiltonian (1), with the normal ordering as indicated, is fashioned after that of the Coulomb gauge QED or QCD, in which only the Coulomb interaction term has been retained; in that case the potential would be $V(\vec{r}) = \alpha/|\vec{r}|$ (positive!), corresponding to the attraction in the fermion-antifermion charge zero (singlet) channel. When this is the case, (1) can be alternatively considered as the full QED or QCD Hamiltonian reduced to the fermionic sector of the Fock space, with no transverse photons (gluons). Needless to say, the model is not Lorentz-invariant.

In general, the potential V is meant to describe an effective low energy four-fermion interaction, with an appropriate UV cut-off; correspondingly, the fermion mass m in (1) should be understood to be UV renormalized. We will clarify this point later.

Models based on the Hamiltonian (1) have been studied in a number of papers [2–4] in the case $m = 0$, when (1) is strictly chiral invariant. In [2, 3] the normal-ordered version $:H:$ of the Hamiltonian was used, while the differences between this and the original Hamiltonian were discussed by the Orsay group (particularly the first two papers of Ref. [4]). In the present article we generalize to the case $m \neq 0$ where, as we shall see, the question of normal ordering is even more important. (After this work was completed, there appeared a paper by Stokar [5], in which the normally ordered Hamiltonian with $m \neq 0$ is assumed; we comment on that in Sec. 4.)

2. The variational principle

In the interaction term in (1) the "charge" densities ϱ are individually normal-ordered, their product, however, is not: it contains terms like $(b^+b)\,(b^+b)$, etc. If we now bring H to the normal-ordered form, we get from the anticommutation relations not merely a c-number, but also an expression bilinear in the fields, which is just the self-energy Hamiltonian:

$$H(m) = \,:H(m):\, + H_{\text{self}}(m) + c\text{-number}, \tag{3}$$

and

$$H_{\text{self}}(m) = \int d^3x d^3y V(\vec{x}-\vec{y}) : \overline{\psi}(\vec{x})\gamma_0 S_m(\vec{x}-\vec{y})\gamma_0\psi(\vec{y}):, \tag{4}$$

where

$$S_m(\vec{x}) = \sum_{\vec{q}} e^{i\vec{q}\cdot\vec{x}} \frac{m-\vec{q}\cdot\vec{\gamma}}{2\varepsilon_q}$$

with $\varepsilon_q = \sqrt{m^2+\vec{q}^2}$ is the free fermion propagator.

The exact ground states of the Hamiltonians H or $:H:$ are, of course, not known. Approximate solutions can be obtained by choosing a certain form of the trial wave function and minimizing the expectation value of the Hamiltonian. The standard choice is the one proposed by Nambu and Jona-Lasinio [6]:

$$|\chi\rangle = (\text{normalization}) \exp \{\sum_{\vec{p},h} \chi_p b^+_{\vec{p}h} d^+_{-\vec{p}h}\} |0\rangle. \tag{5}$$

Here and in the following $p = |\vec{p}|$, and $|0\rangle$ is the perturbative ("empty") vacuum, by definition annihilated by the operators b and d. Physically, (5) is a coherent state of pairs consisting of a fermion and an antifermion of opposite three-momenta and same helicities. These, in analogy to the superconductivity, are usually called "Cooper pairs". A single Cooper pair, averaged over helicities and momenta, with the weight χ_p, as in Eq. (5), is a $J^{CP} = 0^{++}$ state. When the helicity states are converted to the usual orbital angular momentum and total spin basis, such a pair has $L = S = 1$; incidentally, in the case of massless fermions interacting via a vector potential, this state is exactly degenerate in energy with the $L = S = 0$, 0^{-+} state.

The physics behind Eq. (5) is simple: if the interaction is strong enough, fermion-antifermion bound states of *negative* total energy may exist. Hence the energy of the system is lowered by creating more and more such bound states, until they fill up the whole space. If the interactions between different pairs are not very strong, it is then reasonable to approximate the ground state by the coherent state of independent Cooper pairs.

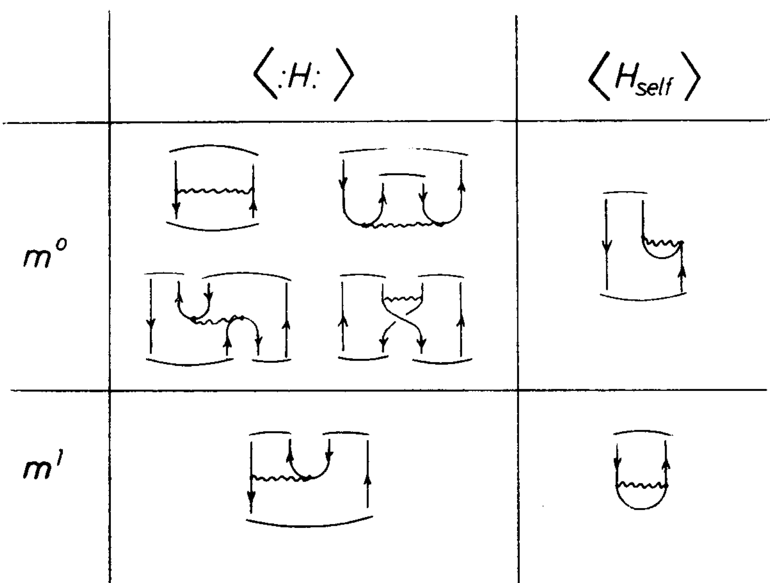

Fig. 1. Diagrams contributing to the expectation value $\langle\chi|H(m)|\chi\rangle$, split into contributions to the normal-ordered Hamiltonian and the self-energy part, and into terms of order m^0 and m^1. The wavy line is the potential V, and the arcs indicate Cooper pairs. Obvious time-reversed and charge-conjugated diagrams have to be added

When evaluating the expectation value of H or $:H:$ in the state $|\chi\rangle$ we obtain, apart from the kinetic energy contribution, several interaction terms, which can be represented diagrammatically as in Fig. 1. The contributions of order m^0 (i.e. nonvanishing when $m = 0$) include scattering of a Cooper pair into another pair with a different internal momentum, creation (or annihilation) of two pairs, and elastic scattering (via annihilation or fermion exchange) of two pairs into the same state of two pairs; there is also a self-energy contribution to the kinetic energy of a single pair.

Next come terms proportional to m: one pair going into two pairs or vice versa, and (originating from H_{self}) creation (or annihilation) of one pair from the vacuum. (We consider m small, and neglect terms of order m^2 and higher.) If we define chirality = helicity for fermions and chirality = −helicity for antifermions, then a Cooper pair is a chirality = ± 1 system. Therefore, when $m = 0$ and chirality is conserved, only an even number of pairs can be created or annihilated. In the terms involving an odd number of pairs chirality must change by one unit at one of the vertices; this costs a factor m.

Let us limit ourselves for a moment to the case $m = 0$. Evaluating $\langle\chi|H(0)|\chi\rangle$, and imposing the condition for the minimum,

$$\frac{\delta}{\delta\chi_p}\langle\chi|H(0)|\chi\rangle = 0,$$

we obtain a nonlinear equation for χ_p,

$$2p\chi_p + \sum_{\vec{q}} V_{\vec{p}-\vec{q}}\left[\hat{p}\cdot\hat{q}\chi_p - \frac{\chi_q}{1+\chi_q^2}(1-\chi_p^2+2\hat{p}\cdot\hat{q}\chi_p\chi_q)\right] = 0, \tag{6}$$

where $\hat{p} = \vec{p}/p$. If $:H:$ is used, the first term in the square brackets has to be omitted. In Eq. (6) we have taken χ_p real, which, for a given $|\chi_p|$, always minimizes the energy [3]. Still, as was also noticed in [3], Eq. (6) is invariant under the transformation $\chi_p \to -\chi_p$, and has two solutions, $\pm\chi_p$.

Here we come to an important point. For $m = 0$ we have two solutions, differing by the sign. They give opposite signs to the fermion condensate,

$$\langle\chi|\bar{\psi}(0)\psi(0)|\chi\rangle = \sum_{\vec{p}} \frac{2\chi_p}{1+\chi_p^2}. \tag{7}$$

This arbitrariness in sign is not an accident. It is simply a consequence of the chiral invariance of $H(0)$ or $:H(0):$ and of the fact that the scalar density $\bar{\psi}\psi$ is *not* chiral-invariant; it transforms into a linear combination of itself and the pseudoscalar density

$$e^{i\alpha Q_5}\bar{\psi}\psi e^{-i\alpha Q_5} = \cos 2\alpha\,\bar{\psi}\psi + \sin 2\alpha\,\bar{\psi}^i\gamma_5\psi, \tag{8}$$

where

$$Q_5 = \int d^3x\bar{\psi}(\vec{x})\gamma_0\gamma_5\psi(\vec{x})$$

is the axial charge. From Eq. (8) it follows that only $\langle\bar{\psi}\psi\rangle^2+\langle\bar{\psi}i\gamma_5\psi\rangle^2$ is chirally invariant and well-defined; and since the expectation value of $\bar{\psi}i\gamma_5\psi$ in the state (5) automatically vanishes, the value of $\langle\bar{\psi}\psi\rangle$ is fixed, but only up to a sign.

In other words, the sign of $\langle\bar{\psi}\psi\rangle$ is not determined simply because, for $m = 0$, the eigenstates of the Hamiltonian, or the trial states, connected by the chiral transformation are degenerate in energy. So, let us take now $m > 0$. This breaks the chiral invariance and lifts the degeneracy. In our case one of the states characterized by $\pm\chi_p$ will have a lower energy and be a better approximation to the ground state; this will give a definite sign to $\langle\bar{\psi}\psi\rangle$.

To see what happens, let us assume that the mass is small (compared to the UV cut-off, which is the only other dimensionful parameter in the model), and write

$$H(m) = H(0)+\Delta H+O(m^2)$$

(and the same for $:H:$). The expectation value of ΔH can be evaluated e.g. from the diagrams of Fig. 1, with the result

$$\langle\chi|\Delta H|\chi\rangle = 2m\sum_{\vec{p},\vec{q}} V_{\vec{p}-\vec{q}}\,\frac{\hat{p}\cdot(\vec{p}-\vec{q})}{\varepsilon_p\varepsilon_q}\,\frac{\chi_p(1-\chi_q^2)}{(1+\chi_p^2)(1+\chi_q^2)}\,.$$

But, since ΔH is a small perturbation, we can take for χ_p the solution of Eq. (6), corresponding to $m = 0$. Hence, using also Eq. (7), we simply get

$$\langle\chi|\Delta H|\chi\rangle = m\sum_{\vec{p}}\frac{2\chi_p}{1+\chi_p^2} \equiv m\langle\chi|\bar{\psi}\psi|\chi\rangle. \tag{9}$$

This implies that the state with $\langle\bar{\psi}\psi\rangle$ *negative* is energetically favoured.

If $:H:$ is taken instead of H, the result is

$$\langle\chi|\Delta:H:|\chi\rangle = -4m\sum_{\vec{p},\vec{q}} V_{\vec{p}-\vec{q}}\,\frac{\hat{p}\cdot(\vec{p}-\vec{q})}{\varepsilon_p\varepsilon_q}\,\frac{\chi_p\chi_q^2}{(1+\chi_p^2)(1+\chi_q^2)} \tag{10}$$

Here again χ_p can be taken to be the solution to the unperturbed problem with $:H(0):$. Numerical solutions are known [3] for a modified Coulomb potential: $|\chi_0| = 1$ and, at least for some reasonable range of parameters, $|\chi_p|$ monotonically decreases towards zero with growing p. Let us assume this and look, in Eq. (10), at the sum over $\vec{q}$ with fixed $\vec{p}$ and $|\vec{p}-\vec{q}|$. Since $\chi_q^2/\varepsilon_q(1+\chi_q^2)$ monotonically decreases, q will tend to be small, and therefore $\vec{p}-\vec{q}$ will tend to be parallel to $\hat{p}$. Consequently, the sum in Eq. (10) will have the same sign as χ_p. Taking into account the additional minus sign in Eq. (10) it follows that to lower the energy we have to take positive χ_p, which will give $\langle\bar{\psi}\psi\rangle$ *positive*!

On the other hand, we have the Gell-Mann, Oakes and Renner (GMOR) [7] relation

$$f_\pi m_\pi^2 \simeq -2m\langle\Omega|\bar{\psi}\psi|\Omega\rangle \tag{11}$$

between the pion mass, the pion weak decay constant, the current quark mass and the expectation value of $\bar{\psi}\psi$ in the true ground state $|\Omega\rangle$. Eq. (11) implies that $\langle\bar{\psi}\psi\rangle$ must be *negative*, evidently in agreement with the result obtained from H and in conflict with the result from $:H:$.

3. The general analysis

In the last Section we considered an approximate ground state $|\chi\rangle$ determined by minimizing $\langle\chi| : H(m) : |\chi\rangle$ within the class of trial functions (5). We found, for reasonable potentials V, that $\langle\chi|\bar{\psi}\psi|\chi\rangle > 0$. Strictly speaking, this does not necessarily contradict the relation (11) because, after all, $|\chi\rangle$ is only an approximation to the exact ground state $|\Omega\rangle$. Our result would then imply that this approximation is so poor that it even fails to reproduce the correct sign of $\langle\bar{\psi}\psi\rangle$. We shall show, however, that there is another, more essential reason for the failure. Namely we find that the mass m in the *normal-ordered* Hamiltonian $:H(m):$ is *not* the same "current" fermion mass that appears in relation (11). In other words, Eq. (11) does not hold when m appearing there is the mass parameter of $:H(m):$. On the other hand, there are no problems with the original $H(m)$.

Let us start with writing down explicit expressions for ΔH and $\Delta :H:$. A simple way to do this is to note that ΔH and $\Delta :H:$ must involve a vertex at which chirality changes by one unit (cf. the discussion of the diagrams of Fig. 1), i.e. a product of operators $b^{+}_{\vec{p}+\frac{1}{2}}b_{\vec{q}-\frac{1}{2}}$, or $b^{+}_{\vec{p}+\frac{1}{2}}d^{+}_{\vec{q}+\frac{1}{2}}$, etc. Now, under the chiral rotation, as defined in Eq. (8), $b_{\vec{p}h} \to \exp(\pm i\alpha)b_{\vec{p}h}$, etc., the sign depending on the chirality. Hence the chirality changing terms in ΔH will acquire factors $\exp(\pm 2i\alpha)$, or, when we add two transforms with the angles α and $-\alpha$, the factor $2\cos 2\alpha$. Consequently,

$$H - \tfrac{1}{2}e^{i\alpha Q_5}He^{-i\alpha Q_5} - \tfrac{1}{2}e^{-i\alpha Q_5}He^{i\alpha Q_5} = (1-\cos 2\alpha)\Delta H + O(m^2).$$

Expanding for small α we get

$$\Delta H = \tfrac{1}{4}[[H, Q_5], Q_5] \tag{12}$$

and, of course, an analogous expression for $\Delta :H:$.

Evaluating the commutators we obtain

$$\Delta H = \sum_{\vec{p},h} m(d_{-\vec{p}h}b_{\vec{p}h} + b^{+}_{\vec{p}h}d^{+}_{-\vec{p}h}) \equiv \int d^3x m\bar{\psi}(\vec{x})\psi(\vec{x}), \tag{13}$$

i.e. exactly the same term which appears in the Hamiltonian (1), also in agreement with Eq. (9). It is now evident that the state with $\langle\bar{\psi}\psi\rangle < 0$ is energetically favourable.

Furthermore, sandwiching Eq. (12) between the ground states gives

$$\langle\Omega|Q_5[H(m)-E_\Omega]Q_5|\Omega\rangle = -2m\langle\Omega|\int d^3x\bar{\psi}(\vec{x})\psi(\vec{x})\,|\Omega\rangle. \tag{14}$$

When the left hand side (l.h.s.) is saturated by the single pion states, this reduces to the GMOR formula (11). Even more generally, the l.h.s. is positive-definite, which implies that $\langle\Omega|\bar{\psi}\psi|\Omega\rangle$ must be negative (or zero, when chiral symmetry is not spontaneously broken).

If we now evaluate $\Delta:H:$, the result is different, essentially because H_{self} does not commute with Q_5. We find

$$\Delta : H : = \sum_{\vec{p},h} (1-F_p)m(d_{-\vec{p}h}b_{\vec{p}h}+b^{+}_{\vec{p}h}d^{+}_{-\vec{p}h}) \tag{15}$$

with

$$F_p = \sum_{\vec{q}} V_{\vec{p}-\vec{q}}/2\varepsilon_q,$$

or, symbolically,

$$\Delta:H: = \int d^3x\text{``}(1-F)m\bar{\psi}\psi\text{''}.$$

As before, we have (now $|\Omega\rangle$ is the ground state of $:H(m):$)

$$\langle\Omega|Q_5[:H(m): -E_\Omega]Q_5|\Omega\rangle = -2\langle\Omega|\Delta : H : |\Omega\rangle \geqslant 0. \tag{16}$$

But, if the potential is sufficiently strong (and it must be sufficiently strong to yield the spontaneous chiral symmetry breaking), F_p may exceed 1, at least in the important region of momenta $\vec{p}$. Hence $\langle\Delta:H:\rangle < 0$ does *not* imply $\langle\bar{\psi}\psi\rangle < 0$. This is why we found $\langle\chi|\bar{\psi}\psi|\chi\rangle > 0$ in the previous Section.

Let us finally try to clarify the physical meaning of Eq. (15). The kinetic energy part of $H(m)$ is obviously

$$\sum_{\vec{p},h} \varepsilon_p(b^{+}_{\vec{p}h}b_{\vec{p}h}+d^{+}_{\vec{p}h}d_{\vec{p}h}).$$

Adding the self-energy terms amounts, as can be checked, to the replacement

$$\varepsilon_p \to \varepsilon_p + \sum_{\vec{q}} V_{\vec{p}-\vec{q}} \frac{m^2+\vec{p}\cdot\vec{q}}{2\varepsilon_p\varepsilon_q} \simeq (1+F_p)\varepsilon_p \tag{17}$$

(the last approximation results from $\vec{p} \simeq \vec{q}$ in the sum). It is this quantity which should be interpreted as the radiatively corrected, "physical" fermion energy in the normal vacuum (this must not be confused with the effective energy, or mass, which the fermion acquires through interactions with the fermion condensate in the chirally non-symmetric vacuum). Since the model is not Lorentz-invariant, the energy of Eq. (17) is not exactly of the form $\sqrt{m_R^2+p^2}$. However, for small p, Eq. (17) is approximately equivalent to the lowest order mass renormalization

$$m \to (1+F_p)m.$$

At the same time the "current" fermion mass which appears in Eqs. (13) and (14) is still the original "bare" mass m introduced in $H(m)$.

Things are different with the normally ordered $:H(m):$. Here, by construction, self-energy corrections are absent (to the lowest order), and the renormalized, "physical", fermion mass in the normal vacuum is simply m. On the other hand, the "current" fermion mass appearing in Eq. (15) is $(1-F_p)m$, i.e., it is the "de-renormalized" mass, in which the renormalization is undone, or, in other words, simply the bare mass.

The situation is summarized in the table below

	Current mass	"Physical" mass
$H(m)$	m	$(1+F_p)m$
$:H(m):$	$(1-F_p)m$	m

As stated before, the current mass is meant to be the mass appearing in the GMOR formula (or its analogues (14), (16)), in the expression for the divergence of the axial current (or the commutator $[H, Q_5]$), etc. The "physical" mass in understood to be the mass the "physical" on-shell fermion would have in the normal vacuum. I.e., it includes the perturbative radiative corrections, but no nonperturbative effects caused by the fermion condensate. The most important message is simply that m in $:H(m):$ is *not* the current mass.

In view of this result there appears to be no good reason to employ $:H:$ instead of H. The original motivation seems to have been, as discussed in [4], to avoid the Coulomb UV divergences present in the self-energy. But, when $m \neq 0$, these divergences would appear anyway, this time in the current mass (F_p would be divergent for a pure Coulomb potential V).

4. *Concluding remarks*

We hope to have clarified the meaning of the fermion masses appearing in the model Hamiltonian $H(m)$ of Eq. (1) and in $:H(m):$. A natural question is how is that related to the full theory, Lorentz-invariant, and without an UV cut-off.

Now the UV renormalization must be done. As a consequence of adding counter--terms, the full QCD Hamiltonian will now contain a mass term $m_B(M)\bar{\psi}\psi$ with a bare mass $m_B(M)$, and a bare coupling constant $g_B(M)$, both depending on some regulator (cut-off) mass M in such a way that physical quantities are M-independent when $M \to \infty$. These $m_B(M)$ and $g_B(M)$ at a given very large M can be taken as parameters fully defining the theory. Of course, they are uniquely related to renormalized parameters $m(\mu)$ and $g(\mu)$, appropriately defined e.g. in terms of exact Green functions at space-like momenta $p^2 = -\mu^2$; these parameters, at a given scale μ, can equally well determine the theory. Actually, if $\mu \gg \Lambda$, Λ being the scale above which $g(\mu) \ll 1$, then $m(\mu) \simeq m_B(\mu)$.

Now, relations like (14) (the GMOR relation) look the same as before, except that on the r.h.s. m is replaced by $m_B(M)$, and the matrix element $\langle\Omega|\bar{\psi}\psi|\Omega\rangle$ involves radiative corrections calculated with the cut-off M (the product must be M-independent, since the l.h.s. is). Hence, the fermion current mass at the (very large) momentum scale M is simply $m_B(M) \simeq m(M)$. Conventionally, one uses a parameter uniquely related to that – the renormalized mass $m(\mu)$ at some fixed scale (say, $\mu = 1$ GeV).

How does this current mass relate to the current mass in the model Hamiltonian? As we have mentioned before, the Hamiltonian (1) is a simplified QCD Hamiltonian with an UV cut-off. The cut-off M has to be taken $M \gtrsim \Lambda$, since above the scale Λ the running coupling constant becomes small. And since we ignore phenomena occurring at momenta large compared to M, we can only require the model Hamiltonian to be approximately equivalent to the full Hamiltonian with the same cut-off M. Consequently, the parameter m in $H(m)$ should be approximately equal to $m_B(M) \simeq m(M)$ of the full theory, i.e. it can indeed be identified with the usual fermion current mass at the cut-off scale $M \gg \Lambda$. From this point of view, the "physical" fermion mass we introduced is also the renormalized mass, but renormalized on the mass-shell, at the small scale of the mass itself.

Let us finally comment on the recent paper [5], which studies the dependence of the fermion condensation on the fermion mass introduced in the Hamiltonian. For a typical Coulomb-type potential it is found there that the condensate disappears when the mass exceeds some critical value $\simeq$ 150 MeV (i.e. roughly the strange quark current mass). However, the Hamiltonian used in [5] is the normal-ordered one, and, according to our analysis, the mass appearing there is the "physical" rather than the current mass. The latter may be then significantly *smaller* than 140 MeV. On the other hand, when H instead of $:H:$ is used, the very occurrence of the spontaneous chiral symmetry breaking may require a stronger potential (to overcome the self-energy effects). But with a stronger potential the condensate is likely to persist for *larger* masses. The situation is therefore unclear and a more careful analysis seems necessary to estimate the critical mass more quantitatively.

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